Implementation of the thermodynamic and phase transition equations of superfluid helium in a CFD software

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Summary

1. **Context**

2. **Objective**

3. **Superfluid Helium (one phase)**

4. **Transition He II – He I (two phase)**

5. **Conclusion**

1. Context

- The design of the next generation of superconducting magnets, cooled by superfluid helium, depends on our ability to simulate heat and mass transfer in these magnets

- Superfluid helium offer:
  - High thermal conductivity
  - Using superconducting magnet at a lower temperature (higher magnetic field)
  - Confined magnets cooling (accelerator magnet…)

2. Objective

To develop a numerical tool for the design of future cryogenic system operating with superfluid helium

- **1st step**: To implement the equations of superfluid helium in Navier-Stokes solver Fluent®
- **2nd step**: To model the superfluid helium phase transition appearing during the quench of a superconducting magnet

3. He II : Theory

Landau [1] and Tisza [2] two-fluid model :

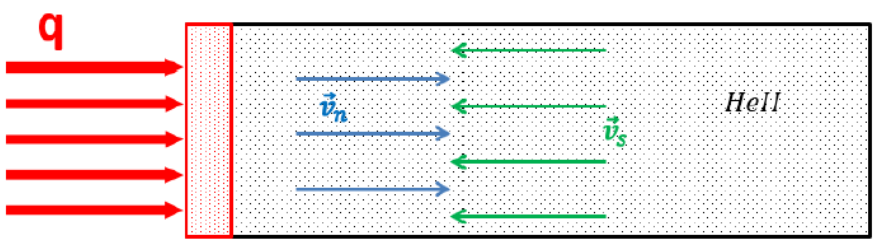
- He II is divided in 2 components:
  - u_s superfluid component
  - u_n normal component
- Normal component transports thermal excitation

Mass equation

$$\rho = \rho_s + \rho_n$$

Momentum equation

$$\rho \boldsymbol{u} = \rho_s \boldsymbol{u_s} + \rho_n \boldsymbol{u_n}$$



Numerical solver used is **ANSYS Fluent® 15.0**

Standard Navier-Stockes equations **+** Terms from the two-fluid equations added in C progamming language

[1] L. Landau,(1949). On the theory of superfluidity, *Physical review*, 75 5 884-885.
[2] L. Tisza (1947). The Theory of Liquid Helium. *Phys. Rev.* **72** (9): 838–854.

3. He II : Numerical model

- Mass equation

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho\, \boldsymbol{u}) = 0$$

- Momentum equation

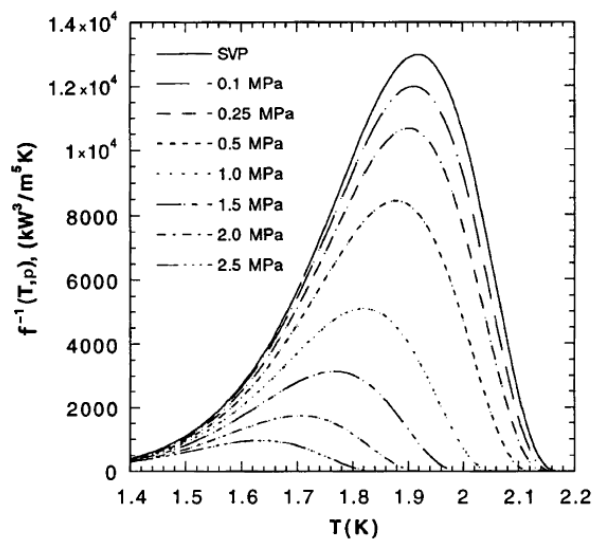
$$\rho \frac{\partial \boldsymbol{u}}{\partial t}$$

$$= -\rho(\boldsymbol{u}\nabla)\boldsymbol{u} - \boldsymbol{\nabla} p - \boldsymbol{\nabla}\left[\frac{\rho_n \rho_s}{\rho}\left(\frac{s}{A\rho_n|\boldsymbol{\nabla} T|^2}\right)^{2/3} \boldsymbol{\nabla} T \boldsymbol{\nabla} T\right]$$

$$+ \eta\left(\boldsymbol{\nabla}^2 \boldsymbol{u} + \frac{1}{3}\boldsymbol{\nabla}(\boldsymbol{\nabla}\cdot\boldsymbol{u}) - \left(\frac{\rho_s^3 s}{A\rho^3 \rho_n |\boldsymbol{\nabla} T|^2}\right)^{\frac{1}{3}}\left[\boldsymbol{\nabla}^2(\boldsymbol{\nabla} T) + \frac{1}{3}\nabla(\boldsymbol{\nabla}\cdot\boldsymbol{\nabla})T\right]\right) + \rho \boldsymbol{g}$$

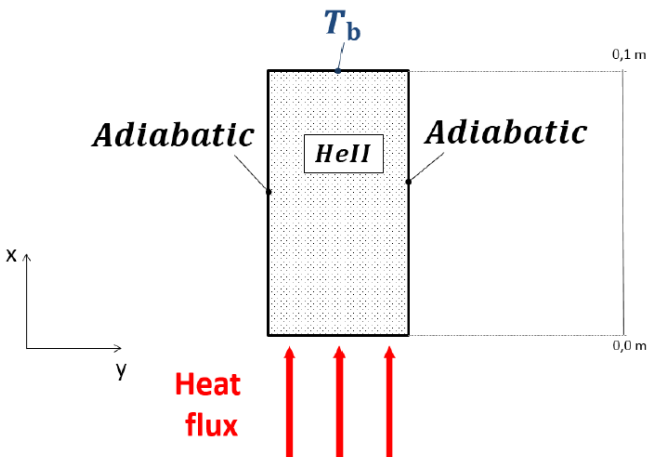
- Heat equation

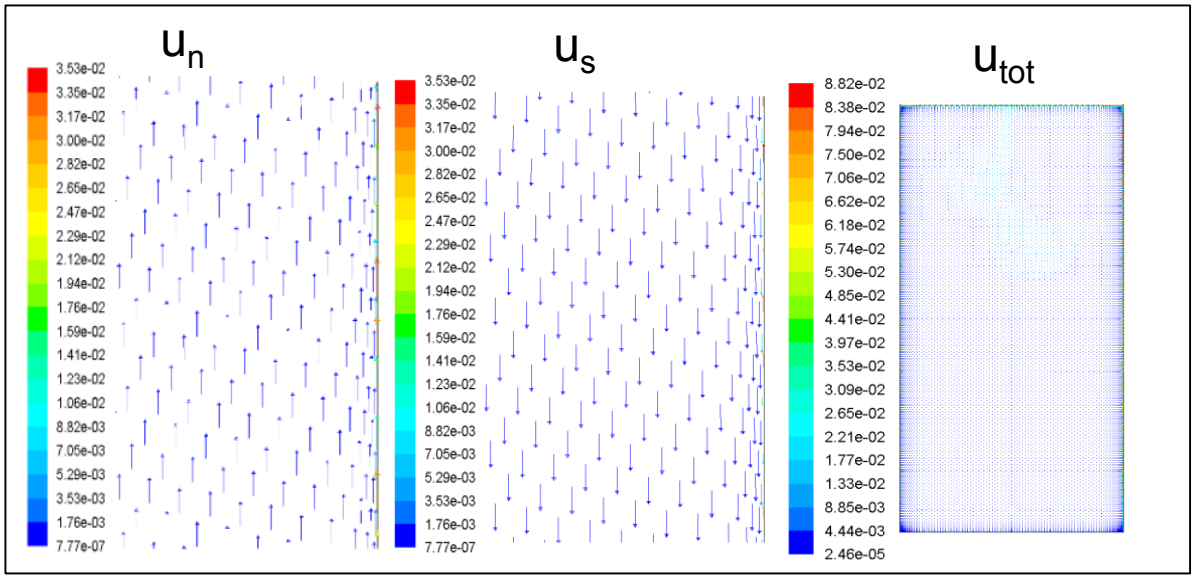
$$\rho \frac{\partial}{\partial t}(c_p T) = -\rho c_p (u \cdot \boldsymbol{\nabla} T) - \boldsymbol{\nabla} \cdot \left(\left(\frac{f(T)}{|\nabla T|^2}\right)^{\frac{1}{3}} \boldsymbol{\nabla} T\right)$$

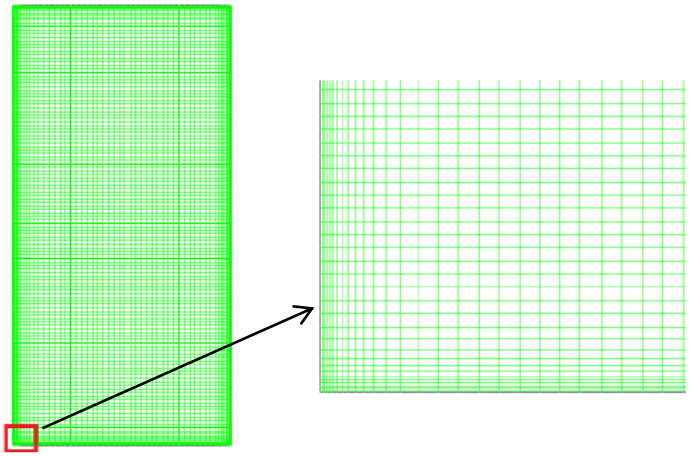


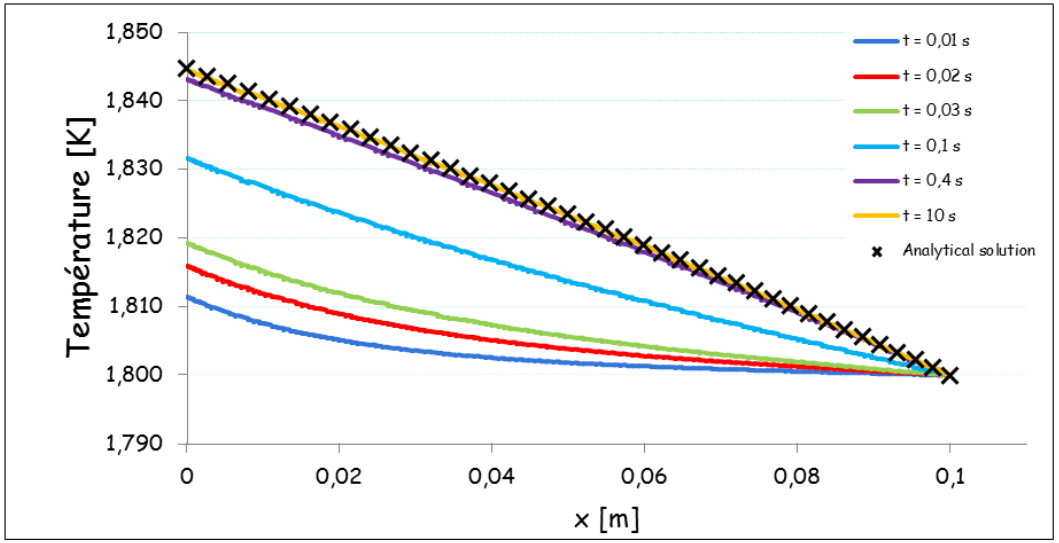
3. He II : Validate the analytic solution

- T_b = 1.80 K
- **Q** = 18000 W.m^{-2}
- **Δt** = 10^{-3} s
- **Mesh** Min 10^{-4} m



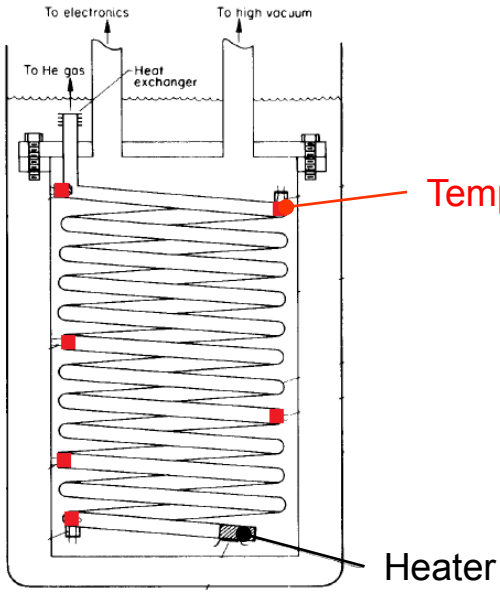




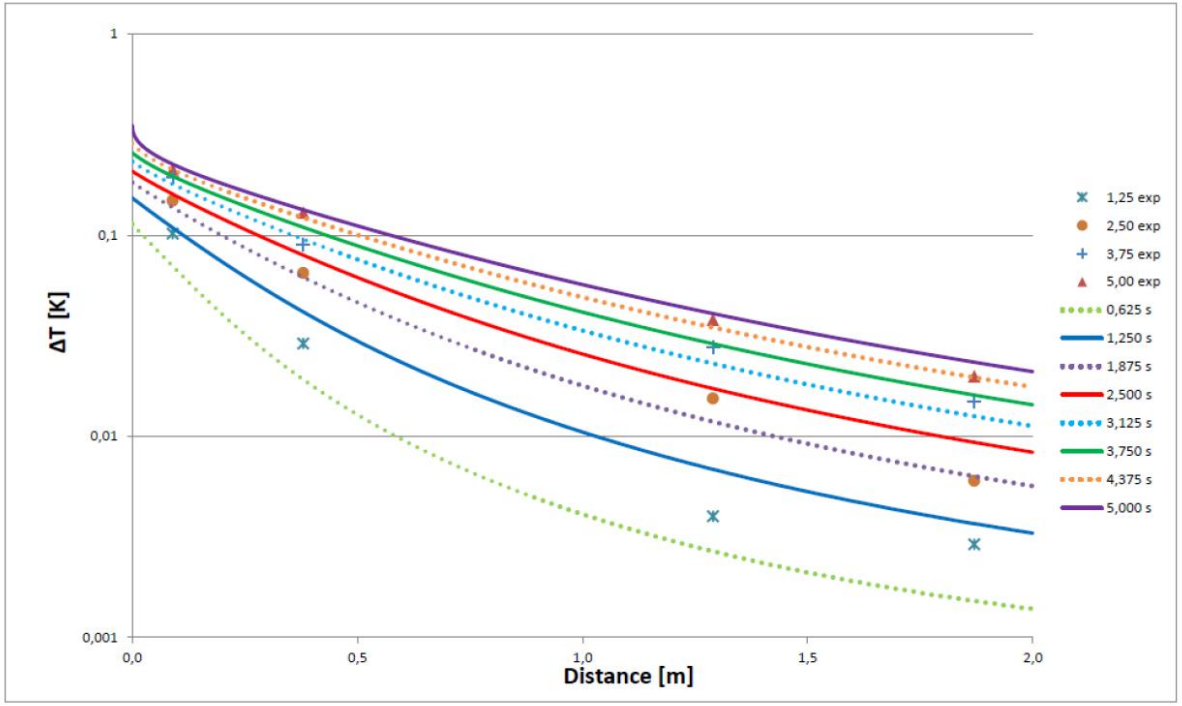


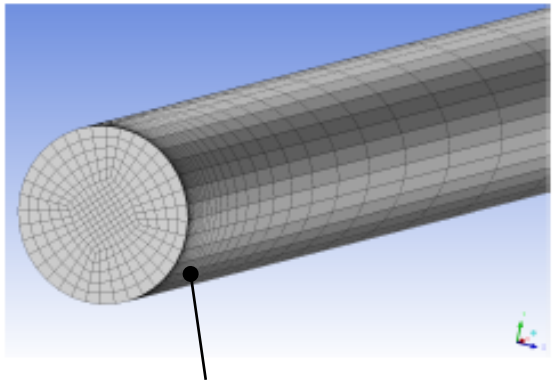
Error < 0,1% with the analytical solution

3. He II : Transient simulation



- **Tube** $Ø_{int}$=9 mm Length = 10 m
- $\mathbf{T_b}$ = 1.802 K
- **Q** = 2.22 W.cm^{-2}
- **Δt** = 10^{-4} s
- **Mesh** Min 10^{-4} m





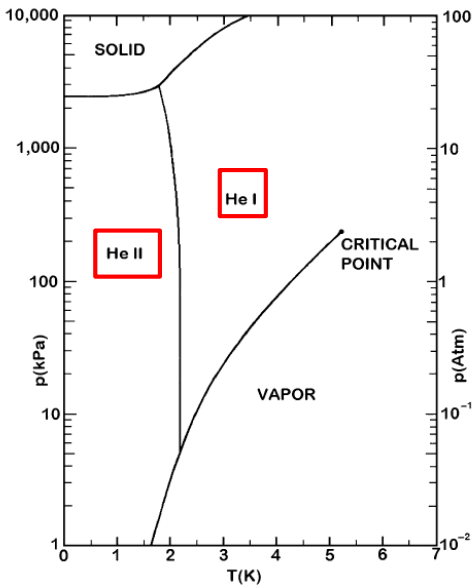
Adiabatic wall

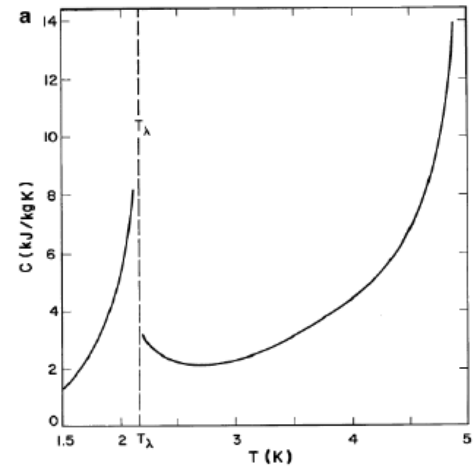
[3] S.W. Van Sciver. Transient heat transport in He II. Cryogenics, 1979.

4. Superfluid transition: Theory

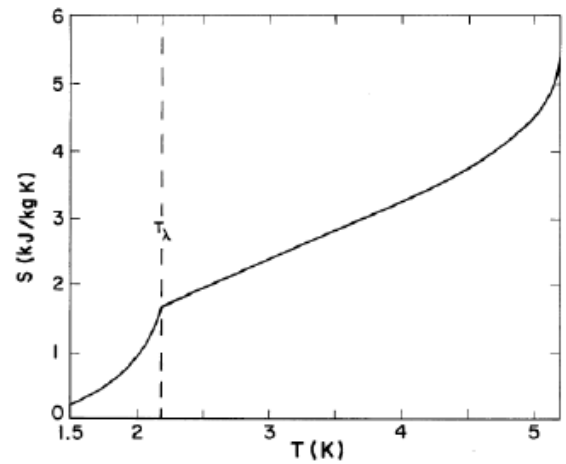
Second order transition (or lambda line)

- $T_\lambda = 2.168\ K$
- No latent heat exchange
- Infinite value of C_p at T_λ (Enthalpy formulation)





(b) Heat capacity



(a) Entropy

4. Superfluid transition: Numerical model (1/2)

- Method: **VOF (Volume of Fluid)**

Volumique fraction α_i

- $\sum \alpha_i = 1$
- Average physical properties

Temperature $T_m = \alpha\, T_1 + (1-\alpha)\, T_2$

if $\alpha_i = 1$ only i phase is present

if $\alpha_i = 0$ i phase isn't present

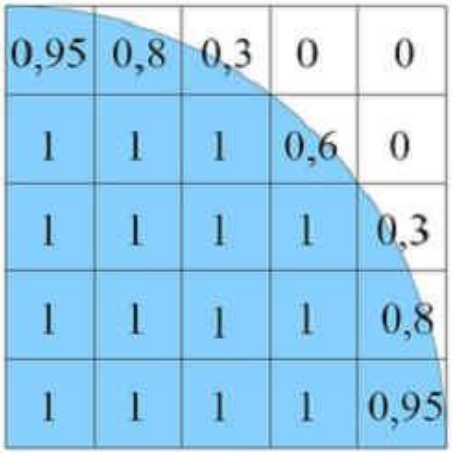
$sif\ 0 \leq \alpha_i \leq 1$ Multiple phases are present

- Avantages :

  – Identification/creation of the interface

  – Mass exchange between phases

  – Heat conservation

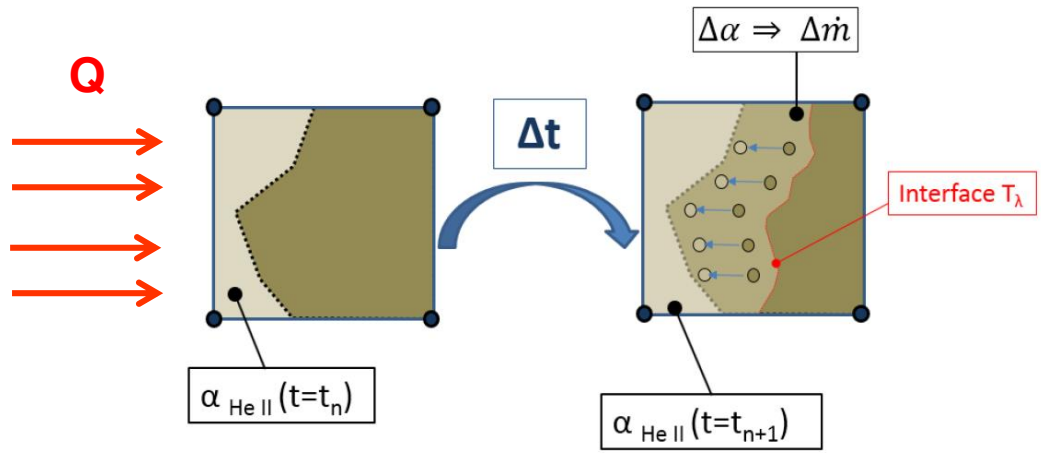


4. Superfluid transition: Numerical model (2/2)

- Mass transfer created for second order He II / He I transition :

Knowing the average temperature and the temperature gradient in the cell

Calculating the volume fraction of He I appeared at each time in the cell



- Evaporation/condensation model implemented in Fluent for liquid/gas transition :

If $T_l > T_{sat}$ $$\dot{m}_{l\to v} = coeff * \alpha_l \rho_l \frac{(T_l - T_{sat})}{T_{sat}},$$

If $T_v < T_{sat}$ $$\dot{m}_{v\to l} = coeff * \alpha_v \rho_v \frac{(T_v - T_{sat})}{T_{sat}},$$

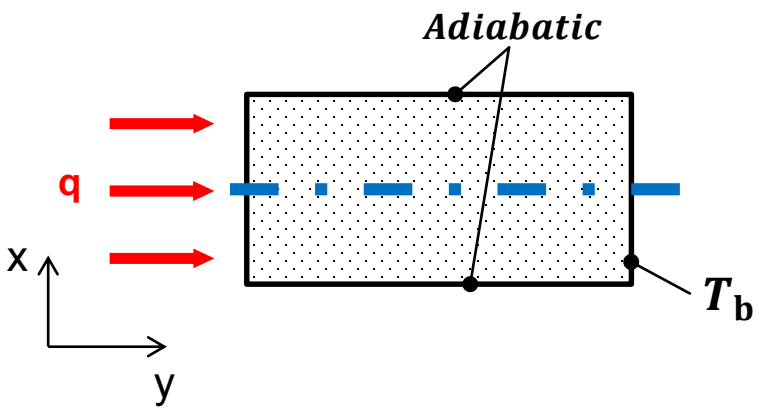
$$coeff = \frac{Cp_l * T_{sat}}{L_v}.$$

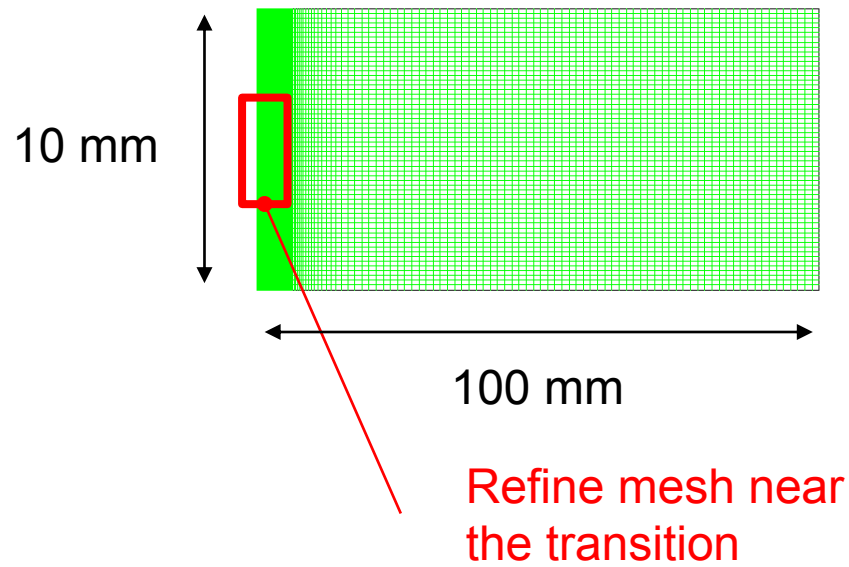
[4] S. Pascali. Numerical study of heat and mass transfer in superfluid helium. Intership report, CEA-Saclay, 2014.

4. Superfluid transition: 1st simulation

1st 2D simulation of the He II /He I transition without helium gas apparition:

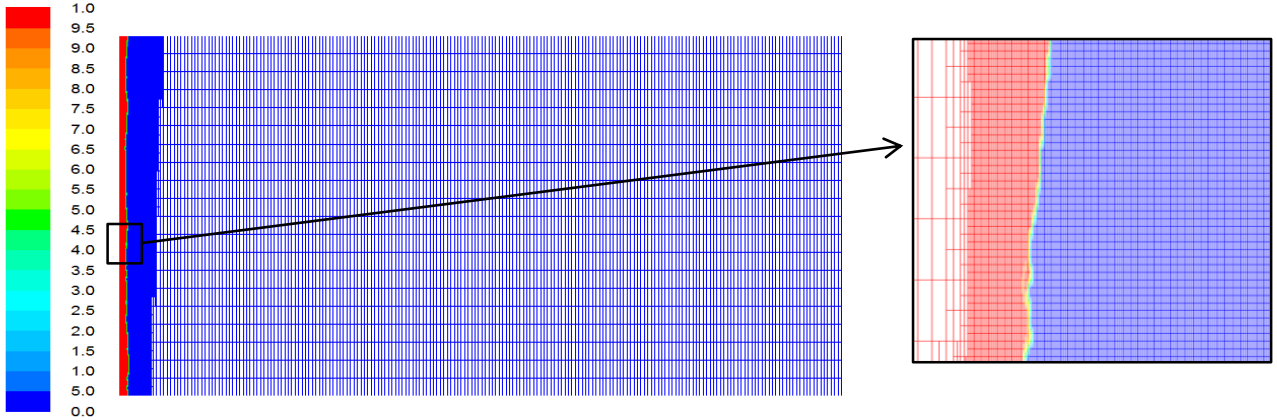
- **T_b** = 2.155 K (proche de T_λ)
- **Q** = 5000 W.m^{-2}
- **Mesh** min 10^{-6} m
- $\Delta t = 10^{-6}$ s
- No gravity effect

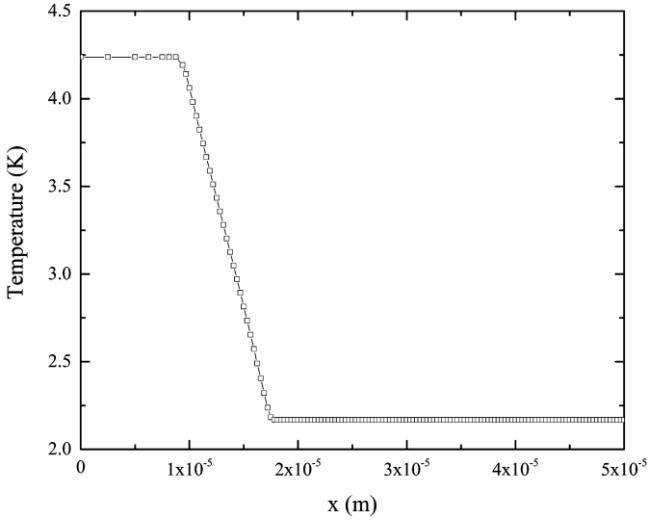


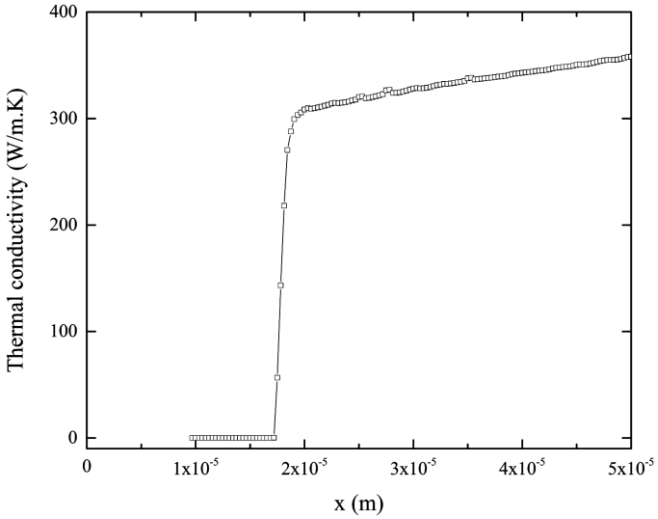


4. Superfluid transition: Results (1/2)

- He I apparition and He II disparition
- Significant variation in the thermal conductivity
- Very thin layer of liquid He I ($3x10^{-6}$ m)



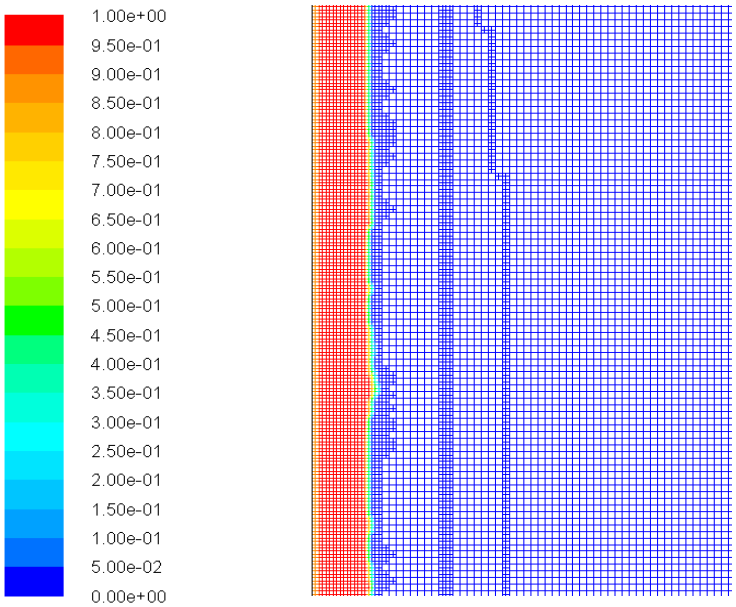


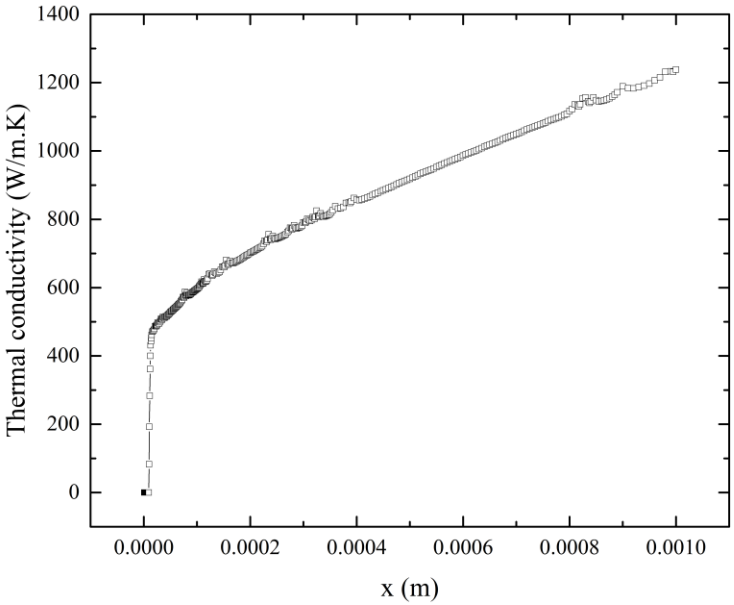


4. Superfluid transition: Evaporation / condensation

Evaporation/condensation model added to the simulation:

- T_b=1.8 K and q=100 kW/m² (increase the helium gas apparition)
- Thermal conductivity still stable close to the He II / He I transition
- Computation is too slow (2 weeks of calculation for a 10^{-6} m thickness of helium gas near the heater)





5. Conclusion

- The Navier Stockes transient equations were implanted in the Fluent code
- The second order transition He II / He I with the VOF method was implemented in the Fluent code
- Simulation results:
  - One phase : Good agreement with the analytical results and transient experimental data
  - Phase transition: 1st results consistent with the theory but the calculation is very sensitive to mesh dimensions and time step
- Future work :
  - Decrease phase transition calculations time
  - Realize an experiment to validate the results obtained for the transition