Simulation of Longitudinal Beam Dynamics Problems in Synchrotrons
Lecture 1

Numerical Methods of Longitudinal Beam Dynamics

Helga Timko

CERN, BE-RF

Inverted CERN School of Computing, 23-24 February 2015

Contents

Why model longitudinal beam dynamics?

INTRODUCTION

What is a beam?

- **Particle beam = collection of particles, usually confined in space**
  - In accelerator physics we use mainly charged beams, which can be manipulated via electromagnetic fields
  - Beam particles are selected to have the same charge state; particles with the same q/m move in the same way (Lorentz):

$$\frac{d}{dt}(\gamma\vec{v}) = \frac{q}{m}\left(\vec{E} + \vec{v} \times \vec{B}\right) \quad (1)$$

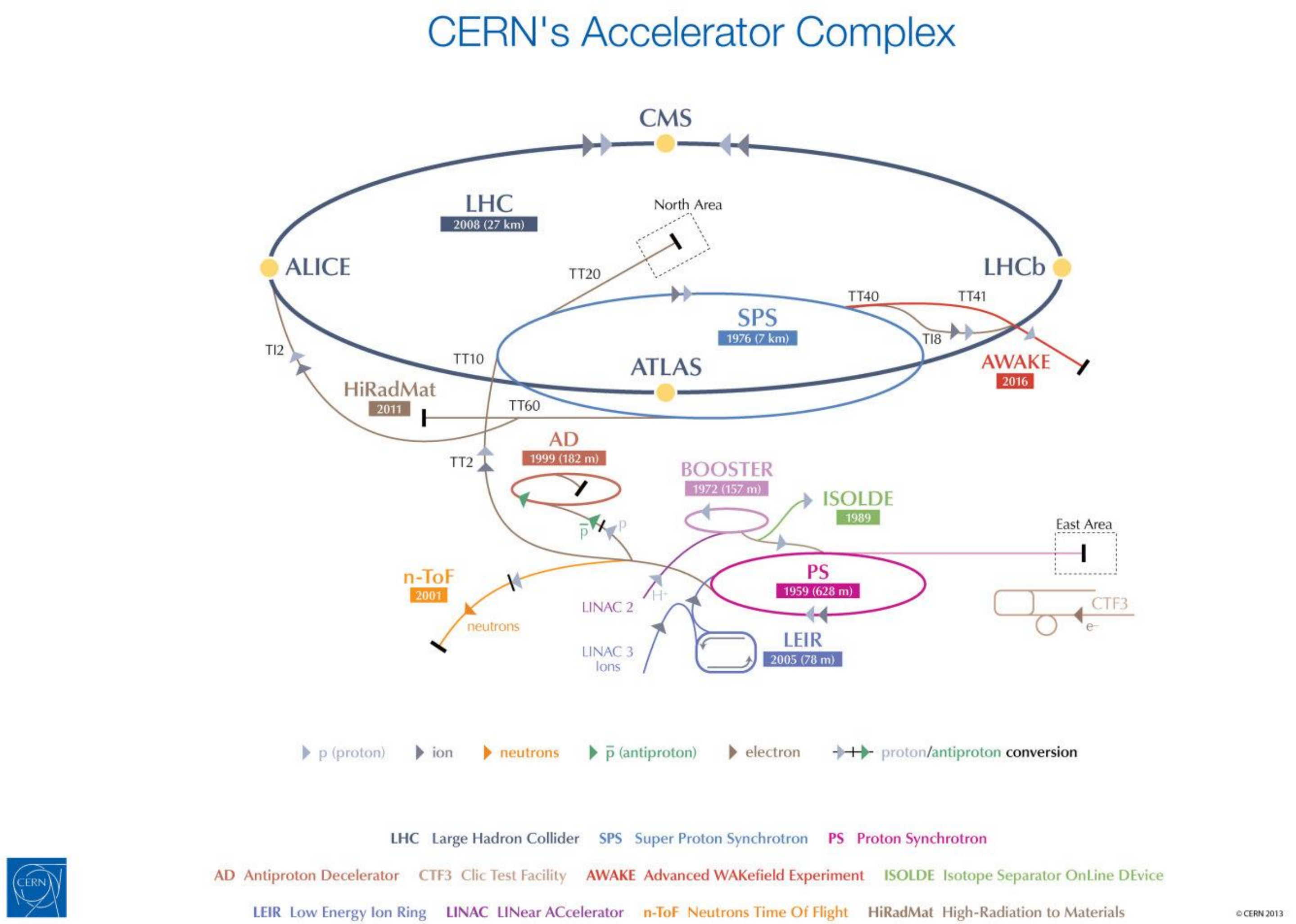
- **A beam is essentially a plasma**
  - In a plasma, spatial confinement is ensured by quasi-neutrality (many species, global charge is zero) and boundary conditions
  - In a beam, spatial confinement is ensured by the RF potential in the cavities (longitudinal) and magnetic focussing (transverse)

CERN School of Computing

Beams at CERN

- **Various beams are being used at CERN for different purposes**
  - LHC and injectors: p (H^+), $^{208}Pb^{54+}$, $^{40}Ar^{11+}$
  - ISOLDE: large range of radioactive isotopes
  - AD and ELENA decelerators: anti-protons, anti-hydrogen
  - …
- **LHC proton beam production**
  - Source → Linac2 (Linac4) → PSB → PS → SPS → LHC
  - Extraction kinetic energies or momenta: 50 MeV (160 MeV) → 1.4 GeV (2 GeV) → 26 GeV/c → 450 GeV/c → 7 TeV
- **LHC ion beam production**
  - Source → Linac3 → LEIR → PS → SPS → LHC

The CERN Accelerator Complex



CERN Beams Department

- **A large range of activities:**
  - Accelerator operation, production & delivery of particle beams
  - Controls infrastructure for all accelerators
  - Accelerator systems
  - Machine and detector alignment
  - Hadron sources
  - Beam instrumentation
  - Accelerator physics studies and teaching activities
    - CERN Accelerator School (CAS): http://cas.web.cern.ch/cas/

CERN School of Computing

Acceleration and synchrotrons

- **Lorentz equation $\Rightarrow$ electric field accelerates, magnetic field changes the direction of velocity**
- **Two ways to accelerate particles**
  - Linacs: no bending magnets needed, but single passage through each cavity
  - Circular accelerators: repetitive passage through the cavities, but strong magnets are required to close the trajectory
    - LHC: 8 cavities/beam $\rightarrow$ **accelerate**; 1232 dipole magnets $\rightarrow$ **bend** with up to 8.33 T; 392 quadrupole magnets $\rightarrow$ **focus** transversely
- **We'll focus on synchrotrons**
  - Magnetic field strength synchronised with the beam energy
  - Cavities' RF frequency and phase is synchronised as well

Acceleration

- **Using DC voltage**
  - Linear potential, uniform electric field

- **Using RF voltage**
  - Need to arrive in the right RF phase to gain energy

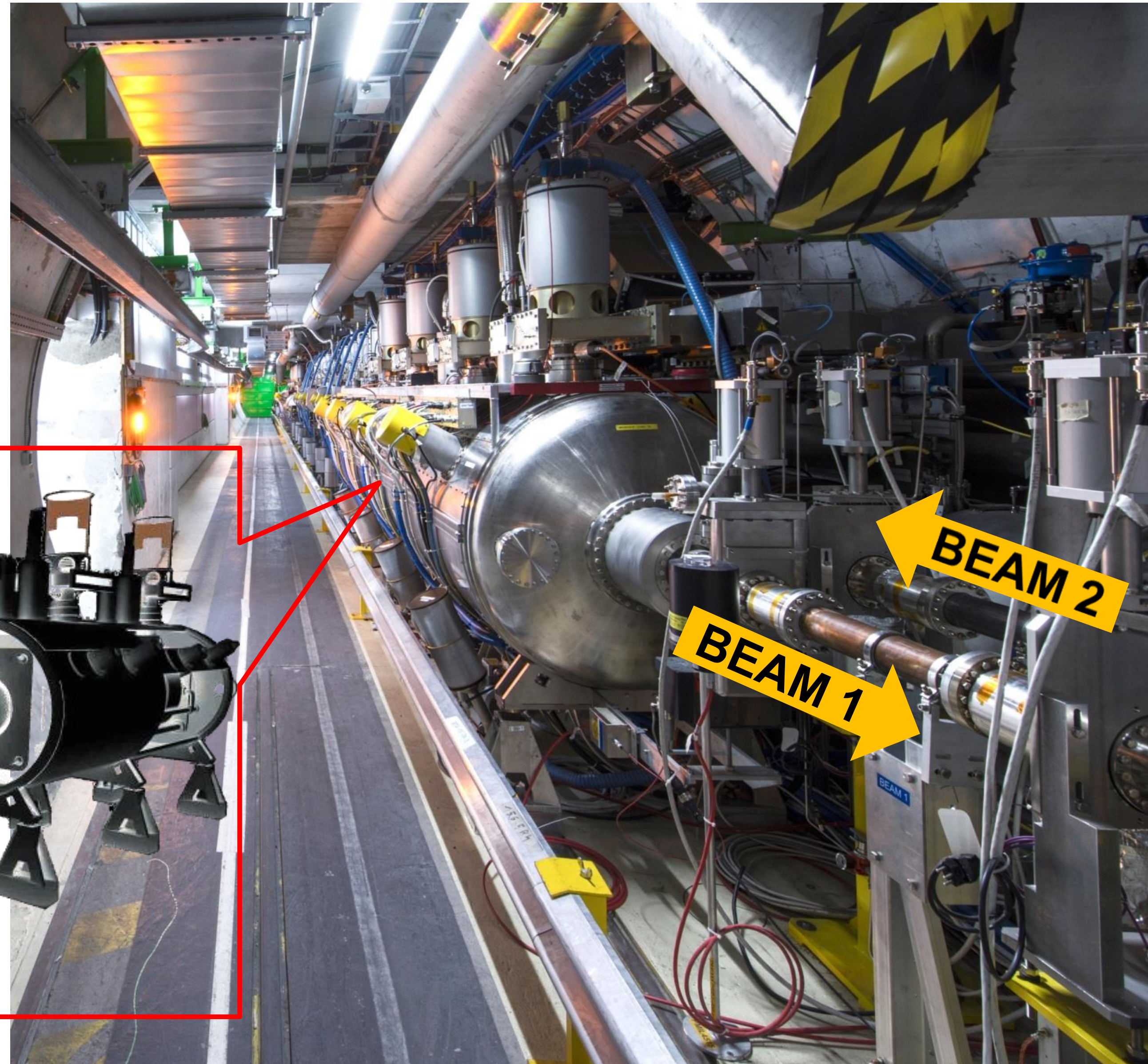
CERN School of Computing

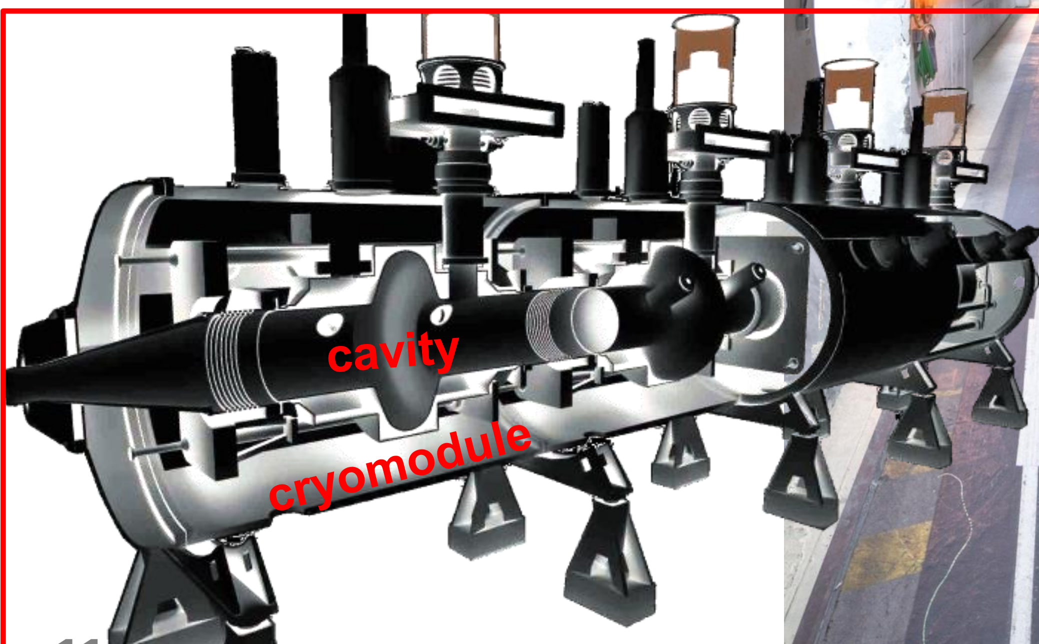
A simple linear accelerator

CERN School of Computing

A synchrotron: the LHC (1)

- **All accelerating cavities are placed in Point 4**
- **Straight section**

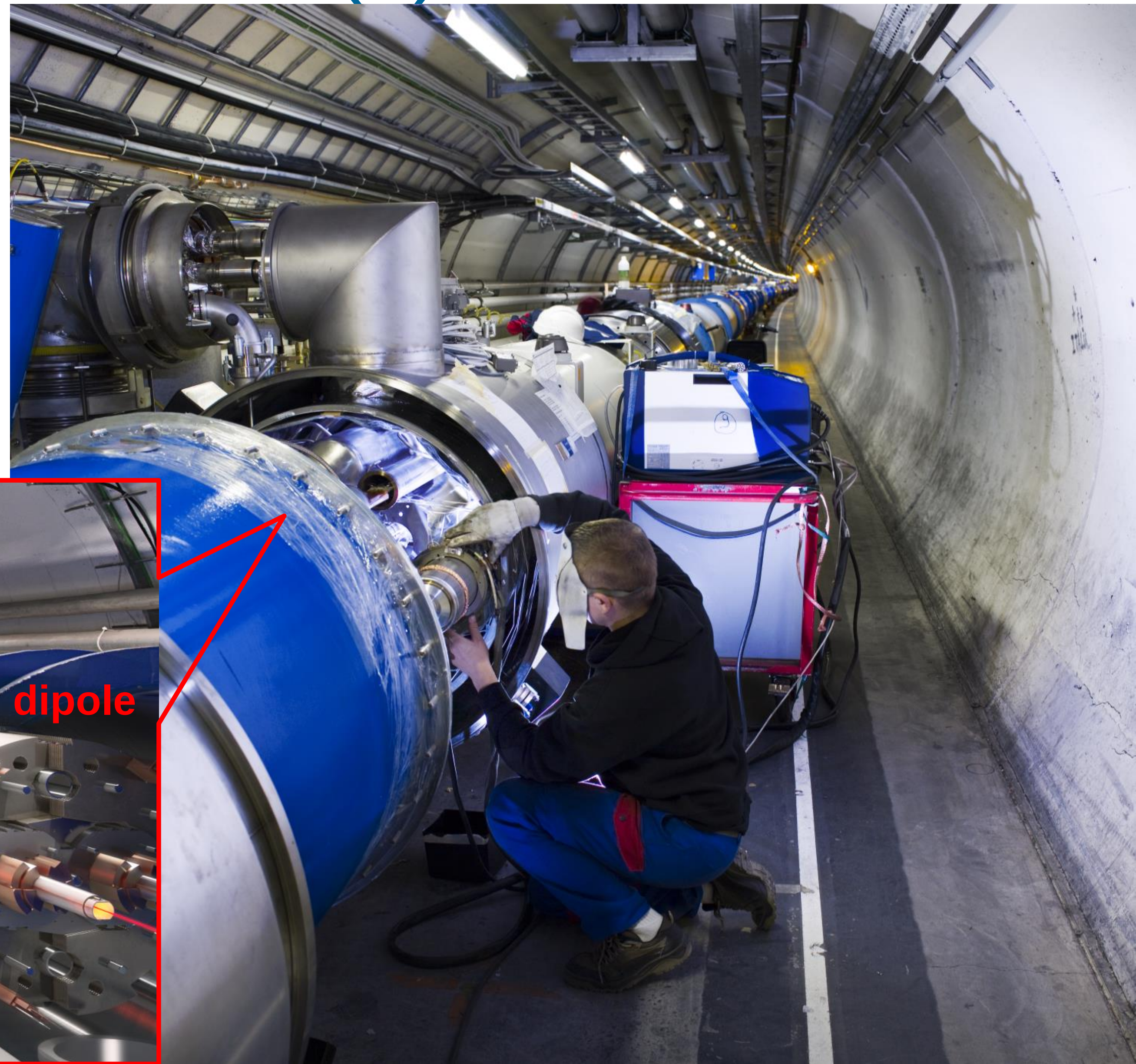




CERN School of Computing

A synchrotron: the LHC (2)

- **Dipole and quadru-pole magnets are placed all along**
- **So-called arcs**



Longitudinal beam dynamics

- **Longitudinal = along, transverse = across the beam pipe**

- **Why do we care about the beam evolution over time?**
  - To design our machines
  - To control/manipulate the beam (splitting, shaping…)
    - PSB: 1 bunch (800 ns) in each of 4 rings, LHC: 2808 bunches (1.2 ns)
  - To ensure a certain beam quality and safe operation
  - Beam instabilities can lead to
    - Uncontrolled emittance (beam size) blow-up → deterioration of beam quality, loss of luminosity
    - Beam losses → loss of luminosity and radiation/safety issue (full LHC beam stores 350 MJ, can drill a hole into the beam pipe!)

- **Complex machines, numerical models needed!**

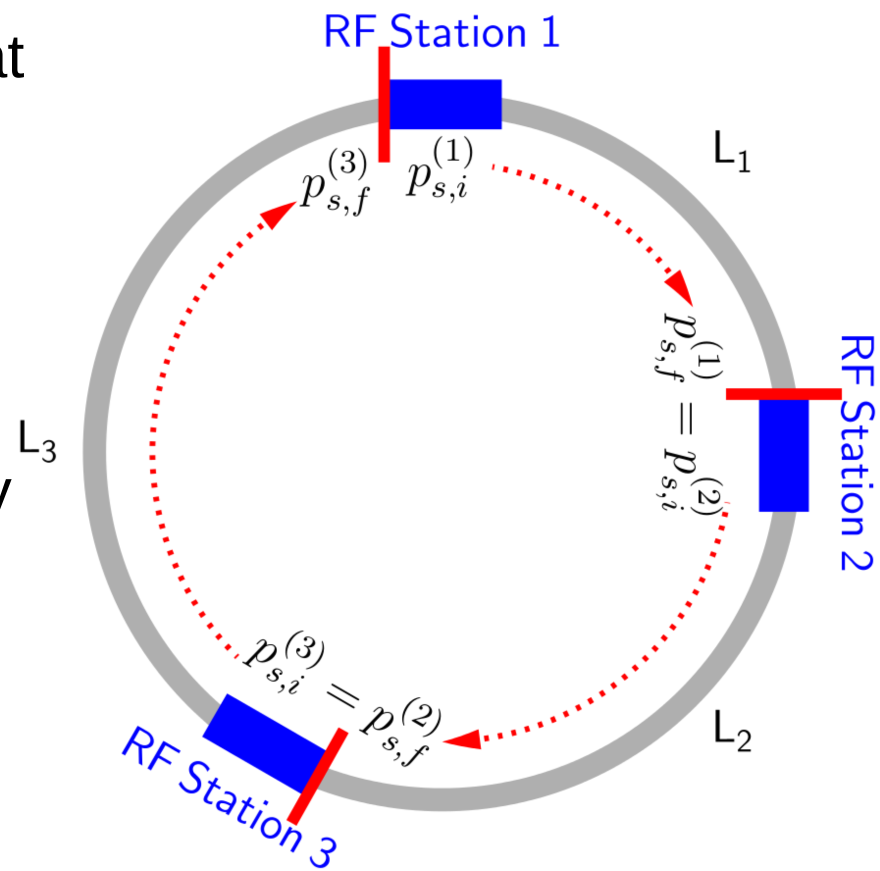
Motion of beam particles in the RF potential

EQUATIONS OF MOTION

CERN School of Computing

Synchrotron model

- **RF stations**
  - Contain cavities (may operate at different harmonics)
  - Keep the beam bundled
  - Described via energy kicks
- **Magnetic arcs**
  - Magnetic field to keep trajectory
  - RF synchronised to B-field
  - Arrival time to next station described via phase drifts
- **We'll focus on single-station machines (OK for protons/ions)**



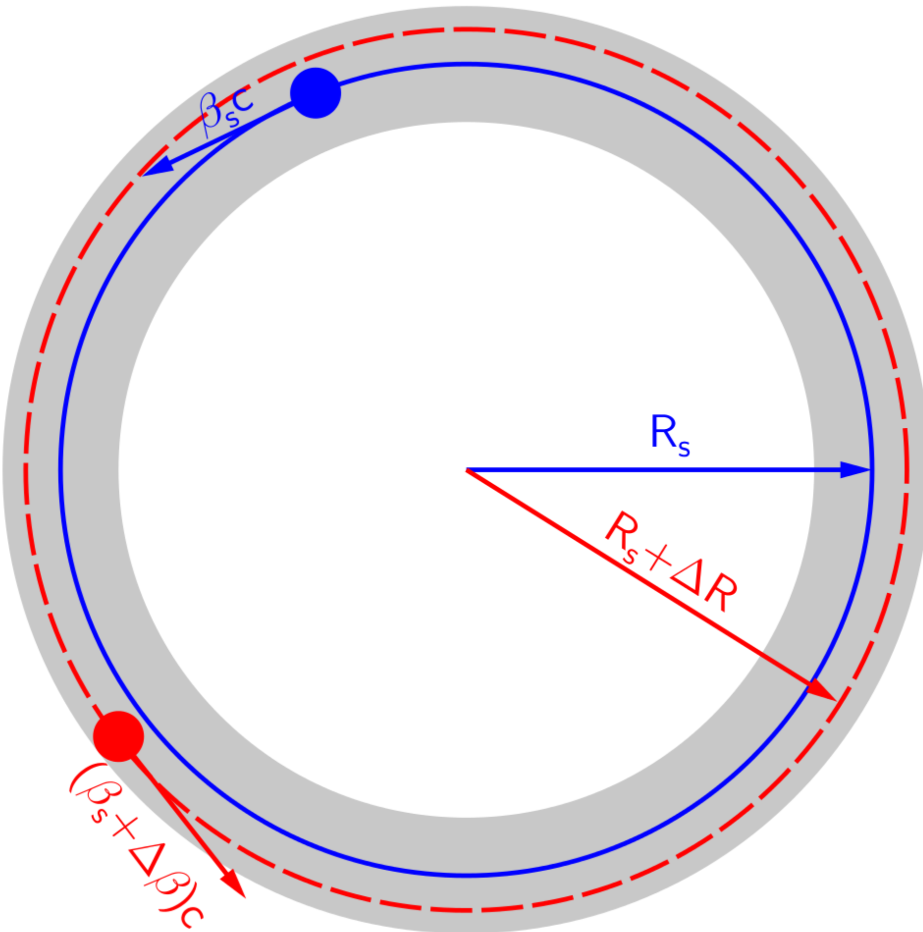
CERN School of Computing

The concept of slippage (1)

- **Ideally, all particles would exactly have the synchronous energy, arrive at the same time to the accelerating cavity, and thus see the same, synchronous RF phase**
  - The 'dynamics' would be quite simple
  - But this is actually not a stable system (see collective effects)

- **In reality, particles can have a slight energy/momentum offset**
  - Typical relative momentum offset, typically $\boldsymbol{\delta = \mathcal{O}\left(10^{-3} - 10^{-4}\right)}$
  - For the synchronous particle, one turn takes $\boldsymbol{T_0 = \frac{2\pi R_s}{\beta_s c}}$, where $\boldsymbol{2\pi R_s}$ is the length of the synchronous orbit and $\boldsymbol{\beta_s = \frac{v_s}{c}}$
  - For an off-momentum particle, one turn takes $\boldsymbol{T = \frac{2\pi R}{\beta c}}$, since the particle has a different velocity and also a different orbit

Slippage



- **By definition, the synchronous particle always returns after one turn to exactly the same position**
- **Note that we are interested in deviations from the synchronous particle, described in phase space**
  - Coordinate system fixed to the synchronous particle
  - Non-inertial
  - Possible change of coordinate system each turn

CERN School of Computing

The concept of slippage (2)

- **If $\delta > 0 \Rightarrow$ the particle travels faster, $\delta < 0 \Rightarrow$ travels slower than the synchronous particle**
  - Intuitively, the effect is stronger for low energies and becomes negligible in the ultra-relativistic limit $\beta \rightarrow 1$

- **How the orbit changes, depends on the magnetic lattice design of the machine, which determines the so-called *transition gamma* γ_T (or *transition energy* $\gamma_T mc^2$) $\frac{1}{{\gamma_T}^2} \equiv \frac{1}{R_s}\frac{dR}{d\delta}$**

- **Below transition, $\gamma < \gamma_T$, speed 'wins' $\Rightarrow$ higher energy particles have higher rev. frequency $\omega \equiv \frac{\beta c}{R} > \frac{\beta_s c}{R_s} \equiv \omega_0$ for $\delta > 0$**

- **Above transition, $\gamma > \gamma_T$, orbit 'wins' $\Rightarrow$ higher energy particles have lower rev. frequency $\omega < \omega_0$ for $\delta > 0$**

CERN School of Computing

The concept of slippage (3)

- **Hence, the associated 'frequency slippage' depends both on the machine (γ_T) and the momentum offset (δ):**

$$\frac{\Delta\omega}{\omega_0} \equiv -\eta(\delta)\delta \approx -\left(\eta_0 + \eta_1\delta + \eta_2\delta^2 + \cdots\right)\delta \quad (2)$$

- **$\eta(\delta)$ is the so-called slippage factor**

- **η_i are machine-dependent constants**
  - Depend on β_s, γ_s, and 'momentum compaction factors' α_i

- **Often the first approximation is sufficient,**

$$\frac{\Delta\omega}{\omega_0} \approx -\eta_0\delta = -\left(\frac{1}{{\gamma_T}^2} - \frac{1}{{\gamma_s}^2}\right)\delta \quad (3)$$

Energy EOM: 'kick'

- **A particle passing the cavity receives an energy kick* of**

$$E^{n+1} = E^n + eV^n \sin\varphi^n \quad (4)$$

where φ^n is the RF phase when the particle crosses the cavity

- **Relative to the synchronous particle, the energy offset is**

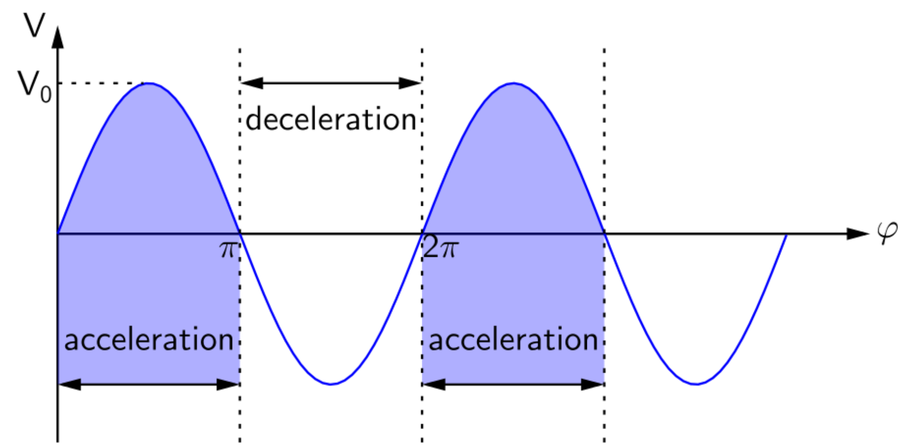
$$\Delta E^{n+1} = \Delta E^n + eV^n \sin\varphi^n - \\ -\left(E_s{}^{n+1} - E_s{}^n\right) \quad (5)$$

RF acceleration synchro-nous w/ magnetic ramp

RF acceleration within the bunch

* N.B. the sinusoidal term is to be replaced by a sum of sinusoids in the case of multiple RF harmonics

Radio-frequency (RF) cavity voltage



Condition for RF acceleration:

$$eV^n \sin\varphi_s{}^n = \left(E_s{}^{n+1} - E_s{}^n\right) > 0$$

$$\therefore\ \varphi_s \in (0, \pi)$$

Stationary energy: $\varphi_s = 0$ **or** π

Phase EOM: 'drift'

- **Assume a particle that has a time delay and energy offset w.r.t. to the synchronous particle** $(\Delta t^n, \Delta E^n)$
  - The first update the energy $\Delta E^n \rightarrow \Delta E^{n+1}$ according to Eq. 5
- **Next turn, the particle crosses the cavity with a time delay of**

$$\Delta t^{n+1} = \Delta t^n + \frac{2\pi}{\omega} - \frac{2\pi}{\omega_s} \quad (6)$$

- **The corresponding RF phase is**

$$\Delta\varphi^{n+1} = \frac{{f_{RF}}^{n+1}}{{f_{RF}}^{n}} \Delta\varphi^n + 2\pi h \left(\frac{1}{1 - \eta(\delta^{n+1})\delta^{n+1}} - 1 \right) \quad (7)$$

- **Note that Eqs. 5 & 7 for kick and drift are exact**
- **Usual approximation:** $\Delta\varphi^{n+1} = \frac{{f_{RF}}^{n+1}}{{f_{RF}}^{n}} \Delta\varphi^n - 2\pi h \eta_0 \delta^{n+1} \quad (8)$

Synchronous phase

- **Small amplitude oscillations**

$$(5) \Leftrightarrow \Delta\dot{E} = \frac{eV\omega_0}{2\pi}(\sin\varphi - \sin\varphi_s)$$

$$\Delta\dot{E} \approx \frac{eV\omega_0 \cos\varphi_s}{2\pi}\Delta\varphi$$

$$(8) \Leftrightarrow \Delta\dot{\varphi} = \frac{h\eta_0\omega_0}{\beta^2 E}\Delta E$$

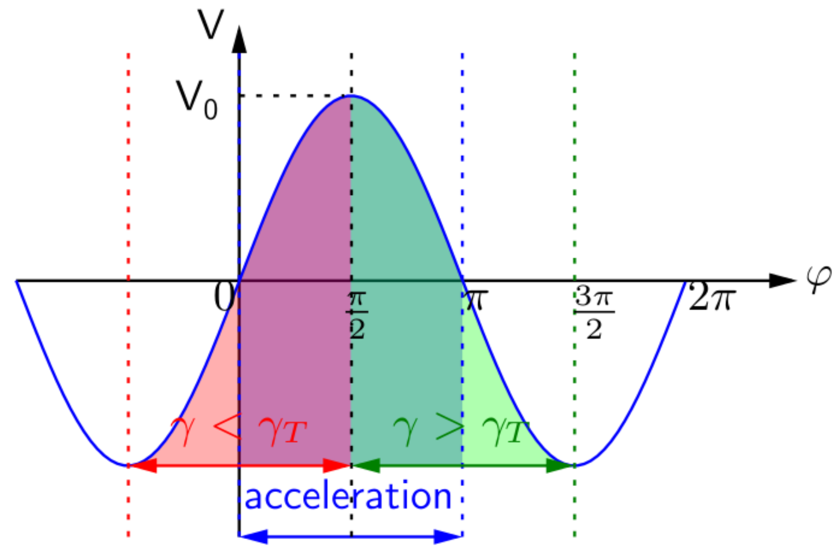
- **Second derivative (same for $\Delta\varphi$):**

$$\Delta\ddot{E} = \frac{eVh\eta_0\cos\varphi_s}{2\pi\beta^2 E}{\omega_0}^2\Delta E$$

- **Simple harm. osc. if $\eta_0 \cos\varphi_s < 0$**
- **Synchrotron oscillation frequency**

$$\omega_s \equiv \sqrt{\frac{eVh\eta_0\cos\varphi_s}{2\pi\beta^2 E}}\,\omega_0$$

Synchronous phase of RF voltage



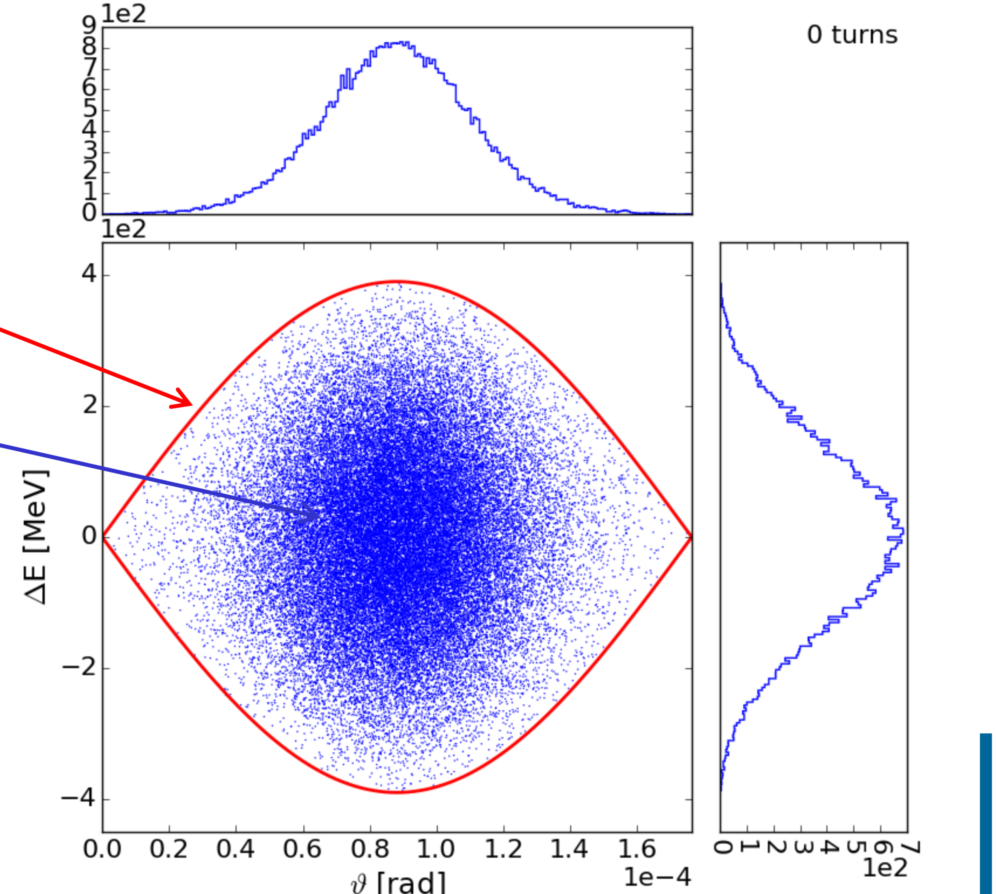
Condition for longitudinal stability, i.e. beam to stay bunched/bundled:

$$\therefore\ \varphi_s \in \left[0, \frac{\pi}{2}\right) \text{ if } \gamma < \gamma_T$$

$$\therefore\ \varphi_s \in \left(\frac{\pi}{2}, 0\right] \text{ if } \gamma < \gamma_T$$

Some terminology…

- **The RF potential that confines the beam determines the 'bucket'**
  - Contour: 'separatrix'
- **The beam that is inside a bucket is a 'bunch'**
- **The complete beam we call a 'bunch train'**
  - Can consist of 'batches' with larger spacing in-between

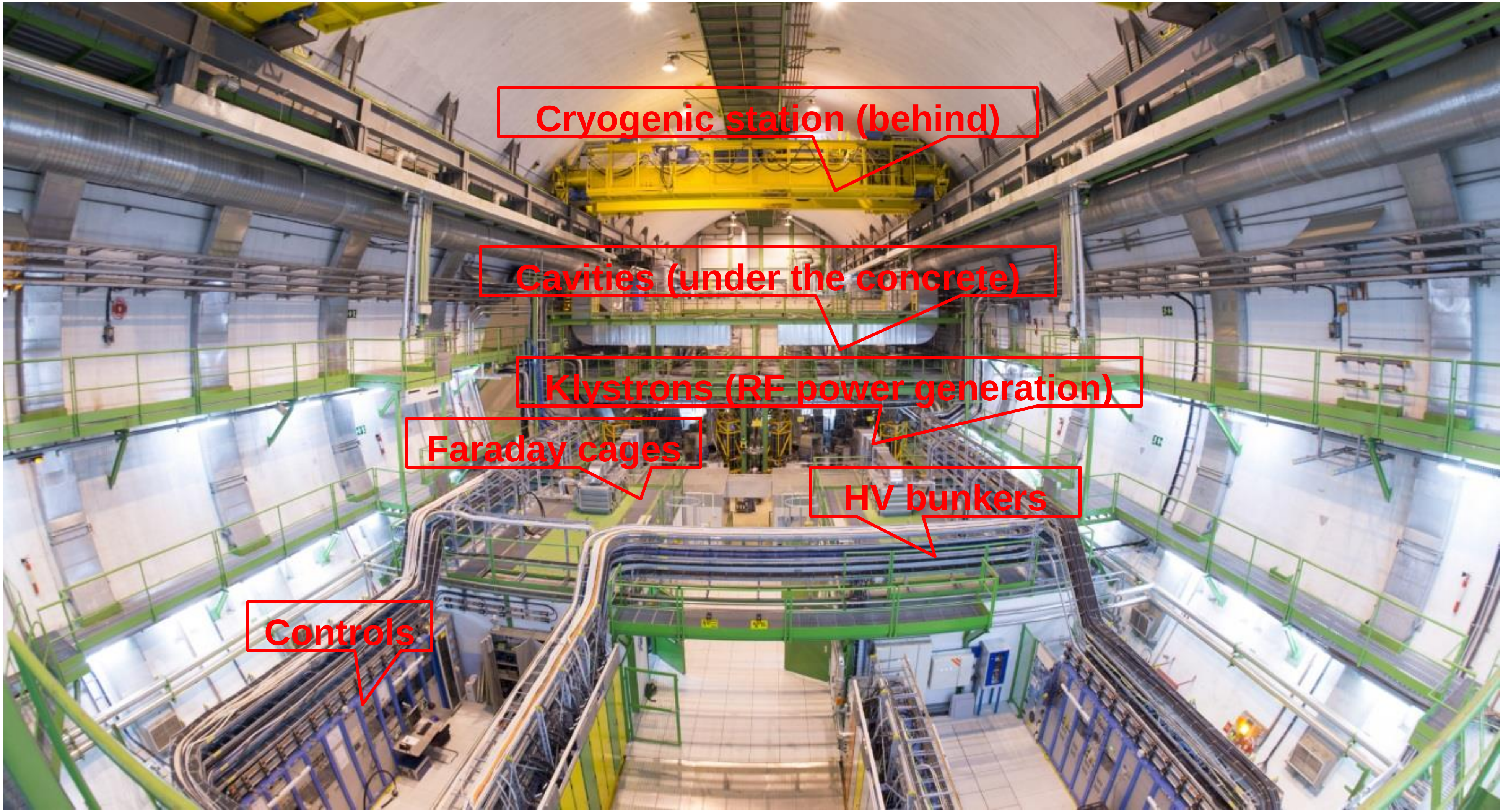


A few examples of how to shape bunches

RF MANIPULATIONS

LHC Point 4 – RF acceleration, 2005

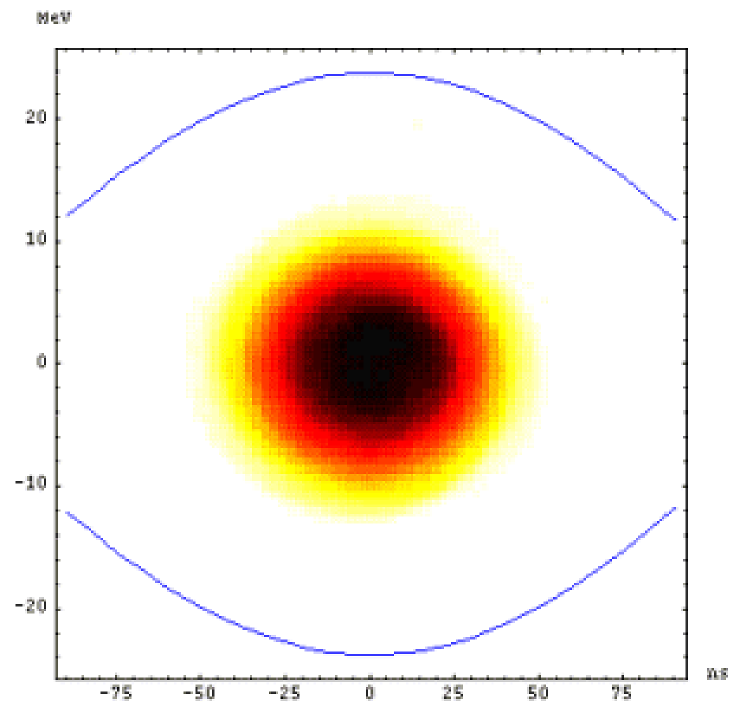
LHC Point 4 – RF acceleration, 2012



Bunch splitting

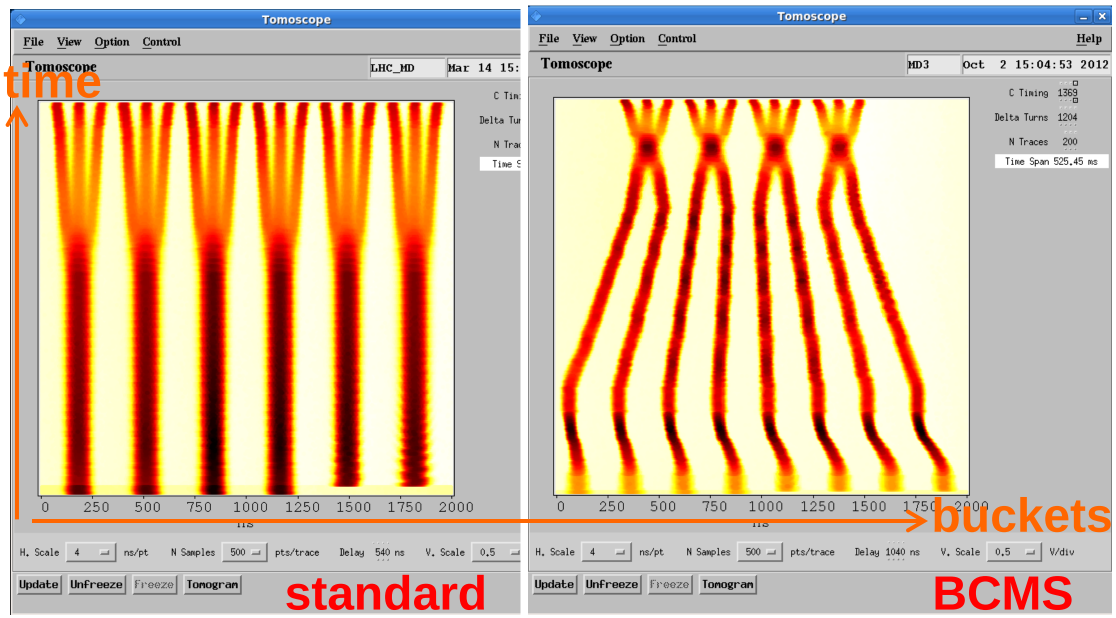
- **Bunch splitting is used several times in the PS, to increase the number of bunches and shorten their bunch length**
  - PS bucket: 100 ns, LHC bucket: 2.5 ns!
- **How it's done: add slowly (adiabatically) a higher-harmonic component to the RF potential**
- **Numerical challenge: what is a bucket in this case? In multi-RF, what are the synchronous phase(s)?**

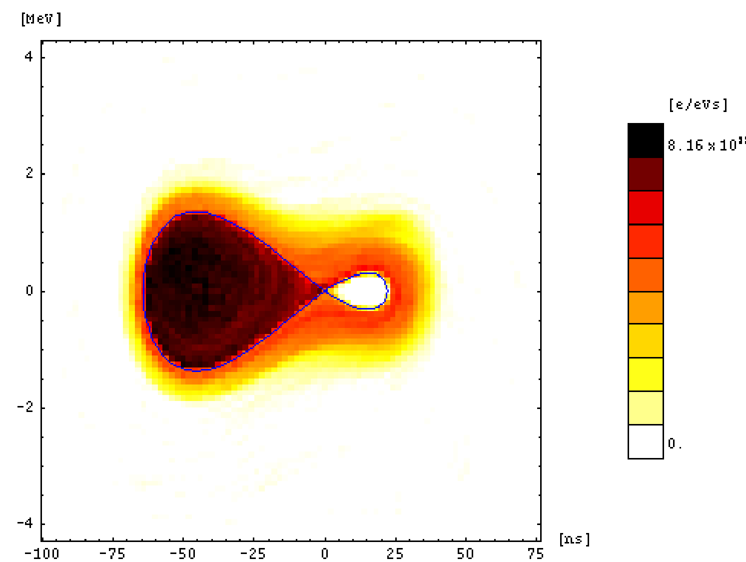
PS bunch splitting (measured via tomography)



Bunch merging

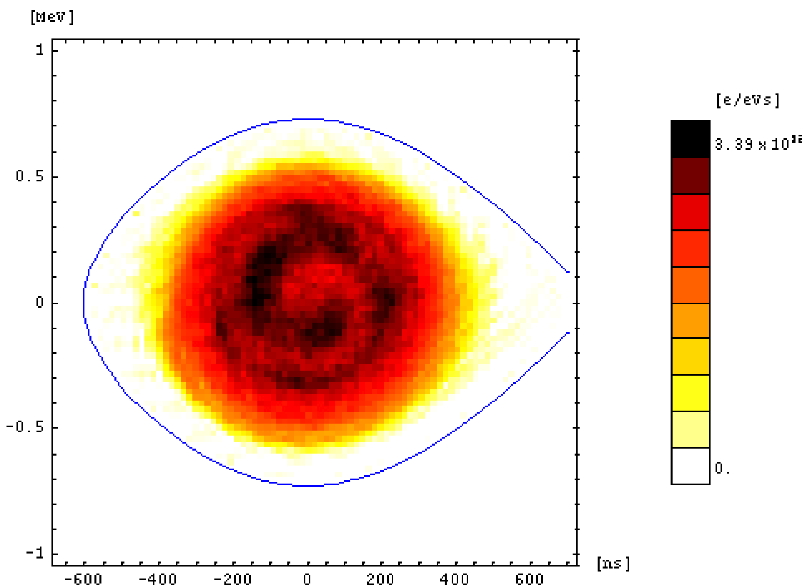
- **The opposite of bunch splitting**
  - Used in the PS ‘BCMS’ beam production scheme to produce brighter beams for the LHC
  - BCMS = batch compression, merging, and splitting

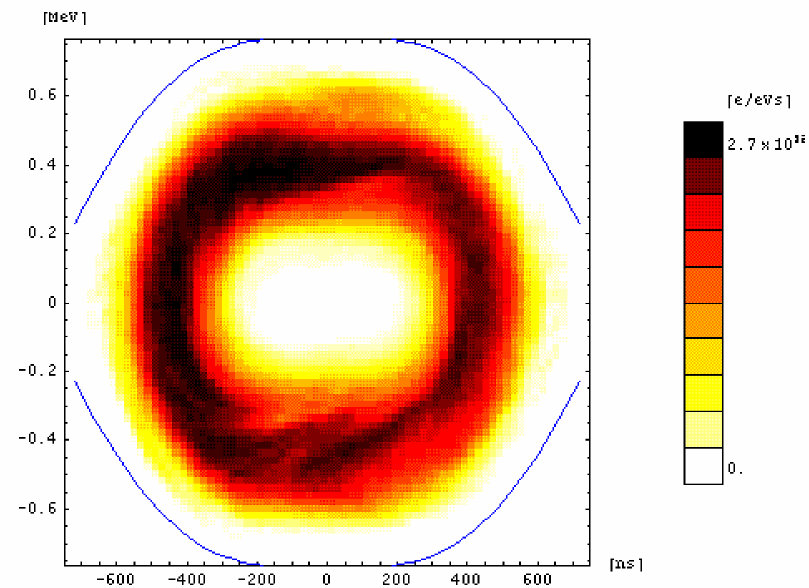




Phase-space shaping

- **Phase-space 'painting' through intentional filamentation (mixing)**
  - Mismatched bunch
- **Hollow bunches to produce a flat line density**

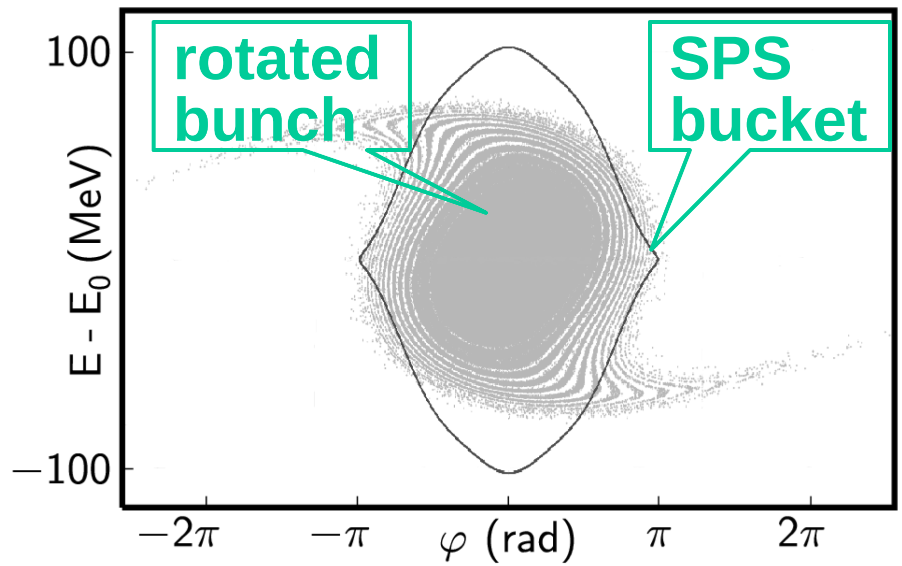




Bunch rotation in phase space

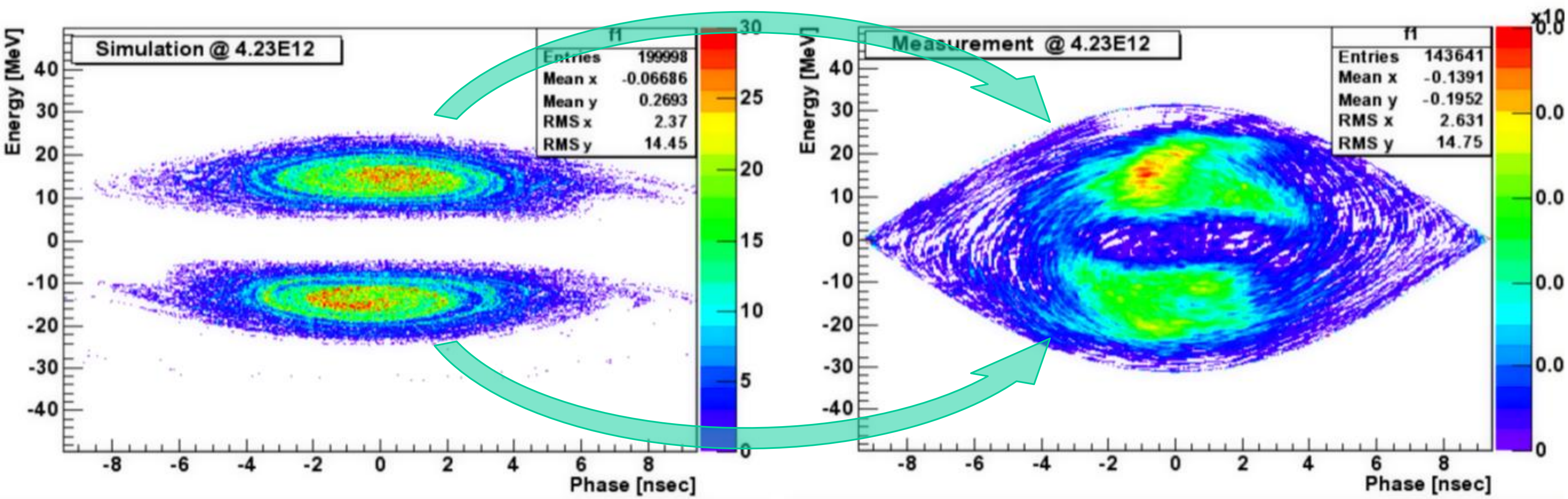
- **Through a sudden (non-adiabatic) voltage increase, the bunch starts to rotate in phase space**
  - A bunch that has initially a long bunch length and small momentum spread will have after $\frac{1}{4}T_s$ a short bunch length and large momentum spread

Used in the PS just before extraction to fit the long PS bunches into the SPS bucket



Slip stacking

- **Merging two bunch trains to increase flux by slipping them in phase/position relative each other**
  - Need 2 different RF systems at different frequencies: one train is decelerated, one accelerated, and finally both are captured
  - Operational at Fermilab, planned for SPS ion operation to halve bunch spacing



Interaction of the beam and its surroundings

COLLECTIVE EFFECTS

CERN School of Computing

Space charge

- **Space charge refers to the 3D charge density distribution $\rho(\vec{x})$**
- **In a real bunch, particles will not only feel the external RF potential, but also the Coulomb interaction with neighbouring particles, which generates an electric potential as well**
  - Negligible for ultra-relativistic beams
- **Ideally, one would need to solve the 3D Poisson equation $\nabla^2\phi = -\nabla\cdot\vec{E} = -\frac{\rho}{\varepsilon_0}$, using many particles and taking into account the external RF potential and the beam pipe (in BCs)**
  - In computational plasma physics this is usually done using the particle-in-cell (PIC) method
  - In beam dynamics, we try to avoid such expensive methods
  - Instead, we model the space charge effect through effective potentials, as an impedance

CERN School of Computing

Machine impedance

- **Talking of which it suddenly becomes very complicated…**
- **The accelerator is built of many conducting elements: beam pipes, cavities, kickers, etc., having a frequency-dependent complex (hermitian) impedance $Z(f)$**
  - The impedance is a machine property
- **Can be described via its Fourier-transform the wake field $W(t)$; in longitudinal plane: $W_{\parallel}(t > 0) = \frac{1}{2\pi}\int_{-\infty}^{\infty} d\omega\, e^{i\omega t} Z_{\parallel}(\omega)$**
- **There is a whole 'zoo' of analytical impedance models for**
  - Resistive wall, space charge, broadband-resonators, etc.
- **Often, however, we need more accurate data**
  - Electromagnetic simulations, e.g. CST Studio

CERN School of Computing

Beam-machine interaction

- The beam, having a time-dependent current density $j(t)$ will interact with the impedance
- Just like in an electric circuit, an electric potential $\phi = jZ$ will be induced
- Hence, the effective potential according to which the beam will move is the sum of the RF and beam-induced potential
- The modified energy equation becomes

$$\Delta E^{n+1} = \\ = \Delta E^{n} + eV^{n} \sin \varphi^{n} - \left({E_s}^{n+1} - {E_s}^{n}\right) - \\ - e^2 \underbrace{\int_{-\infty}^{t^{n+1}} d\tau\, \lambda(\tau) W_{\parallel}(t^{n+1} - \tau)}_{\text{convolution of bunch line density and wake}} \quad (9)$$

beam

machine

convolution of bunch line density and wake

CERN School of Computing

Modelling collective effects

- **Due to the convolution, the motion of a particle ‘behind’ depends on the induced voltage of the particles ‘in front’**
- **The energy kick due to collective effects can be modelled**
  - In frequency domain: kick = impedance $\times$ bunch spectrum
  - In time domain: kick = convolution of wake and bunch profile
  - Numerically, both methods have pros and cons ($\rightarrow$ Lecture 2)
- **Usually it is more effective to discretise the bunch profile using slices and calculate collective effects separate from the EOMs every turn**
  - The slicing has to be chosen such that all frequencies playing a role are resolved $\boldsymbol{\Delta t} < \frac{1}{\boldsymbol{f}}$

CERN School of Computing

The concept of Landau damping

- **Now if an impedance sits e.g. at a frequency that matches the synchrotron frequency of the bunch, collective oscillations can be excited, the bunch can become unstable**

- **In fact, even without any machine impedance, purely due to space charge, a bunch with no momentum spread (i.e. all particles oscillating with ω_s) is unstable as well**

- **It is due to the momentum spread in the bunch, and the resulting 'tune spread' in synchrotron oscillation frequency $\omega_s = \omega_s(\varphi)$ that the bunch is stable. This natural damping of oscillations is called Landau damping.**

- **There are many types of beam instabilities and each of them has its own intensity threshold. Thresholds can be increased through controlled damping, at least up to some level…**

Low-power level RF loops

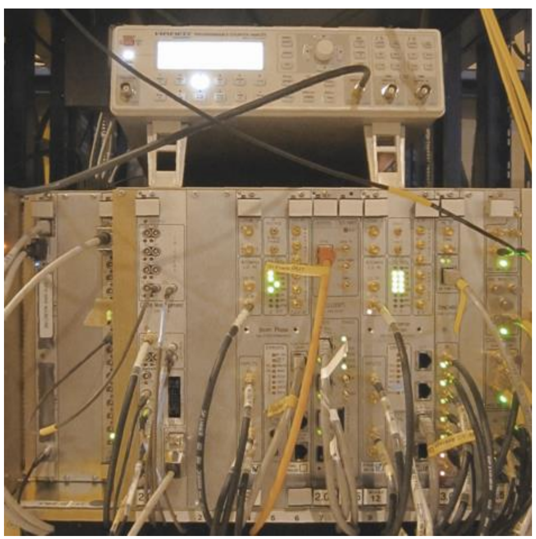
LLRF

CERN School of Computing

What are LLRF loops?

- **LLRF loops are control electronics that act on-line on RF voltage, phase, and frequency with the main purpose of**
  - Monitor and correct the applied RF w.r.t. the designed RF
  - Correct for noise or collective effects
  - Stabilise the beam by damping collective oscillations
- **For shielding, LLRF equipment is installed in 'Faraday cages'**

LHC Beam control



LHC Faraday cage

CERN School of Computing

Examples

- **Keeping cavity V and φ as programmed**
  - Cavity loop: Slow feedback
  - One-turn-delay feedback to compensate for beam loading
  - Feed-forward (optional): to reduce the errors in the correction
- **Synchro and radial loops: keep the bunch centred**
  - Synchro loop: synchronises RF with magnetic field ($\omega_{RF} = h\omega_0$)
  - Radial loop: corrects transverse position
- **Beam phase loop: correct differences btw. RF and beam phase**
  - Damp injection oscillations, phase noise, etc.
- **Longitudinal/transverse dampers: designed to damp instabilities (longitudinal/transverse oscillations)**

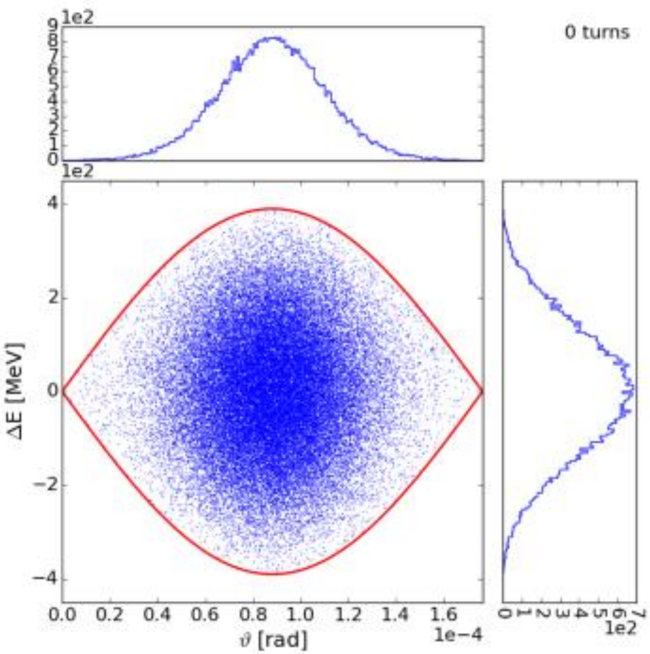
How to model LLRF loops?

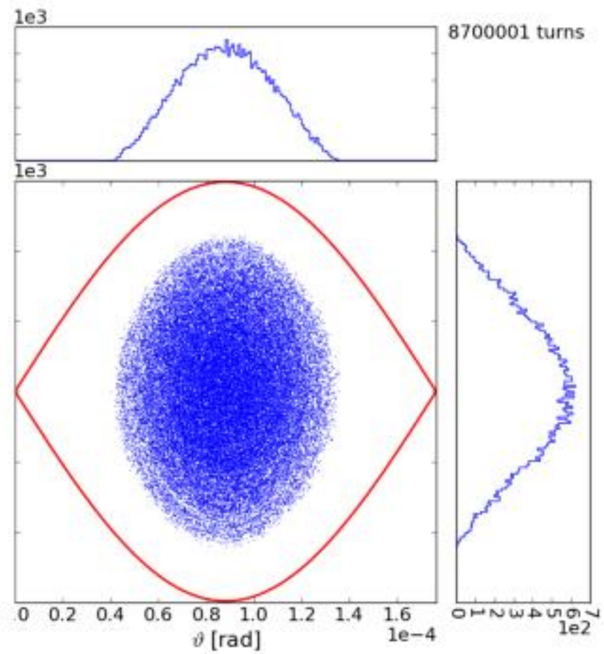
- **Some loops are 'built in' in the numerical model: we assume**
  - Cavity V and φ as programmed (e.g. cavity loop, FB, FF)
  - RF frequency as programmed (e.g. synchro loop)
- **Other loops have to be modelled via their action**
  - Applying transfer functions on the particle coordinates (e.g. transverse damper)
  - Changing the EOMs (phase loop)
  - Modelling the electronics (loop gain, bandwidth, etc.)
- **In general, we need to take into account frequency and phase shifts in the EOMs**

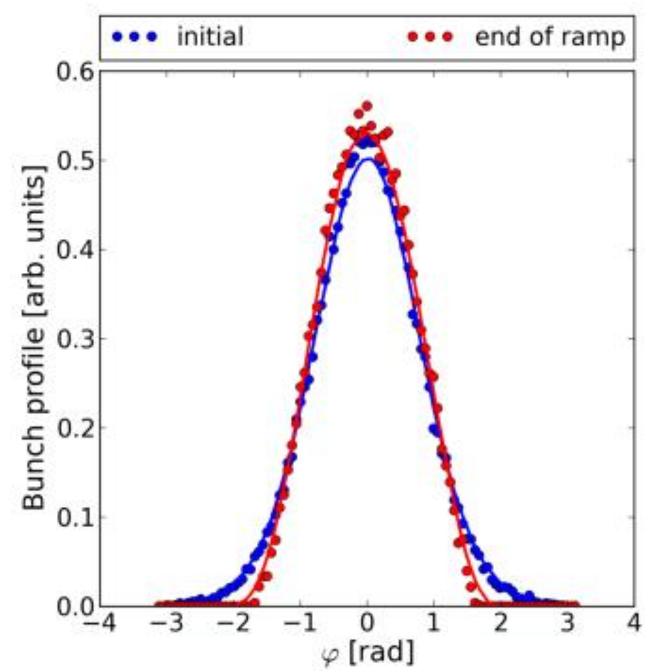
A ‘real’ example

- **Simulation of the LHC acceleration ramp with controlled emittance blow-up and phase loop**
  - Natural bunch shrinkage during ramp leads to loss of Landau damping; controlled phase noise injection used for blow-up

Before ramp, 450 GeV **After ramp, 4 TeV**







Are end-to-end simulations possible?

COMPLEXITY OF NUMERICAL MODELS

End-to-end simulations?

- **So-called end-to-end simulations are used to calculate beam dynamics in linacs, from one end to the other**
  - Possible because one can use mapping from one stage to another, and the investigated phenomena are very different
- **Could we do something similar for synchrotrons, say simulate the beam dynamics from injection to PSB till physics in LHC?**
- **What do you think? What is your feeling for the complexity of the problem?**

A (very) rough runtime estimate (1)

- **How long would it take to simulate 'just' the LHC full beam with intensity effects?**
- **A reasonable simulation that was done:**
  - Single bunch
  - 50,000 particles, 100 slices
  - Acceleration ramp: 8,700,000 turns (11 minutes real time)
  - Phase loop and noise injection for controlled emittance blow-up
  - No intensity effects
  - Runtime: **3 days on a single CPU**

A (very) rough runtime estimate (2)

- **Intensity effects (might even need much more particles/slices)**

  $\times$ 2 $\rightarrow$ **6 days**

- **Full LHC cycle: injection (1 h) + ramp (11 min.) + physics (8 h)**

  $\times$ 50 $\rightarrow$ **300 days ~ 1 year**

- **In physics: beam-beam interaction, 6D phase space**

  $\times$ 2-10? $\rightarrow$ **2 years**

- **Full beam**

  $\times$ 2808 $\rightarrow$ **5600 years**

- **Coupled bunch effects**

  $\times$ 2-10? $\rightarrow$ **>10000 years (≫ LHC lifetime!)**

CERN School of Computing

Take-home messages

- **What is a bunch and how can we accelerate it?**
- **How does the magnetic field relate to the cavities' RF frequency in a synchrotron?**
- **What kind of basic effects (building blocks) do we need to consider in a numerical model?**

References

- CERN BLonD Beam Longitudinal Dynamics code: http://blond.web.cern.ch
- Machine operation websites accessible via: https://espace.cern.ch/be-dep/OP/default.aspx
- S.Y. Lee: Accelerator Physics, World Scientific, Singapore, 2012
- A.W. Chao: Physics of collective beam instabilities in high energy accelerators, Wiley, New York, 1993
- S. Hancock: PS RF manipulations measured with the PS tomograph https://tomograp.web.cern.ch/tomograp/examples.html
- S. Hancock: PS BCMS production scheme https://indico.cern.ch/event/238152/session/2/contribution/9/material/slides/1.pdf
- H. Timko: Studies on PS bunch rotation and PS-SPS transmission http://journals.aps.org/prstab/pdf/10.1103/PhysRevSTAB.16.051004
- Slip stacking: Anckenbrandt (1981), MacLachlan (2001), Seiya et al. (2008)