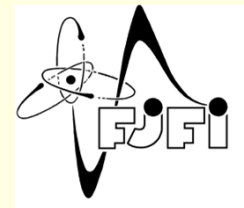


Photons walking the line

Igor Jex

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Crete, August 2015

Dept. Physics, FNSPE CTU in Prague



In collaboration

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- T. Kiss, B. Kollar (Budapest)
- E. Andersson (Edinburgh)
- G. Alber (Darmstadt)
- V. Potoček, A. Gábris, M. Štefaňák, C. Hamilton, J. Novotný, H. Lavička (Prague)

Outline

- Motivation
- Formulation of the problems
- Classification of walks
- Implementation of walks
- Walks with perturbations
- Walks on random graphs
- Percolation
- Outlook



Motivation

- Classical walks are useful
- Quantum walks are interesting
- Exploration of the power of quantum walks
- Practical implications
- Influence of external perturbation

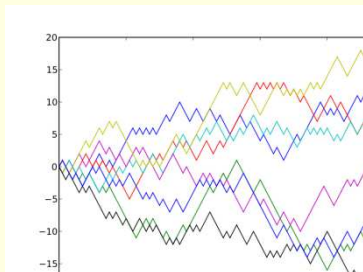
The Problem of The Random Walk

Karl Pearson to Nature 1905 (july 27)

Can any of your readers refer me to a work wherein I should find a solution of the following problem, or failing the knowledge of any existing provide me with an original one? I should be extremely grateful for aid in the matter.

A man starts from point O and walks l yards in a straight line, he then turns through any angle whatever and walks another l yards in a second straight line. He repeats this process n times. I require the probability that after n of these stretches he is at a distance between r and $r+\delta r$ from his starting point, O.

The problem is one of considerable interest, but I have only succeeded in obtaining an integrated solution for two stretches. I think, that a solution ought to be found, if only in the form of a series of powers of r/n , when n is large.



Quite an interesting issue

matter how many degrees of freedom the resonator possesses, or what the form of its potential energy. Indeed, according to this argument, equation (2) is proved for any dynamical system, e.g. the molecules of a gas.

It is, however, known that equation (2), with Planck's meaning of k , is true if, and only if, the energy of each dynamical system is expressible as the sum of two squares. It can, indeed, be shown directly that this latter condition is exactly the condition that Prof. Planck's assumed basis of probability calculations shall be a legitimate basis, i.e. shall be independent of the time. Happily, this condition of the energy being a sum of two squares may be supposed to be satisfied by Planck's resonators, so that we may regard equation (1) as true for such resonators. The equation has, however, no physical meaning, owing to the presence of the arbitrary small quantity ϵ , and can acquire a physical meaning only by putting $\epsilon=0$. It then leads merely to equation (2), which can be obtained much more readily from the theorem of equipartition.

Taking u to be the law of radiation, when ν is the reciprocal of the period of vibration, Planck introduces from his first paper the equation

$$u = (8\pi\nu^2/c^3)U \dots \dots \dots (3)$$

which in combination with equation (2) would lead to the law of radiation,

$$(8\pi k/c^3)T\nu^2 d\nu \dots \dots \dots (4)$$

and this, on replacing ν by c/λ , becomes

$$8\pi kT\lambda^{-4} d\lambda \dots \dots \dots (5)$$

which agrees with my own result. Planck arrives at equation (3) by the help of his assumption of "naturliche Strahlung," but I believe it will be found that this "assumption" is capable of immediate proof by the methods of statistical mechanics. Except for this, and the other differences already stated, the way in which expression (5) has been reached in the present letter is identical, as regards underlying physical conceptions, with the way in which it has been obtained by Lord Rayleigh and myself.

Planck does not reach expression (5) at all, as he does not pass from equation (1) to equation (2). Instead of putting $\epsilon=0$, he puts $\epsilon=h\nu$, where h is a constant, and this leads at once to his well known law of radiation. It will now be clear why Planck's formula reduces to my own when $h=\infty$. For taking $h=\infty$ is the same thing as taking $\nu=0$, or $\epsilon=0$.

The relation $\epsilon=h\nu$ is assumed by Planck in order that the law ultimately obtained may satisfy Wien's "displacement law," i.e. may be of the form

$$\nu^3 f(T/\nu) d\nu \dots \dots \dots (6)$$

This law is obtained by Wien from thermodynamical considerations on the supposition that the energy of the ether is in statistical equilibrium with that of matter at a uniform temperature. The method of statistical mechanics, however, enables us to go further and determine the form of the function $f(T/\nu)$: it is found to be $8\pi k(T/\nu)$, so that Wien's law (6) reduces to the law given by expression (4). In other words, Wien's law directs us to take $\epsilon=h\nu$, but leaves h indeterminate, whereas statistical mechanics gives us the further information that the true value of h is $h=0$. Indeed, this is sufficiently obvious from general principles. The only way of eliminating the arbitrary quantity ϵ is by taking $\epsilon=0$, and this is the same as $h=0$.

Thus it comes about that in Planck's final law

$$\frac{8\pi k}{\lambda^5} \frac{1}{e^{hc/\lambda kT} - 1} d\lambda \dots \dots \dots (7)$$

the value of h is left indeterminate; on putting $h=0$, the value assigned to it by statistical mechanics, we arrive at once at the law (5).

The similarities and differences of Planck's method and my own may perhaps be best summed up by saying that the methods of both are in effect the methods of statistical mechanics and of the theorem of equipartition of energy, but that I carry the method further than Planck, since Planck stops short of the step of putting $h=0$. I venture to express the opinion that it is not legitimate to stop short at this point, as the hypotheses upon which Planck has worked lead to the relation $h=0$ as a necessary consequence.

Of course, I am aware that Planck's law is in good agreement with experiment if h is given a value different from zero, while my own law, obtained by putting $h=0$, cannot possibly agree with experiment. This does not alter my belief that the value $h=0$ is the only value which it is possible to take, my view being that the supposition that the energy of the ether is in equilibrium with that of matter is utterly erroneous in the case of ether vibrations of short wave-length under experimental conditions.

J. H. JEANS.

On the Spontaneous Action of Radium on Gelatin Media.

SINCE my communication to NATURE on the subject of the experiments in which I have been for some time past engaged, my attention has been directed to the fact that M. B. Dubois, in a speech at Lyons last November, stated that he had obtained some microscopic bodies by the action of radium salts on gelatin bouillon which had been rendered "aseptic," but in what manner it is not stated.

I write to direct attention to the fact, as also to add that M. Dubois's experiments were quite unknown to me.

Moreover, the theory that some elementary form of life, far simpler than any hitherto observed, might exist and perhaps be brought about artificially by "molecular and atomic groupings and the groupings of electrons"—in virtue of some inherent property of the atoms of such substances as radium—was pointed out in my article on the "Radio-activity of Matter" in the *Monthly Review*, November, 1903, whilst the experiments which I have been carrying out to verify this view have been for a long time known in Cambridge.

Although I did not make a speech on the subject, I demonstrated the growths to many people at the Cavendish and Pathological laboratories early in the Michaelmas Term last year.

So momentous a result as it seemed required careful confirmation, and much delay was also caused in taking the opinions of various men of science before I ventured to write to you upon the subject.

That M. Dubois's experiments have been made quite independently I do not entertain the slightest doubt.

Some critics have suggested that these forms I have observed may be identified with the curious bodies obtained by Quincke, Lehmann, Schenck, Leduc and others in recent times, and by Rainey and Crosse more than half a century ago; but I do not think, at least so far as I can at present judge, that there is sufficient reason for so classifying them together. They seem to me to have little in common except, perhaps, the scale of being to which as microscopic forms they happen to belong.

JOHN BUTLER BURKE.

The Problem of the Random Walk.

CAN any of your readers refer me to a work wherein I should find a solution of the following problem, or failing the knowledge of any existing solution provide me with an original one? I should be extremely grateful for aid in the matter.

A man starts from a point O and walks l yards in a straight line; he then turns through any angle whatever and walks another l yards in a second straight line. He repeats this process n times. I require the probability that after these n stretches he is at a distance between r and $r + \delta r$ from his starting point, O.

The problem is one of considerable interest, but I have only succeeded in obtaining an integrated solution for two stretches. I think, however, that a solution ought to be found, if only in the form of a series in powers of $1/n$, when n is large.

KARL PEARSON.

The Gables, East Ilsley, Berks.

British Archæology and Philistinism.

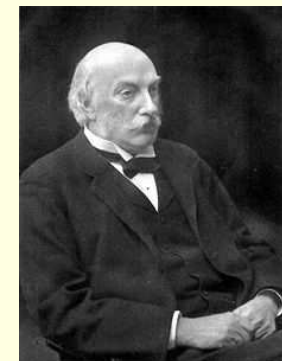
AT the end of the second week in July two contracted skeletons were found in a nurseryman's grounds near the famous British camp at Leagrave, Luton. Both were greatly contracted; one, on its right side, had both arms straight down, one under the body the other above; the other skeleton lay upon its left side, with the left hand

Classical walks

This problem proposed by prof. Pearson in the current number of NATURE is the same as that of the composition of n iso-periodic vibrations of unit amplitude and of phases distributed at random, considered in *Phil. Mag.*, x. p. 73, 1880; xlvii, p.246, 1899. If n be very great, the probability sought be

$$2 \exp(-r^2/n) r dr .$$

Probably methods similar to those employed in the papers referred to would avail for the development of an appropriate expression applicable when n is only moderately great.



J. Strutt, Rayleigh (1842-1919)

Pearson's answer

while Mr. Fletcher, paymaster, and Dr. Simpson are collecting the insects and land plants. I may say at once that the latter are of the type which one would expect to find on purely oceanic islands, but their distribution from island to island is interesting, as well as their preferences for sand or rock, drought or moisture, &c., most of the islands having definite zones with their peculiar plants.

It is really as yet too early to say anything about the reefs here, as there are one or two places which I have not yet been able to visit. What strikes one, however, very forcibly is the comparative absence of life on them. Of course there are in places plenty of corals, but the number of species is quite limited. There is a fair number of the usual *Alcyonaria*, but Sponges, Hydroïds, and Tunicates are very few in species and in quantity. *Turbellaria* are very rare, while Molluscs, Echinoderms, and Crustacea are few in species and, except certain common forms, not numerous. Psychodera we have obtained, as well as a few Sipunculids, but *Amphioxus* and *Thalassema* we have not found. At Minikoi in two tides I have brought to the camp as great a variety of animals as Cooper and I have obtained here working ten tides up to the present. Indeed, life here is strictly limited in variety, and, when the marine collections have been fully worked up, one is inclined to anticipate, even so early, that some definite light will be thrown on the distance to which the larvae of marine animals can cross the open ocean, on the distribution, in fact, of marine animals. The same, too, is true as well of the marine plants, mullipores alone being common.

I am now endeavouring to work up the physical conditions of the atoll so as to find, if possible, whether there is any physical cause for the comparative paucity of free-living animals. I am sending Cooper in the ship tomorrow to Diego Garcia, where he will have four or five days while she is coaling to examine the land and reefs. I remain here, but I hope by the time of his return, in about twelve days, to have finished my work and to move on to Peros Banhos, while the *Sealark* is sounding between the banks and round the Chagos Archipelago."

The Problem of the Random Walk.

I HAVE to thank several correspondents for assistance in this matter. Mr. G. J. Bennett finds that my case of $n=3$ can really be solved by elliptic integrals, and, of course, Lord Rayleigh's solution for n very large is most valuable, and may very probably suffice for the purposes I have immediately in view. I ought to have known it, but my reading of late years has drifted into other channels, and one does not expect to find the first stage in a biometric problem provided in a memoir on sound. From the purely mathematical standpoint, it would still be very interesting to have a solution for n comparatively small. The sections through the axis of Lord Rayleigh's frequency surface for n large are simply the "cocked hat" or normal curve of errors type; for $n=2$ or 3 they do not resemble this form at all. For $n=2$, for example, the sections are of the form of a double U, thus U U, the whole being symmetrical about the centre vertical corresponding to $r=0$, but each U itself being asymmetrical. The system has three vertical asymptotes. It would be interesting to see how the multiplicity of types for n small passes over into the normal curve of errors when n is made large.

The lesson of Lord Rayleigh's solution is that in open country the most probable place to find a drunken man who is at all capable of keeping on his feet is somewhere near his starting point! K. PEARSON.

Proposed Magnetic and Allied Observations during the Total Solar Eclipse on August 30

In response to my appeal for simultaneous magnetic and allied observations during the coming total solar eclipse, cooperative work will be conducted at stations distributed practically along the entire belt of totality and also at outside stations, nearly every civilised nation participating.

These observations will afford a splendid opportunity for further testing the results already obtained. All those

NO. 1867, VOL. 72]

who are able to cooperate are invited to participate in this important work.

The scheme of work proposed embraces the following:—
(1) Simultaneous magnetic observations of any or all of the elements according to instruments at the observer's disposal, every minute from August 29, 22h., to August 30, 4h., Greenwich mean astronomical time.

[To ensure the highest degree of accuracy attainable, the observer should begin work early enough to have everything in complete readiness in proper time. See precautions taken in previous eclipse work as explained in the journal *Terrestrial Magnetism* (vol. v., p. 148, and vol. vii., p. 16). It is essential, as shown by past experience, that the same observer make the readings throughout the entire interval.]

(2) At magnetic observatories, all necessary precautions should be taken so that the self-recording instruments will be in good operation, not only during the proposed interval, but also for some time before and after, and eye readings should be taken in addition wherever it is convenient.

[It is recommended that, in general, the magnetographs be run on the usual speed throughout the interval, and that, if a change in the recording speed be made, every precaution possible be taken to guard against instrumental changes likely to affect the continuity of the base lines.]

(3) Atmospheric electricity observations should be made to the extent possible by the observer's equipment and personnel at his disposal.

(4) Meteorological observations in accordance with the observer's equipment should be made at convenient periods (as short as possible) throughout the interval. It is suggested that, at least, temperatures be read every fifth minute (directly after the magnetic reading for that minute).

(5) Observers in the belt of totality are requested to take the magnetic reading every fifteen seconds during the time of totality, and to read temperatures as frequently as possible.

(6) At those stations where the normal diurnal variation cannot be obtained from self-recording instruments, it is desirable to make the necessary observations for this purpose on as many days as possible before and after the day of the eclipse, and to extend the interval of observations given above if conditions permit. In general, those who will have self-recording instruments have decided to run them for at least eight days before and after the day of the eclipse.

It is hoped that observers will send full reports of their work to me as soon as possible for incorporation in the complete monograph on this subject to be published by the Carnegie Institution of Washington.

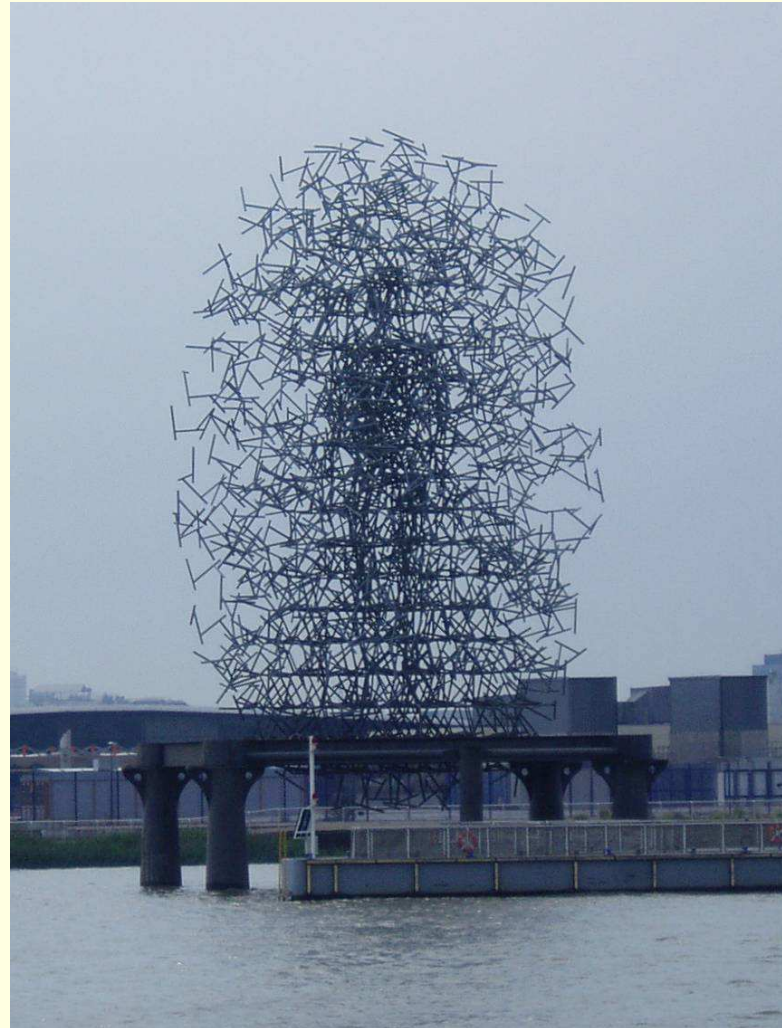
L. A. BAUER,
Department Terrestrial Magnetism, Carnegie
Institution, Washington, D.C., July 15.

British Fruit Growing.

IN your remarks on p. 207 (July 27) on the above subject, you mention "the diversity of yield from farms in the same neighbourhood . . . due presumably to differences of shelter and aspect." It is a remarkable thing that, so far as I know, nothing has ever been done to find out and publish the most suitable localities, as regards soil and climate, for orchard planting. It is a question of very great complexity, and can only be dealt with properly by officials appointed for that purpose; but its importance in fruit culture is so obvious that a considerable expenditure would be well repaid. Few people have any idea of the great climatic differences in localities within even a few hundreds of yards!

This house is on the south slope of the long range of Lower Greensand hills which runs parallel with the Chalk range the whole length of Kent from west to east. At this point the slope rises steeply from 200 feet above sea-level to 500 feet, my house being about 350 feet. I have carefully observed the effects of frost, &c., for the last six years, and it appears to me that the variations in temperature in the vertical limits mentioned are much greater than would be expected. Up to the 400-foot contour line the climate is singularly equable, which is proved not only by daily thermometrical observations, but by the

Simple quantum transition



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Journal of Experimental Psychology

VOL. II, No. 1.

FEBRUARY, 1917

ON THE MEASUREMENT OF VISUAL STIMULATION INTENSITIES

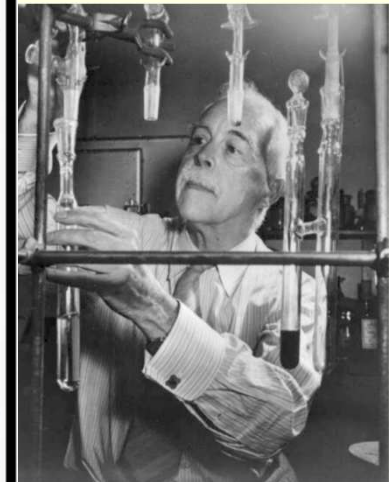
BY LEONARD T. TROLAND

*Nela Research Laboratory, National Lamp Works of General Electric Company,
Nela Park, Cleveland, O.*

I. INTRODUCTION

It must probably be admitted that there are few fields of science in which definite quantitative results are obtainable, which have been more carelessly cultivated than that of visual psycho-physiology. The literature of visual research is truly monumental, the ascertained qualitative facts are legion, and yet the laws of vision are few and vague.

In an extensive monograph on the laws of color adaptation, yet to be published, the present writer has expressed his intensity measures throughout in terms of a unit involving the pupillary area, and has proposed that this unit, called the *photon*, be adopted as the standard means of specifying the photometric intensity of visual stimulation conditions.

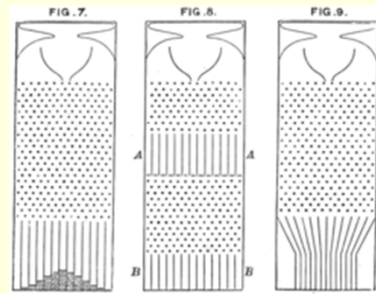
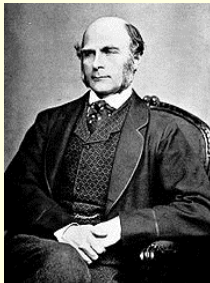
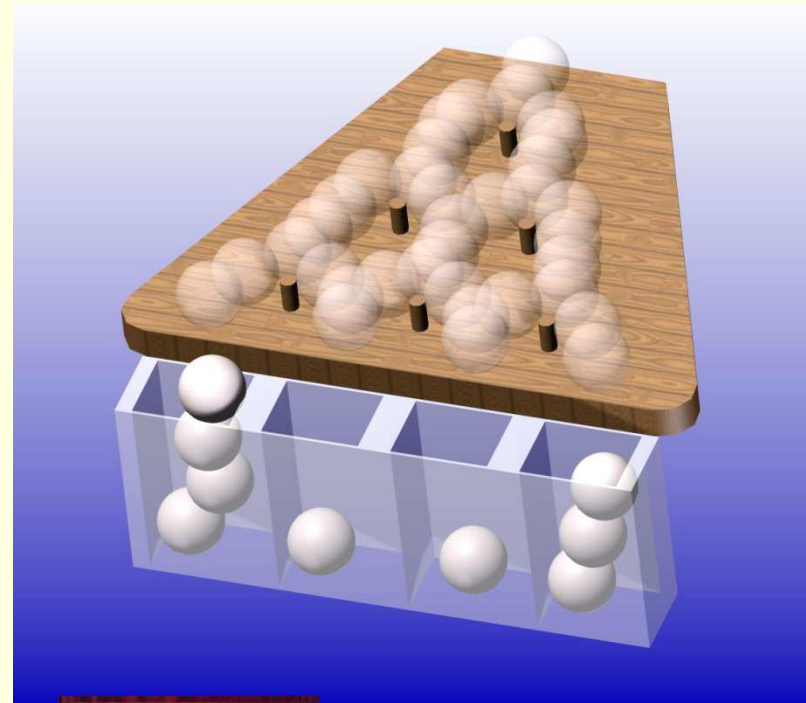
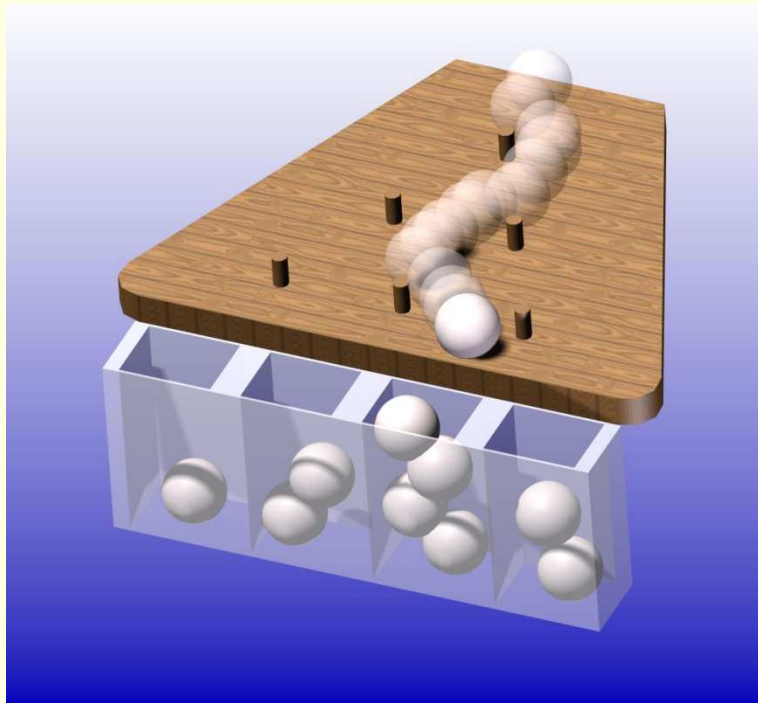


Gilbert N. Lewis
1875-1946

First -- American physicist and
-- French physiologist, René Wurmser (1890-1993), fourth -- July 1926 French physicist F. Wolfers (ca. 1890-1971)

17-1933) - 1921, third

From classical to quantum walks



Quantum walks

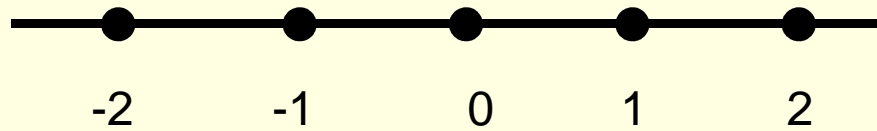
PHYSICAL REVIEW A

VOLUME 48, NUMBER 2

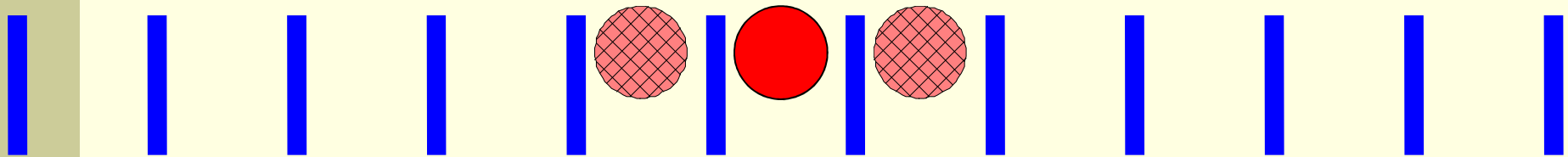
AUGUST 1993

Quantum random walks

Y. Aharonov,^{*} L. Davidovich,[†] and N. Zagury[†]



Discrete quantum walks



Requirements on a Quantum Walk:

- Coherent superposition of positions
- Unitary time evolution

▶ Particles with an inner degree of freedom

Quantum walks

$$U = S \cdot (C \otimes I)$$

$$\mathcal{H} = \mathcal{H}_C \otimes \mathcal{H}_P$$

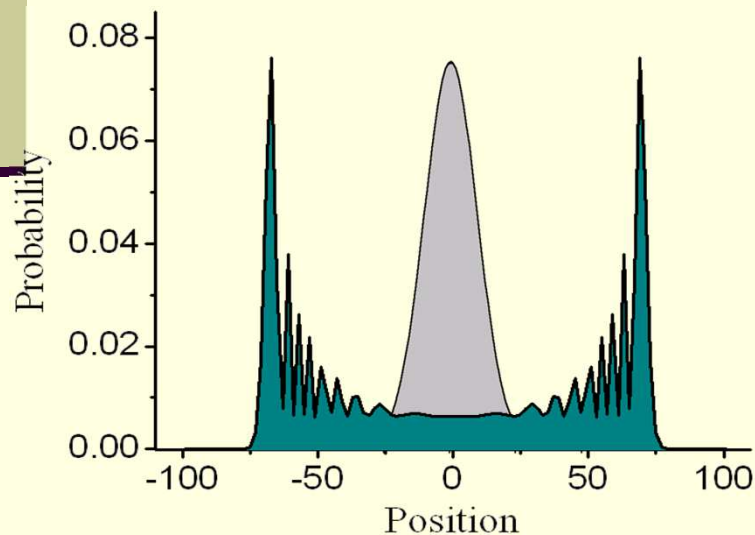
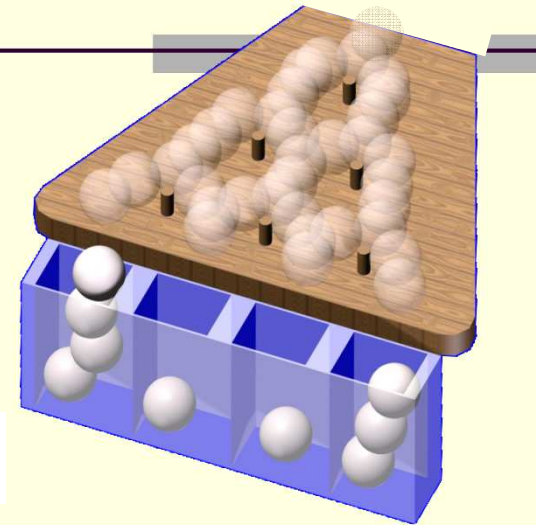
Acts on internal state

Coin dependent displacement

(1D walk: 2-level system)

$$|x, \downarrow\rangle \rightarrow |x - 1, \downarrow\rangle$$

$$|x, \uparrow\rangle \rightarrow |x + 1, \uparrow\rangle$$



Dependence with:

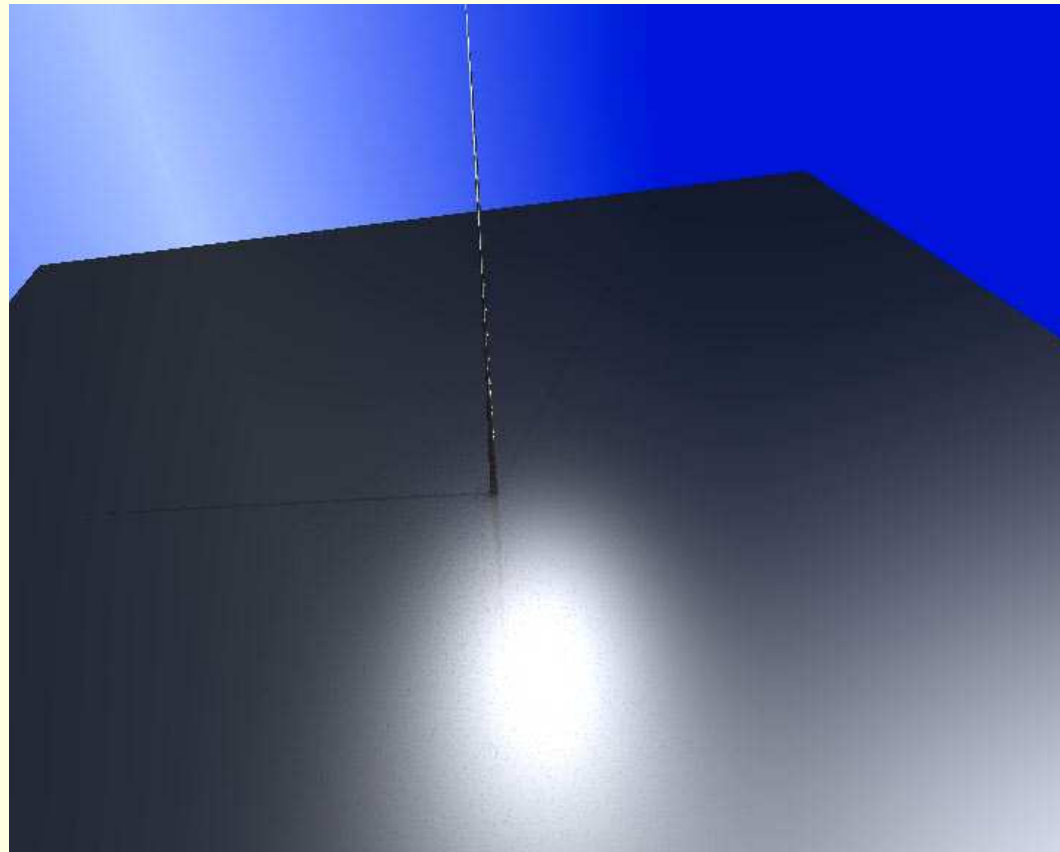
- Initial state
- Coin operator

$$\sigma^2 \sim N^2$$

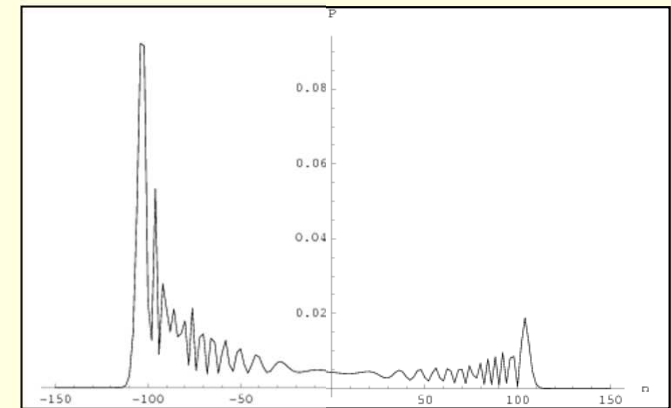
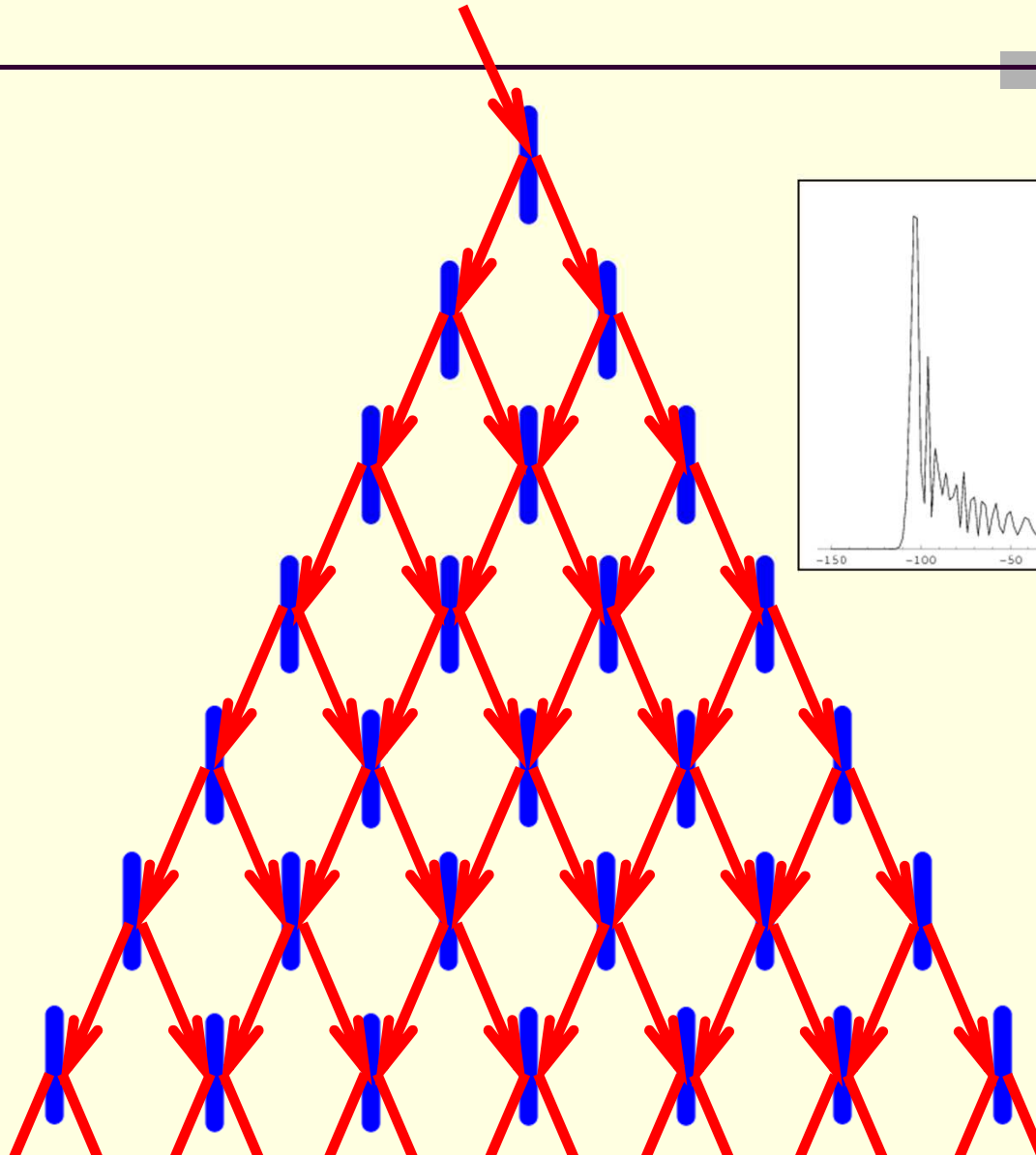
J. Kempe, *Contemp. Phys.* **44** (4), 307 (2003).

V.M. Kendon, *Phil. Trans. R. Soc. A* **364**, 3407 (2006).

Simple example – 2D quantum walk



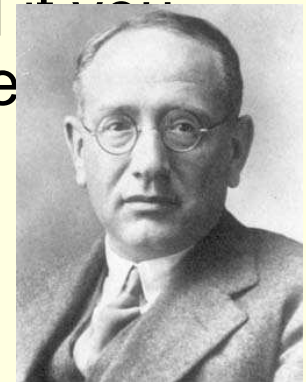
Optical Galton board



Classification of walks

G. Pólya -- hungarian mathematician (13 Dec 1887, Budapest (Hungary) - 7 Sept 1985, Palo Alto (USA))
worked on probability, analysis, number theory, geometry, combinatorics and mathematical physics, highly interested in teaching natural sciences to students

A GREAT discovery solves a great problem, but there is a grain of discovery in the solution of any problem. Your problem may be modest, but if it challenges your curiosity and brings into play your inventive faculties, and if you solve it by your own means, you may experience tension and enjoy the triumph of discovery.



G. Pó



Über eine Aufgabe der Wahrscheinlichkeitsrechnung betreffend die Irrfahrt im Straßennetz.

Von

Georg Pólya in Zürich.

1. Ich beziehe den d -dimensionalen Raum auf ein rechtwinkliges Koordinatensystem. Ich betrachte diejenigen Punkte, deren Koordinaten x_1, x_2, \dots, x_d sämtlich ganzzahlig sind, und solche Verbindungsgeraden dieser Punkte, die einer der d Koordinatenachsen parallel sind. Die Gesamtheit dieser Geraden bildet das d -dimensionale *Geradennetz*, und die Punkte mit ganzzahligen Koordinaten, die man gewöhnlich als Gitterpunkte bezeichnet, sollen die *Knotenpunkte* des Netzes heißen. In jedem Knotenpunkte kreuzen sich d zueinander rechtwinklige Geraden des Netzes, und jede Gerade wird durch die daraufliegenden Knotenpunkte in gleiche Stücke von der Länge 1 geteilt. Auf dem Geradennetz soll ein Punkt auf Geratewohl herumfahren. D. h. an jeden neuen Knotenpunkt des Netzes angelangt, soll er sich mit der Wahrscheinlichkeit $\frac{1}{2d}$ für eine der möglichen $2d$ Richtungen entscheiden. Der Bestimmtheit halber wollen wir uns vorstellen, daß der herumwandernde Punkt zur Zeit $t = 0$ im Anfangspunkt des Koordinatensystems seine Irrfahrt beginnt, und daß er sich mit der Geschwindigkeit 1 bewegt. In der Zeit t beschreibt er einen Zickzackweg von der Länge t , in jedem ganzzahligen Zeitpunkt $t = 0, 1, 2, 3, \dots$ passiert er einen Knotenpunkt und fällt eine vom Zufall geleitete Entscheidung unter $2d$ gleichmöglichen Richtungen.

Für $d = 1$ haben wir eine, in gleiche Segmente geteilte, unbegrenzte Gerade und die geometrische Darstellung des „Wappen-oder-Schrift“-Spiels vor uns. Die Wappenseite einer Münze soll einem Spieler eine Geldeinheit Gewinn einbringen, die Schriftseite einen ebenso großen Verlust; der jeweilige Stand von Gewinn und Verlust soll als positiver bzw. negativer Abstand an einer Geraden von einem festen Ausgangspunkte aus durch eine bewegliche Marke registriert werden. Nach jedem Wurf verschiebt

nough for physics
osophy.

n.

Pólya number – the problem

Basic problem -- what is the probability for a random walker to return to its starting position

characteristics of random walks considered

- the walker leaves its actual position in each step with certainty
- probability for each direction is the same $1/(2d)$ with d being the dimensionality of the walk grid

Definition of Pólya number

Definition of the Pólya number

$$P = 1 - \frac{1}{\sum_{t=1}^{\infty} p_o(t)}$$

$p_o(t)$ is the probability the walker returns to the origin after t steps

The value of the Pólya number gives two classes

- $P = 1$ - recurrent
- $P < 1$ - transient

link to the meeting problem for two walkers

Classical walks and dimension

Pólya showed (1921) that $p_o(t) \sim 1/t^{d/2}$ and hence

- the walk is recurrent for $d=1, 2$
- the walk is transient for $d > 2$

When is a walk recurrent?

- It is recurrent if the probability at the origin $p_o(t)$ does not decay faster than $1/t$

$$\lim_{t \rightarrow \infty} (t \cdot p_o(t)) > 0$$

Pólya number for quantum walks

$$P = 1 - \bar{P}$$

$$\bar{P} = \prod_{t=1}^{\infty} (1 - p_o(t)) \approx K_c \exp\left(-\int_{t_c}^{\infty} p_o(t) dt\right)$$

$$p_o(t) = \|\psi(0, t)\|^2$$

$$\psi(0, t) = \sum_{j=1}^d I_j(t) \quad I_j(t) = \int_{K^d} \frac{dk}{(2\pi)^d} e^{i\omega_j(k)t} f_j(k)$$

$$f_j(k) = (\psi(0, 0), v(k)) \cdot v(k)$$

Controlling the Pólya number

$$\psi(0,t) = \sum_{j=1}^d I_j(t) \quad I_j(t) = \int_{K^d} \frac{dk}{(2\pi)^d} e^{i\omega_j(k)t} f_j(k)$$

The recurrence of QW is determined by

- the initial state $\psi(0,0)$
- the coin C (determines the spectrum)
- the topology of the walk determined by the displacements

This is in great contrast to the classical RW, where the recurrence behavior is uniquely determined by the dimension of the RW.

Implications

The asymptotic behavior of $I_j(t)$ is affected only by the saddle points of the phase $\diamond_j(k)$ and their degeneracy

- if there are no saddle points - $I_j(t)$ decay exponentially
- if there is a finite number of non-degenerate saddle points then of $I_j(t) \sim 1/t^{d/2}$ and hence

$$p_o(t) \sim 1/t^d$$

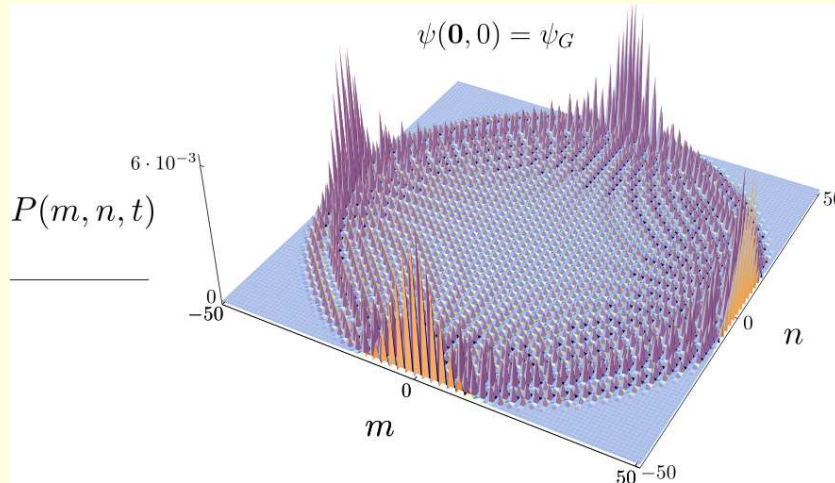
and the 1-D walk should be recurrent

- when $\diamond_j(k)$ does not depend explicitly on some k_i we can split the integral and after evaluation

$$p_o(t) \sim 1/t^{d-n}$$

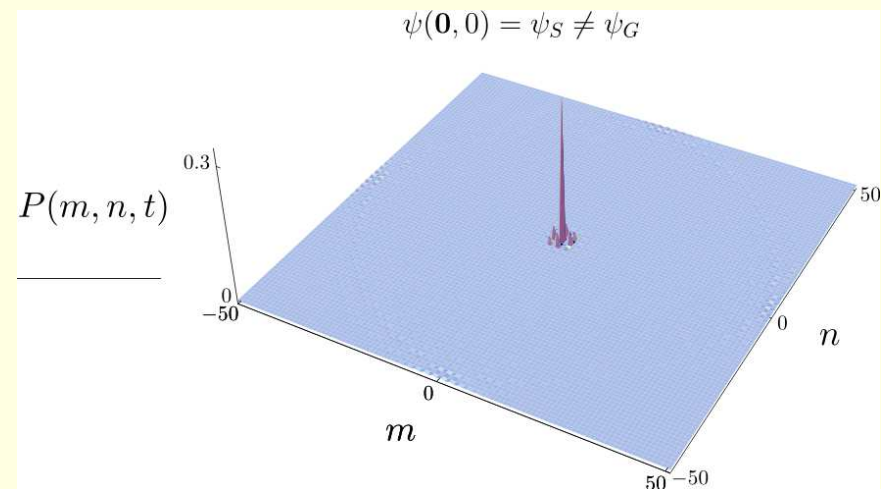
and we can find localization (in higher dimensions we can slow down the decay at the origin)

Example of localization I - Grover walk

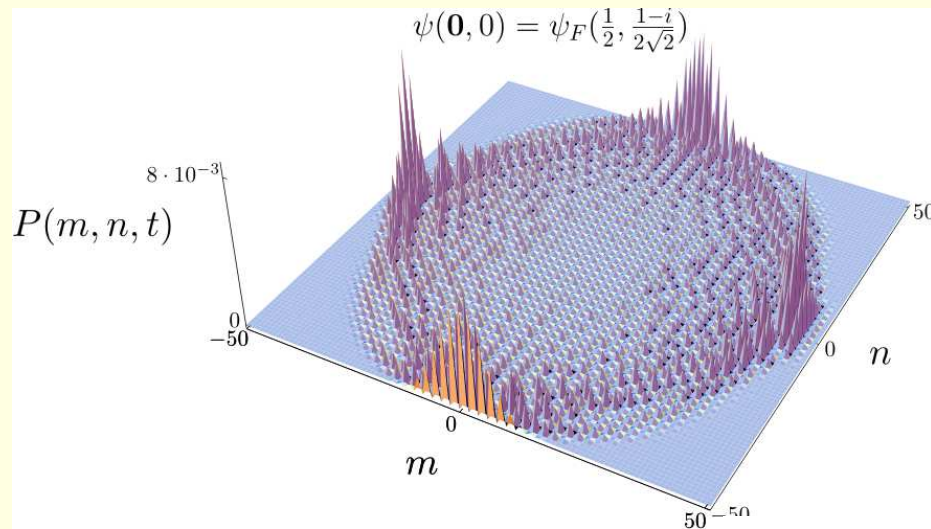


$$G = \frac{1}{2} \begin{pmatrix} -1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 \\ 1 & 1 & -1 & 1 \\ 1 & 1 & 1 & -1 \end{pmatrix}$$

The Grover walk exhibits localization and is for certain initial states recurrent

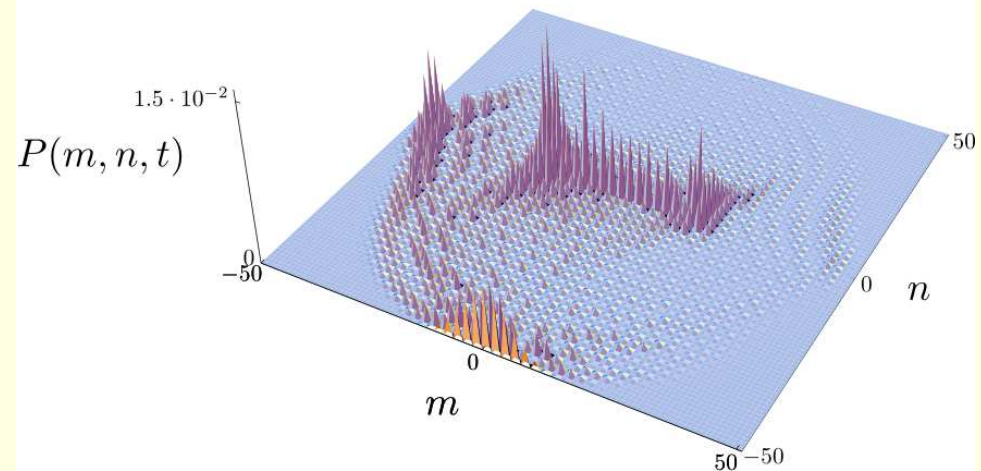


Example of localization II - Fourier walk



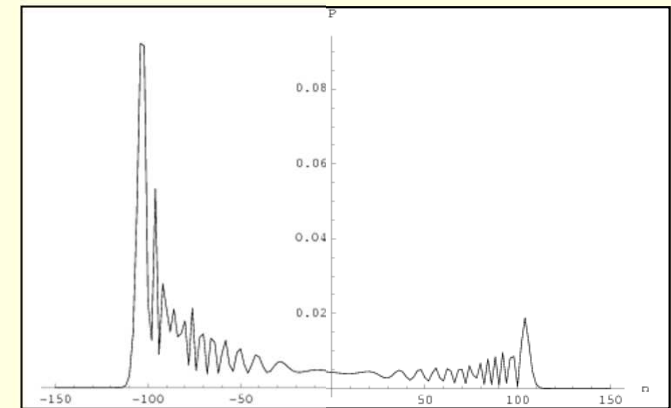
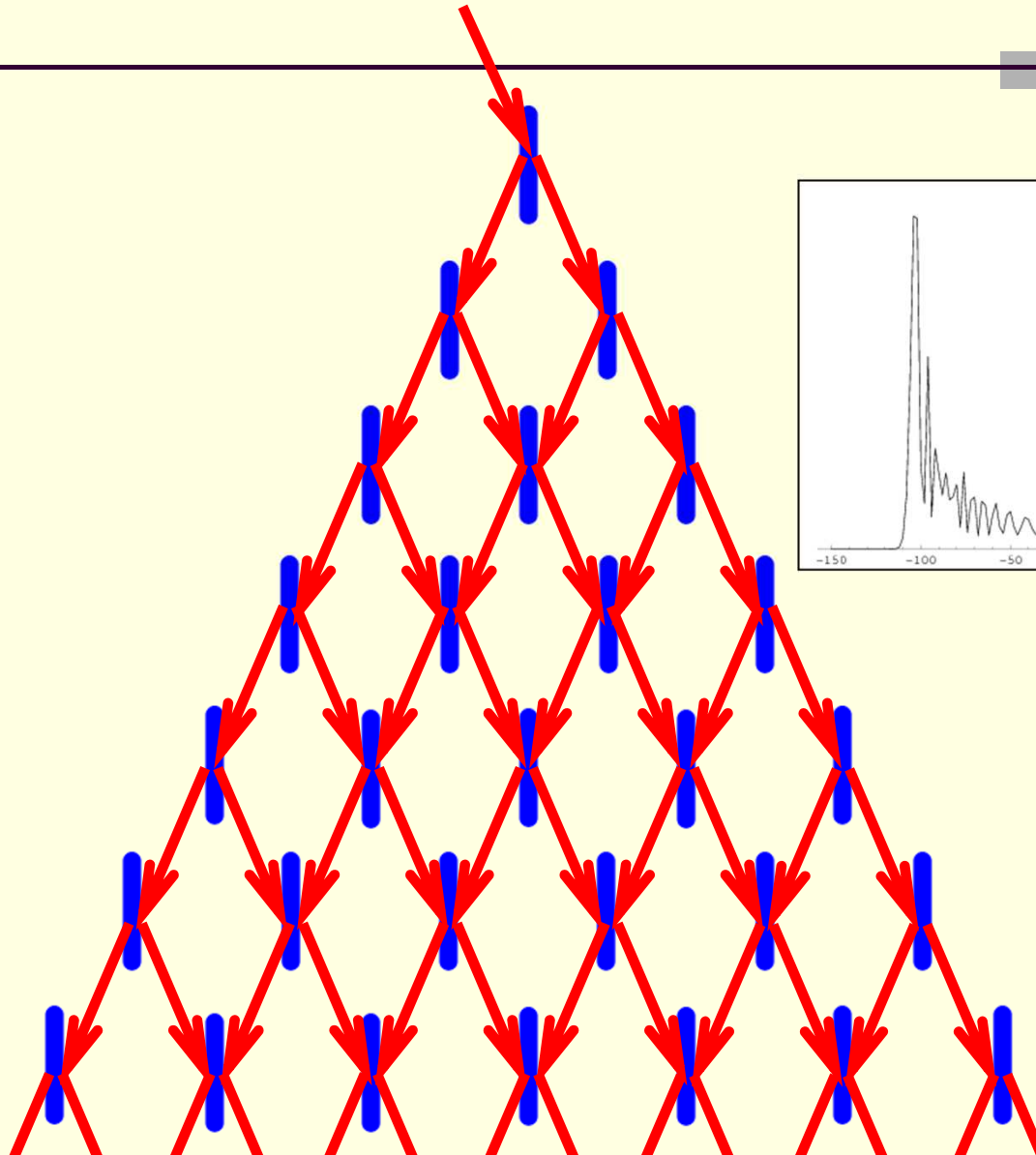
$$F = \frac{1}{2} \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & i & -1 & -i \\ 1 & -1 & 1 & -1 \\ 1 & -i & -1 & i \end{pmatrix}$$

$$\psi(\mathbf{0}, 0) = (1, 0, 0, 0)^T$$

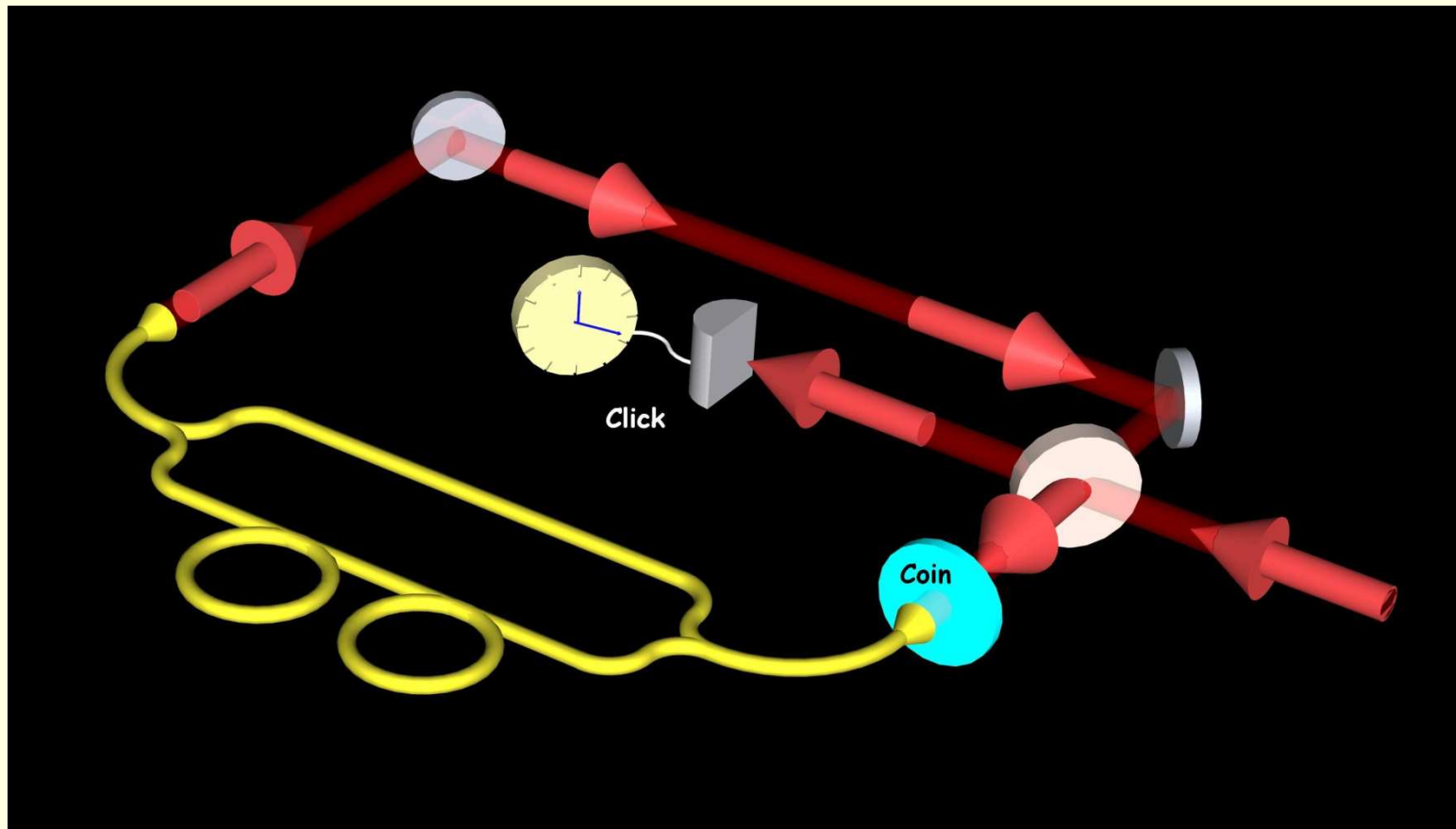


The Fourier walk does not exhibit localization but is for certain initial states recurrent

Optical Galton board

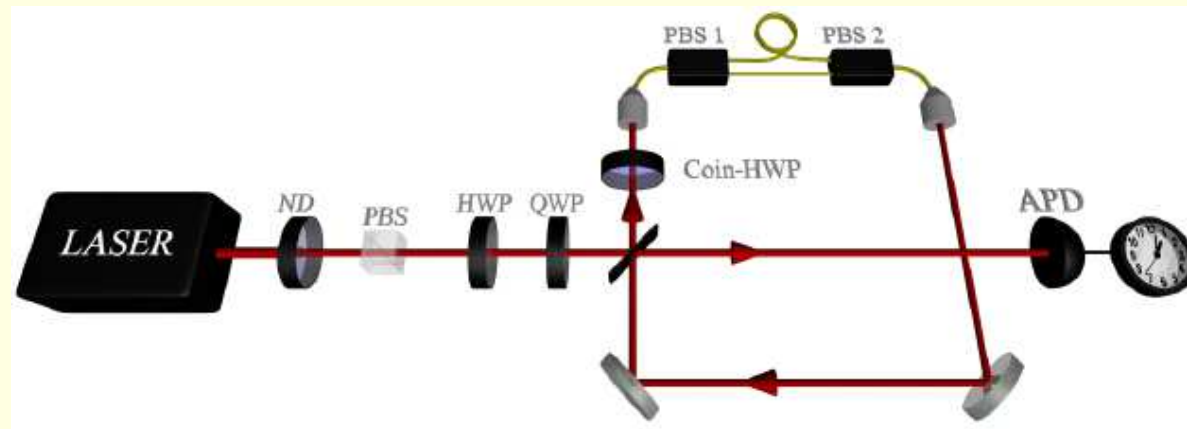
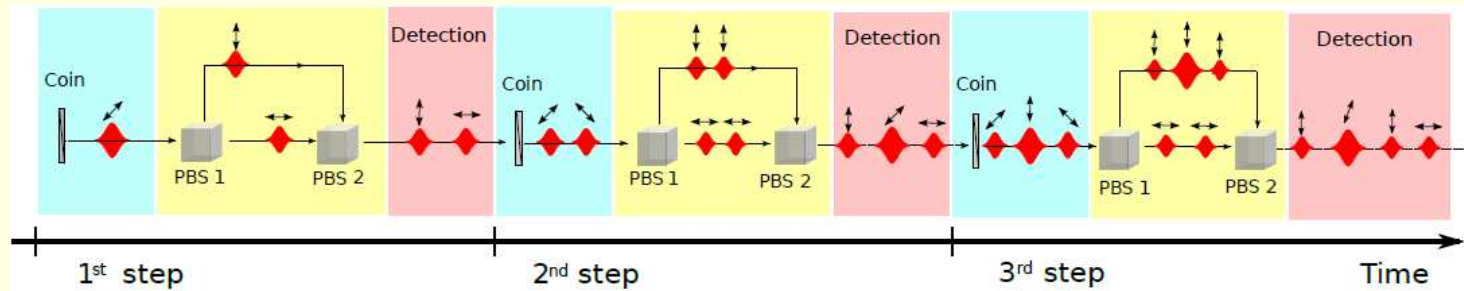


An implementation



H. Schmitz, R. Matjeschk, Ch. Schneider, J. Glueckert, M. Enderlein, T. Huber, T. Schaetz Phys. Rev. Lett. **103**, 090504 (2009),
M. Karski, L. Forster, Jai-Min Choi, A. Steffen, W. Alt, D. Meschede, A. Widera Science 325, 174 (2009)

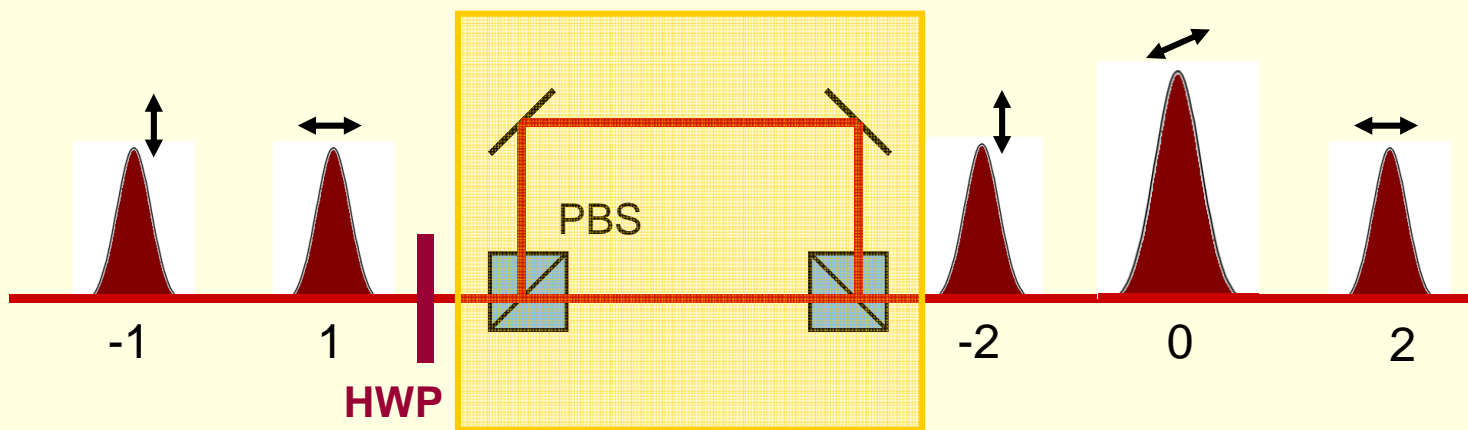
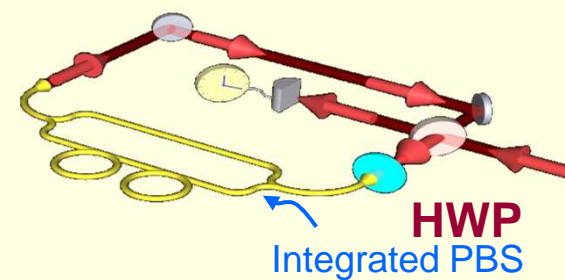
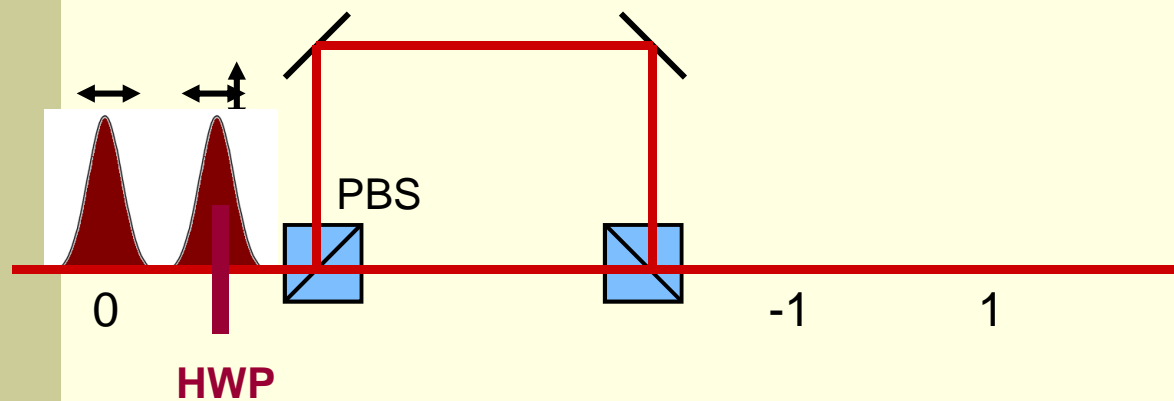
The mechanism I



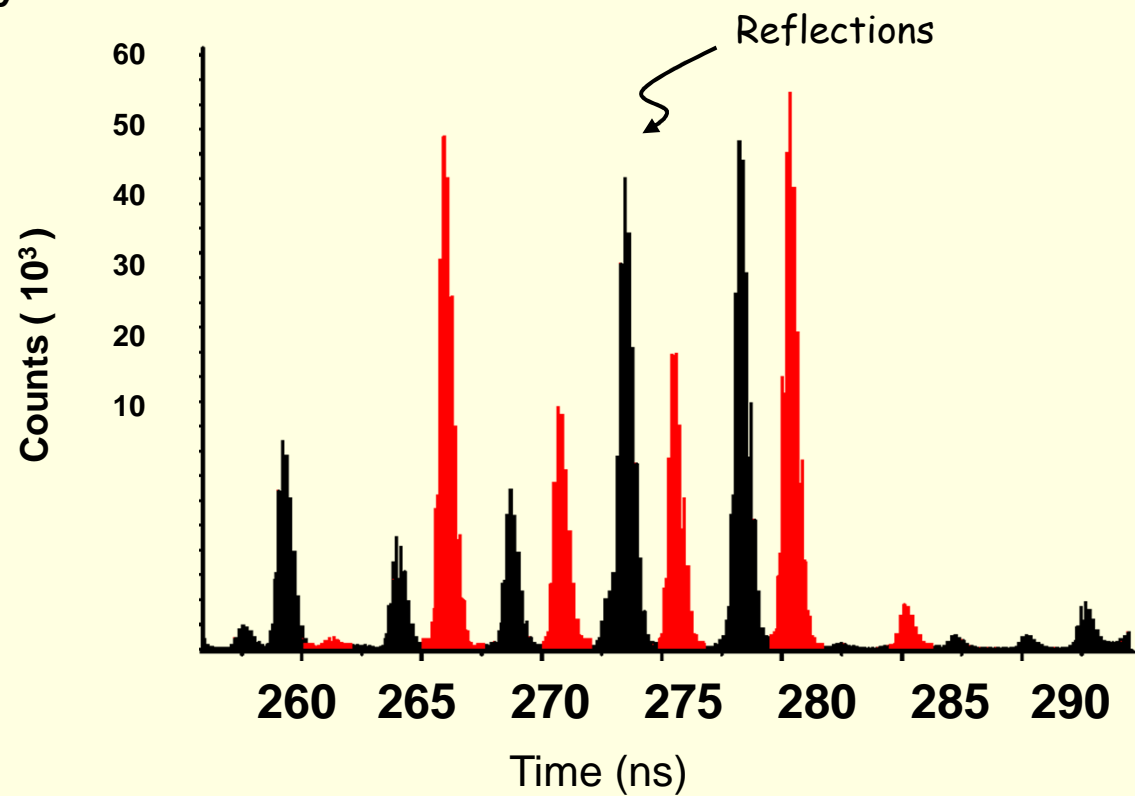
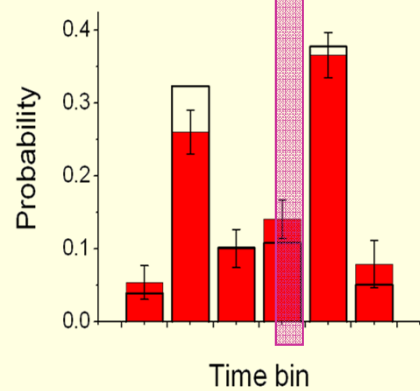
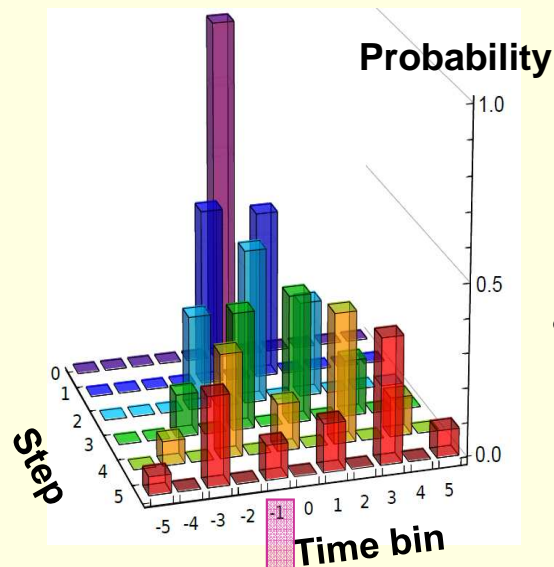
A. Schreiber, K. N. Cassemiro, V. Potocek, A. Gabris, P. J. Mosley, E. Andersson, I. Jex, Ch. Silberhorn
Phys. Rev. Lett. **104** (2010) 05502

The mechanism II

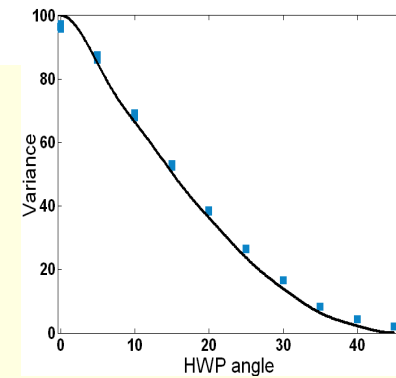
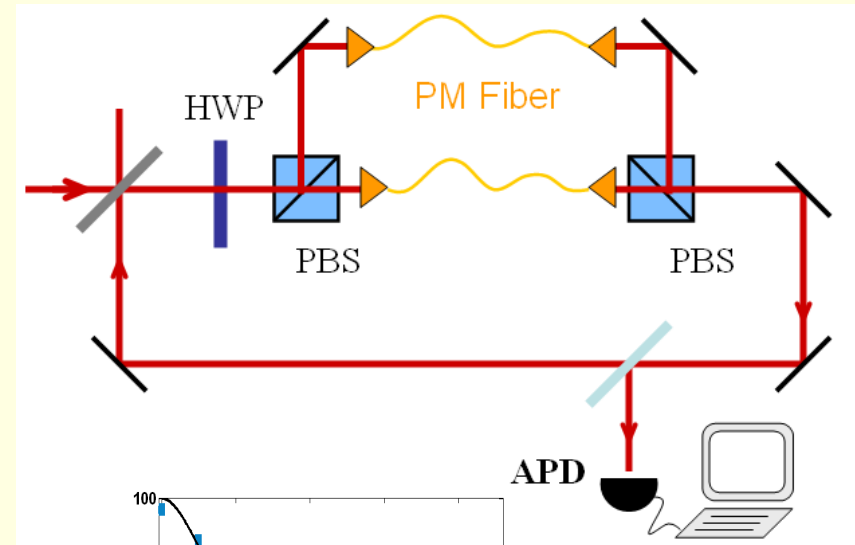
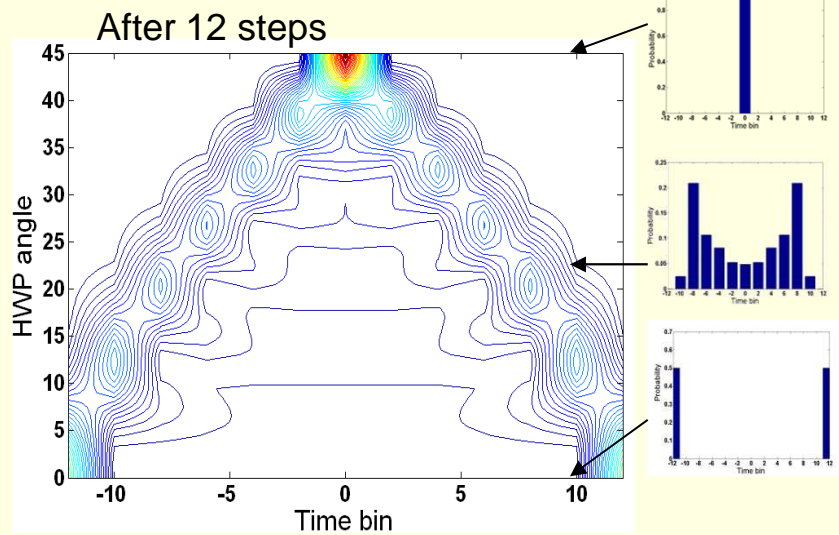
Coin: **Polarization** degree of freedom



First results

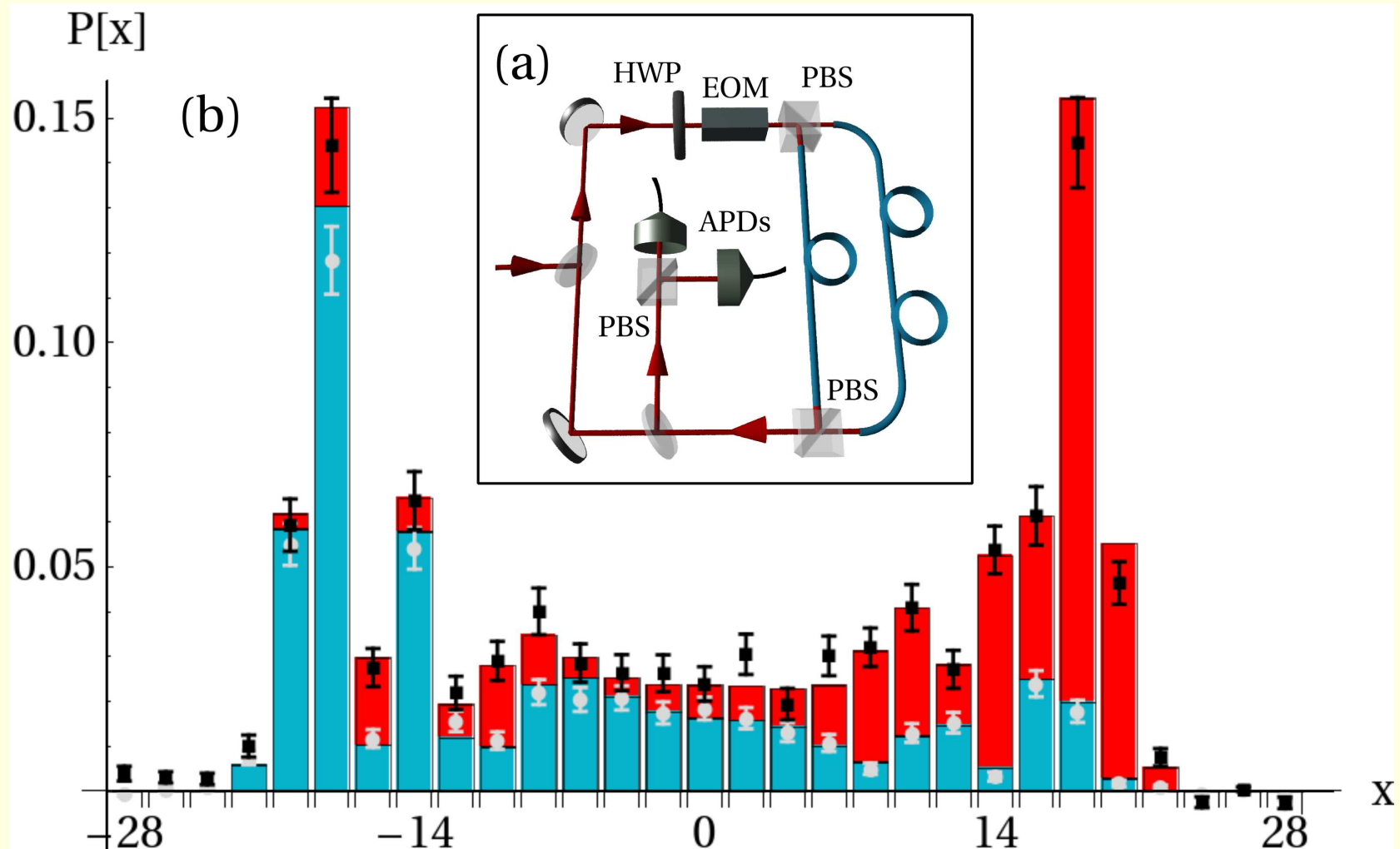


Improved setup – coin control

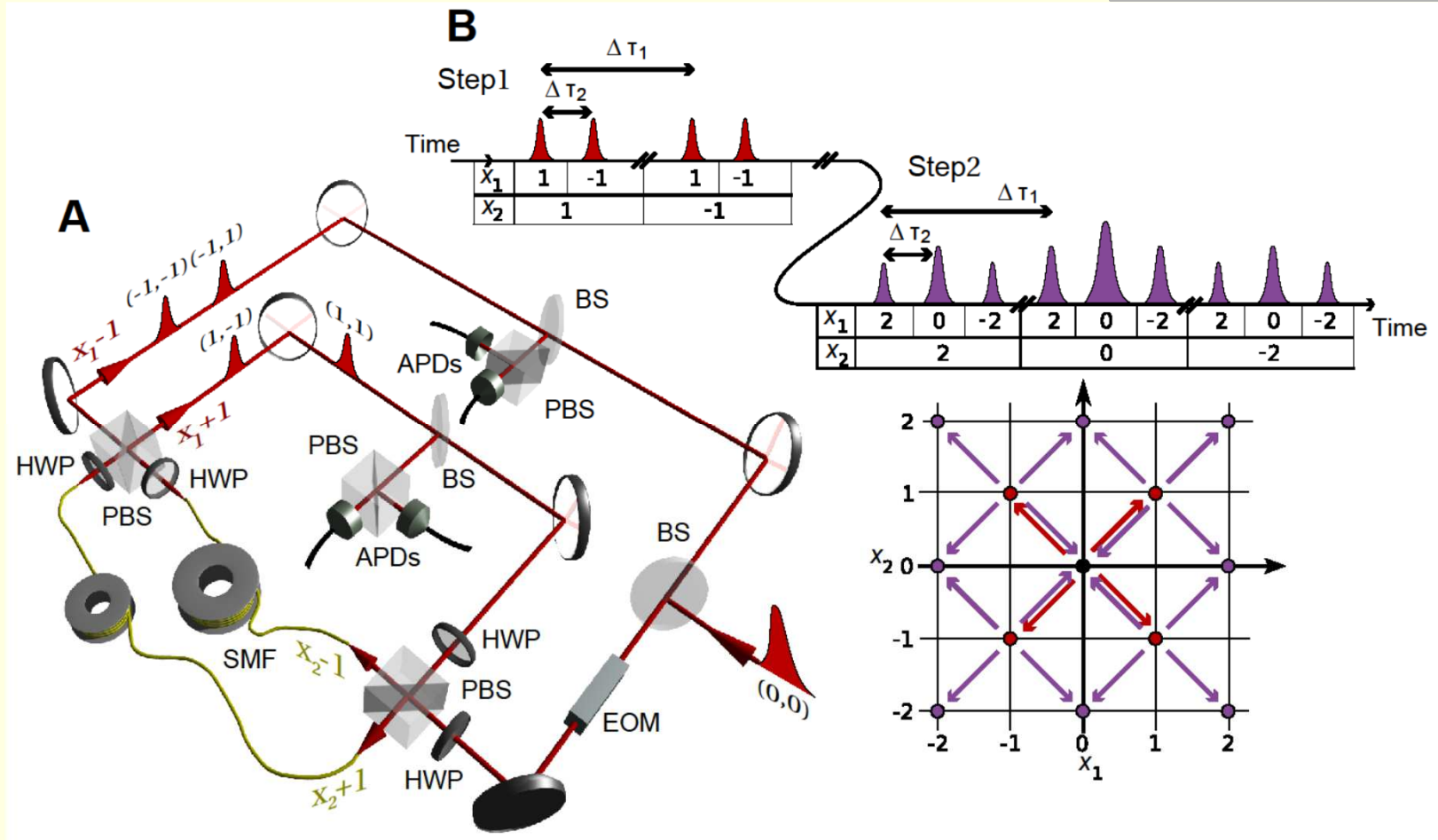


After 10 steps

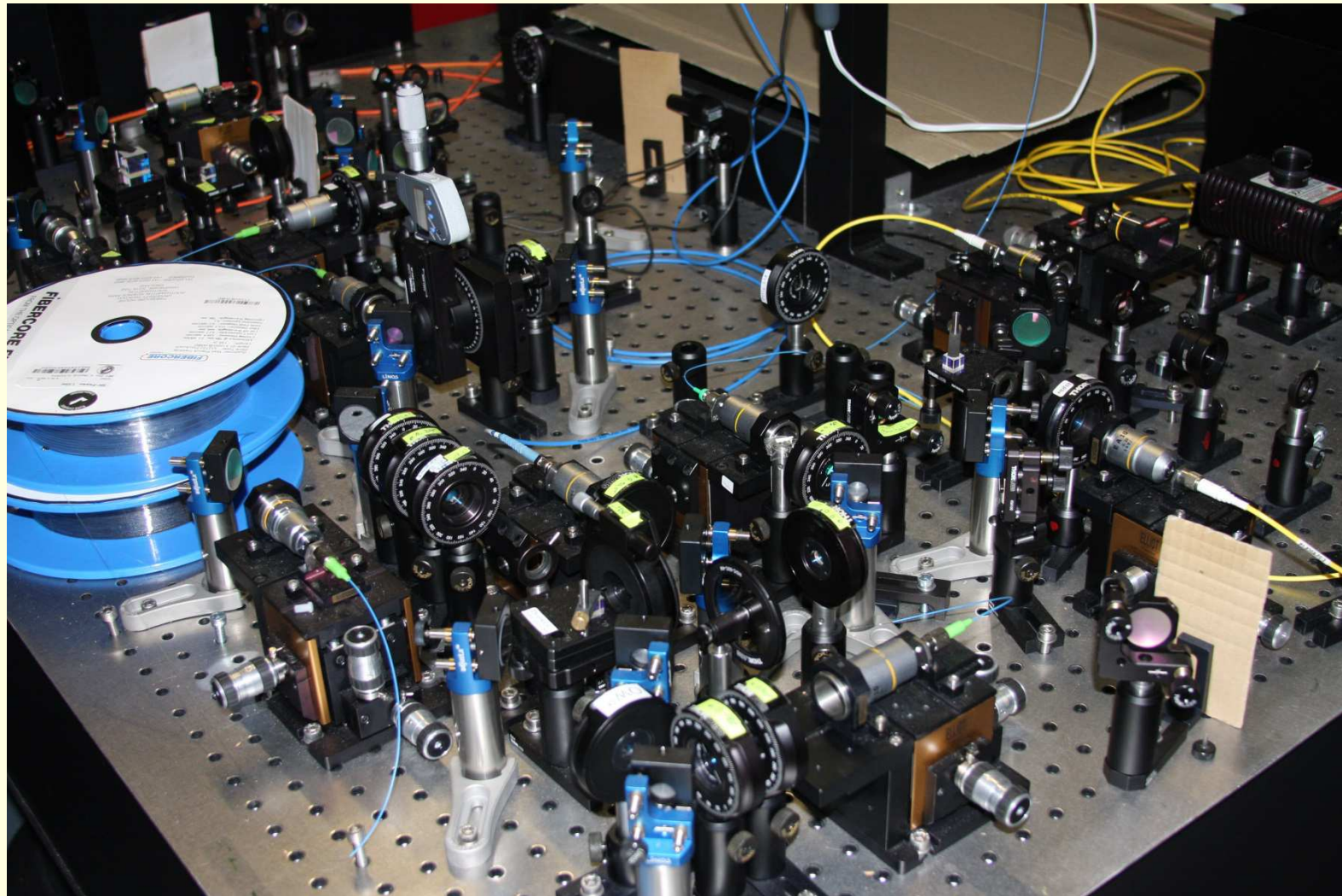
Advanced experiments



The new experiment



The actual experiment



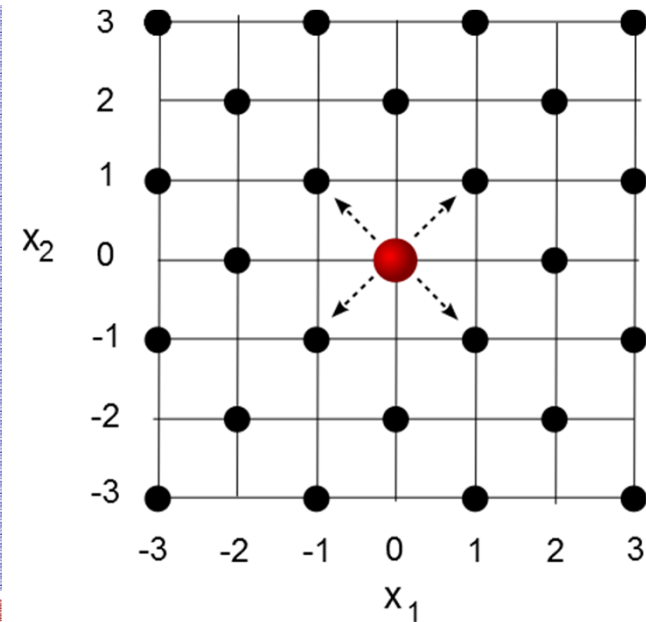
Discrete-time QW in 2D

$$|\psi_n\rangle = \sum_{x_1, x_2, c_1, c_2} a_{x_1, x_2, c_1, c_2} |x_1, x_2\rangle \otimes |c_1, c_2\rangle$$

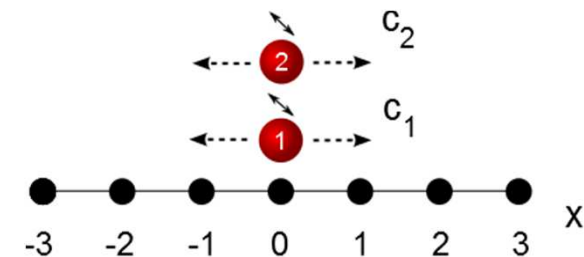
Position and coin state for
dimension 1 and 2

Coin operation: U(4)

Step operation: Shift in x_1 - and x_2 -**direction**



=



Two-Particle QW in 1D

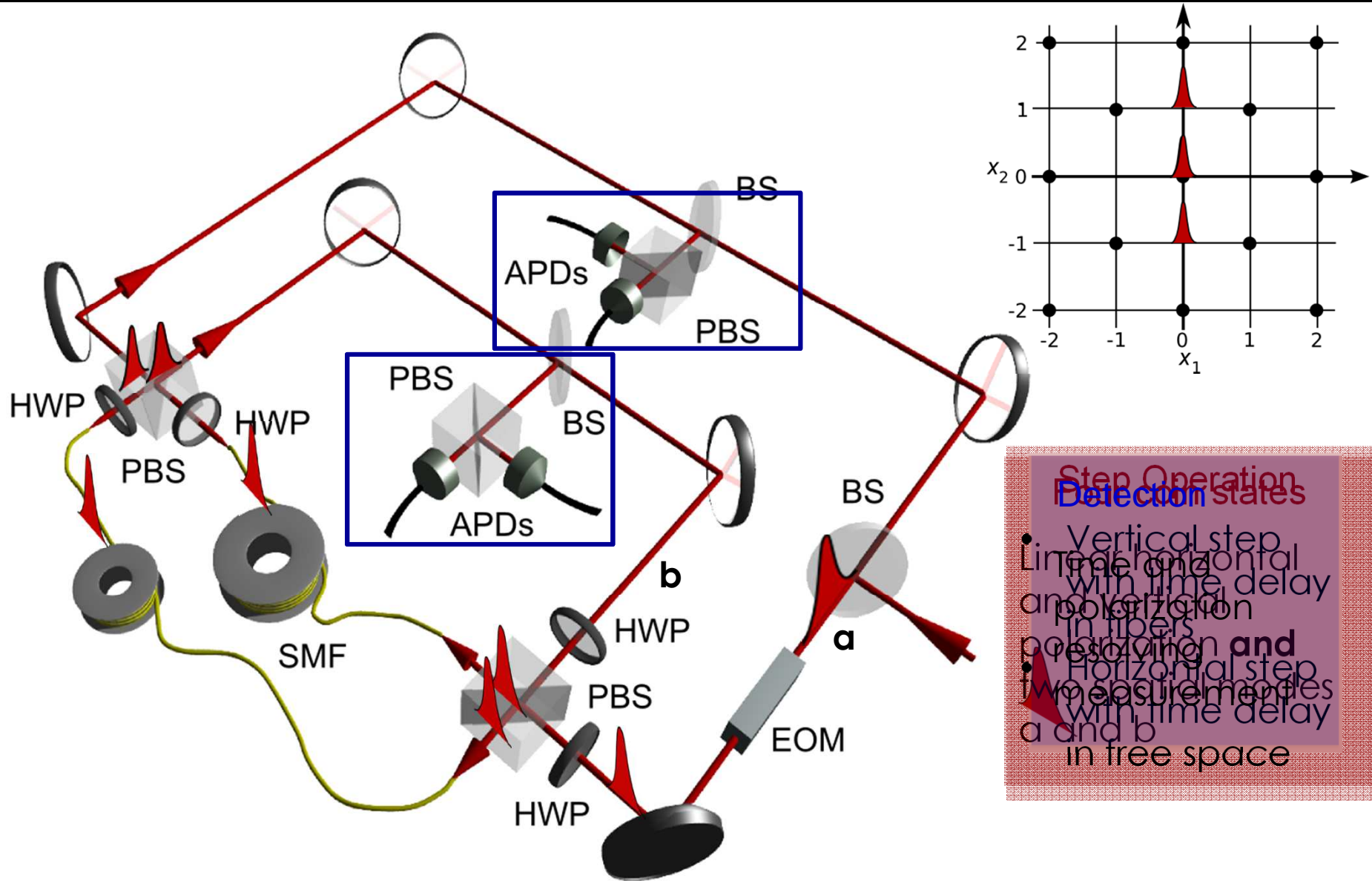
$$|\psi_n\rangle = \sum_{x_1, x_2, c_1, c_2} a_{x_1, x_2, c_1, c_2} |x_1, x_2\rangle \otimes |c_1, c_2\rangle$$

Position and coin state for
particle 1 and 2

Coin operation: U(4)

Step operation: Shift for **particle 1 and 2**

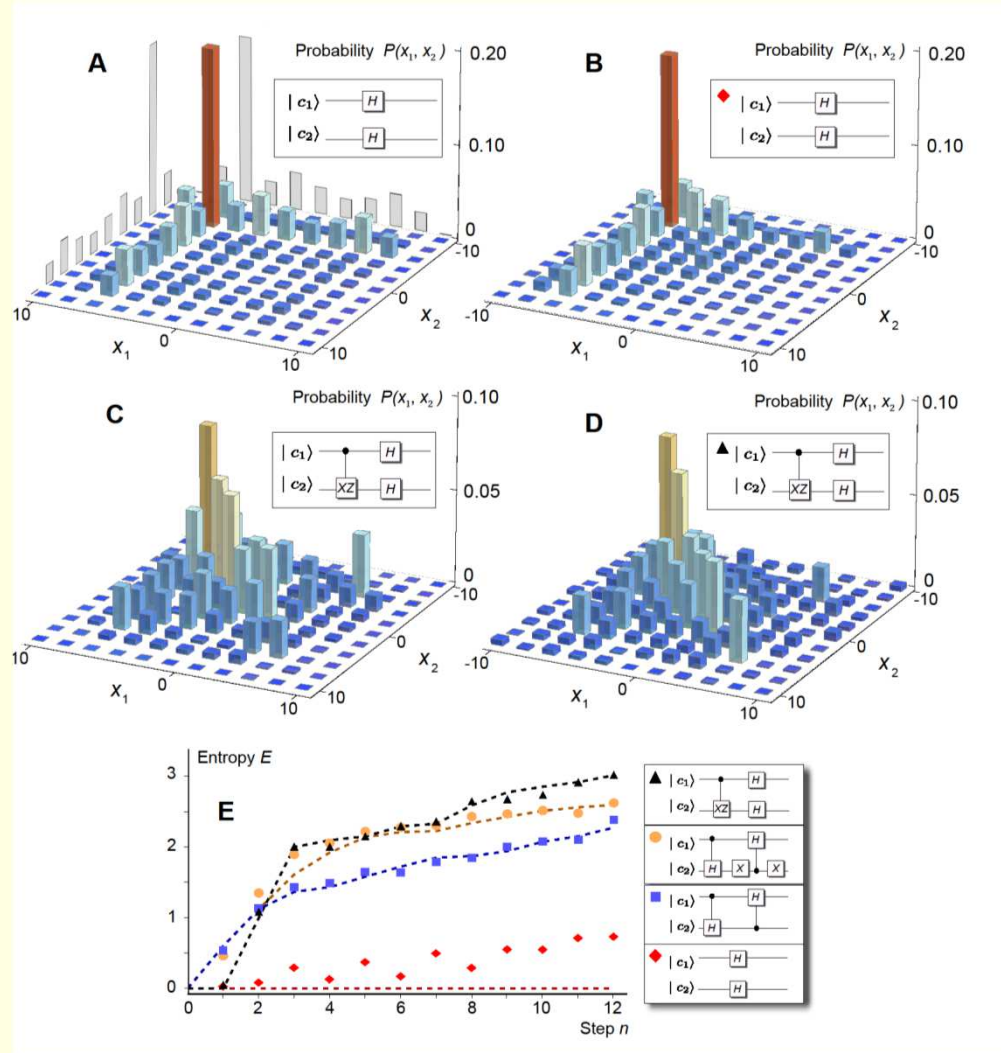
2D Implementation



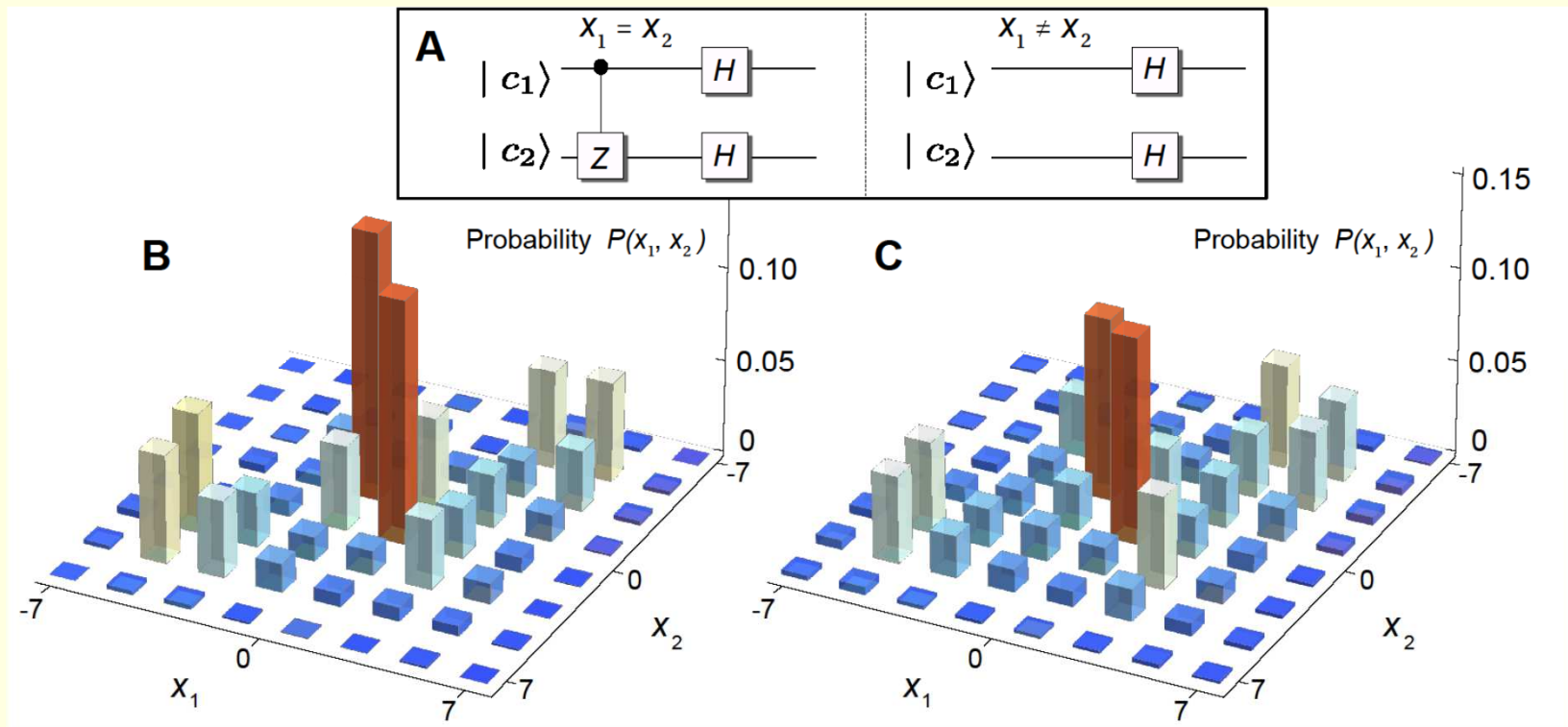
Step Operation
Detection states

- Vertical step
 Line of horizontal
 with time delay
 and polarization
 in fibers
- Horizontal step
 two measurements
 with time delay
 a and b
 in free space

Results I

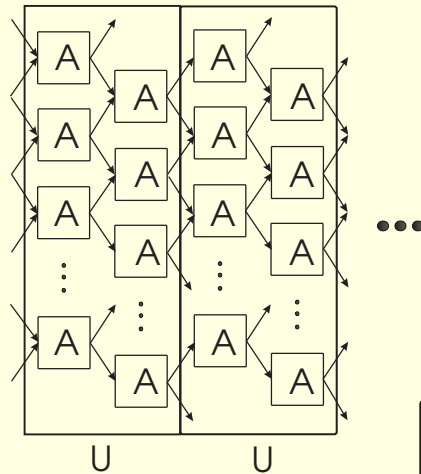
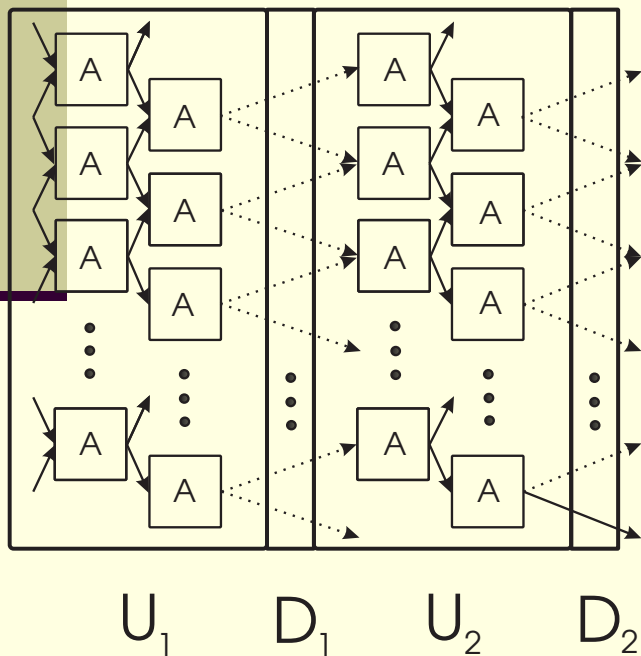


Results II



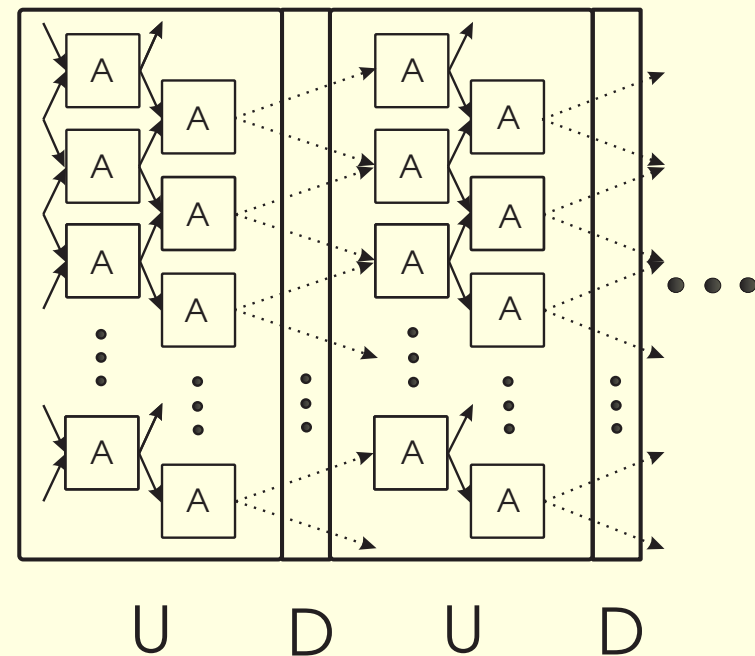
Quantum walks with errors

Continuous error change

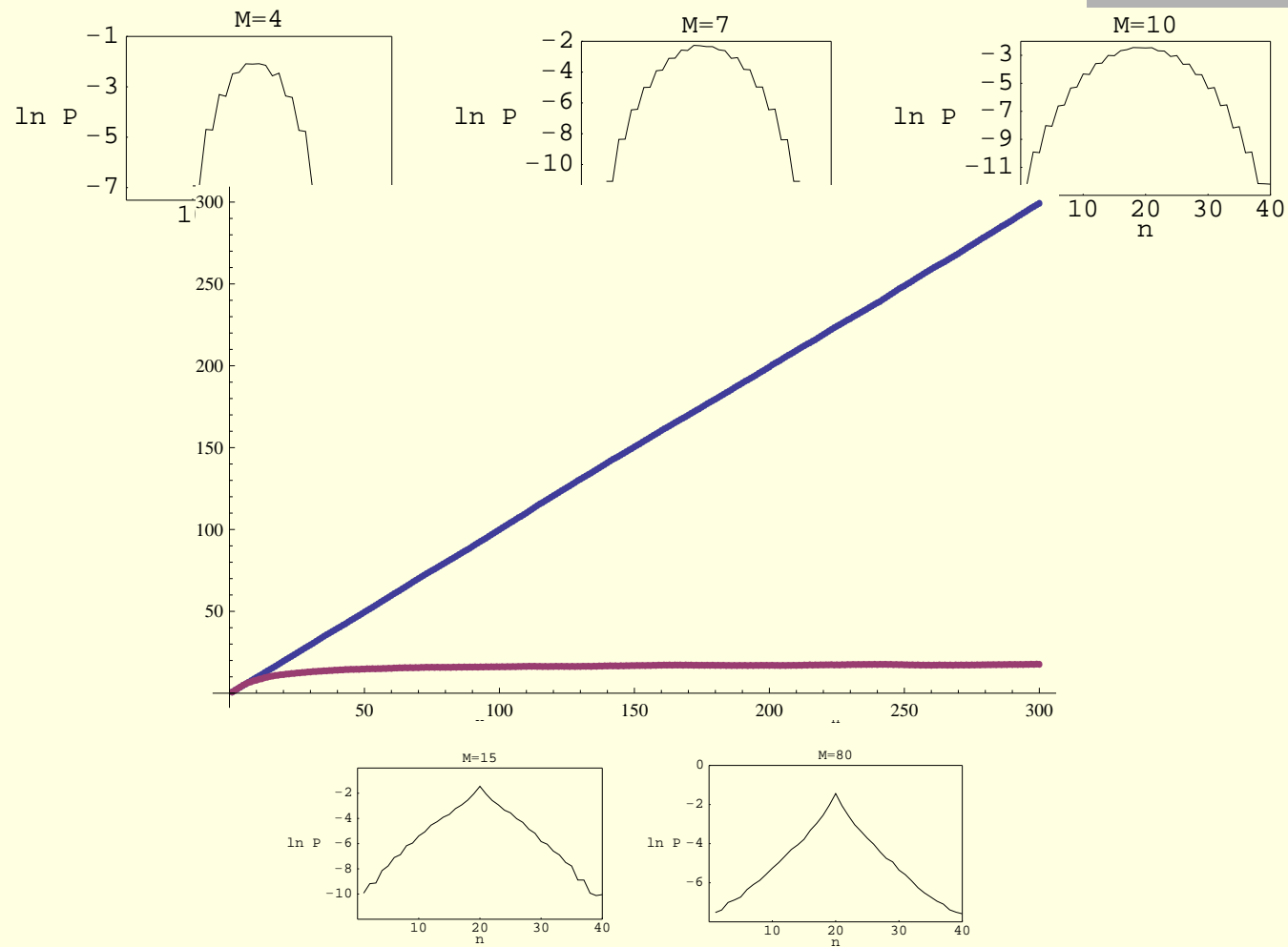


Ideal case

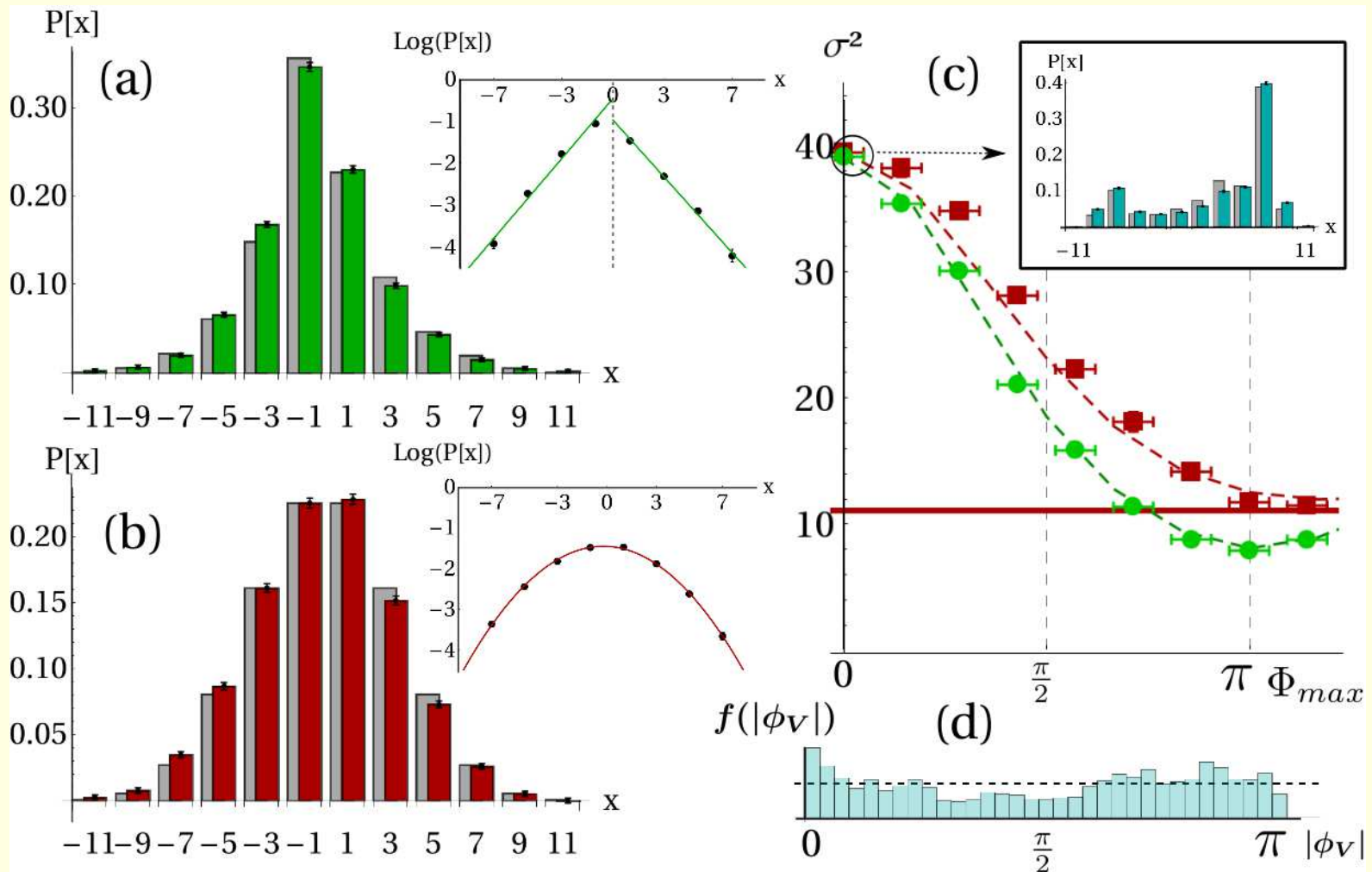
Repeated error



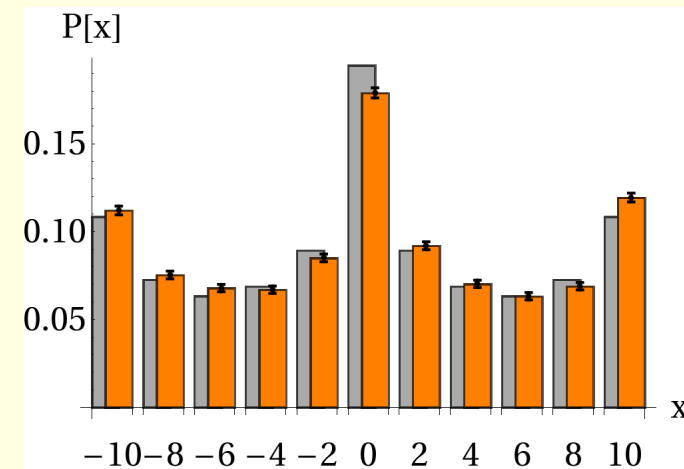
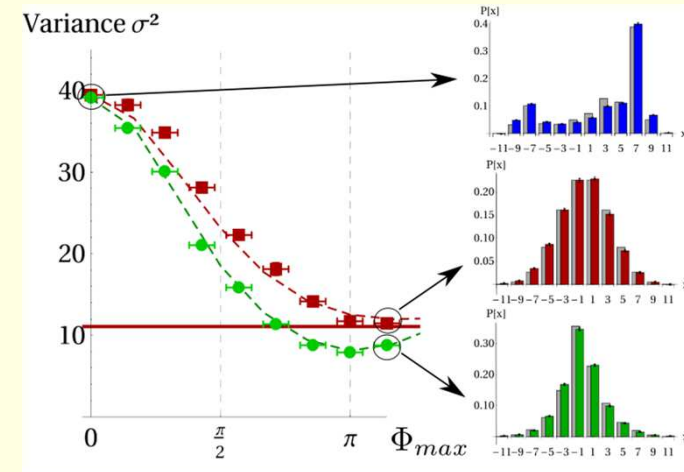
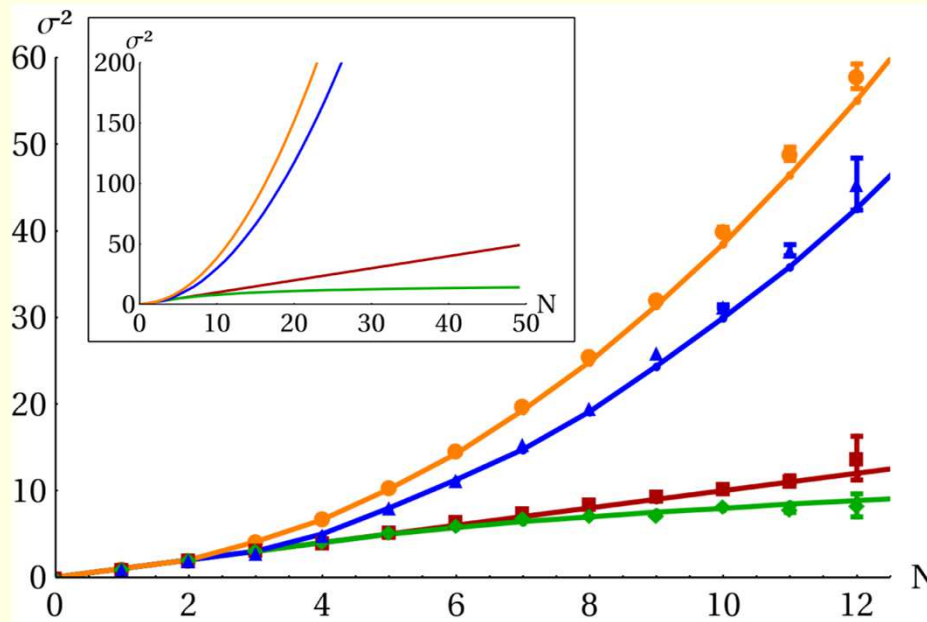
Effect -localization



Randomization I

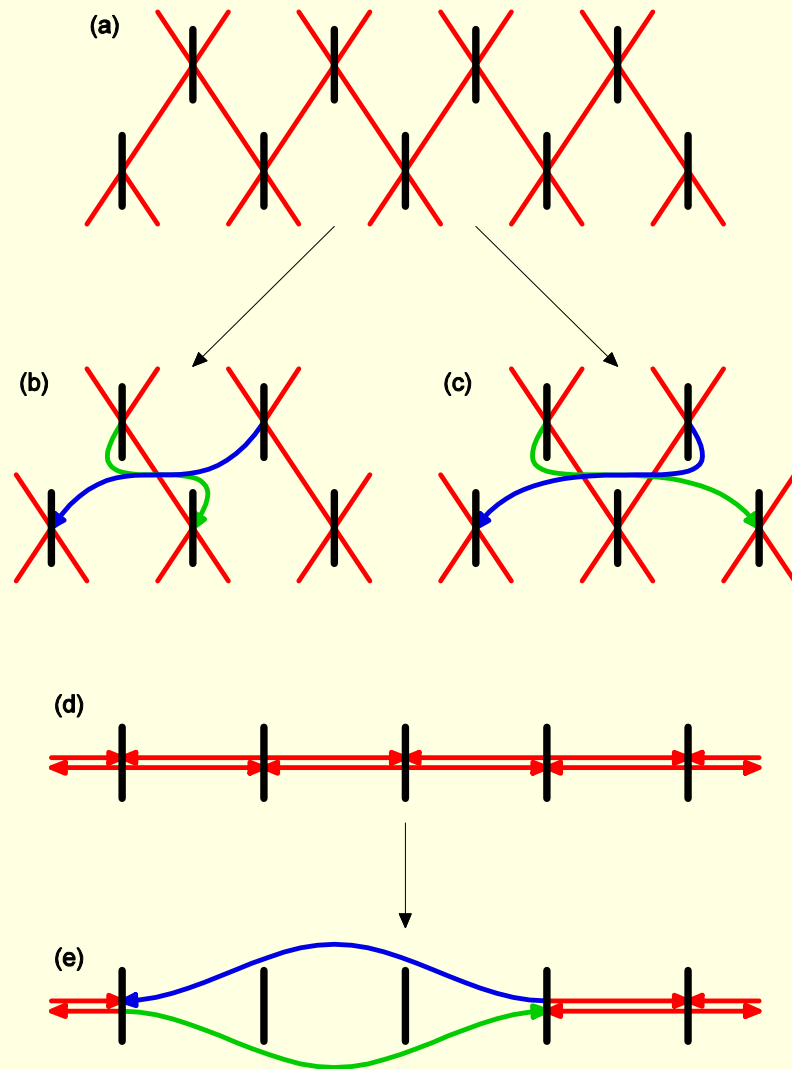


Randomization II

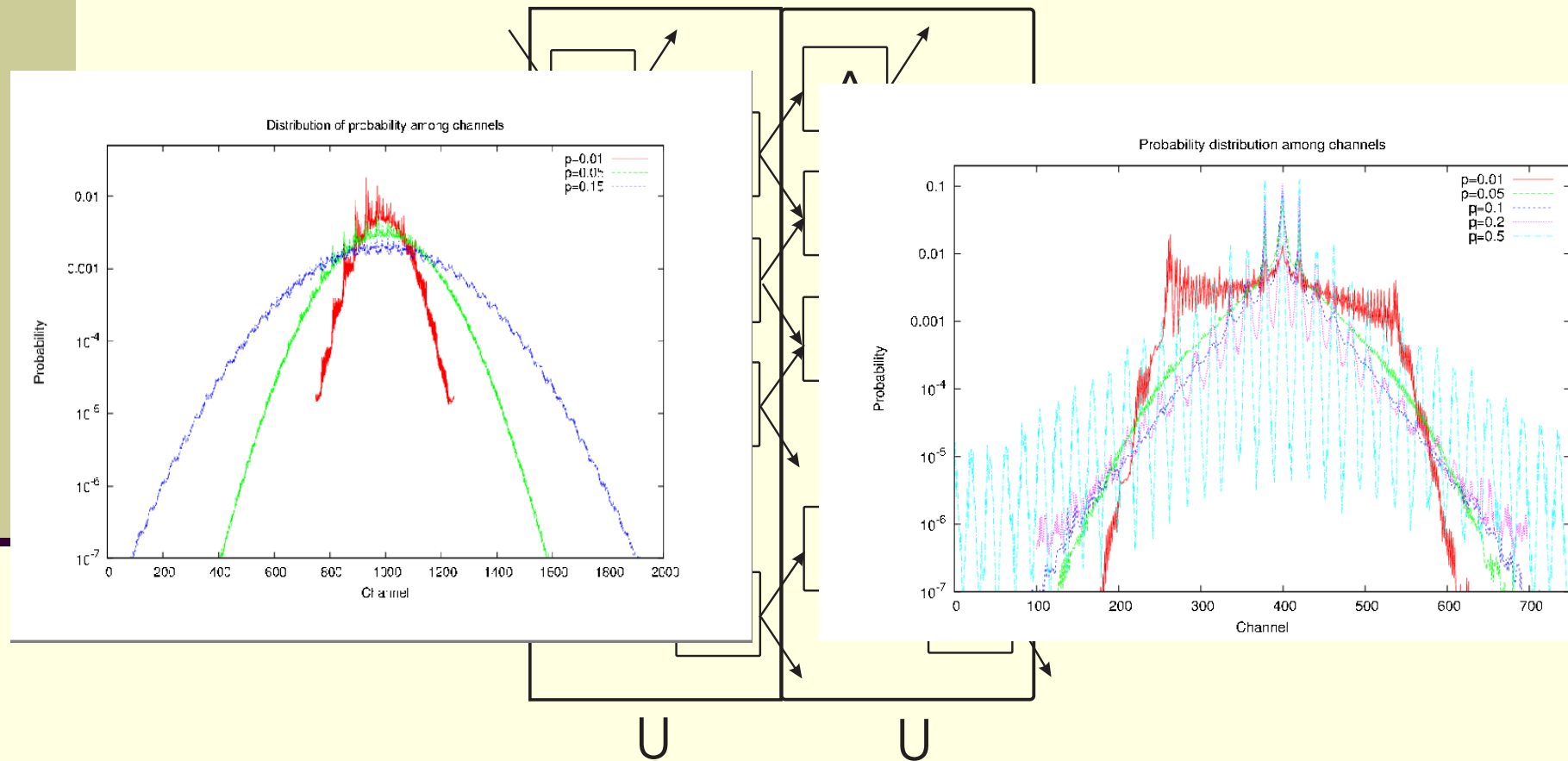


A. Schreiber, K. N. Cassemiro, V. Potoček, A. Gábris, I. Jex, and Ch. Silberhorn Phys. Rev. Lett. **106** (2011) 180403

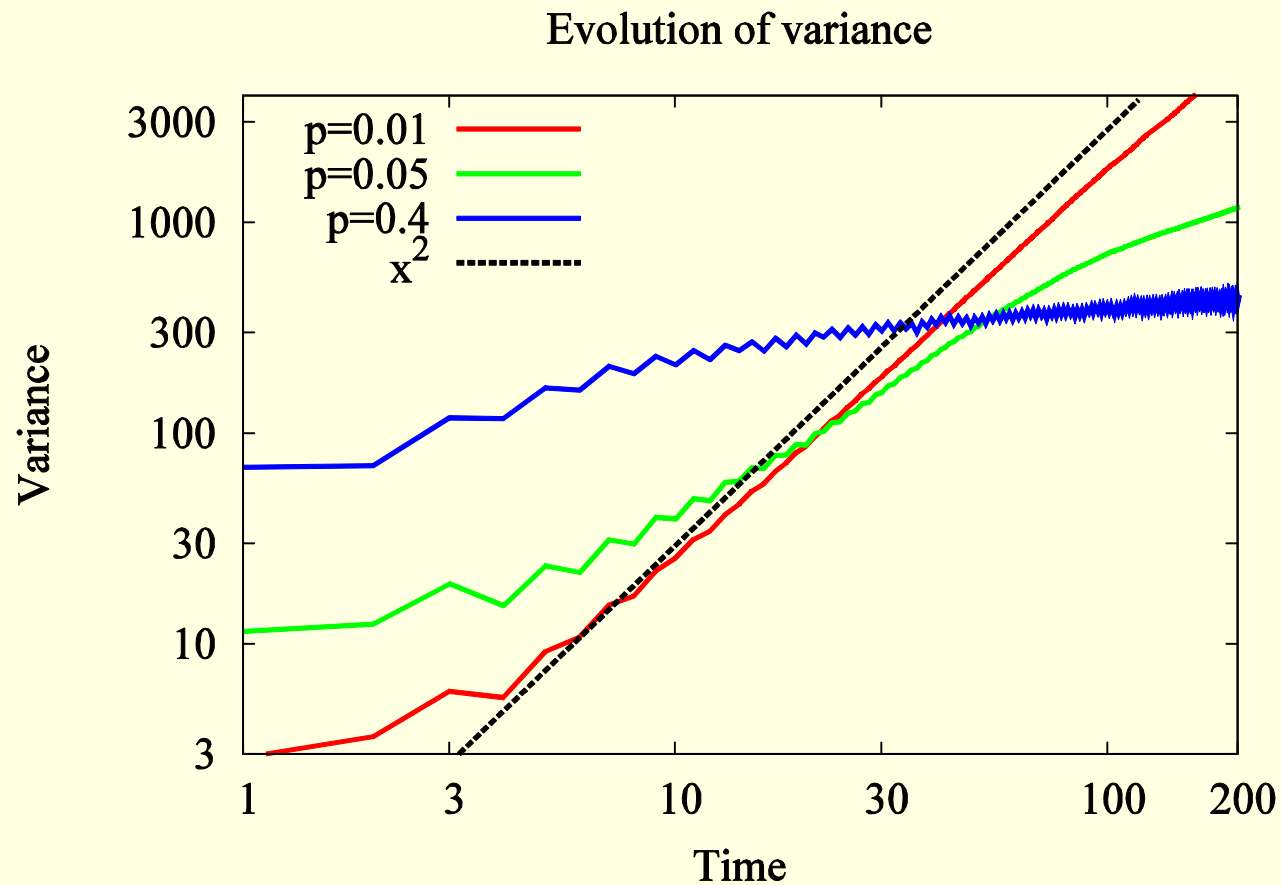
Localization and jumps



Localization and jumps

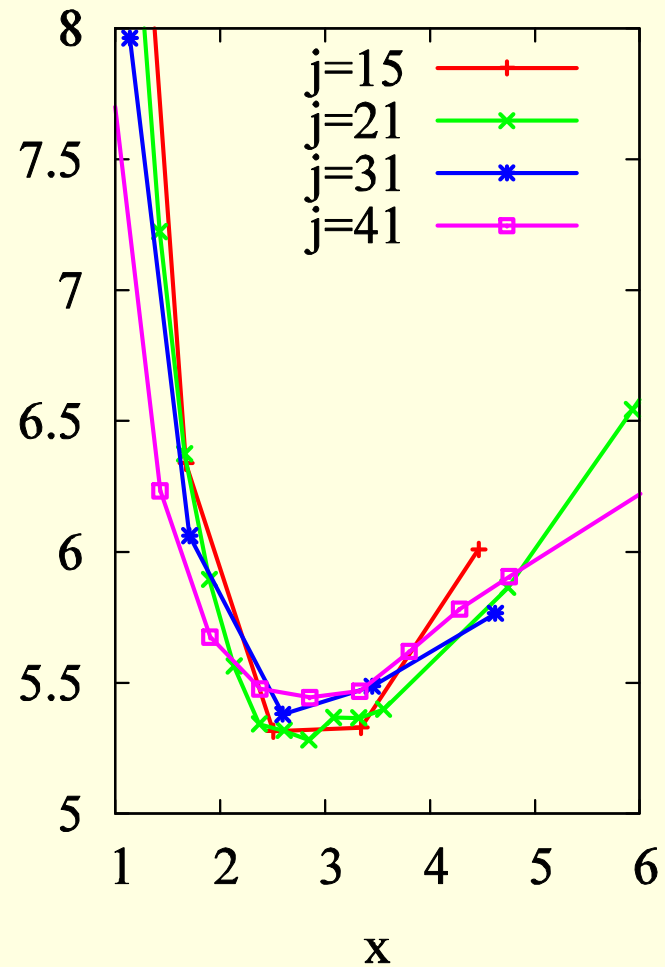
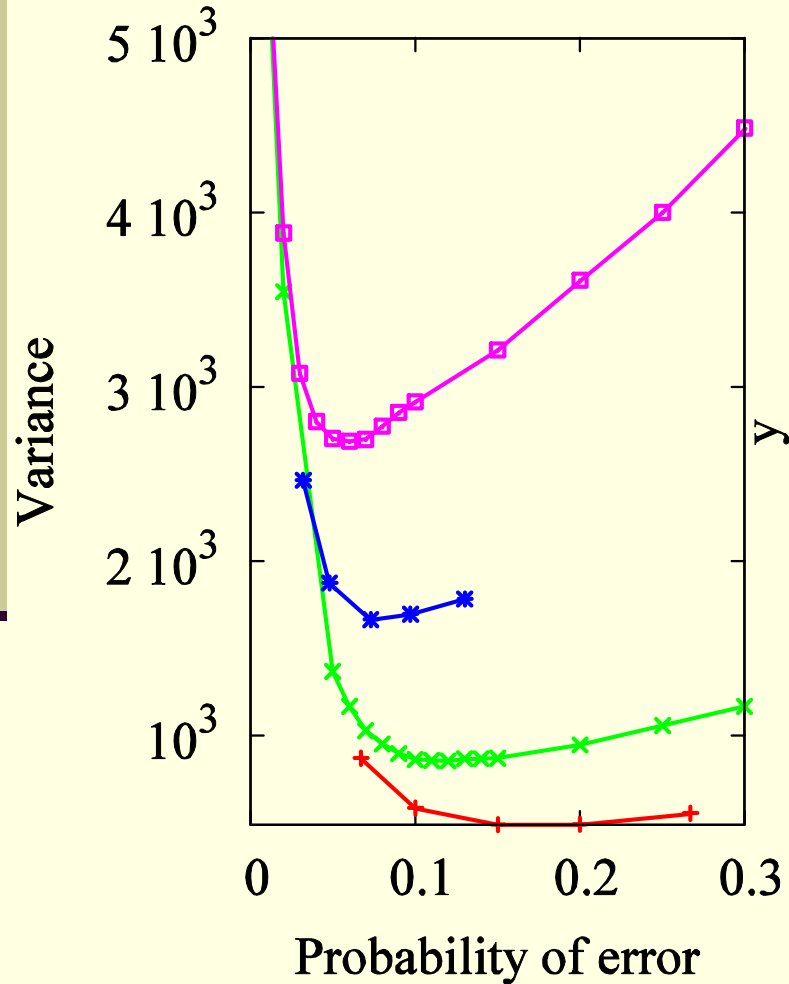


Localization and jumps



Localization and jumps

Dependence of variance on error $X=p j^\alpha$, $Y=j^{-\beta}$ $\text{Var}(x)$, $\alpha=1.04, \beta=1.67$



Open quantum dynamics and quantum walks

- Time evolution of the density matrix:
 - unitary evolution
 - coupling to the external system
 - randomness in the (unitary) evolution

$$\begin{aligned}\dot{\hat{\rho}} = & -i[\hat{H}, \hat{\rho}] + \\ & \sum h_{mn} \left(2X_n \hat{\rho} X_m^+ - \hat{\rho} X_m^+ X_n - X_m^+ X_n \hat{\rho} \right) + \\ & \sum \Lambda_i (U_i^+ \hat{\rho} U_i - \hat{\rho})\end{aligned}$$

A. S. Holevo, Statistical Structure of Quantum Theory (Springer, Berlin, 2001)

R. Alicki, K. Lendi, Quantum Dynamical Semigroups and Applications (Springer, Berlin, 1987)

Formulation of the problem

- N sites and a single coin
- Graph describing possible steps
- Set of unitary operations U_i describing walks
- Set of probabilities p_i
- Iterative applications of the operation

$$\Phi(\hat{\rho}) = \sum_{i \in I} p_i \hat{U}_i \hat{\rho} \hat{U}_i^\dagger$$

$$\Phi^{(n)}(\hat{\rho}) = \Phi(\Phi(\Phi(\dots(\Phi(\hat{\rho}))))))$$

Percolation on graphs

- Each configuration is characterized by

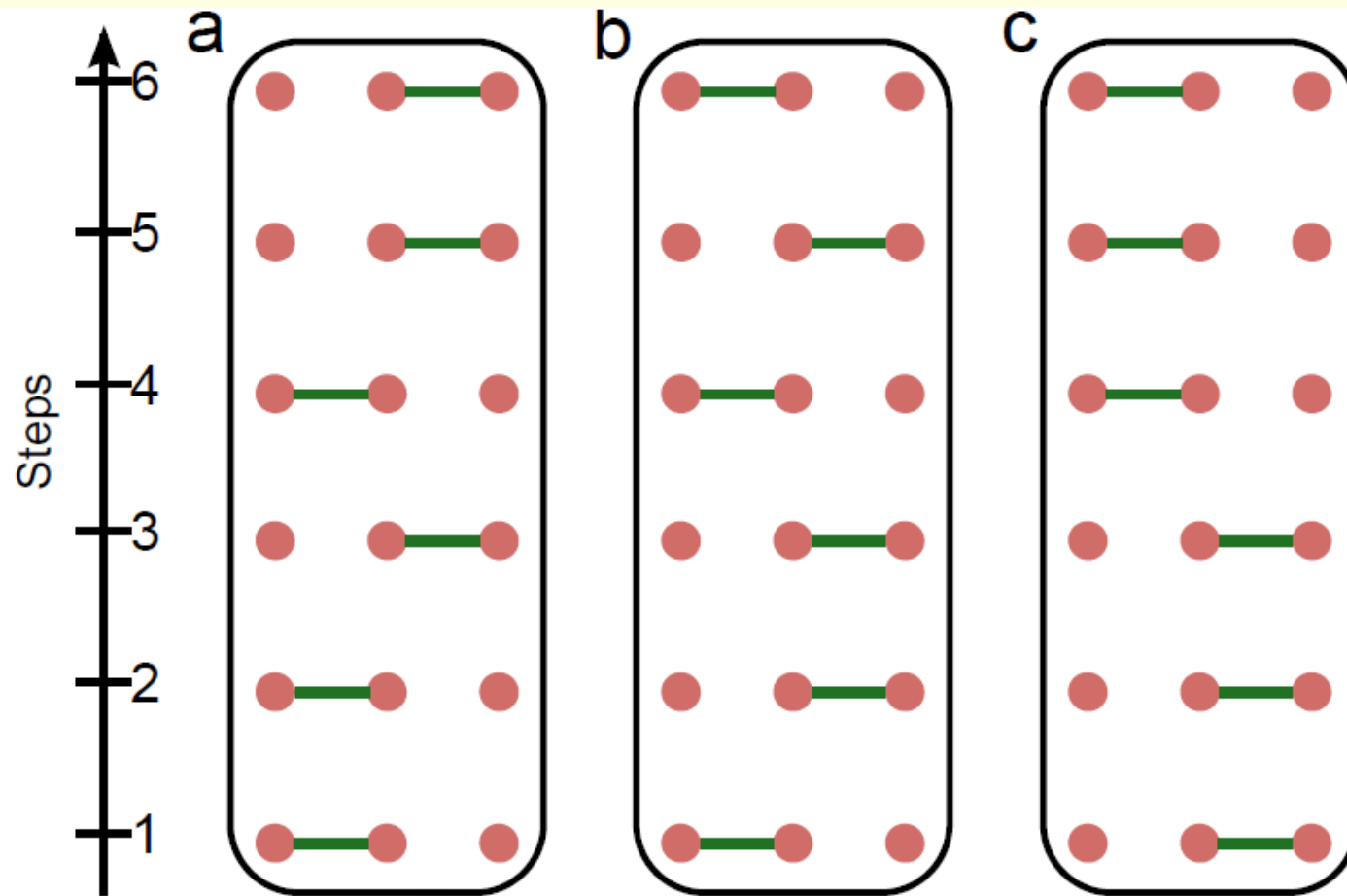
$$U_{\kappa} = S_{\kappa} \cdot (I_P \otimes C)$$

$$S_{\kappa} = \sum_v \left(\sum_{(v, v \oplus d) \in K} |v \oplus d\rangle\langle v| \otimes |d\rangle\langle d| + \sum_{(v, v \oplus d) \notin K} |v\rangle\langle v| \otimes R|d\rangle\langle d| \right)$$

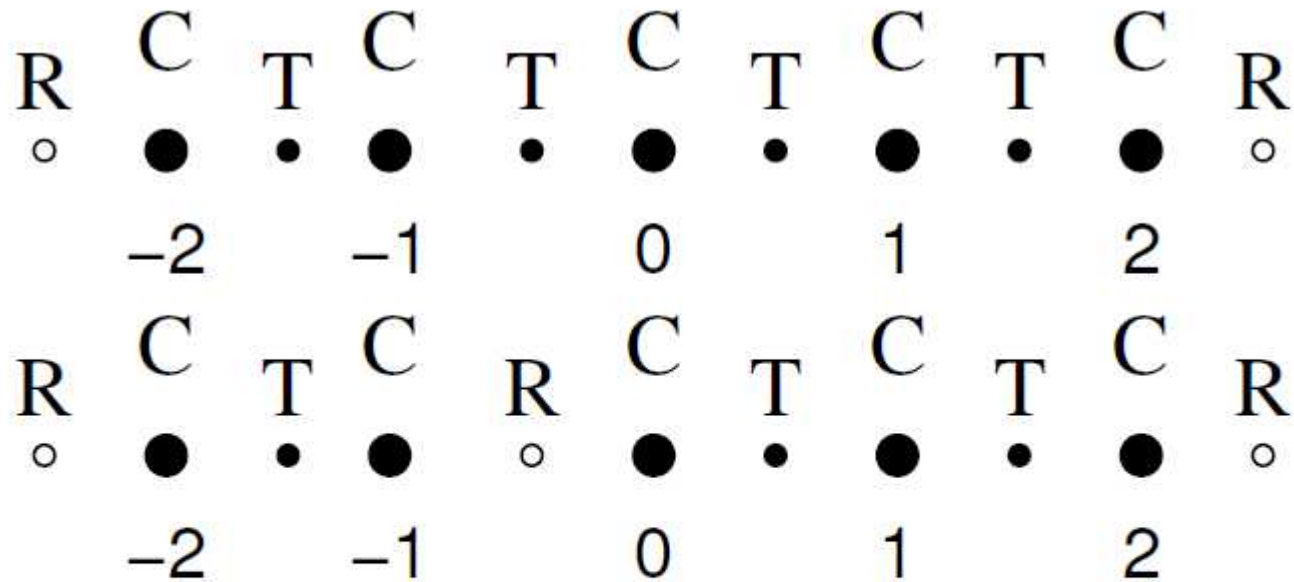
- Time evolution reads

$$\hat{\rho}(n+1) = \sum_{\kappa} p_{\kappa} \hat{U}_{\kappa} \hat{\rho}(n) \hat{U}_{\kappa}^{\dagger} \equiv \Phi(\hat{\rho}(n))$$

Percolated walk -- implementation

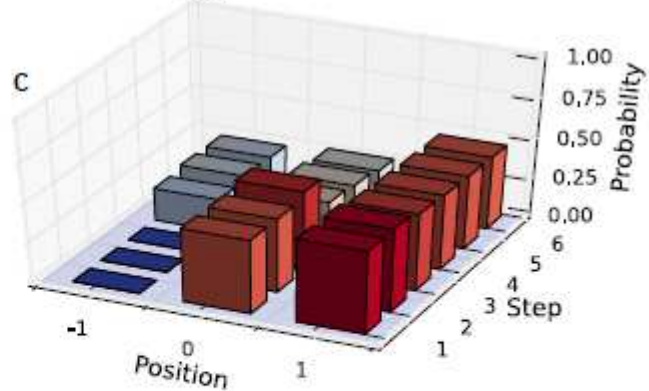
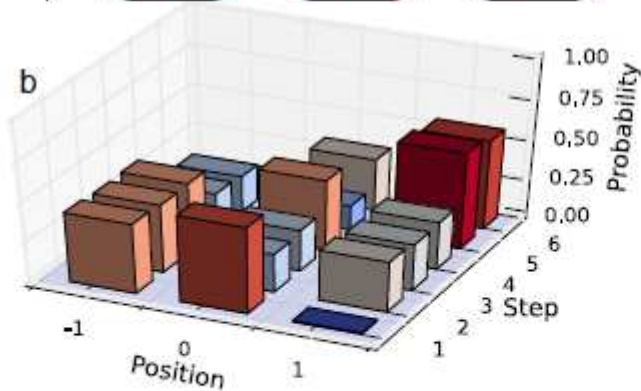
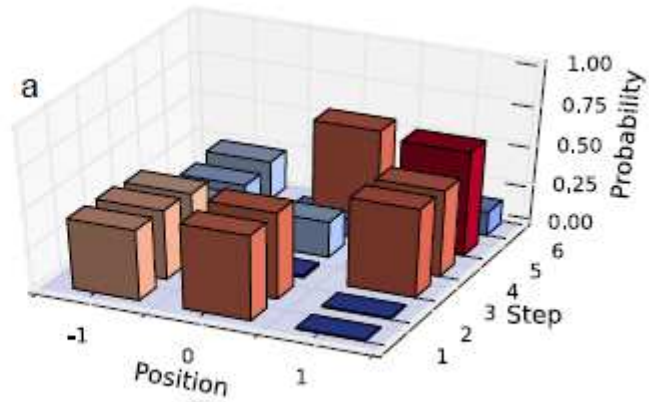
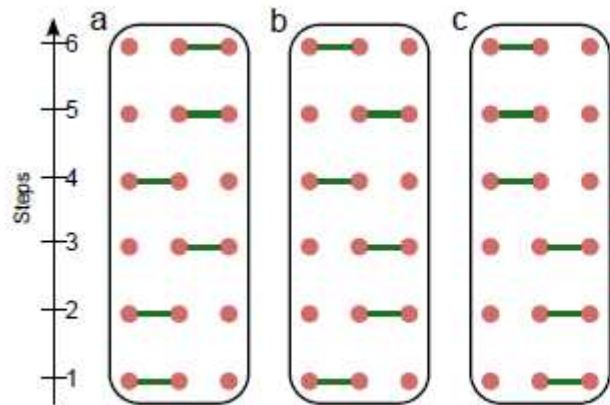


Implementation with walk primitives



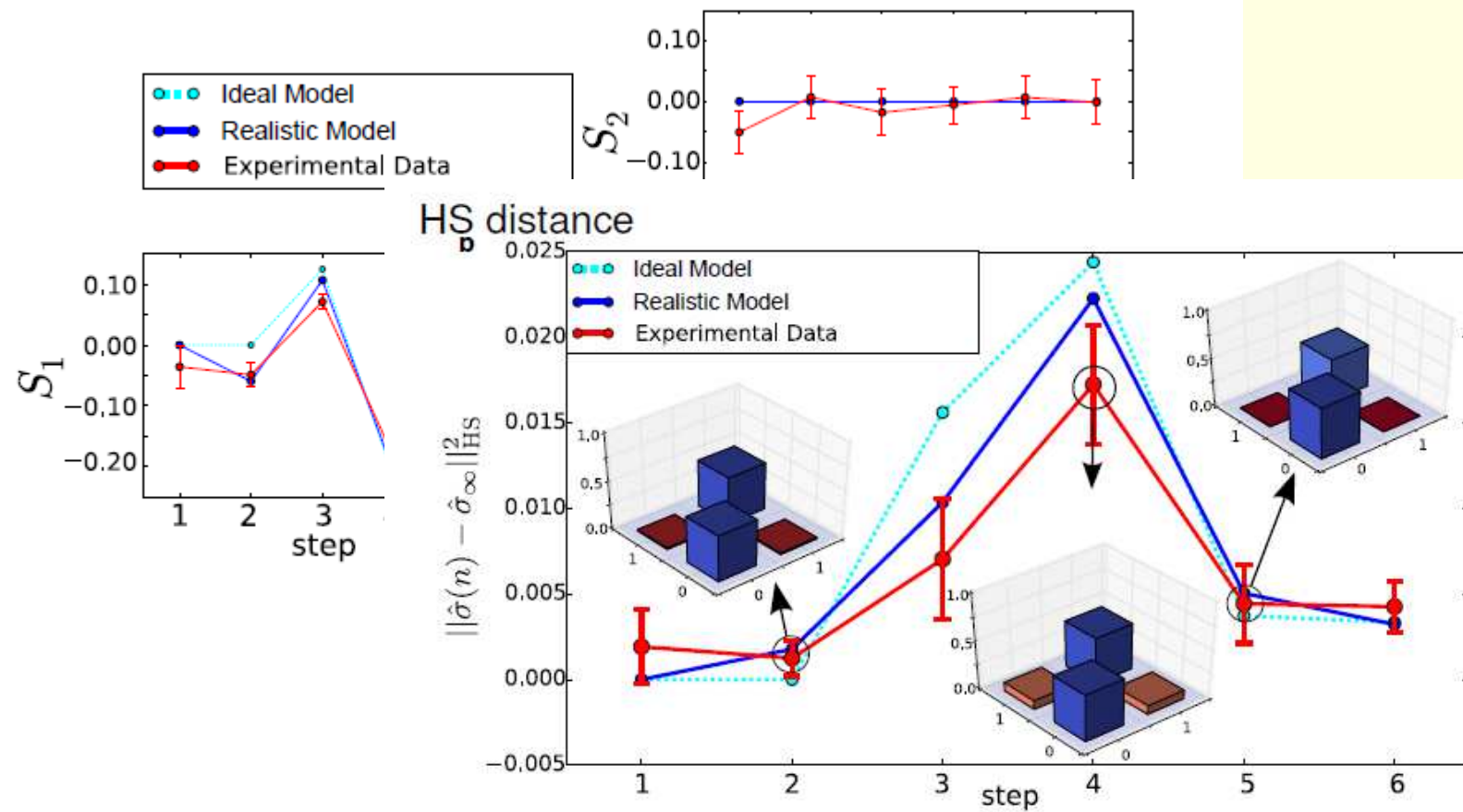
$$\hat{T} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \hat{R} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}.$$

Photons walking on graphs



Measurement results

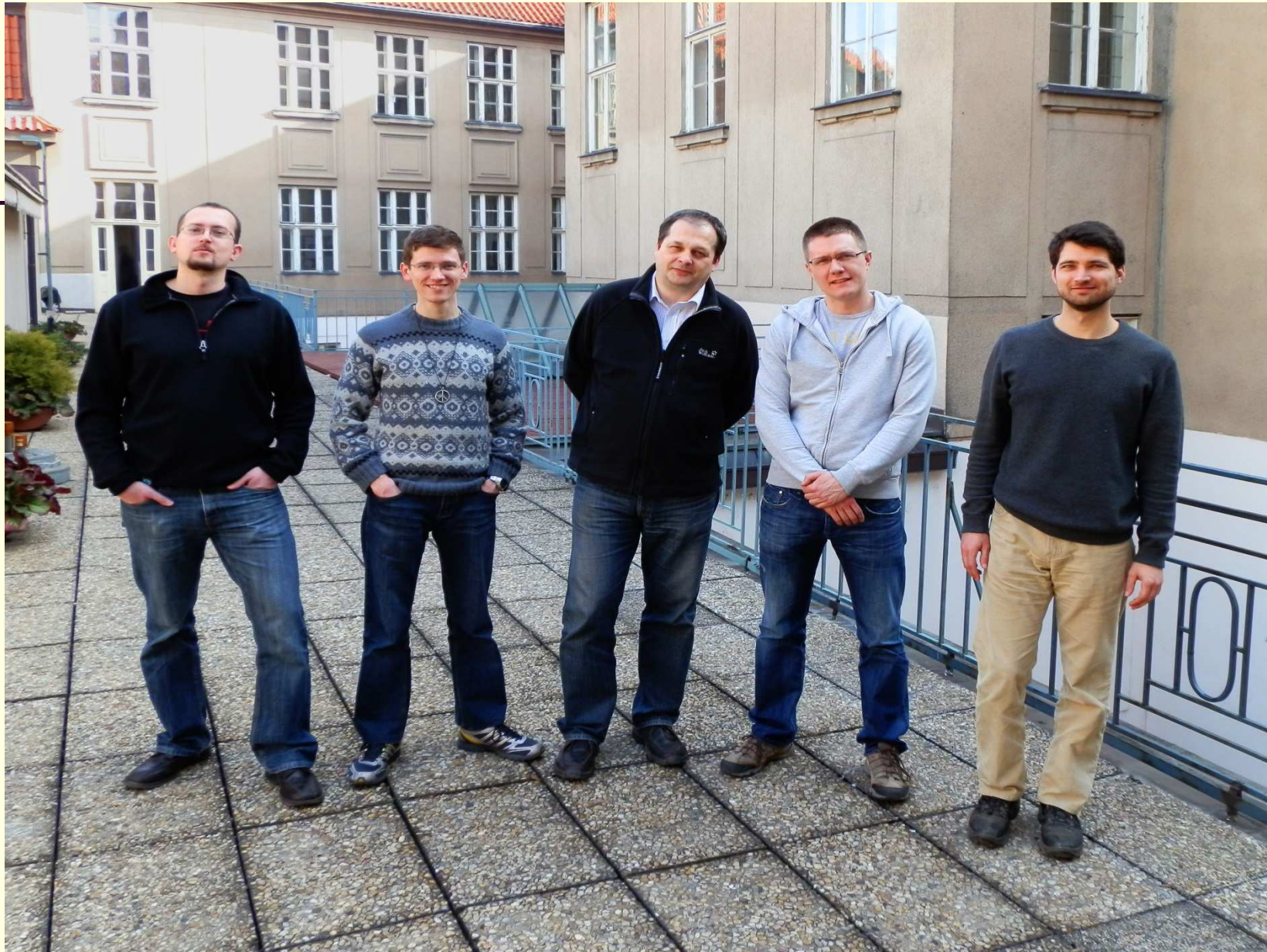
Full tomography on the coin state
a



Summary

- Quantum walks and its properties
- Classification of quantum walks
- Quantum walks with imperfections
- Quantum walks and its realizations





Structure theorem

Asymptotic states are characterized by

$$\Phi(\hat{\rho}_\infty) = \hat{U}_i P(\hat{\rho}) \hat{U}_i^+ = \sum_{|\lambda|=1} \lambda \text{Tr}(\hat{X}_\lambda^+ \hat{\rho}_\infty) \hat{X}_\lambda$$

$$\hat{\rho}_\infty = P(\hat{\rho})$$

$$P(.) = \sum_{|\sigma|=1} \text{Tr}(\hat{X}_\sigma^+ (.)) \hat{X}_\sigma$$

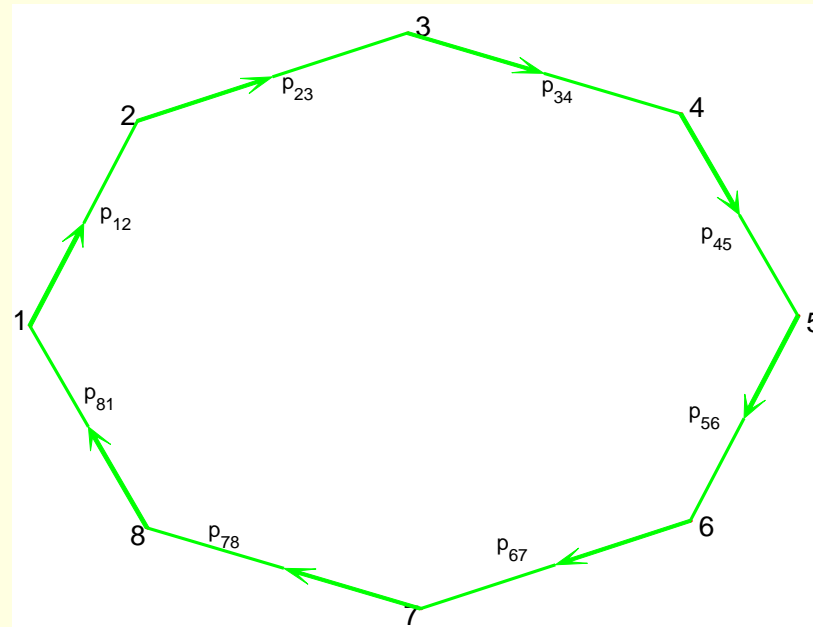
The needed operators are determined using the relations

$$\hat{U}_i \hat{X}_\lambda \hat{U}_i^+ = \lambda \hat{X}_\lambda \quad \text{Tr}(\hat{X}_\lambda^+ \hat{X}_{\lambda'}) = \delta_{\lambda\lambda'}$$

The asymptotic state does not depend on the set of probabilities p_i
It depends on the topology of the network and the set of operators U

Simple application

- p_{kl} probability that a link is missing



Asymptotics

$$(I_P \otimes C) X_\lambda (I_P \otimes C^+) = \lambda S_K X_\lambda S_K^+$$

$$S_K X_\lambda S_K^+ = S_{K'} X_\lambda S_{K'}^+$$

These basic relations lead to

$$(I_P \otimes C) X_\lambda (I_P \otimes C^+) = \lambda X_\lambda$$

1D asymptotics

- We assume the coin

$$C(\alpha, \beta) = \begin{pmatrix} i e^{-i\alpha} \sin \beta & \cos \beta \\ \cos \beta & i e^{i\alpha} \sin \beta \end{pmatrix}$$

- The eigenvalues for the attractor space is a subset of $1, \exp(i2\beta), \exp(-i2\beta)$
- The attractor space has the following properties

1D asymptotics - line

- The dimension of the space for $\text{Exp}(i2\beta)$ is one
- The dimension of the space for $\text{Exp}(-i2\beta)$ is one
- The dimension of the space for 1 is three
- As the reflection operator we have chosen $R = \sigma_x$

1D asymptotics - circle

- For $N=2k+1$ and $\alpha=l\pi/N$ the eigenvalue is 1 and has dimension 2
- For $N=2k$ and $2l\pi/N$ the attractor space equals to that of the line
- For $N=2$ and $\alpha \neq l\pi$ the attractor space is two dimensional (eigenvalue 1)
- In all other cases only the trivial solution is available

- As the reflection operator we have chosen $R=\sigma_x$

Implications

- The position distribution is in all cases uniform
- The asymptotics of the line and the circle can be different
- The number of positions is important (possibility of identification)
- The choice of the coin is nontrivial
- The coin can be used to generate limit cycle instead of a limit state

Open questions

- Experiments (walks in higher dimensions)
- Percolation in higher dimensions (nonuniform)
- Convergency rate
- Graph identification