

# Lagrangian of the model

Lagrangian of Gross-Neveu model looks like

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Zeeman term can be written as

$$g = 2 \implies \sigma_s \mu_B H \bar{\psi}^s \gamma^0 \psi^s = \mu_B H \bar{\psi}^1 \gamma^0 \psi^1 - \mu_B H \bar{\psi}^2 \gamma^0 \psi^2$$

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We can define two matrices  $\gamma^5$  analogs of (3+1) dimensional space-time

$$\gamma^3 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}, \quad \gamma^5 = \gamma^0 \gamma^1 \gamma^2 \gamma^3 = i \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix}, \quad (4)$$

# Symmetry

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It is occurred that this symmetry can be dynamically broken and fermion can acquire mass

## AB phase

Show that in  $R^d \times S^1$  space-time vector potential along compactified dimension can not be gauged away.

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Gauge transform is

$$\begin{aligned} \psi &\rightarrow e^{ie\alpha} \psi \\ A_\mu &\rightarrow A_\mu - \partial_\mu \alpha \end{aligned} \tag{7}$$



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Since boundary condition is imposed on  $\psi$ , take periodic boundary condition  $\psi(x_1, \dots, x_d, x_{d+1} + L) = \psi(x_1, \dots, x_d, x_{d+1})$ ,  $\alpha$  should yield a condition:

$$\alpha(x_1, \dots, x_d, x_{d+1} + L) = \alpha(x_1, \dots, x_d, x_{d+1}) + 2\pi en \quad (8)$$

where  $L$ -length of compactified dimension.

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we will get that system interact with vector potential and not with strength tensor of magnetic field So we call corresponding phase Aharonov-Bohm phase.

# Boundary conditions

We impose the boundary condition

$$\psi(x + L) = e^{2\pi i\alpha}\psi(x) \quad (10)$$

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$\alpha = \frac{1}{2}$  -anti-periodic boundary condition



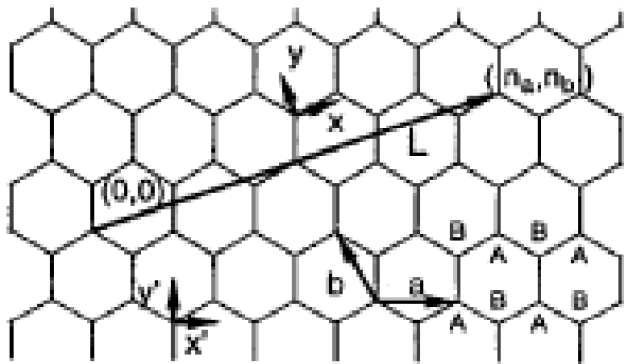


Рис.: Lattice

Boundary condition for graphene is a little more complicated, it is different for different Dirac points

$$\psi_K(x + L) = \psi_K(x)e^{2\pi i(\phi - \frac{\nu}{3})} \quad (11)$$

$$\psi_{K'}(x + L) = \psi_{K'}(x)e^{2\pi i(\phi + \frac{\nu}{3})}$$

$$\psi = \begin{pmatrix} \psi^{AK} \\ \psi^{BK} \\ \psi^{BK'} \\ \psi^{AK'} \end{pmatrix} = \frac{1}{L} \sum_{n=-\infty}^{\infty} e^{(\frac{i x}{R})(n+\phi)} \frac{1}{2\pi} \int d^2 p e^{i p x} \begin{pmatrix} \psi_n^{AK} e^{i(\frac{x}{R})\nu/3} \\ \psi_n^{BK} e^{i(\frac{x}{R})\nu/3} \\ \psi_n^{BK'} e^{-i(\frac{x}{R})\nu/3} \\ \psi_n^{AK'} e^{-i(\frac{x}{R})\nu/3} \end{pmatrix}$$

## Equivalent action

$$Z(\bar{\eta}, \eta) = N' \int [d\bar{\psi}][d\psi] \exp\left(i \int (\mathcal{L} + \bar{\eta}\psi + \bar{\psi}\eta) d^3x\right),$$
$$S[\bar{\psi}, \psi, \sigma] = \int d^2x (i\bar{\psi}\gamma^\mu \partial_\mu \psi - \sigma\bar{\psi}\psi - \frac{N}{2G}\sigma^2) \quad (13)$$

Perform Hubbard-Stratonovich transformation

$$Z(\bar{\eta}, \eta) = \int D[\bar{\psi}(x)]D[\psi(x)]e^{iS[\bar{\psi}, \psi]} = \quad (14)$$
$$= \int D[\bar{\psi}(x)]D[\psi(x)]D[\sigma(x)]e^{iS[\bar{\psi}, \psi, \sigma]}$$

Auxiliary Lagrangian has the form

$$\mathcal{L}(\bar{\psi}, \psi, \sigma) = i\bar{\psi}\gamma^\mu \partial_\mu \psi - \sigma\bar{\psi}\psi - \frac{N}{2G}\sigma^2. \quad (15)$$

Euler-Lagrange equation for scalar field is

$$\sigma = -\frac{G}{N}\bar{\psi}\psi. \quad (16)$$

$$Z[\eta, \bar{\eta}, J] = \int D[\psi(x)] D[\bar{\psi}(x)] D[\sigma(x)] \exp \left( i \int dx \mathcal{L}(\bar{\psi}, \psi, \sigma) + \int dx \bar{\eta} \psi + \int dx \bar{\psi} \eta + \int dx \sigma J \right) \quad (17)$$

$N : J = O(N)$ .

Integrate over fermion fields we obtain

$$Z[\eta, \bar{\eta}, J] = \int D[\sigma(x)] \exp \left( iN \int dx \left( -\frac{1}{2G} \sigma^2 - i \text{tr} \ln D - \frac{1}{N} \bar{\eta} D^{-1} \eta \right) + \int dx \sigma J \right)$$

$$\langle \sigma \rangle = -iG \text{tr} \frac{1}{i\gamma\partial - \sigma} + J + \frac{1}{N} \bar{\eta} \eta, \quad (19)$$

Generating functional of connected Green functions are

$$e^{iW(\bar{\eta}, \eta, J)} = Z(\bar{\eta}, \eta, J) = N' \int [d\bar{\psi}][d\psi] \exp\left(i \int (\mathcal{L} + \bar{\eta}\psi + \bar{\psi}\eta + \sigma J) d^3x\right),$$

Definition of classical average fields are

$$\sigma(x) = \frac{\delta W}{\delta J(x)} = \langle \sigma \rangle + o\left(\frac{1}{N}\right) \quad (20)$$

$$\psi(x) = \frac{\delta W}{\delta \bar{\eta}(x)} = D^{-1} \eta \quad (21)$$

$$\bar{\psi}(x) = \frac{\delta W}{\delta \eta(x)} = -\bar{\eta} D^{-1} \quad (22)$$

having performed Legendre transformation effective action could be derived

$$\Gamma[\psi, \bar{\psi}, \sigma] = W[\eta, \bar{\eta}, J] - \int \bar{\psi} \eta - \int \bar{\eta} \psi - \int \sigma J = \quad (23)$$

$$\Gamma[\psi, \bar{\psi}, \sigma] = \int dx \left( \bar{\psi} \gamma^\mu \partial_\mu \psi - \sigma \bar{\psi} \psi - \frac{N}{2G} \sigma^2 - iN \text{tr} \ln(i\gamma \partial - \sigma) \right) \quad (24)$$

$$\frac{\delta \Gamma}{\delta \psi} = \bar{\eta}, \quad \frac{\delta \Gamma}{\delta \bar{\psi}} = -\eta, \quad \frac{\delta \Gamma}{\delta \sigma} = -J \quad (25)$$

Taking the sources to null we get vev of the  $\psi, \bar{\psi}, \sigma$  fields satisfying condition of extremum of effective potential

$$\frac{\delta \Gamma}{\delta \psi} = 0, \quad \frac{\delta \Gamma}{\delta \bar{\psi}} = 0, \quad \frac{\delta \Gamma}{\delta \sigma} = 0 \quad (26)$$

For fermions we get Dirac equation

$$i\gamma^\mu \partial_\mu \psi - \sigma \psi = 0 \quad (27)$$

Suppose in order to maintain Lorentz invariance that

$$\psi = 0, \quad \bar{\psi} = 0.$$

# Effective potential

We obtain the effective action of field

$$\Gamma[\sigma] = \int d^3x \left( -\frac{N}{2G} \sigma^2 - iN \text{tr} \ln(i\gamma\partial - \sigma) \right) \quad (28)$$

Defining effective potential as

$$V_{\text{eff}} = -\frac{\Gamma}{\int d^3x} = -\frac{\Gamma}{TV}$$

$$V_{\text{eff}} = \frac{N}{2G} \sigma^2 + i\frac{1}{TV} N \text{tr} \ln(i\gamma\partial - \sigma) \quad (29)$$



$$V_{eff} = \frac{\sigma^2}{2G} - \frac{1}{\beta L} \sum_s \sum_{n=-\infty}^{\infty} \times \quad (30)$$

$$\times \int_{-\infty}^{\infty} \frac{dp}{2\pi} \ln \left[ p_0^2 + \left( \frac{2\pi}{L} \right)^2 (n + \phi + \alpha)^2 + p_1^2 + \sigma^2 \right].$$

$$\phi = \frac{eL^2 H}{8\pi^2}$$

magnetic flux

To consider theory at finite temperature and density we need to substitute integration to summation over Matsubara frequencies

$$p \rightarrow \frac{2\pi}{\beta} \left( l + \frac{1}{2} \right) - i\mu, \quad \int_{-\infty}^{\infty} \frac{dp_0}{2\pi} \rightarrow \frac{1}{\beta} \sum_{n=-\infty}^{\infty} \quad (31)$$

Eff potential at finite temperature is

$$V_{\text{eff}} = \frac{\sigma^2}{2G} - \frac{1}{\beta L} \sum_s \sum_l \sum_{n=-\infty}^{\infty} \times \quad (32)$$

$$\times \int_{-\infty}^{\infty} \frac{dp_1}{2\pi} \ln \left[ \left( \frac{2\pi}{\beta} \left( l + \frac{1}{2} \right) - i\mu_s \right)^2 + \left( \frac{2\pi}{L} \right)^2 (n + \phi)^2 + p_1^2 + \sigma^2 \right].$$

There is a very usefull formula, using it we can separate contribution of compactification

$$\sum_{n=-\infty}^{\infty} \ln\left(1 + \frac{b^2}{(n + \alpha)^2 + a^2}\right) = \quad (33)$$

$$\sum_{n=-\infty}^{\infty} (\ln((n + \alpha)^2 + a^2 + b^2) - \ln((n + \alpha)^2 + a^2)) = \quad (34)$$

$$= \int_{-\infty}^{\infty} d\tau \ln\left(1 + \frac{b^2}{\tau^2 + a^2}\right) +$$

$$\ln \frac{1 - 2 \cos(2\pi\alpha) e^{-2\pi\sqrt{a^2+b^2}} + e^{-4\pi\sqrt{a^2+b^2}}}{1 - 2 \cos(2\pi\alpha) e^{-2\pi\sqrt{a^2}} + e^{-4\pi\sqrt{a^2}}}$$

Effective potential could be obtained in the relatively short form

$$V_{\text{eff}} = \frac{N}{\pi} \left( \frac{\sigma^3}{3} - \frac{\sigma^2 \sigma_0}{2} \right) + V_{\mu T} + \quad (35)$$
$$+ \frac{4N}{\beta L} \sum_{n=1}^{\infty} \sum_{l=-\infty}^{\infty} \frac{\sigma_l}{n} K_1(L\sigma_l n) \cos(2\pi\alpha n)$$

where is used the notation

$$\Delta_l = \sqrt{\left( \frac{2\pi}{\beta} \left( l + \frac{1}{2} \right) - i\mu \right)^2 + \Delta^2}$$

$K_{\mu}(x)$  modified Bessel function of the second kind

Contribution due to finite temperature and chemical potential is given by

$$V_{\mu T} = -\frac{N}{\pi\beta^3} Li_3(-e^{-\beta\sigma+\beta\mu}) + -\frac{N}{\pi\beta^3} Li_3(-e^{-\beta\sigma-\beta\mu}) + \quad (36)$$

$$-\frac{N\sigma}{\pi\beta^2} Li_2(-e^{-\beta\sigma+\beta\mu}) + -\frac{N\sigma}{\pi\beta^2} Li_2(-e^{-\beta\sigma-\beta\mu}) = \quad (37)$$

where polylogarithm was used which is defined

$$Li_\nu(z) = \sum_{k=1}^{\infty} \frac{z^k}{k^\nu} \quad (38)$$

We need renormalization of the coupling constant

$$\frac{1}{g(\mu)} = \frac{1}{G} - \frac{\Lambda}{\pi} + \frac{\mu}{\pi^2} = \frac{1}{g} + \frac{\mu}{\pi^2}. \quad (39)$$

Running of the coupling

$$\frac{1}{g(\mu)} = \frac{1}{g(\mu_0)} + \frac{1}{\pi^2}(\mu - \mu_0). \quad (40)$$

$$V_{\text{eff}} = V_L + V_0 + V_{\mu T} \quad (41)$$

$$V_L = 2\text{Re}\left(\frac{1}{\pi L^3} \text{Li}_3(e^{-L\sigma+2\pi i\phi}) + \frac{\sigma}{\pi L^2} \text{Li}_2(e^{-L\sigma+2\pi i\phi})\right) \quad (42)$$

$$V_0 = \frac{N}{\pi} \left( \frac{\sigma^3}{3} - \frac{\sigma^2 \sigma_0}{2} \right) \quad (43)$$

$$V_{\mu T} = -\frac{2}{\beta L} \sum \int \frac{dp_1}{2\pi} (\ln(1 + e^{\beta E_{np_1}^+}) + \ln(1 + e^{\beta E_{np_1}^-})) \quad (44)$$

$$E_{np_1}^{\pm} = E_{np_1} \pm \mu \quad (45)$$

$$V_{\text{eff}} = \frac{N}{\pi} \left( \frac{\sigma^3}{3} - \frac{\sigma^2 \sigma_0}{2} \right) + \frac{2N}{\pi L^3} \text{Li}_3(e^{-L\sigma+2\pi\alpha}) + \frac{2N}{\pi L^3} \text{Li}_3(e^{-L\sigma-2\pi\alpha}) + \quad (46)$$

$$\begin{aligned} & \frac{2N\sigma}{\pi L^2} \text{Li}_2(e^{-L\sigma+2\pi\alpha}) + \frac{2N\sigma}{\pi L^2} \text{Li}_2(e^{-L\sigma-2\pi\alpha}) \\ & - \frac{2N}{\pi\beta L} \sum_{\sigma_n < \mu} \int \frac{dp_1}{2\pi} \ln \left( 1 + e^{(-\beta\sqrt{p_1^2 + \Delta_n^2} + \beta\mu)} \right) + \\ & + \frac{2N}{\pi\beta L} \sum_{\sigma_n > \mu} \sum_{m=1}^{\infty} (-1)^m \frac{\sigma_n}{m} K_1(\beta\sigma_n m) e^{\beta\mu m} \\ & + \frac{2N}{\pi\beta L} \sum_{n=-\infty}^{\infty} \sum_{m=1}^{\infty} (-1)^m \frac{\sigma_n}{m} K_1(\beta\sigma_n m) e^{-\beta\mu m} \end{aligned}$$

where  $\Delta_n = \sqrt{\left(\frac{2\pi}{L}\right)^2(n + \alpha)^2 + \sigma^2}$



Full magnetization is given by

$$m = -\left(\frac{\partial V_{\text{eff}}}{\partial \mu} \frac{g\mu_B}{2} + \frac{\partial V_{\text{eff}}}{\partial \phi} \frac{eL^2}{8\pi^2}\right) \quad (47)$$

$$n_s = \frac{1}{L} \sum_{n=-\infty}^{\infty} \int \frac{dp_1}{2\pi} \left( \frac{\text{sh}(\beta\mu_s)}{\text{ch}(\beta\mu_s) + \text{ch}(\beta E_{np_1})} \right) \quad (48)$$

where

$$E_{np_1} = \sqrt{p_1^2 + \left(\frac{2\pi}{L}\right)^2 (n + \alpha)^2 + \sigma^2}$$

Magnetization is given by

$$m = \mu_B(n_1 - n_2)$$

# Representation of spinors

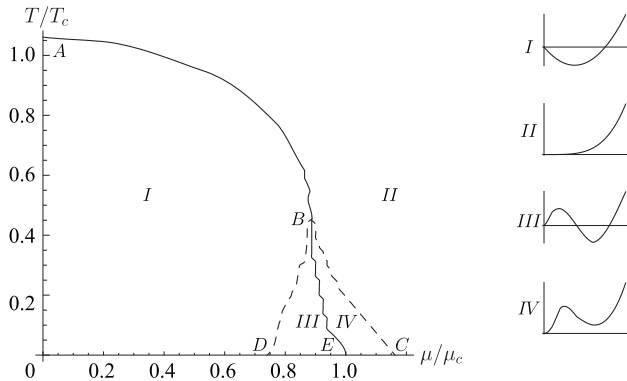
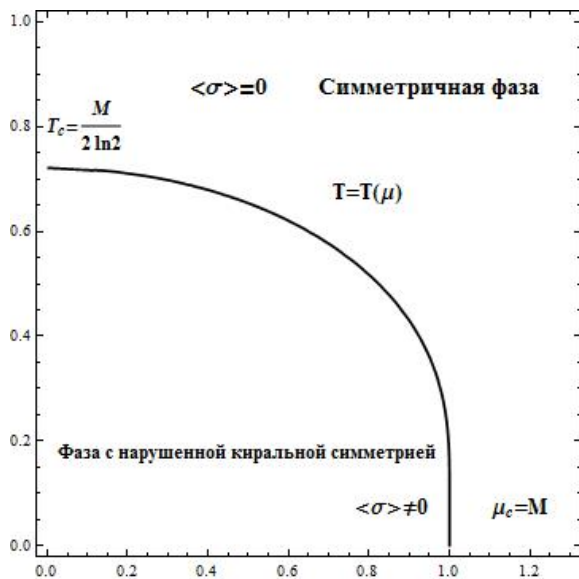


Рис.:  $T$ - $\mu$  phase diagram for  $L=1.2L_c$

# Plots



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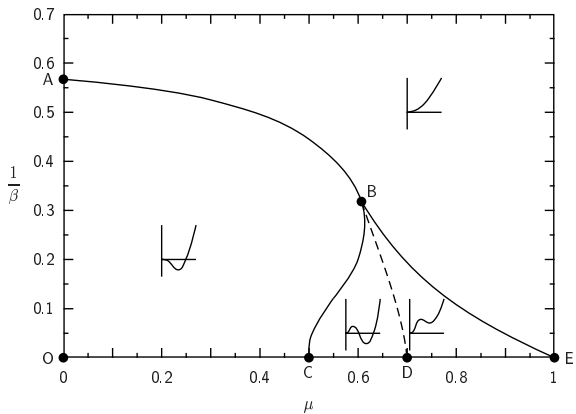


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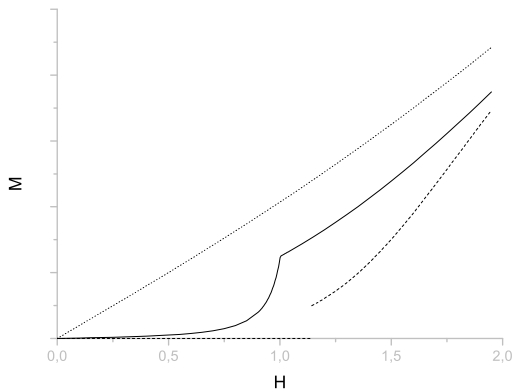


Рис.: Dependence of Zeeman magnetization on magnetic field  
 $T = \frac{1}{3} T_c$  (dashed line),  $T = \frac{4}{5} T_c$  (solid line),  $T = \frac{3}{2} T_c$  (dotted line)

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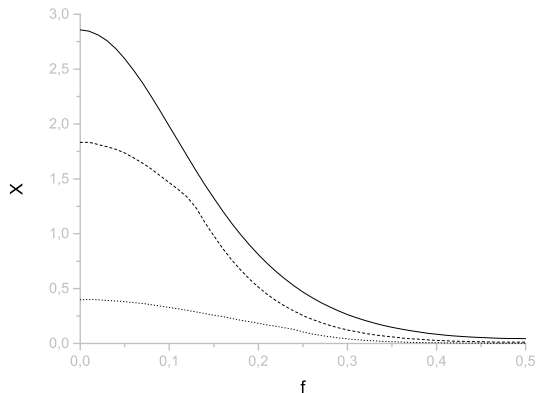


Рис.: magnetic susceptibility on flux  $T = 3/2 T_c$  (solid line),  $T = 5/4 T_c$  (dashed line).  $T = T_c$  (dotted line)

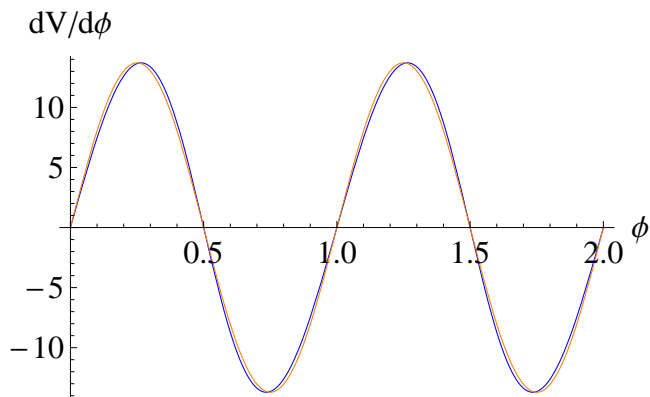


Рис.: Aharonov-Bohm magnetization on flux  $L \gg L_c, \beta \gg \beta_c$

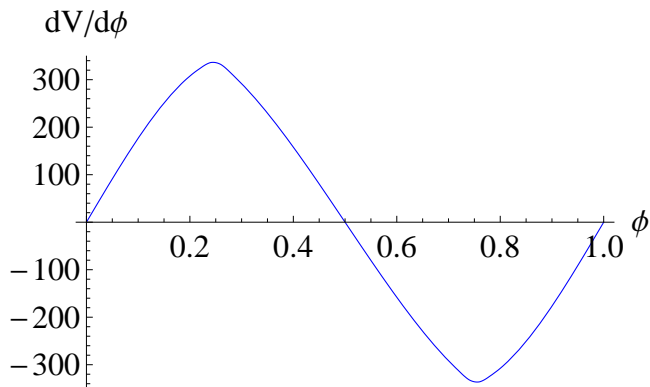


Рис.: AB magnetization on magnetic flux  $L = 0,5L_c$ ,  $\beta = 3\beta_c$



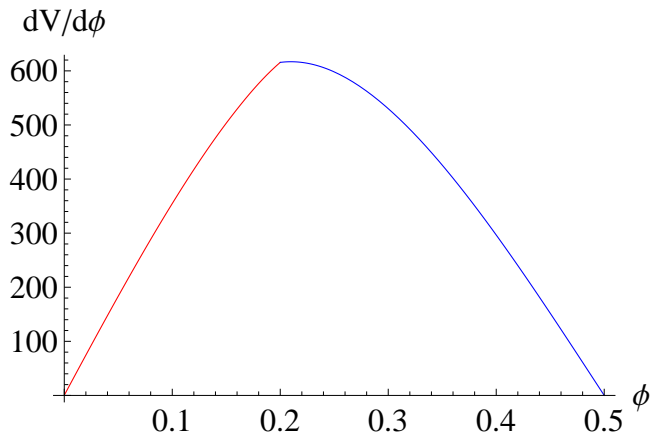


Рис.: T- $\mu$  phase diagram for  $L=1.2L_c$

# Plots

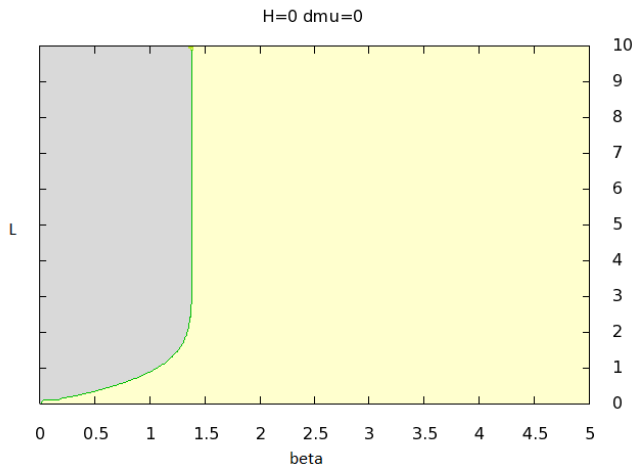


Рис.: L -T phase diagram for H=0

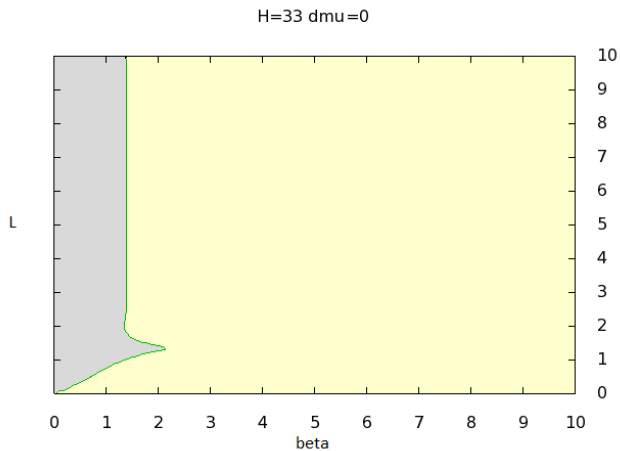


Рис.: L -T phase diagram for H=33

# Plots

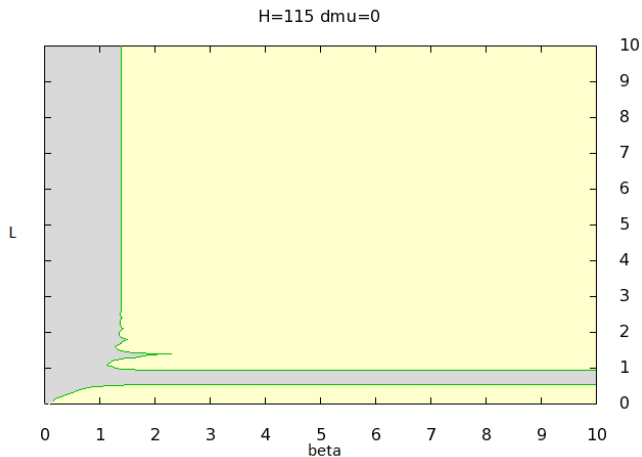


Рис.: L -T phase diagram for H=115

# Plots

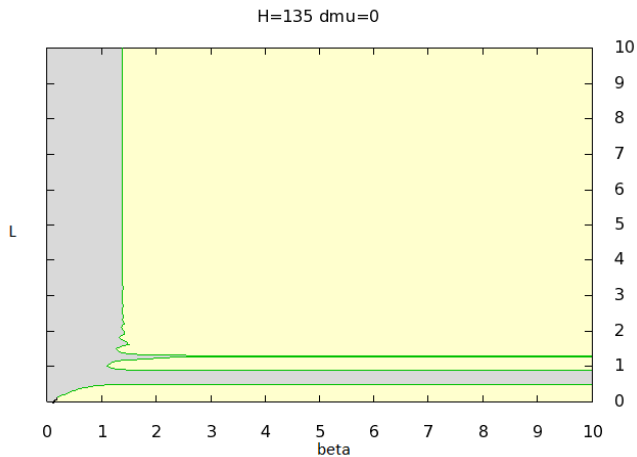


Рис.: L -T phase diagram for H=135

# Plots

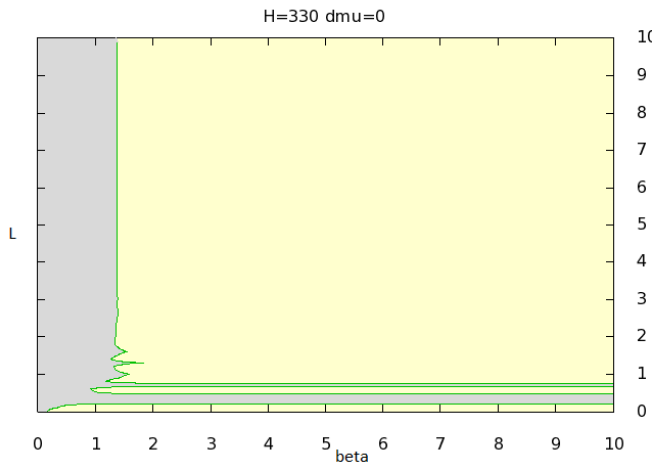


Рис.: L -T phase diagram for H=0

# Plots

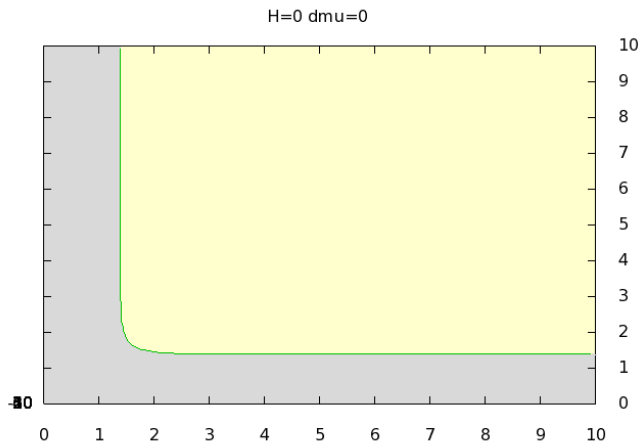


Рис.: L -T phase diagram for  $H=0$ ,  $\alpha = \frac{1}{2}$

# Plots

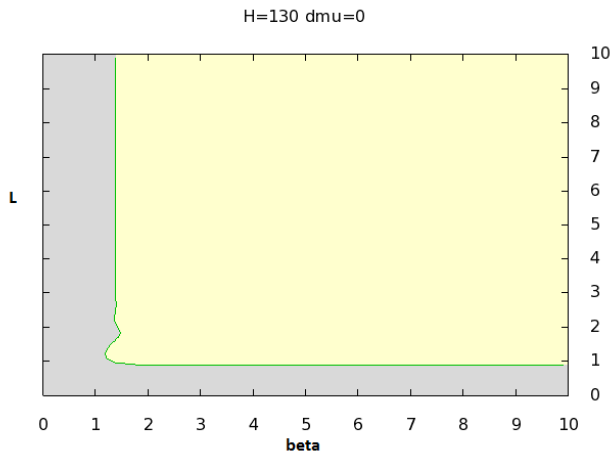


Рис.: L -T phase diagram for H=130,  $\alpha = \frac{1}{2}$



# Plots

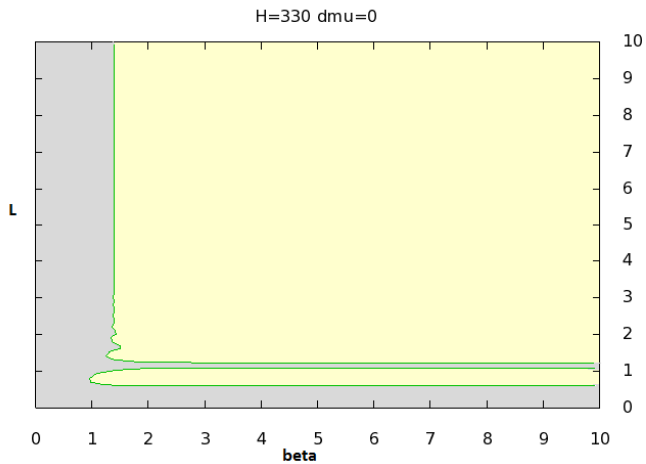


Рис.: L -T phase diagram for H=330,  $\alpha = \frac{1}{2}$

# Summary

Gross-Neveu model with one compactified spatial dimension (i.e., a cylinder) under the influence of the magnetic field parallel to the axis of the cylinder with consideration for the finite temperature and chemical potential.

Two Zeeman interaction with spins of the electrons and AB phase due to flux through cylinder can be accounted for in (2+1)

We hope this result can shed light on carbon nanotubes under influence of external magnetic field

**Thanks for the attention**