Implementation of the thermodynamic and phase transition equations of superfluid helium in a CFD software

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Summary

1. Context

2. Objective

3. Superfluid Helium (one phase)

4. Transition He II – He I (two phase)

5. Conclusion
1. Context

• The design of the next generation of superconducting magnets, cooled by superfluid helium, depends on our ability to simulate heat and mass transfer in these magnets.

• Superfluid helium offer:
  – High thermal conductivity
  – Using superconducting magnet at a lower temperature (higher magnetic field)
  – Confined magnets cooling (accelerator magnet….)
2. Objective

To develop a numerical tool for the design of future cryogenic system operating with superfluid helium

- **1st step**: To implement the equations of superfluid helium in Navier-Stokes solver Fluent®

- **2nd step**: To model the superfluid helium phase transition appearing during the quench of a superconducting magnet
3. He II: Theory

Landau [1] and Tisza [2] two-fluid model:

- He II is divided in 2 components:
  - $u_s$ superfluid component
  - $u_n$ normal component
- Normal component transports thermal excitation

\[
\begin{align*}
\text{Mass equation} & : & \rho = \rho_s + \rho_n \\
\text{Momentum equation} & : & \rho \mathbf{u} = \rho_s \mathbf{u}_s + \rho_n \mathbf{u}_n
\end{align*}
\]

Numerical solver used is ANSYS Fluent® 15.0

Standard Navier-Stockes equations + Terms from the two-fluid equations added in C programming language

3. He II : Numerical model

- Mass equation
  \[
  \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{u}) = 0
  \]

- Momentum equation
  \[
  \rho \frac{\partial \mathbf{u}}{\partial t} = -\rho (\mathbf{u} \nabla) \mathbf{u} - \nabla p - \nabla \left[ \frac{\rho_n \rho_s}{\rho} \left( \frac{s}{A \rho_n |\nabla T|^2} \right)^{2/3} \nabla T \nabla T \right] + \eta \left( \nabla^2 \mathbf{u} + \frac{1}{3} \nabla (\nabla \cdot \mathbf{u}) \right) - \left( \frac{\rho_s^3 s}{A \rho^3 \rho_n |\nabla T|^2} \right)^{1/3} \left[ \nabla^2 (\nabla T) + \frac{1}{3} \nabla (\nabla \cdot \nabla T) \right] + \rho g
  \]

- Heat equation
  \[
  \rho \frac{\partial}{\partial t} (c_p T) = -\rho c_p (\mathbf{u} \cdot \nabla T) - \nabla \cdot \left( \left( \frac{f(T)}{|\nabla T|^2} \right)^{1/3} \nabla T \right)
  \]
3. He II : Validate the analytic solution

- $T_b = 1.80 \, K$
- $Q = 18000 \, W.m^{-2}$
- $\Delta t = 10^{-3} \, s$
- Mesh Min $10^{-4} \, m$

Error < 0,1% with the analytical solution
3. He II : Transient simulation

- **Tube** $\varnothing_{\text{int}} = 9 \text{ mm}$ Length = 10 m
- $T_b = 1.802 \text{ K}$
- $Q = 2.22 \text{ W.cm}^{-2}$
- $\Delta t = 10^{-4} \text{ s}$
- **Mesh** Min $10^{-4} \text{ m}$

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**Adiabatic wall**

**Heater**

Temperature sensors

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4. Superfluid transition: Theory

Second order transition (or lambda line)

- $T_\lambda = 2.168 \, K$
- No latent heat exchange
- Infinite value of $C_p$ at $T_\lambda$ (Enthalpy formulation)
4. Superfluid transition: Numerical model (1/2)

• Method: VOF (Volume of Fluid)

Volumique fraction $\alpha_i$

- $\sum \alpha_i = 1$
- Average physical properties
  
  Temperature $T_m = \alpha T_1 + (1 - \alpha) T_2$

if $\alpha_i = 1$ only i phase is present

if $\alpha_i = 0$ i phase isn't present

$s i f \ 0 \leq \alpha_i \leq 1$ Multiple phases are present

• Avantages :

  – Identification/creation of the interface
  – Mass exchange between phases
  – Heat conservation
4. Superfluid transition: Numerical model (2/2)

- Mass transfer created for second order He II / He I transition:

Knowing the average temperature and the temperature gradient in the cell

Calculating the volume fraction of He I appeared at each time in the cell

- Evaporation/condensation model implemented in Fluent for liquid/gas transition:

\[
\text{If } T_i > T_{\text{sat}} \quad \dot{m}_{l\rightarrow v} = \text{coeff} \times \alpha_i \rho_i \frac{(T_i - T_{\text{sat}})}{T_{\text{sat}}} \times \frac{L_v}{C_p_i},
\]

\[
\text{If } T_v < T_{\text{sat}} \quad \dot{m}_{v\rightarrow l} = \text{coeff} \times \alpha_v \rho_v \frac{(T_v - T_{\text{sat}})}{T_{\text{sat}}} \times \frac{L_v}{C_p_i}.
\]

\[
\text{coeff} = \frac{C_p_i \times T_{\text{sat}}}{L_v}.
\]

4. Superfluid transition: 1\textsuperscript{st} simulation

1\textsuperscript{st} 2D simulation of the He II /He I transition without helium gas apparition:

- $T_b = 2.155 \text{ K (proche de } T_{\lambda})$
- $Q = 5000 \text{ W.m}^{-2}$
- Mesh min $10^{-6} \text{ m}$
- $\Delta t = 10^{-6} \text{ s}$
- No gravity effect

Refine mesh near the transition
4. Superfluid transition: Results (1/2)

- He I apparition and He II disparition
- Significant variation in the thermal conductivity
- Very thin layer of liquid He I ($3 \times 10^{-6}$ m)
4. Superfluid transition: Evaporation / condensation

Evaporation/condensation model added to the simulation:

- $T_b=1.8$ K and $q=100$ kW/m² (increase the helium gas apparition)
- Thermal conductivity still stable close to the He II / He I transition
- Computation is too slow (2 weeks of calculation for a $10^{-6}$ m thickness of helium gas near the heater)
5. Conclusion

• The Navier Stockes transient equations were implanted in the Fluent code

• The second order transition He II / He I with the VOF method was implemented in the Fluent code

• Simulation results:
  – One phase: Good agreement with the analytical results and transient experimental data
  – Phase transition: 1st results consistent with the theory but the calculation is very sensitive to mesh dimensions and time step

• Future work:
  – Decrease phase transition calculations time
  – Realize an experiment to validate the results obtained for the transition