

# Exact, Finite, Model Renormalization of Non-Perturbative, Gauge-Invariant, Realistic QCD.

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# Schwinger Generating Functional for QCD

a) Starting Point: Schwinger Generating Functional (GF) for QCD, with gluon operators in an Arbitrary (Relativistic) Gauge.

b) Re-arrange this GF in terms of a “Reciprocity Relation”, and a “Gaussian Linkage Operation”; and the GF now depends upon two functionals of A,

$$\mathcal{Z}_{\text{QCD}}[j, \bar{\eta}, \eta] = \mathcal{N} e^{-\frac{i}{2} \int \frac{\delta}{\delta A} \cdot \mathbf{D}_c^{(0)} \cdot \frac{\delta}{\delta A}} \cdot e^{-\frac{i}{4} \int \mathbf{F}^2 + \frac{i}{2} \int A \cdot (-\partial^2) \cdot A} \cdot e^{i \int \bar{\eta} \cdot \mathbf{G}_c[A] \cdot \eta + \mathbf{L}[A]} \Big|_{A = \int \mathbf{D}_c^{(0)} \cdot j}$$

Quark line

$$G_c(x, y|A) = [m + \gamma \cdot (\delta - igA\tau)]^{-1}$$

Virtual quark loop

$$L[A] = \ln[1 - i\gamma A\tau_c[0]]$$

c) Insert Halpern's half-century-old representation for the  $\mathbf{F}^2$

Halpern's Representation

$$e^{-\frac{i}{4} \int F^2} = N \int d[\chi] e^{\frac{i}{4} \int \chi^2 + \frac{i}{2} \int F \cdot \chi}$$

$$\chi_{\mu\nu}^a = -\chi_{\nu\mu}^a$$

The next two steps were overlooked for decades:

1

2

# 1

## GAUGE INVARIANCE BY GAUGE INDEPENDENCE

Trivial re-arrangement can now be made to formally insure gauge-invariance, Even though the GF still apparently contains gauge-dependent gluon propagators.

Functional derivatives on Generative Functional,  $Z_{QCD}$ , to pull down quarks and gluons  $\rightarrow$

$$Z_{QCD}[j, \bar{\eta}, \eta] = N \int d[\chi] e^{\frac{i}{4} \int \chi^2} e^{\mathcal{D}_A^{(0)}} \cdot e^{\frac{i}{2} \int \chi \cdot F + \frac{i}{2} \int A \cdot (-\partial^2) \cdot A} e^{i \int \bar{\eta} \cdot G_c[A] \cdot \eta + L[A]} \Big|_{A=\int D_c^{(0)} \cdot j}$$

2n-point functions

(HMF, YG, TG, YMS, Annals of Physics, 2012)

$$= N \int d[\chi] e^{\frac{i}{4} \int \chi^2} e^{\mathcal{D}_A^{(0)}} e^{\frac{i}{2} \int \chi \cdot F + \frac{i}{2} \int A \cdot (D_c^{(0)})^{-1} \cdot A} G_c(1|gA) G_c(2|gA) e^{L[A]} \Big|_{A=0}$$

$$e^{\mathcal{D}_a F_1[A]} = \exp\left[\frac{i}{2} \int \bar{Q} \cdot D_c^{(0)} \cdot (1 - \bar{K} \cdot D_c^{(0)})^{-1} \cdot \bar{Q} - \frac{1}{2} \text{Tr} \ln(1 - D_c \cdot \tilde{K})\right] \\ \cdot \exp\left[\frac{1}{2} \int A \cdot \bar{K} \cdot (1 - D_c^{(0)} \cdot \bar{K})^{-1} \cdot A + i \int \bar{Q} \cdot (1 - \bar{K} \cdot D_c^{(0)})^{-1} \cdot A\right]$$

$$D_c^{(0)} \cdot (1 - \bar{K} \cdot D_c^{(0)})^{-1} \\ = D_c^{(0)} \cdot [1 - (\hat{K} + (D_c^{(0)})^{-1}) \cdot D_c^{(0)}]^{-1} \\ = -(\tilde{K}_{\mu\nu}^{ab} + g f^{abc} \chi_{\mu\nu}^c)^{-1} = -\hat{\mathbf{K}}^{-1}$$

Gluon Bundle



$$\begin{aligned}
e^{\mathcal{D}^A} F_1[A] F_2[A] &= \exp\left[-\frac{i}{2} \int \bar{Q} \cdot \hat{\mathbf{K}}^{-1} \cdot \bar{Q} + \frac{1}{2} \text{Tr} \ln \hat{\mathbf{K}} + \frac{1}{2} \text{Tr} \ln(-D_c^{(0)})\right] \\
&\cdot \exp\left[\frac{i}{2} \int \frac{\delta}{\delta A'} \cdot \mathbf{D}_c^{(0)} \cdot \frac{\delta}{\delta A'}\right] \\
&\cdot \exp\left[\frac{i}{2} \int \frac{\delta}{\delta A'} \cdot \hat{\mathbf{K}}^{-1} \cdot \frac{\delta}{\delta A'} - \int \bar{Q} \cdot \hat{\mathbf{K}}^{-1} \cdot \frac{\delta}{\delta A'}\right] \cdot (e^{\mathcal{D}^A} F_2[A'])
\end{aligned}$$

$$\begin{aligned}
e^{\mathcal{D}^A} F_1[A] F_2[A] &= N \exp\left[-\frac{i}{2} \int \bar{Q} \cdot \hat{\mathbf{K}}^{-1} \cdot \bar{Q} + \frac{1}{2} \text{Tr} \ln \hat{\mathbf{K}}\right] \\
&\cdot \exp\left[\frac{i}{2} \int \frac{\delta}{\delta A} \cdot \hat{\mathbf{K}}^{-1} \cdot \frac{\delta}{\delta A} - \int \bar{Q} \cdot \hat{\mathbf{K}}^{-1} \cdot \frac{\delta}{\delta A}\right] \cdot \exp(L[A])
\end{aligned}$$

**No gauge dependence.**

(H.M.Fried, *Modern Physics Letters A*, 2013)

$-\hat{\mathbf{K}}^{-1}$  or also written as  $(f \cdot \chi)^{-1}$



**All gluons exchanges summed!**

2

# Use Faddeev Representation for $G[A]$ and $L[A]$

$$\begin{aligned}
 G_c(x, y|A) &= i \int_0^\infty ds e^{-ism^2} e^{-\frac{1}{2}\ln(2h)} \int d[u] e^{\frac{i}{4} \int_0^s ds' [u'(s')]^2} \delta^{(4)}(x - y + u(s)) \\
 &\times \left[ m - \gamma_\mu \frac{\delta}{\delta u'_\mu(s)} \right] \mathcal{N}_\Omega \mathcal{N}_\Phi \int d[\alpha] \int d[\Xi] \int d[\Omega] \int d[\Phi] \left( e^{i \int_0^s ds' [\alpha^a(s') - i\sigma_{\mu\nu} \Xi_{\mu\nu}^a(s')] \tau^a} \right)_+ \\
 &\times e^{-i \int ds' \Omega^a(s') \alpha^a(s') - i \int ds' \Phi_{\mu\nu}^a(s') \Xi_{\mu\nu}^a(s')} \\
 &\times e^{-ig \int ds' u'_\mu(s') \Omega^a(s') A_\mu^a(y - u(s')) + ig \int ds' \Phi_{\mu\nu}^a(s') F_{\mu\nu}^a(y - u(s'))}, \\
 &1 = \int d[\alpha] \delta[\alpha^a(s') + g u'_\mu(s') A_\mu^a(y - u(s'))], \\
 &1 = \int d[\Xi] \delta[\Xi_{\mu\nu}^a(s') - g F_{\mu\nu}^a(y - u(s'))]
 \end{aligned}$$

**Gaussian**

$$\begin{aligned}
 L[A] &= -\frac{1}{2} \int_0^\infty \frac{ds}{s} e^{-ism^2} e^{-\frac{1}{2}\ln(2h)} \times \mathcal{N}_\Omega \mathcal{N}_\Phi \int d^4x \int d[\alpha] \int d[\Omega] \int d[\Xi] \int d[\Phi] \\
 &\times \int d[v] \delta^{(4)}(v(s)) e^{\frac{i}{4} \int_0^s ds' [v'(s')]^2} \\
 &\times e^{-i \int ds' \Omega^a(s') \alpha^a(s') - i \int ds' \Phi_{\mu\nu}^a(s') \Xi_{\mu\nu}^a(s')} \cdot \left( e^{i \int_0^s ds' [\alpha^a(s') - i\sigma_{\mu\nu} \Xi_{\mu\nu}^a(s')] \tau^a} \right)_+ \\
 &\times e^{-ig \int_0^s ds' v'_\mu(s') \Omega^a(s') A_\mu^a(x - v(s')) - 2ig \int d^4z (\partial_\nu \Phi_{\nu\mu}^a(z)) A_\mu^a(z)} \\
 &\times e^{+ig^2 \int ds' f^{abc} \Phi_{\mu\nu}^a(s') A_\mu^b(x - v(s')) A_\nu^c(x - v(s'))}
 \end{aligned}$$

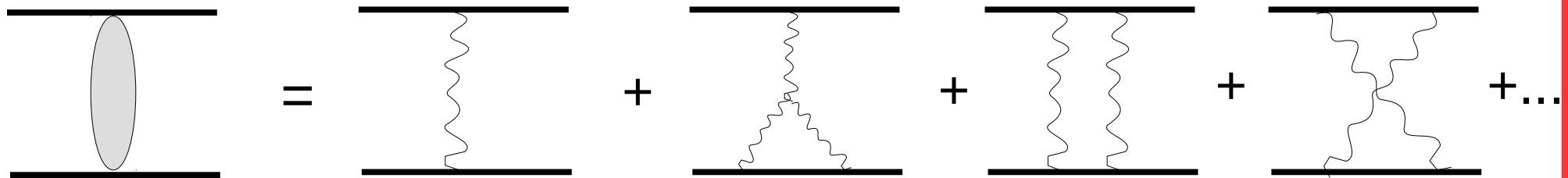
**Gaussian**

H. M. Fried, Y. Gabellini, T. Grandou, Y.-M. Sheu, *Eur. Phys. Journal C* **65** (2010).

H. M. Fried, Y. Gabellini, T. Grandou, and Y.-M. Sheu, *Ann. Phys.* 338 (2013)

All the Gaussian Linkage operations can then be carried through exactly, corresponding to the summation of all gluons exchanged between any pair of quark (and/or antiquark) lines, and including cubic and quartic gluon interactions.

Define a “Gluon Bundle” (GB) as the Sum over all gluon exchanges between any pair of quark lines,



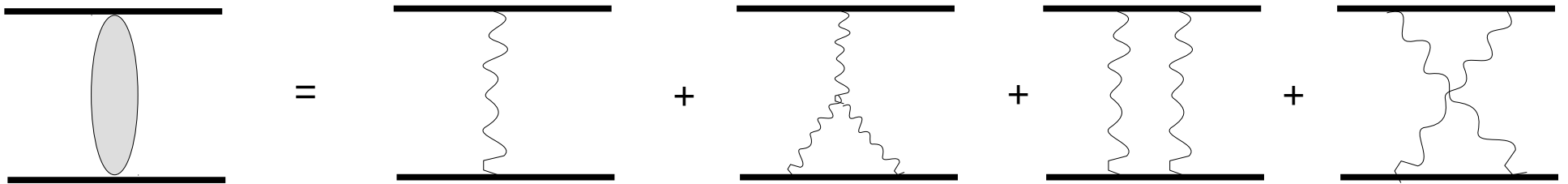
THE RESULT:

Explicit cancellation of all gauge-dependent gluon propagators, with resulting GF exhibiting Manifest Gauge Independence.. and one finds a new, exact property of Non-Perturbative, Gauge-Invariant QCD,

$\mathcal{D}_A^{(0)}$  Is gaussian functional operation,  $G[A]$  and  $L[A]$  are gaussian in Fradkin representation. Functional derivatives operations with respect to sources  $\bar{\eta}, \eta$  can be performed exactly. This produces sum of all Feynman graphs corresponding to the exchange of infinite number of gluons between quarks.

# New QCD Property: Effective Locality (EL)

Define a “Gluon Bundle” (GB) as the Sum over all gluon exchanges between any pair of quark lines,



The space-time coordinates of both ends of a GB are equal, modulo small Uncertainties in their transverse coordinates.

What this means is that, at high energies, the Halpern FI can be reduced to sets of Ordinary integrals, yielding a vast simplification in the calculation of all QCD Correlation functions.

(Pencil and paper + desktop computer can now replace huge, Multi-processing lattice calculations.)

# How to introduce transverse quark fluctuations from First Principles?

We believe we know how to do this, but work still underway.

What we have done is to introduce phenomenological transverse fluctuation amplitudes for Every quark-gluon vertex, replacing the usual gluon-quark current interaction at the same space-time point,

$$\int d^4x \bar{\psi}(x) \gamma_\mu A_\mu^a(x) \tau_a \psi(x) \quad \text{by}$$
$$\int d^2x'_\perp \int d^4x a(x_\perp - x'_\perp) \bar{\psi}(x') \gamma_\mu \tau_a A_\mu^a(x) \psi(x'),$$

With  $a(x_\perp - x'_\perp)$  real and symmetric, and  $x'_\mu = (x'_\perp, x_L, x_0)$ .

(HMF, YG, TG, PHT, YMS, Annals of Physics, 2014)

The probability of finding two quarks separated by a transverse (or impact parameter) distance is then:

$$\varphi(b) = \int \frac{d^2q}{(2\pi)^2} e^{iq \cdot b} |\tilde{a}(q)|^2.$$

How to choose  $\varphi(b)$ ? It is directly related to quark binding; how does  $\varphi(b)$  produce  $V(r)$ , the  $q - \bar{q}$  binding potential whose lowest bound state represents the pion?

First try: A Gaussian,  $\varphi(b) \approx e^{-(\mu b)^2}$  where  $\mu^{-1}$  sets the scale of transverse fluctuations. Then, all absurdities of correlation functions disappear. But this distribution is “too symmetric”, and gives a zero  $V(r)$ .

Second try: A “deformed” Gaussian,  $\varphi(b) = \varphi(0)e^{-(\mu b)^{2+\xi}}$  with  $\xi$  a “deformation parameter”, real and small.

(HMF,YG,TG,YMS, Annals of Physics, 2014)

A straight-forward calculation yields, for small  $\xi$

$$V(r) \approx \xi \mu (\mu r)^{1+\xi}$$

## CONCERNING TRANSVERSE QUARK FLUCTUATIONS

Starting from conventional, quark-field-operator equations of motion, one can define “IN” and “OUT” operator fields,  $\psi_{out}^{in}(x)$  as in any Abelian Theory; QCD just has more complicated interaction, no?

But this assumption is wrong. For decades we have known that all asymptotic quark states are hadronic Bound states of quarks; and for such a bound state we can specify longitudinal and time Coordinates, but not transverse coordinates, since they are always fluctuating.

NB: The conventional “static quark” approximation used in lattice and other model Binding-potential calculations in all non-perturbative Amplitudes are plagued with divergences because the “static quarks” are not physical.

Without taking such “transverse imprecision” into account, all non-perturbative Amplitudes are plagued with absurdities.

# Mass of pion from QCD

Substituting this potential into a Schrödinger binding equation, using the “quantic” approximation, then yields  $\mu \sim m_\pi, \xi \approx 0.1$ .

This is sensible, since the max. fluctuations should be  $\leq$  than  $m_\pi^{-1}$ .

Our result encompasses two different lattice calculations,  $V \sim r$  and  $V \sim r \ln(r)$ . But all lattice and other model calculations of  $q - \bar{q}$  binding correspond to an amplitude containing only one of the two Casimir SU(3) invariants,  $C_2, C_3$ ; our amplitude contains both.

(T.Grandou EuroPhys. L. 107, 2014)

(H.M.Fried, T.Grandou, R.Hoffman, arXiv:1504:05502)

What method do we use to pass from  $\varphi(b)$  to  $V(r)$  ?

Imagine that a  $q$  and a  $\bar{q}$  are scattering at high energy. One can write an Eikonal approximation, valid in the limit of  $s \gg |t|$ , for the conventional scattering amplitude. (Details for QCD eikonals were worked out by HMF. YG, JA, and BMcK in two papers circa 1983. )

# Eikonal to Potential

It has been well-known for a half-century that, assuming a specific  $V(r)$ , in ordinary QM, or in Abelian QFTs, the corresponding eikonal function  $E(\mathbf{b})$  is given by

$$E(b) = \gamma(s) \int_{-\infty}^{+\infty} dz_L V(\vec{b} + \hat{p}_L z_L),$$

where  $\gamma(s)$  is a constant depending on CM energy and the type of interaction.

We can write the non-perturbative amplitude corresponding to a GB exchanged between a  $q$  and a  $\bar{q}$ ; and we see that the Eikonal limit of this amplitude has  $E(\mathbf{b})$  defined in terms of  $\varphi(b)$ , and proportional to:  $\ln\{\varphi(b)\}$ . Here,  $E(\vec{b}) = E(b)$ , and  $V(\vec{r}) = V(r)$ .

Our method: Calculate the 2-D Fourier transform  $\tilde{E}(k_{\perp}^2)$  of  $E(b)$ .

Extend  $k_{\perp}^2 \rightarrow k_{\perp}^2 + k_L^2 = \vec{k}^2$ , so that we now have  $\tilde{E}(\vec{k}^2)$ ; and then calculate the 3-D transform of this  $\tilde{E}(\vec{k}^2)$ , which will yield  $V(r)$ .

# Pion's mass from QCD

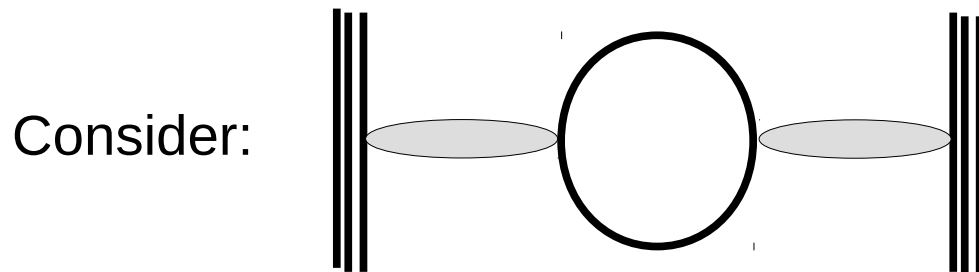
No static approximation required! Our analysis gives  $E(b)$  explicitly, in terms of  $\varphi(b)$ , so that we can calculate  $V(r)$  for any choice of  $\varphi(b)$ . The minimal bound state energy representing the pion shows that most of the pion mass comes from the gluons forming the GB, and relatively little from the quark masses.

# Nuclear Physics from QCD

If you look up Nuclear forces on Wikipedia, you'll find the statement that there exists no derivation on the basis of QCD.

Here is the first (to our knowledge) example of nucleon binding (for a Model deuteron) from basic QCD. The Model neglects electrical charge, and nucleon spins (which can always be added in ); this is a Qualitative model, describing the essence of Nuclear Physics.

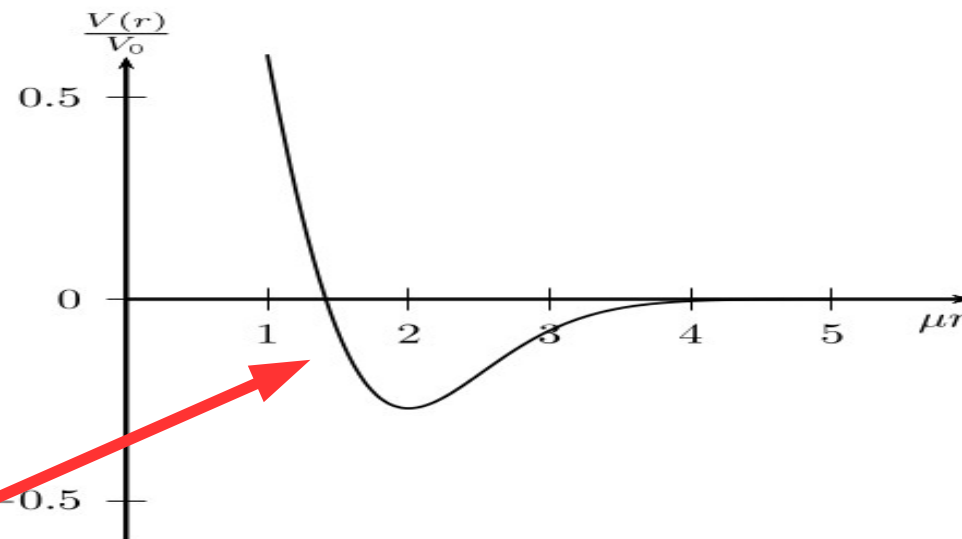
Question of Scale: Quark binding takes place for  $r_{ij} \sim m_{\pi}^{-1}$ , but nucleon binding takes place at 2, or 3, or 4 times that distance. How to achieve this?



This requires extraction and regularization of the logarithmic UV divergence of the loop, which contributes two essential features:

- a) It “stretches”, so that distances larger than  $m_{\pi}^{-1}$  can easily enter.
- b) It provides a crucial change of sign for the effective n-n binding potential.

*H.M. Fried et al. / Annals of Physics 338 (2013) 107–122*



**Fig. 3.** Nucleon potential with  $V_0 = c(\frac{g_R^2}{4\pi})\mu$ .

This sign change can be the basis of nucleon-binding to form nuclei. Expect (and hope) that nuclear physicists will employ such effective potentials to discuss heavier nuclei

# Virtual Closed-Quark Loops and their interactions with GBs.

All the basic, “radiative correction” structure of non-perturbative QCD comes from interacting closed-quark-loops with GBs. How can this be efficiently described? I will try to do this in words, describing the functional operations that need to be performed.

A single 'dressed' quark has an amplitude proportional to

$$N \int d[\chi] e^{\frac{i\chi^2}{4}} (\det(gf \cdot \chi))^{-\frac{1}{2}} e^{\hat{\mathcal{D}}_A} \cdot G_c[A] e^{L[A]} \Big|_{A \rightarrow 0},$$

While two scattering quarks are described by

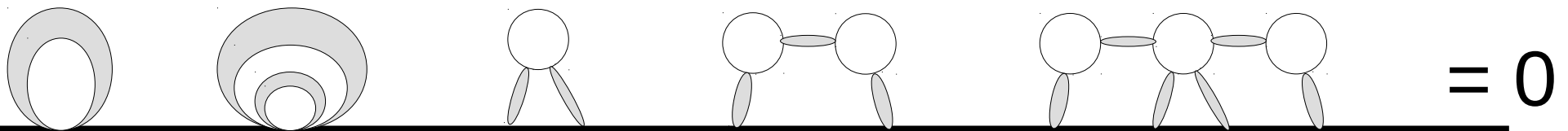
$$N \int d[\chi] e^{\frac{i\chi^2}{4}} (\det(gf \cdot \chi))^{-\frac{1}{2}} e^{\hat{\mathcal{D}}_A} G_c^{(1)}[A] G_c^{(2)}[A] e^{L[A]} \Big|_{A \rightarrow 0},$$

$$\hat{\mathcal{D}}_A = \frac{i}{2} \int \frac{\partial}{\partial A} (gf \cdot \chi)^{-1} \frac{\partial}{\partial A}$$

And  $\chi(x)$  is the Halpern functional variable originally used to represent  $e^{\frac{i}{4} \int d^4x F^2(x)}$

Every GB exchanged is represented by the linkage operator connecting the two  $G_c[A]'$ s to each other, and the  $G_c[A]'$ s to  $L[A]$  .

And here the relative simplicity of non-perturbative QCD shows itself clearly, for all of its “self-energy” graphs vanish, either by the asymmetry of the  $(f \cdot \chi)^{-1}$  color and coordinate indices, or by explicit loop integration.



What is the  $Z_q$  of a (non-perturbative) quark?  $Z_q = 1$

Non-Perturbative QCD is far simpler than QED!

# QCD Renormalization

The 'radiative corrections' of QCD enter when there is momentum transfer between one quark and another quark; and the procedure may occur when the momentum transfer passes through intermediate GBs and/or closed quark loops.

For simplicity, let us suppress possible quark binding into hadrons, and just consider two quarks exchanging momentum transfer in their CM.

A useful technique is the exact Functional Cluster Expansion,

$$e^{\hat{\mathcal{D}}_A} \cdot e^{L[A]} = \exp\left[\sum_{n=1}^{\infty} \frac{Q_n}{n!}\right], \quad Q_n = e^{\hat{\mathcal{D}}_A} (L[A])^n |_{connected}$$

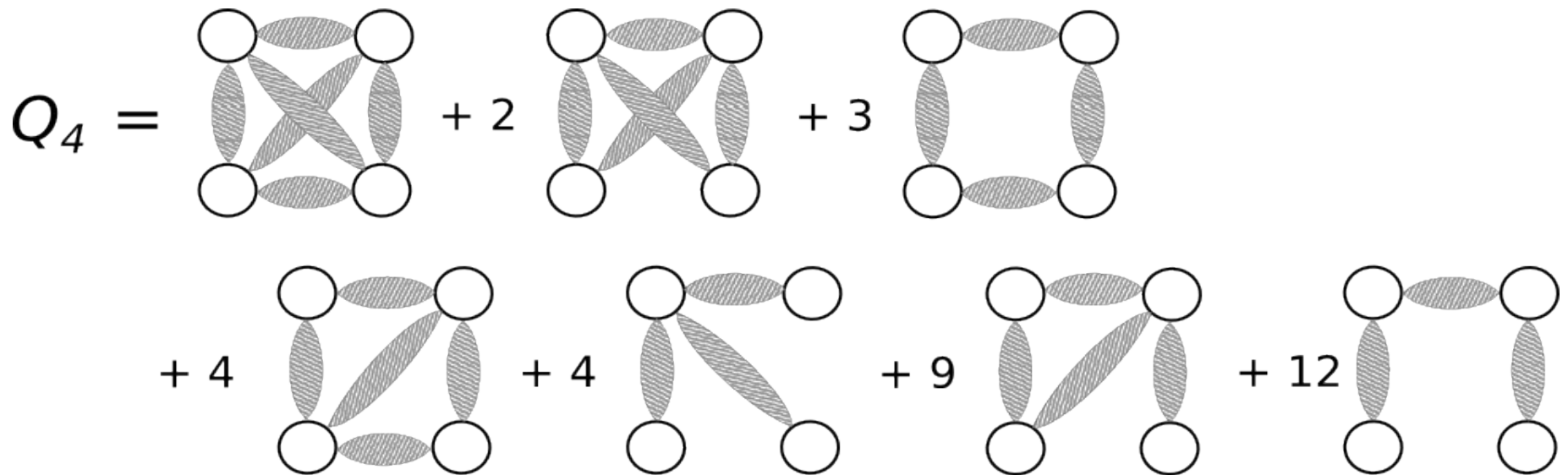
With linkage operator

$$\hat{\mathcal{D}}_A = \frac{i}{2} \int \frac{\partial}{\partial A} \hat{K} \frac{\partial}{\partial A}, \quad (\hat{K})_{\mu\nu}^{ab} = (gf_{abc}\chi_{\mu\nu}^c)^{-1} \leftarrow GB$$

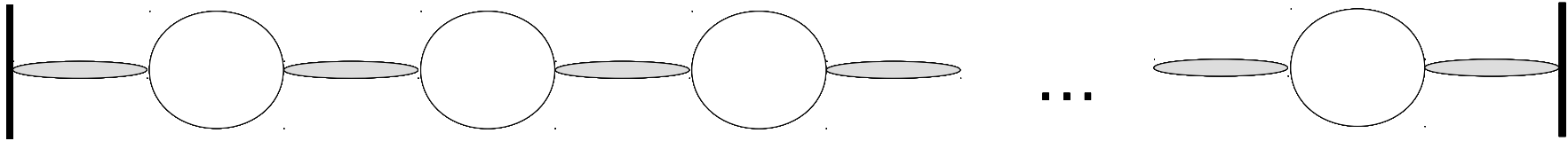
For example,  $Q_1 = e^{\hat{\mathcal{D}}^A} L[A] = \bigcirc + \bigcirc \text{---} \bigcirc + \bigcirc \text{---} \bigcirc + \dots = \bar{L}[A]$

And  $Q_2 = \bar{L}[A](e^{\hat{\mathcal{D}}^A} - 1)\bar{L}[A]$

Things get complicated very quickly; e.g.  $Q_4$  is given by



Even functionally, it is a horrid mess. But, there exists one way of reducing this to an easily-calculated set of 'chain-graph-loops',



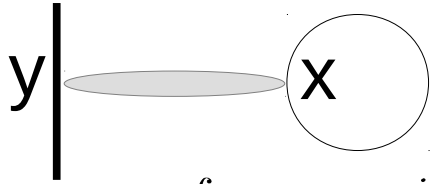
Which form a geometric series that can be summed.

But this depends crucially on the definition of renormalization....

At this point, let's 'take stock' of where we stand. We began with a Theory of quarks and gluons; but the gluons have disappeared, and only their GB sums remain.

What to do? Renormalize the GBs!

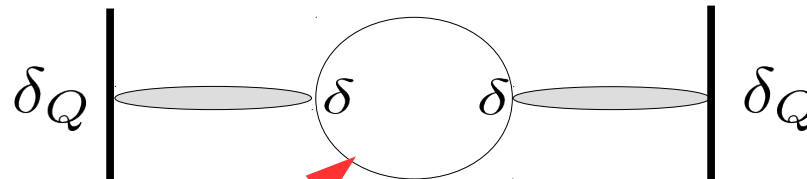
# QCD Renormalization



$$N \int d[\chi] \exp\left[\frac{i}{4} \int \chi^2 + ig \int ds' u'_\mu(s') \Omega_a(s') (f \cdot \chi(w))^{-1} \Big|_{\mu\alpha}^{ab} \int_0^t dt' v_\alpha(t') \hat{\Omega}_b(t') \cdot \int d^2 x'_\perp \int d^2 y'_\perp a(x_\perp - x'_\perp) \cdot a(y_\perp - y'_\perp) \cdot \delta^{(4)}(y' - u(s') - x' + v(t'))\right]$$

Because each quark represents the “physical particle” of QCD, we'll replace the  $\delta$  at each quark site by  $\delta_Q$ , a finite quantity. But where the  $\delta$  connects to the loop – which is a virtual and not a physical particle – it remains  $\delta$ , and (very shortly)  $\rightarrow 0$ .

In this one-loop, 2 GB drawing, there is a net  $\delta^2$  multiplying the loop.



But the loop is proportional to an expected UV log divergence, which we'll call

$$\ell, \quad \ell = \ln(\Lambda/m_\alpha)$$

This loop – as well as every such loop in a chain of such GB loops – produces a factor of

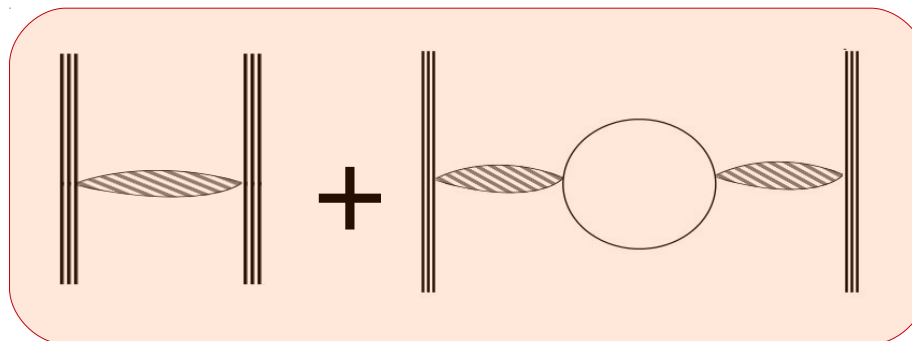
$$\delta^2 \cdot \ell \Big|_{\ell' \rightarrow \infty, \delta \rightarrow 0} = \kappa$$

which we DEFINE to be real, finite, positive number (subsequently determined by experiment).

What does this Model mean? That only the GB chain graphs are non-zero! All other closed loops entering into the Functional Cluster Expansion vanish.

AND, these chain graphs form a geometric series, which can be summed, is everywhere finite, and can be compared with High Energy pp scattering data. It can be used to define a renormalized charge  $g_R(\vec{q}_\perp^2)$ , as combinations of diminishing dependence on  $q_\perp$  arise from multiple factors of the Fourier transform of powers of  $\varphi(b)$ .

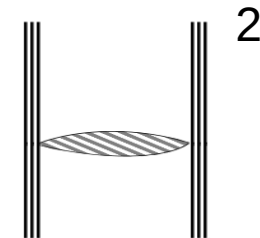
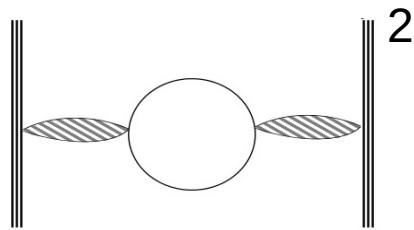
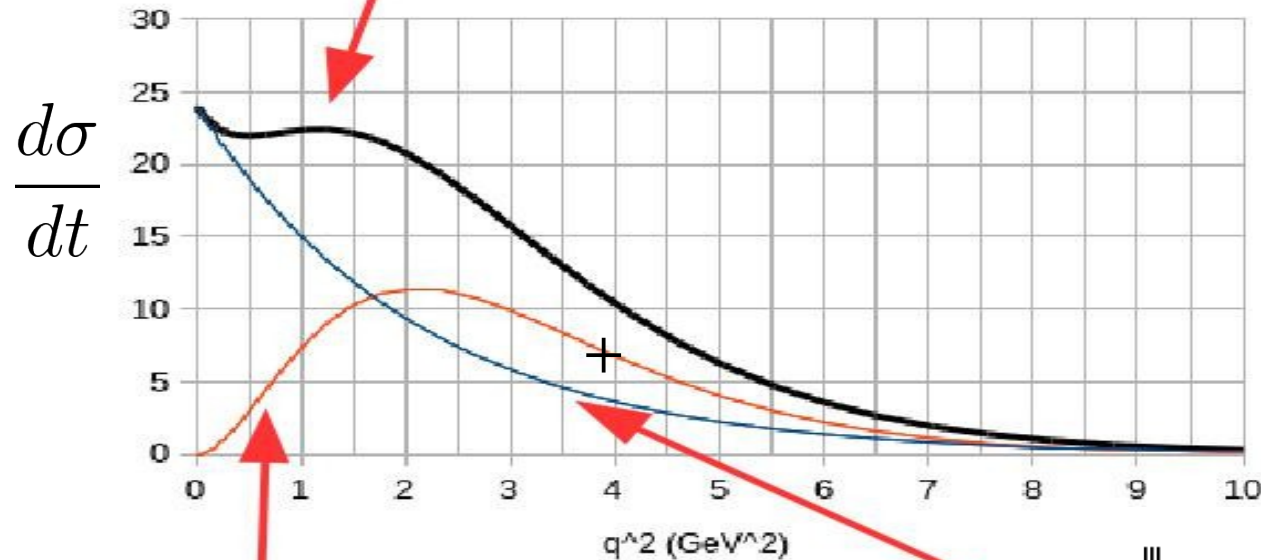
A preliminary approximate analysis suggests a quite close resemblance to scattering data. For the amplitude approximated by its first two terms:



(arxiv 1412.2072)

$$(g\phi)^2 + (g\phi_a)^2 = |g\phi - i g\phi_a|^2$$

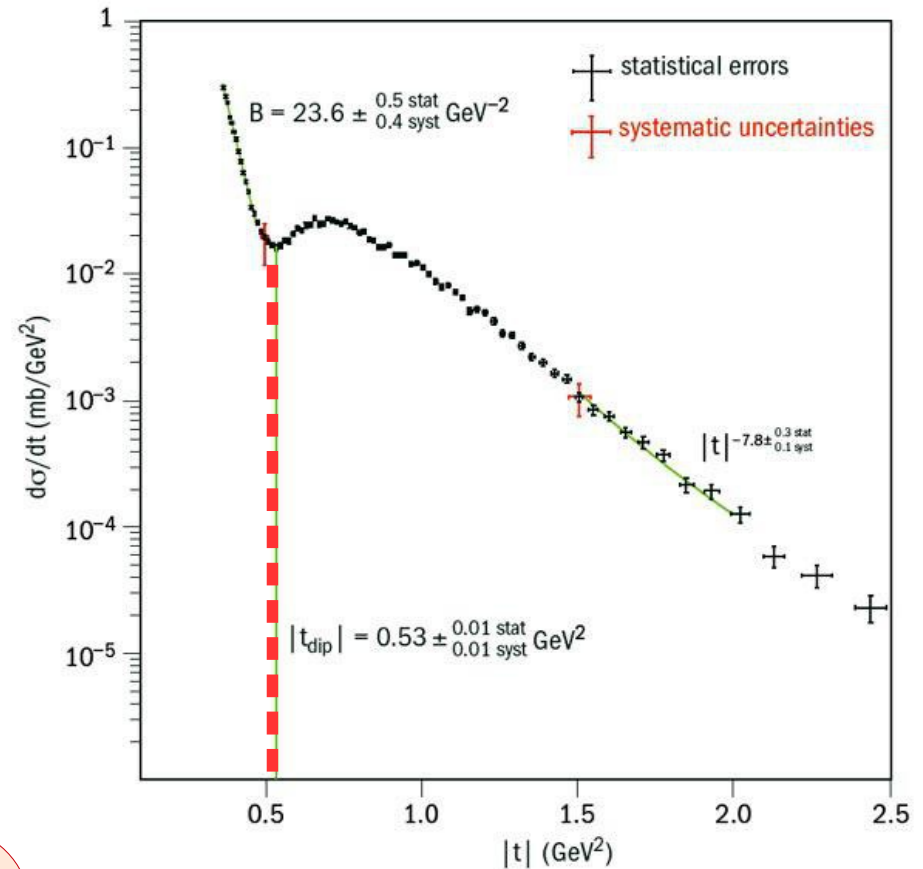
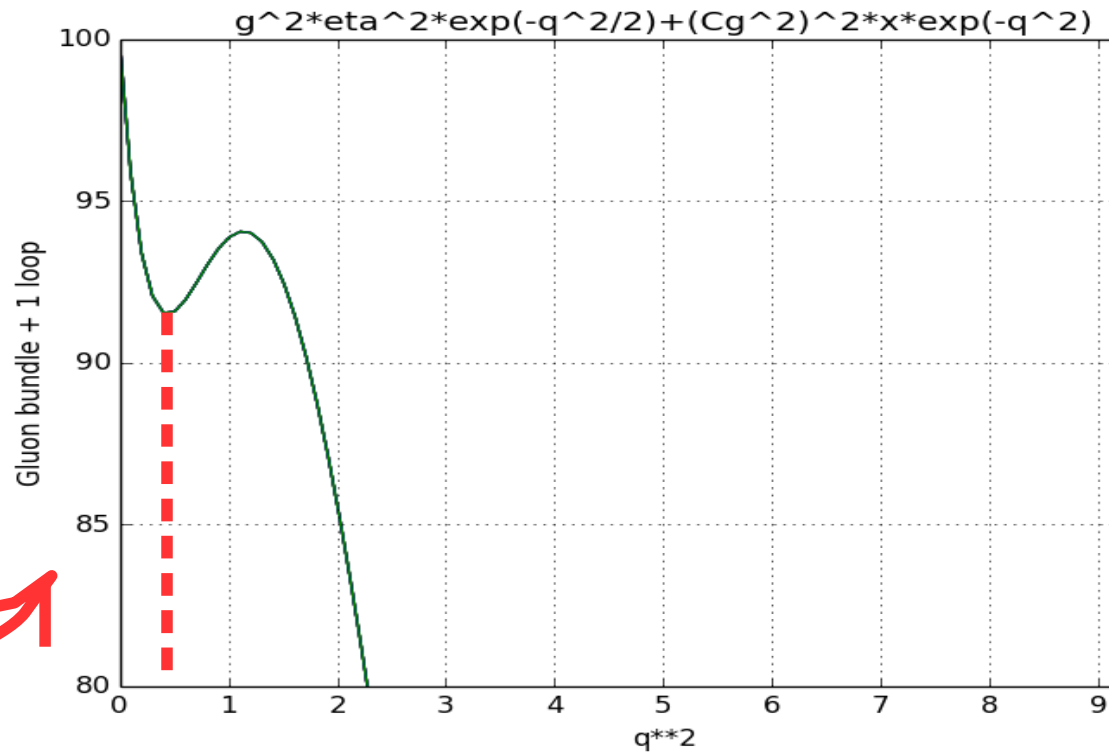
$$(gP)^2 + (gPa)^2$$



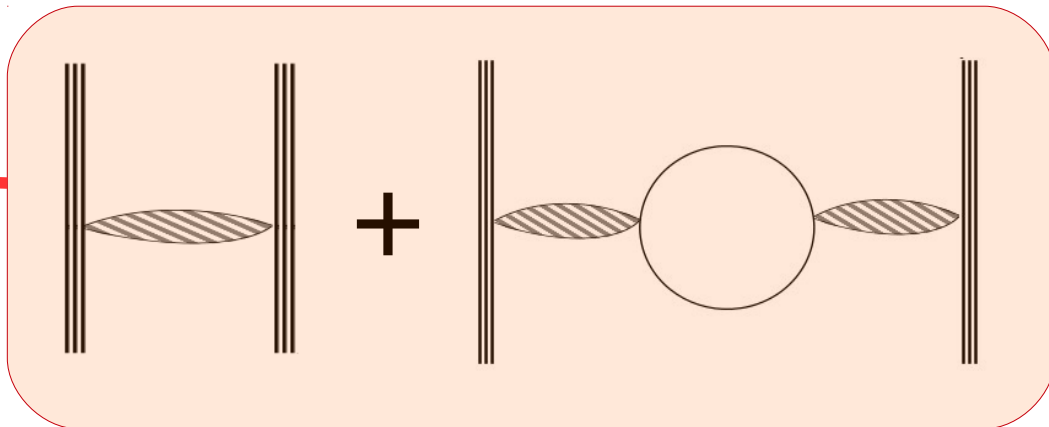
$$\begin{aligned} \mu &= 1\text{GeV}/c^2, & \xi &= 0.1 \\ g &= 10, & \lambda &= 0.18, \\ \kappa &= \delta^2 | = 1 \end{aligned}$$

(H.M.Fried, Y.Gabellini, T.Grandou, Y-Ming Sheu, PHT. Annals of Physics 2015)

# pp scattering diffraction dip at $\sim 0.5\text{GeV}^2$

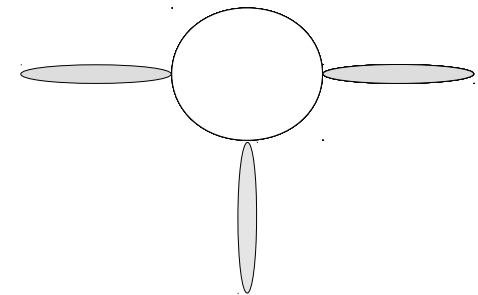


(TOTEM, EPL 101 (2013) 21002)

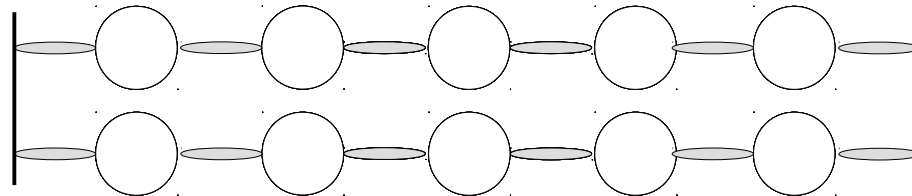


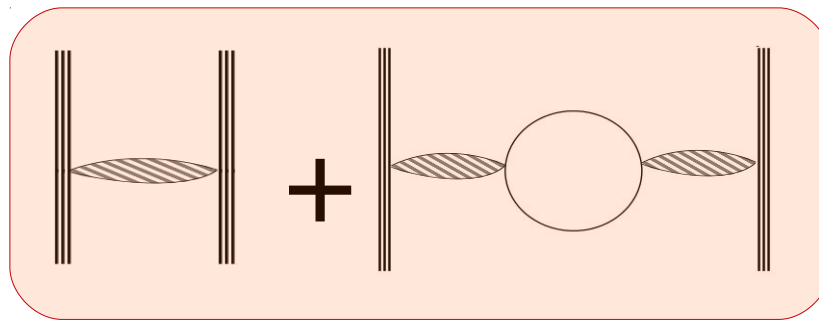
# Parameters to learn from experiments

- g coupling
- $\kappa = \delta^2 \cdot \ell$
- Energy dependence from  $\delta_{Quark} \propto (1/energy)$
- our choice of renormalization



- chain loop interactions. Interactions between chains..





The first two terms here are the ones most relevant in the GeV momentum-transfer region of scattering, it is easy to produce familiar “diffraction dip”; and we have every expectation of being able to produce at least qualitative fits at TeV energies and momentum-transfers.

Perhaps it is time to end this talk... but with the appreciation that we now have at least one, finite, renormalized Model of non-perturbative QCD.

Comparisons with scattering data are now underway...and we hope for the best. Mais nous verrons....

My colleagues and I thank you for your attention!

# THANK YOU!