Numerical Simulation and Experimental Investigation of a Novel Scotch Yoke for a Gifford-McMahon Cryocooler

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Contents

• Introduction
• Simulation and Analysis
  ➢ Conventional and Novel Scotch Yoke
  ➢ Simulation
• Experimental Results and Discussions
  ➢ Scotch Yoke Configuration
  ➢ P-V Work and Cooling Capacity
• Conclusions
• Introduction
Introduction

- Since 1990, the efficiency of a 4K GM cryocooler has been continuously improved by optimizing the operation parameter and valve timing, etc.

- In 2012, a high-efficiency cold head was developed at SHI and the input power was reduced by about 30% compared to a conventional 1W 4K GM cryocooler. (ICEC 24, ICC 17)

- In order to improve the efficiency of the cold head, several novel and effective concepts were proposed and investigated.

- One of the novel concepts was to modify the Scotch yoke to carry out an asymmetric GM cycle.
• Simulation and Analysis
  ➢ Conventional and Novel Scotch Yoke
  ➢ Simulation
Two-stage GM Cryocooler

Compression space

1st seal

Cylinder

2nd regenerator (Pb)

2nd regenerator (HoCu₂)

Motor

Return

Supply

1st displacer

1st regenerator

1st expansion space

2nd displacer

2nd expansion space
- Scotch yoke is used to transfer valve motor rotation to reciprocating displacer motion
Scotch Yoke

- In the novel Scotch yoke, there is a concave part at the upper center of the slide groove and a convex part at the lower center of the slide groove.
Physical Model

- Calculate both the 1\textsuperscript{st} and 2\textsuperscript{nd} stage, simultaneously
- Calculate losses in the clearance and spiral groove
Basic Equations

• Continuity Equation
\[ \frac{dm_i}{dt} = m_i - m_{i+1} \]

• Energy Conservation Equation of Gas Flow
\[ \frac{\partial (m_i h_i)}{\partial t} + (m h)_{f_{i+1}} - (m h)_{f_i} + \alpha_i A_i (T_i - T_{wi}) - V_i \frac{dP_i}{dt} = 0 \]

• Momentum Conservation Equation
\[ \frac{\partial P_i}{\partial x} = -f_r \frac{1}{d_h} \frac{\rho u_i^2}{2} u_i \]

• Energy Conservation Equation of Regenerator Material
\[ V_{wi} c_{wi} \frac{\partial T_{wi}}{\partial t} = -\lambda_{wi} A_{wi} \frac{\partial^2 T_{wi}}{\partial x^2} dx + \alpha_i A_i (T_i - T_{wi}) \]

Xu M Y and Morie T 2012 Cryocoolers 17 pp 253-9
Basic Equations

• 1\(^{st}\) Displacer Movement with a Conventional Scotch Yoke

\[
x_{e1} = S_{ce1} + 0.5 \times S_{e1} \times \left\{1 - \cos \left[ (\alpha - \varphi_{e1}) \times \frac{\pi}{180} \right]\right\}
\]

• 1\(^{st}\) Displacer Movement with a Novel Scotch Yoke in principle

\[
x_{e1} = \begin{cases} 
S_{ce1} + 0.5 \times S_{e1} \times \left\{2.0 - \cos \left[ (\alpha - \varphi_{e1}) \times \frac{\pi}{180} \right] - \cos \left( \frac{\pi}{180} \theta \right) \right\} & 0 < \alpha \leq \beta \\
S_{ce1} + S_{e1} \times \left\{1.0 - \cos \left[ (\alpha - \varphi_{e1}) \times \frac{\pi}{180} \right] \right\} & \beta < \alpha \leq \gamma \\
S_{ce1} + 0.5 \times S_{e1} \times \left\{2.0 - \cos \left[ (\alpha - \varphi_{e1}) \times \frac{\pi}{180} \right] - \cos \left( \frac{\pi}{180} \theta \right) \right\} & 0 < \alpha \leq 180 + \beta \\
S_{ce1} + S_{e1} & 180 + \beta < \alpha \leq 180 + \gamma \\
S_{ce1} + 0.5 \times S_{e1} \times \left\{2.0 - \cos \left[ (\alpha - \varphi_{e1}) \times \frac{\pi}{180} \right] + \cos \left( \frac{\pi}{180} \theta \right) \right\} & 180 + \beta < \alpha \leq 360
\end{cases}
\]
Movement of Roller Bearing & Displacement of Scotch Yoke

[Diagram showing the movement of roller bearings and displacement of a Scotch yoke over different angles.]

- **Displacement (mm)**
  - Asymmetric
  - Conventional

- **Angle (deg)**
  - 0
  - 90
  - 180
  - 270
  - 360

- **Positions: 1, 2, 3, 4, 5, 6, 7, 8**

- **Graph**
  - Blue line: Asymmetric
  - Red line: Conventional

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Simulation Results

<table>
<thead>
<tr>
<th></th>
<th>Cooling capacity</th>
<th>P-V work</th>
<th>Mass flow rate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st stage at 40 K (W)</td>
<td>2nd stage at 4.2 K (W)</td>
<td>1st stage (W)</td>
</tr>
<tr>
<td>Conventional</td>
<td>55.3</td>
<td>1.63</td>
<td>91.9</td>
</tr>
<tr>
<td>Asymmetric</td>
<td><strong>58.9</strong></td>
<td>1.73</td>
<td><strong>96.8</strong></td>
</tr>
</tbody>
</table>
• Experimental Results and Discussions
  ➢ Scotch Yoke Configuration
  ➢ P-V Work and Cooling Capacity
Experimental Results

- The cooling capacity at the first stage at 40 K was improved by about 10%, from 51.5 W to 57.3 W.

<table>
<thead>
<tr>
<th></th>
<th>Temperature without heat load</th>
<th>Temperature with 44W/1.0W heat load</th>
<th>Estimated cooling capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>1st stage (K)</td>
<td>2nd stage (K)</td>
<td>1st stage (K)</td>
</tr>
<tr>
<td>Conventional</td>
<td>22.1</td>
<td>2.99</td>
<td>37.4</td>
</tr>
<tr>
<td>Asymmetric</td>
<td>22.3</td>
<td>3.00</td>
<td>35.9</td>
</tr>
</tbody>
</table>
• Conclusions
A novel GM cycle, called an asymmetric GM cycle, was proposed.

In the GM cycle, the expansion process is longer, while the discharging process is shorter than in a conventional GM cycle.

In order to carry out the GM cycle, a novel Scotch yoke was invented. In the Scotch yoke, there is a concave part at the upper center of the slide groove and a convex part at the lower center of the slide groove.

The effect of the Scotch yoke has been confirmed by numerical simulation and experimental investigation. The cooling capacity at the first stage at 40 K was improved by about 10 %, from 51.5 W to 57.3 W.
Thank you!