A Critical Omission in the Critical State Model

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Trapped field magnets (TFMs), composed of bulk HTS, are able to retain fields much higher than permanent ferromagnets. For applications, quality measures generally improve as $B$ or $B^2$. Therefore, there is broad interest in applications of TFMs.

However, TFMs present added application challenges. They must be cooled below $T_C$ and, if warmed, they must be reactivated.

For FC activation, the critical state model (CSM) predicts $B_A = B_{T,\text{max}}$. However, the field must be kept on for ~ seconds. This results in large energy requirements, detrimental to many applications.

For ZFC activation, the Critical State Model (CSM) predicts $B_A \geq 2 B_{T,\text{max}}$. However, ZFC does not require long cooling time, and a short pulse can be used, greatly reducing energy needs.

As a result, pulsed-ZFC is strongly preferred.

However, a problem remains: it is difficult to achieve $B_A \geq 2 B_{T,\text{max}}$ for high field TFMs.
• E.g., activation/re-activation is a limiting design factor in TFM motors. One optimized design used ~1/3 of the rotor space for activation coils. This caused a 33% loss of the expected power.

• An additional problem is that a pulsed-ZFC activation heats the TFM. This lowers $J_C$ and trapped field maximum, $B_{T,max}$.

• Various world groups have spent decades trying to overcome TFM heating by high fields, in an attempt to obtain full activation.

• E.g., some have developed a 10 pulse, varying-amplitude, varying-temperature sequence in order to approach 80% of full activation.

• We have performed a series of experiments at 77 K to study details of the pulsed-activation process of TFMs.

• In an earlier experiment, on high $J_C$ TFMs, we found a factor of 2 reduction in the required field for pulsed-ZFC activation; i.e., from $B_A \geq 2 B_{T,max}$ to $B_A \approx B_{T,max}$. 
• A follow-up experiment is reported here, using TFMs with a wide range of $J_C$ to explore for regularities of the anomalous behavior, and perhaps insight into the physics.

• As in the previous experiment, the activation field diameter is smaller than the 20 mm TFM.

• The magnet coils have Hiperco-50 cores, to obtain higher applied fields.

★ Finite element calculations, based on CSM, indicate no activation anomalies are expected due to this geometry.

• The field is approximately flat for $0 \leq r \leq 6$ mm, and decreases linearly to zero at the TFM periphery.

Schematic of Experiment
• Between the TFM and the bottom coil there is a 1.4 mm gap, into which 7 Hall probes are placed. These are placed 1.15 mm apart, spanning $1.7 < r < 8.6$ mm of the 10 mm TFM radius.

• A current pulse from capacitive discharge is used. This has a rise time of $\sim 2$ ms, and is $\sim 30$ ms long.

• For the coils used, 500 A provides $B_A \approx 3.3$ T.

• We will use the symbol $B_A$ to represent the maximum of the applied field.

• The TFMs used were melt-textured, single grains of YBCO, 20 mm diameter X 8 mm long.
They contained pinning centers (PCs) with one of two extreme geometries: (1) Broken columnar PCs, (2) “point” PCs.

The spectrum of $J_C$ values used in the new experiment was $5,000 \leq J_C \leq 50,000 \text{ A/cm}^2$.

The earlier experiment showing anomalous results was performed on high $J_C$ ($\sim 50,000 \text{ A/cm}^2$) samples.

Previous to that, a similar experiment, performed on low $J_C$ samples ($\sim 10,000 \text{ A/cm}^2$) showed good agreement with CSM.

We first consider data on trapped field, taken 2 minutes after the 30 ms activating pulse.

We compare the samples of both low and high $J_C$ to the critical state model (CSM).
• CSM requires a smooth rise in trapped field vs. $B_A$. This condition is satisfied for the low $J_C$ sample.
• The high $J_C$ sample exhibits an anomalous giant field leap (GFL) in $B_{T,\text{max}}$. 
• Next we note that CSM requires \( B_A \geq 2 \times B_{T,max} \) in order to fully activate a TFM to its maximum achievable \( B_{T,max} \).

• The low \( J_C \) samples require \( B_A \approx 4.1 \times B_{T,max} \) for full activation, thus satisfying this requirement of CSM. \( (B_A \geq 2 B_{T,max}) \).

• The high \( J_C \) sample is activated to its maximum achievable trapped field when \( B_A \approx 1.6 \times B_{T,max} \), a clear violation of CSM.

• **This violation is very encouraging for applications.**

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• In order to probe GFL more deeply, a study of the time evolution of the HTS field was developed.

• Data were taken every 100 micro-seconds at various values of the pulse height, \( B_A \), of the activating field.

• We denote by \( B_{HTS} \) the field which has penetrated the HTS during activation.
• As $B_A$ is increased, CSM requires $B_{HTS}$ to increase most rapidly at large $r$, and more slowly as $r \to 0$.

• The low $J_c$ sample behaves in accord with CSM.

• The high $J_c$ sample behaves in accord with CSM, until (1) the pulse is at its peak value, and (2) $B_A \geq B_{T,max}$. Then, in $\sim 500 \, \mu s$, the situation rapidly reverses, in contradiction to CSM.
Critical State Model (CSM) Compared to Experiment

• The high $J_C$ samples violate CSM in the following ways:
  
  • Prior to GFL the values of trapped field, $B_T(r)$, are suppressed relative to CSM by a factor of ~6.
  
  • After GFL, $B_T$ is enhanced. I.e., full activation is achieved at $B_A = B_{T,max}$. CSM requires $B_A \geq 2 B_{T,max}$.
  
  • CSM requires smooth increase of HTS field vs. $B_A$. Instead, $B_T$ leaps when the induced $\bar{E} = 0$, $B_A \approx B_{T,max}$.
  
  • High $J_C$ and low $J_C$ samples behave differently. CSM makes no $J_C$ distinction.
  
  • CSM has $B_T$ rise at same rate as $B_A$. Instead, $B_T$ leap occurs very fast (~500 µs). This is 4x faster than $B_A$.  

$E = 0$
The New Experiment

• We planned several experiments to look for regularities in the GFL phenomenon. The first was to study GFL vs. $J_C$.

• We produced samples with a variety of $J_C$ using refined Y211, nuclear recoil, and nuclear fission to make PCs.

• Trapped field was separately measured for each sample by field cooling (FC) in a magnet with $R_{\text{mag}} > R_{\text{TFM radius}}$.

• $B_{T,\text{max}}$ of each sample was measured on an $x,y$ scanner using a Hall probe.

• From these measurements, $J_C$ was calculated.

• We did several experiments to check that the equipment was properly functioning.
• E.g., the measured FC values of $B_{T,max}$ were compared to the pulsed-ZFC measurements of $B_{T,max}$.

• Good agreement with a linear relationship was found. Extrapolation to zero reflects the effect of the Hiperco-50 core.
- We next used data on trapped field at $t = 2 \text{ min.}$ to measure where the leap started ($= B_{\text{Thresh}}$) and where the leap ended, ($= B_{\text{End of leap}}$).
Without recourse to any theory we see:

- The threshold is a decreasing function of $J_C$.
- The end-of-leap is an increasing function of $J_C$.
- The leap phenomenon increases with $J_C$.
- The magnitude of the leap grows to ~2 T at $J_C \sim 50,000 \text{ A/cm}^2$.
- Both point PCs and columnar PCs show the same general GFL behavior.
- Therefore, at least to first order, GFL is independent of pinning center geometry.
• We next considered the data on $B_A/B_{T,\text{max}}$, corrected to the TFM surface.

• Note that for low $J_C$ samples, $B_A/B_{T,\text{max}} \approx 3.2$, a result in agreement with CSM.

• Note that for high $J_C$ samples, $B_A/B_{T,\text{max}} \approx 1.0 \pm 10\%$, a result incompatible with CSM.

★ The special point at 5000 A/cm$^2$ is a finite element calculation based on CSM.
• What is the physics causing GFL?

• We **speculated** with the first GFL observation, that the very large Lorentz force, \( F_L \propto J_C \times B_{HTS} \) may be moving the fluxoids away from the locations required for optimum diamagnetic shielding.

• In this new experiment we can measure \( B_{HTS} \) just prior to the leap (= \( B_{Thresh} \)), and calculate \( J_C \times B_{thresh} \propto F_{L,thresh} \).

• We use the time dependent data to find \( B_{Thresh} \) so that we do not have to correct for unknown creep rate.

• We have data on seven points in r, just prior to GFL. We fit 6 of these with 2 straight lines in order to find the peak value of \( B_{Thresh} \).

• Typical fits are shown in the next slide.

• We use the measured FC value of \( B_{T,max} \) to represent \( J_C \).
• Examples of determinations of $B_{\text{thresh}}$ for samples with $J_C$:
  (a) 14.1 kA/cm$^2$, PCs = Y211
  (b) 36.6 kA/cm$^2$, PCs = n-recoil
  (c) 41.2 kA/cm$^2$, PCs = U/n

• Falling line on right is caused by decreasing values of $B_A(r)$. 
• Using the fitted values of $B_{\text{Thresh}}$ and the FC measurements of $B_{T,\text{max}}$ as a measure of $J_C$, we obtain values of $B_{\text{Thresh}} \times B_{T,\text{max},\text{FC}} \propto B_{\text{Thresh}} \times J_C \propto F_{L,\text{thresh}}$ (the Lorentz force when the leap occurs).
Our Opinions About the Physics of GFL

• We consider the most revealing behavior of the experimental results to be the suppression of $B_{HTS}$ prior to the leap.

• But what is it that limits the increase of penetrated field?

• We postulate that rapid flux leakage causes the limitation.

• We postulate that when $\vec{E} \rightarrow 0$ (i.e., at the peak of the $B_A$ pulse) the Lorentz force frees fluxoids from their shielding location.

• (Clearly, however, $F_L$ may be a cause or a consequence.)

• The postulated fluxoid movement is similar to creep, but is much faster. We describe it as a “fluxoid cascade.”

• The flux loss limits pre-leap $B_T$ to anomalously low values.

• If $F_L$ is indeed causal, we cannot say whether $F_L$ or its derivative is the cause because sample geometry is constant.
• In particular, note that current reverses at $B_{\text{Thresh}}$, and therefore $F_L$ reverses by 180°.

• Thus, at $B_{\text{Thresh}}$, $\Delta F = 2 F_L$. This discontinuity in $F_L$ occurs at the peak value of $B_{\text{HTS}} (= B_{\text{Thresh}})$.

• We favor the large stress due to $\Delta F_L$ as the cause of the fluxoid cascade.

• While the activating field is still on, the fluxoids lost in the cascade are (partially) replaced by fluxoids introduced by $B_A$.

• If only free fluxoids were involved we would expect a rapid increase in $B_{T,max}$ when the pulse begins to decrease, and $\vec{E}$ reverses sign.

• However, from our postulates, we do not see a reason that the GFL is delayed until $B_A = B_{T,max}$.

• Hence, at this point, our explanation is incomplete.
Closing Comments

• CSM was said to postulate:
  – Electric field causes maximum J to flow.
  – Ampere’s Law is valid.

• We believe that a third postulate was implied: fluxoids remain in place when $\vec{E} = 0$.

• We are not alone in noting that CSM requires fluxoid stability.

• In 1962, when C.P. Bean was developing CSM, P.W. Anderson was investigating “creep” [the decrease of $B_T$ with time].

• The Anderson model of creep postulates that thermally activated fluxoids escape from their pinning potential. The fluxoids then move off “guided by $F_L$."

• Anderson, in his seminal paper on flux creep noted, “We have obviously predicted that there is no precise critical state.”
Thus, while CSM is a remarkably useful theoretical aid, we must view it as a very convenient fiction. It has been a useful approximation, because the creep correction is so small.

Based upon our experiments to date, we postulate that $B_T(r)$ is suppressed by a fluxoid cascade caused by increasing $F_L$.

GFL occurs uniquely at $B_A \approx B_{T,max}$, and $\vec{E} \approx 0$. When it does occur, the free fluxoids in the cascade permit it to happen quickly.

However, a field leap would then be expected whenever the induced $\vec{E}$ field switches direction, independent of $B_A$.

Instead, the leap only occurs when $B_A$ is large enough to fully activate the TFM.

Therefore our present model is, at best, incomplete.

Our experiments continue in the hope of resolving this and other very significant anomalies.