The Universal Wave Function Interpretation of String Theory

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Goal

1. What is the Grand Unification Theory to Unify all particles, forces, and space-time?

2. How to make prediction from Grand Unification Theory?

Outline

- Derive **Universal Wave Function Interpretation of String Theory (UWFIST)**
- Application of UWFIST
  - Dark Energy and Dark Matter
  - Prediction of Cosmological Constant
  - Prediction of energy source for inflation and flatness of 4-dimensional space-time

Space-Time Uncertainty Relation

Measurement of Space-Time

Restrictions from Quantum Physics

To measure the time duration $\Delta\tau$, it takes the energy $\Delta E \sim \hbar/\Delta\tau$.

To measure space size $\Delta\sigma$, it takes the momentum $\Delta P \sim \hbar/\Delta\sigma$.

$\hbar = h/2\pi$. $h = 6.626\times10^{-34}$ j•s

Space–Time Uncertainty Relation

Restriction From General Relativity

ΔE will curve the space–time and create a black hole with the horizon on the order of $G\Delta E/c^4$.

The measurable causal region is:

$$\Delta\sigma \geq G\Delta E/c^4.$$

G is the gravitational constant 6.674×10^{-11} $N\cdot m^2/kg^2$ and c is the speed of light 3×10^8 m/s.

–> Uncertainty relation between space $\Delta\sigma$ and time $\Delta\tau$:

$$\Delta\sigma\ \Delta\tau \geq l_p\ t_p \qquad (1)$$

Here l_p is the Planck length, $l_p = (\hbar G/c^3)^{1/2} = 1.616 \times 10^{-35}$ meter.

And t_p is the Planck time, $t_p = (\hbar G/c^5)^{1/2} = 5.39 \times 10^{-44}$ second.

Space Time Quantization

Energy and Time Uncertainty Relation $\Delta H \Delta \tau \geq \hbar$

$$\Downarrow$$

$$[H, \tau] = i\hbar.$$

Momentum and Space Uncertainty Relation $\Delta P \Delta \sigma \geq \hbar$

$$\Downarrow$$

$$[P, \sigma] = i\hbar.$$

Space–Time Uncertainty Relation $\Delta \sigma \Delta \tau \geq l_p t_p$

$$\Downarrow$$

$$[\sigma, \tau] = i\, l_p t_p$$

Quantization of Space-Time

$$[H, \tau] = i\hbar$$

$$\Downarrow$$

$$A_t = i \int_0^t H \, d\tau/\hbar$$

$$\Downarrow$$

$$\Psi(t) = \exp(i \int_0^t H \, d\tau/\hbar) \, \Psi(0).$$

$$[P, \sigma] = i\hbar$$

$$\Downarrow$$

$$A_x = i \int_0^x P \, d\sigma/\hbar.$$

$$\Downarrow$$

$$\Psi(x) = \exp(i \int_0^x P \, d\sigma/\hbar) \, \Psi(0).$$

Quantization of Space-Time

$$[\sigma, \tau] = i l_p t_p$$

$$\Downarrow$$

$$A_s = \int d\tau d\sigma / l_p t_p \ (2)$$

$$\Downarrow$$

$$\psi(L,T) = \exp(i \int_0^T d\tau \int_0^L d\sigma / l_p t_p)\ \psi(0,0) \ (3)$$

This demonstrates when we take gravity into consideration, we need to use "string" action (3) rather than "particle" action $A = \int d\tau$ to obtain a consistent theory.

Universal Wave Function Interpretation of String Theory (UWFIST)

The Difference From Normal String Theory

1. The world-sheet space and time integration of the action is over the causal region. i.e.

$$A_s = \int_0^T d\tau \int_0^L d\sigma \ / \ l_p^2$$

Here T and L are the age and length of the causal horizon, not the length of the string.

2. Introduction of the universal wave function $\Psi(L,T)$

$$\psi(L,T) = \exp(i \int_0^T d\tau \int_0^L d\sigma / l_p t_p) \ \psi(0,0)$$

Proper Space-Time and Target Space-Time

- The proper space and time (σ, τ) form the world sheet.

- The normal observed space-time is a projection from the proper space-time $X^\mu(\sigma,\tau)$

$$X^\mu: (\sigma, \tau) \to X^\mu(\sigma,\tau)$$

World-sheet -> Target Space,

The observed space-time is the target space.

$$A_s = (1/l_p t_p) \int d\tau d\sigma\, g^{1/2} g^{ab}\, \partial_a X_\mu \partial_b X^\mu$$

Universal Wave Function

The Action in the presence of massless background field:

$$A_s = i\alpha \int_0^T d\tau \int_0^L d\sigma\, g^{1/2}(g^{\alpha\beta}G^{\mu\nu}\partial_\alpha X_\mu \partial_\beta X_\nu + \epsilon^{\alpha\beta}B^{\mu\nu}\partial_\alpha X_\mu \partial_\beta X_\nu + 1/4\alpha\, \Phi\, R)$$

The Universal Wave Function:

$$\Psi(X^\mu(L,T), G^{\mu\nu}(L,T), B^{\mu\nu}(L,T), \Phi(L,T)) = \int DX^\mu\, Dg^{\mu\nu}\, DB^{\mu\nu}\, D\Phi\, \exp[i\alpha \int_0^T d\tau \int_0^L d\sigma\, g^{1/2}(g^{\alpha\beta}G^{\mu\nu}\partial_\alpha X_\mu \partial_\beta X_\nu + \epsilon^{\alpha\beta}B^{\mu\nu}\partial_\alpha X_\mu \partial_\beta X_\nu + 1/4\alpha\, \Phi\, R)]$$

Dark Energy and Dark Matter

The existence of vibrations:

$$\exp[i\pi n(\sigma+c\tau)/L], \quad \exp[i\pi n(\sigma-c\tau)/L]$$

These vibrations have:

$$E \approx h/T \;\text{->}\; \text{dark energy}$$
$$m \approx h/Tc^2 \;\text{->}\; \text{dark matter}$$

It takes the lifetime of known universe to detect them

-> dark energy and dark matter!

Weyl Invariance, Hologram, and Equation of Motion

Weyl symmetry: the local rescaling invariance of the world-sheet metric:

$$g'^{\alpha\beta}(\tau, \sigma) = \exp(2\omega(\tau, \sigma))\, g^{\alpha\beta}(\tau, \sigma),$$

$$X'^{\mu}(\tau, \sigma) = X^{\mu}(\tau, \sigma),$$

$$G'^{\mu\nu}(\tau, \sigma) = G^{\mu\nu}(\tau, \sigma),$$

$$B'^{\mu\nu}(\tau, \sigma) = B^{\mu\nu}(\tau, \sigma),$$

$$\Phi'(\tau, \sigma) = \Phi(\tau, \sigma),$$

for arbitrary $\omega(\tau, \sigma)$.

Weyl Invariance, Hologram, and Equation of Motion

The world-sheet is a hologram

↓

The observed universe is a projection from the world-sheet hologram.

↓

Equation of Motion

General Relativity

Vacuum Energy

$E_{n,m} = (n + ½)\, h/T.$

The lowest energy of vacuum fluctuation is
$E_{0n,m} = h/2T.$

Define Vacuum energy as the lowest energy state. The total lowest vacuum energy is:
$E_{vac} = h/2T \times TL/\, l_p^2 = hL/(2\, l_p^2).$

It is proportional to the length of the causal horizon.

Vacuum Energy Density

The lowest vacuum energy density in the 3-dimensional space is:

$$\rho_{4\text{-dim vac}} = E_{vac}/(4\pi L^3/3) = \rho_p \, l_p^2/(2L^2) \qquad (4)$$

Here ρ_p the Planck energy density,

$$\rho_p = E_p/(4\pi l_p^3/3),$$

E_p is the Planck energy $E_p = \hbar/t_p$, t_p and l_p are the Planck time and Planck length $l_p = c\, t_p = (\hbar G/c^3)^{1/2}$.

Vacuum Energy Density

- $\rho_{4\text{-dim vac}}$ is proportional to L^{-2}
- $\rho_{n\text{-dim vac}}$ is proportional to L^{2-n}
- When $L = l_{p}$, $\rho_{4\text{-dim vac}} = \rho_p / 2$.

This large vacuum energy density provides the energy source for inflation, the rapid initial expansion. As universe expands, the vacuum energy density decreases, but is non-zero. Therefore our universe will continue to expand.

Why is our 4-dim Space-Time Flat?

Now let us assume that L is the Hubble radius with L = c/H.

$\rho_{4\text{-dim vac}} = 3H_0^2/(8\pi G) = \rho_c$ (5)

-> 4-dim space-time is naturally flat at all time.

The recent Wilkinson Microwave Anisotropy Probe (WMAP) measurements have led to the conclusion of a flat universe, i.e. our universe's energy density is of the value of critical energy density with only a 0.5% margin of error.

Calculation of Cosmological Constant

Measurement of Cosmological Constant Λ

The non-zero cosmological constant was indicated

1. The distance-red shift relation for Type Ia supernovae indicates the expansion of the universe is accelerating.
2. Cosmic microwave background radiation
3. Recent WMAP measurements

$\Lambda \sim 1.7 \times 10^{-121}$ in Planck units.

Calculation of Cosmological Constant

$$\Lambda_c = 8\pi\ \rho_{vac}$$

$$\Lambda_c = 4\pi\ \rho_p\ t_p^2/t_u^2 \sim 10^{-121}\ \rho_p$$

It is quite amazing that in UWFIST, using the age of the universe and the fundamental constants ħ and G we are able to derive the value of cosmological constant to the same order of magnitude as the observed value.

Conclusion

Universal Wave Function Interpretation of String Theory (UWFIST)

- Dark Energy and Dark Matter
- Prediction of Cosmological Constant
- Prediction of energy source for inflation and flatness of 4-dimensional space-time

UWFIST is worth further investigation.