Cosmology 2

Primordial fluctuations, continued

NExT summer school 2015

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Last time

- Best come from spectrum
- These equations can be integrated with a suitable initial condition
  I haven't told you how to get that — see Dias et al. **arXiv:1502.03125**
- Doesn't involve any approximation beyond tree-level and our ability to compute the initial condition sufficiently accurately
  Initial condition requires slow-roll approximation, but not afterwards
- Computational cost is peanuts
  0.00507s per *k*-mode on my laptop — easily fast enough to include in a parameter-estimation Monte Carlo.
  Analytic estimates aren't the best way to compare to data.
- Freely available codes exist

Spectrum codes (in chronological order)

FieldInf (Ringeval, Martin — FORTRAN)

http://theory.physics.unige.ch/~ringeval/fieldinf.html

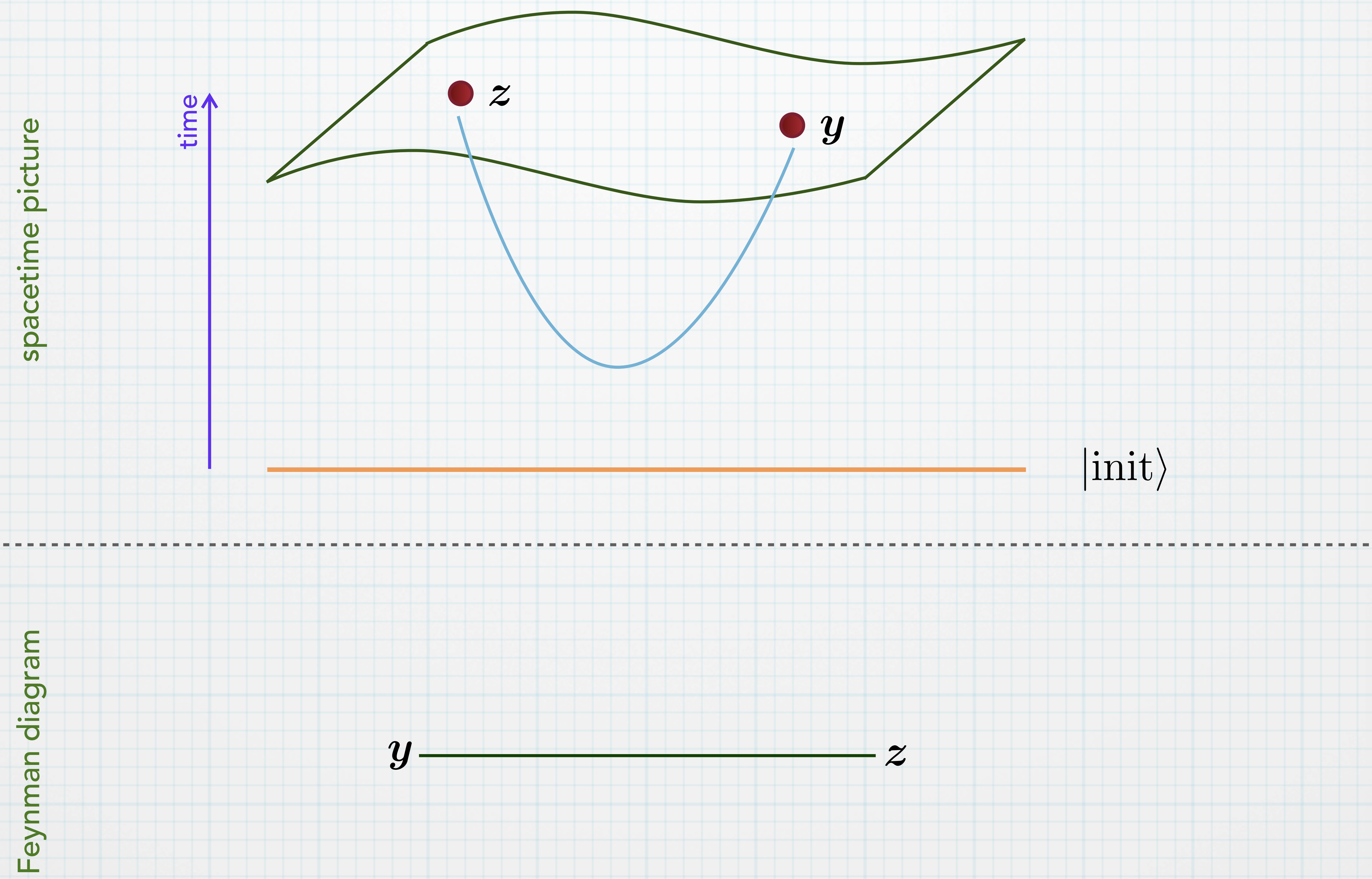
ModeCode, MultiModeCode (Easter, Frazer, Peiris, Price, Xu — FORTRAN)

http://modecode.org | only trivial field-space metric

Sussex & QMUL code (Dias, Frazer, DS — Mathematica)

http://transportmethod.com

The next level of complexity is the bispectrum, which measures three-body interactions



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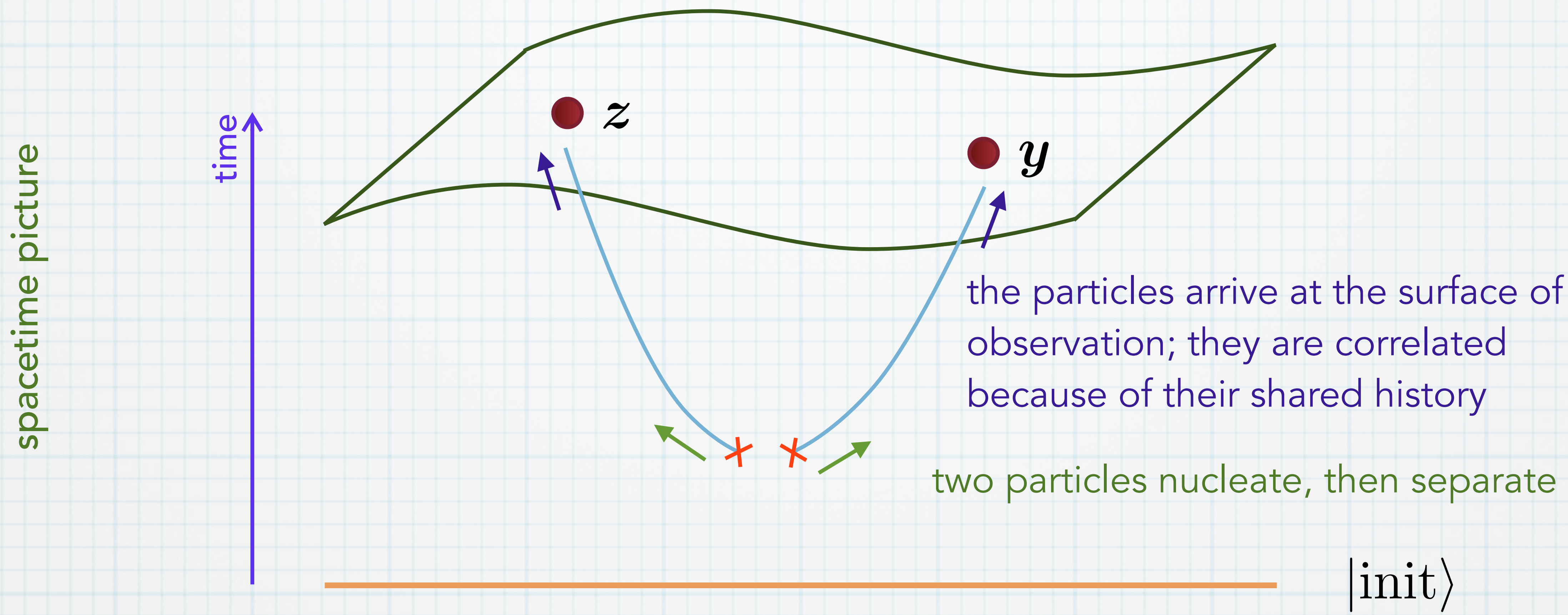

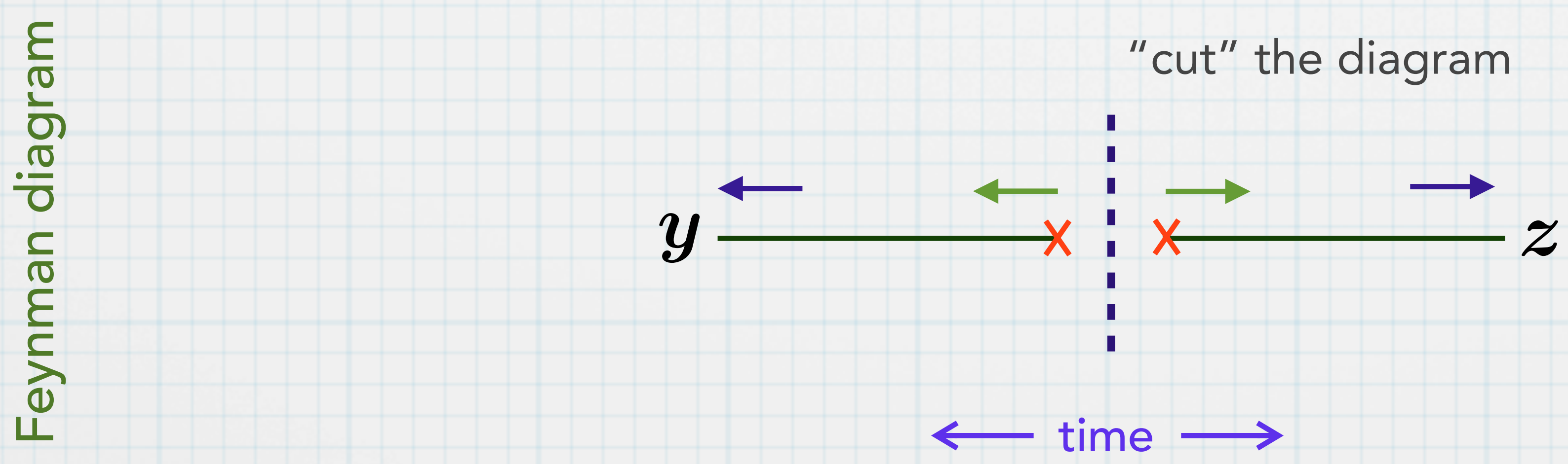

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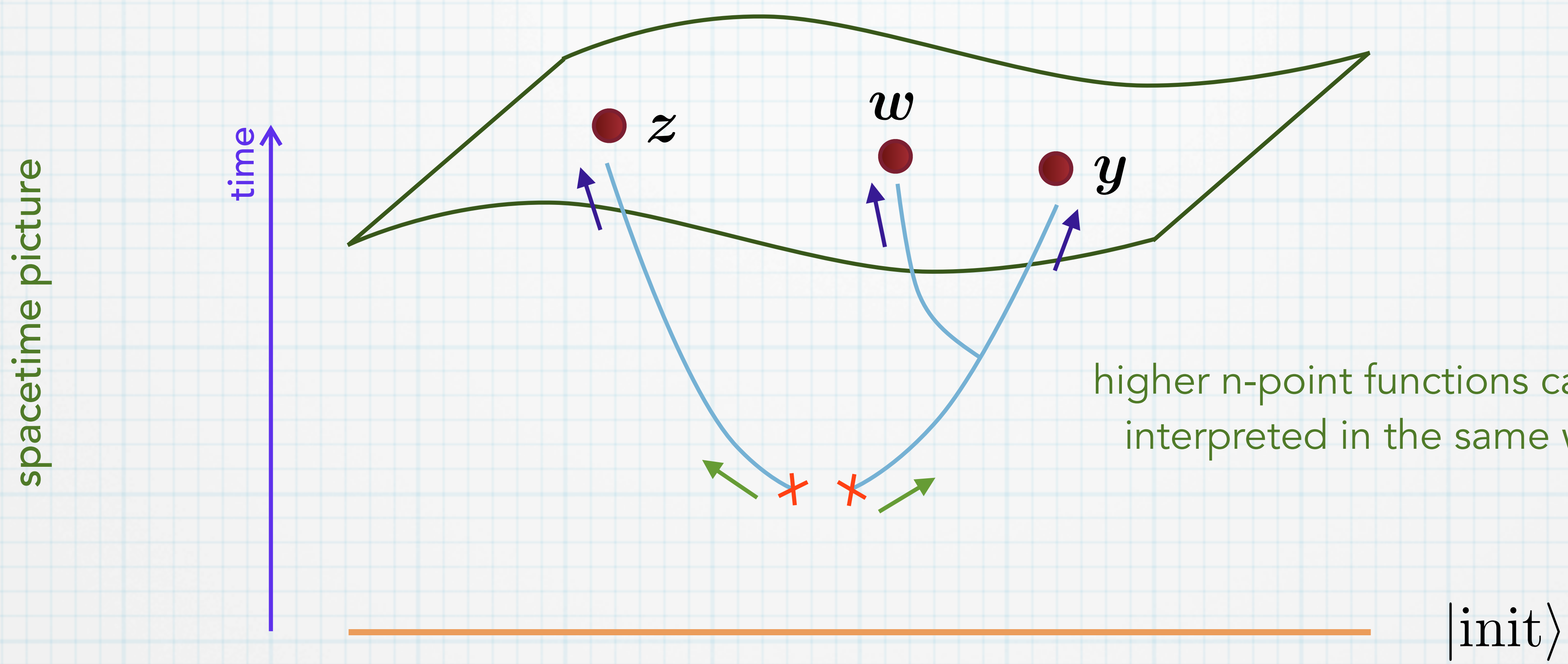
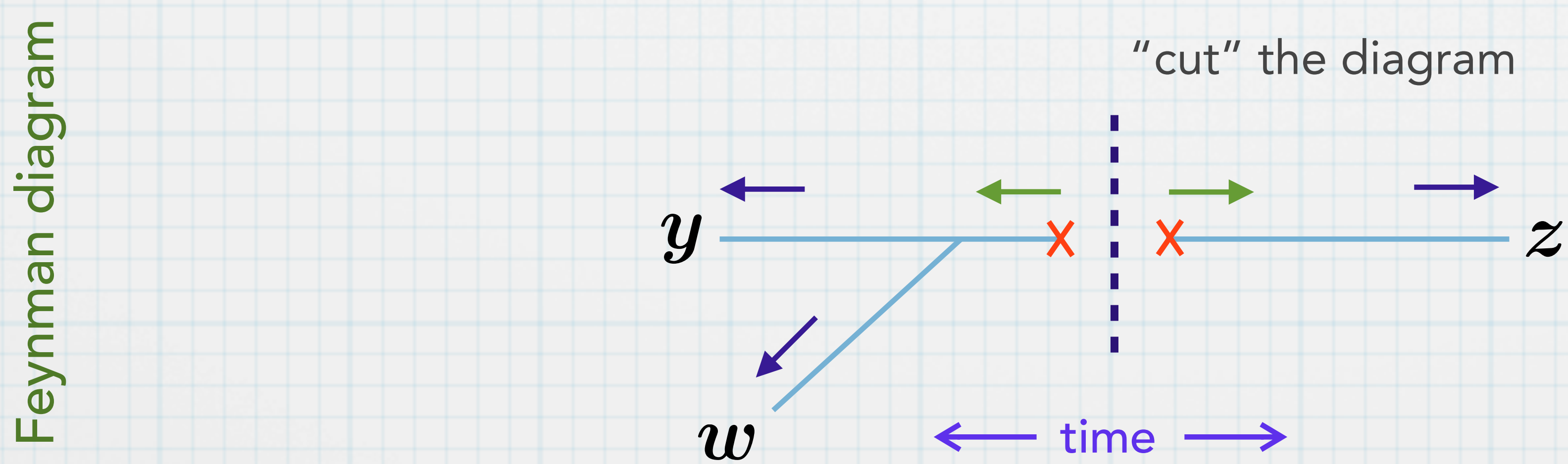




How to compute the bispectrum

The structure is the same as for the spectrum, although the details are more complicated

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\zeta(\boldsymbol{k}_3)\rangle_\tau = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2+\boldsymbol{k}_3)B_\tau(k_1,k_2,k_3)$$

three-point function

Fourier convention

Bispectrum

The evolution equation for the three-point functions is

$$\mathcal{D}_N\langle X^a X^b X^c\rangle = u_{ad}\langle X^d X^b X^c\rangle + u_{ade}\langle X^d X^b\rangle\langle X^e X^c\rangle + \text{cyclic perms}$$

same as 2pf

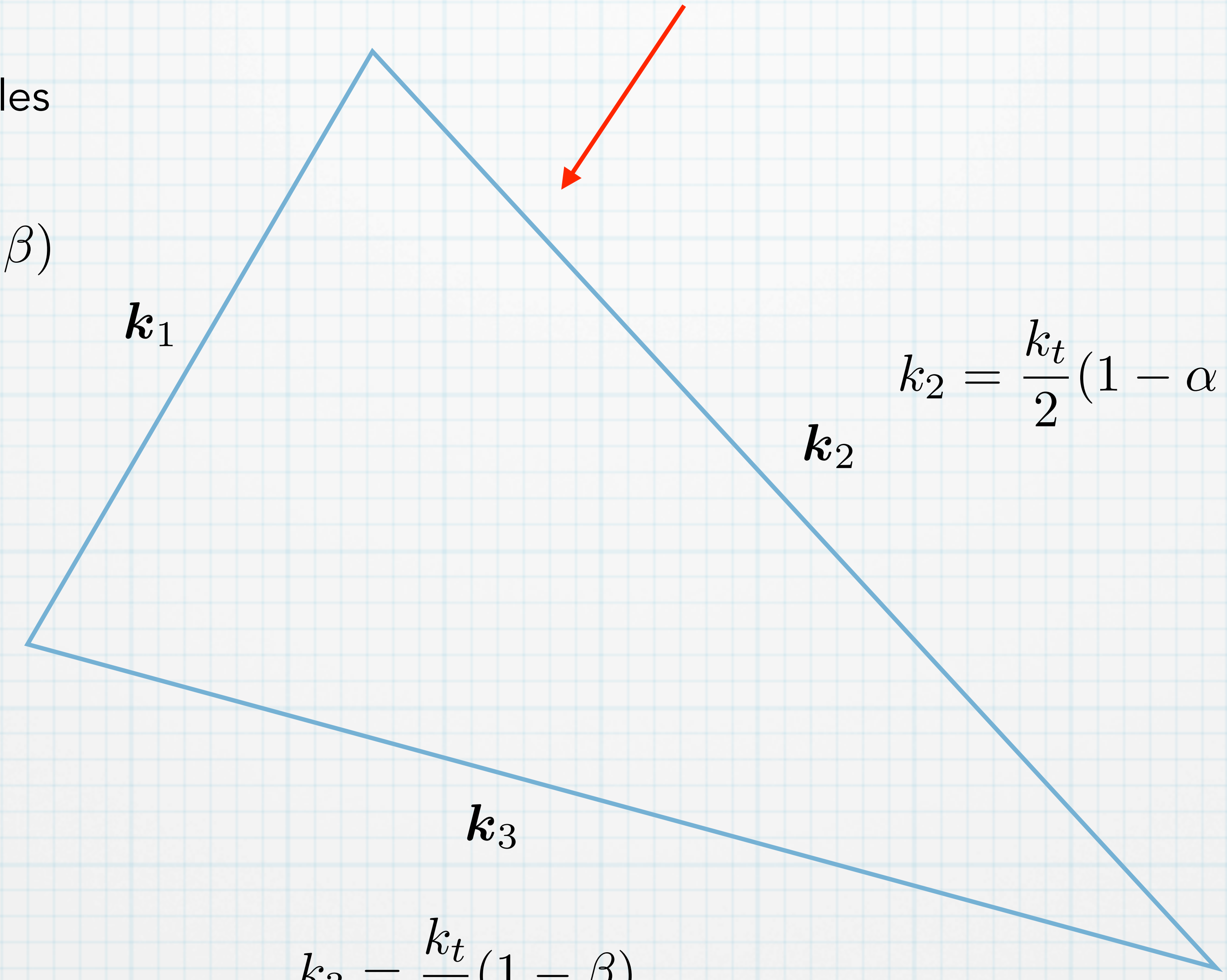
built from 3rd-order terms in Hamiltonian

2-point function sources the 3-point function

$$k_t = \text{perimeter} = k_1 + k_2 + k_3$$

α = 0 means isosceles

$$k_1 = \frac{k_t}{2}(1 + \alpha + \beta)$$

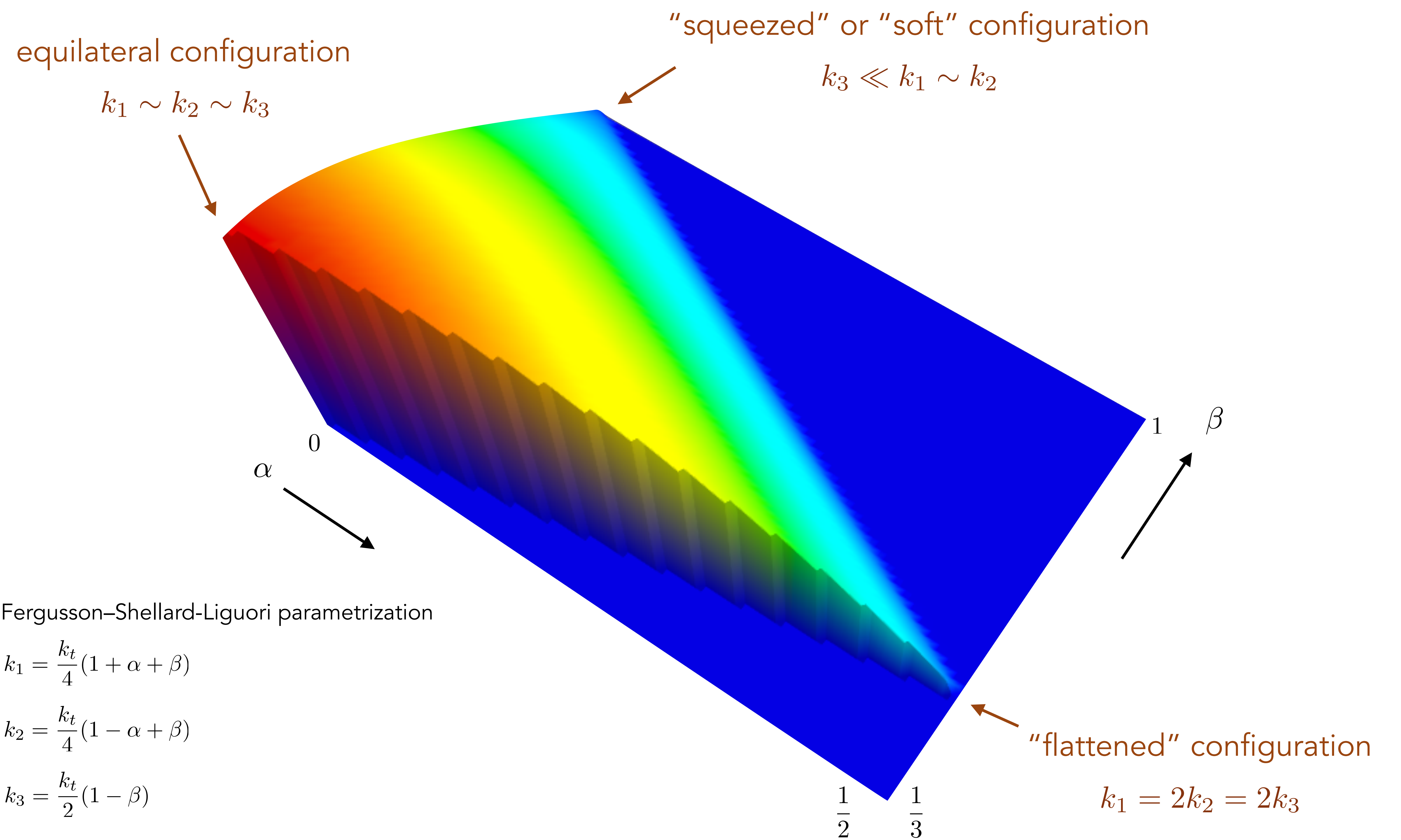


$$k_2 = \frac{k_t}{2}(1 - \alpha + \beta)$$

$$k_3 = \frac{k_t}{2}(1 - \beta)$$

β varies from 0 to 1 as k_3 varies from 0 to $k_t/2$

There is a lot of information in the bispectrum



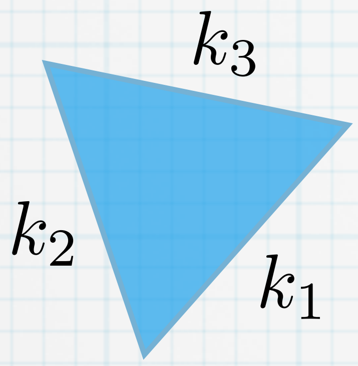
Fergusson–Shellard-Liguori parametrization

$$k_1 = \frac{k_t}{4}(1 + \alpha + \beta)$$

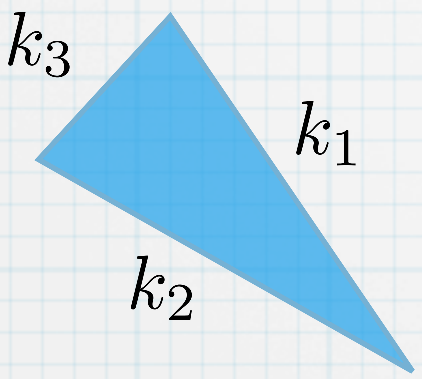
$$k_2 = \frac{k_t}{4}(1 - \alpha + \beta)$$

$$k_3 = \frac{k_t}{2}(1 - \beta)$$

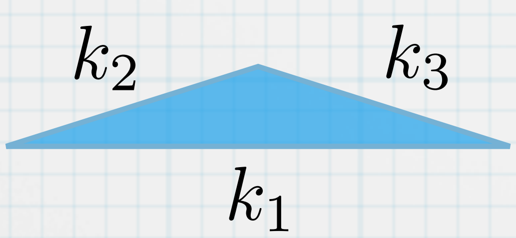
The mode–mode correlation in the bispectrum is diagnostic of the underlying microphysics



- **Equilateral**. Indicates that the fluctuations have strong, nontrivial self-interactions.
  Favours stringy or supergravity scenarios
  Dominantly like–like correlations



- **Squeezed**. Indicates that there are long-range forces which set up correlations, so multiple light modes.
  Dominantly long–short correlations



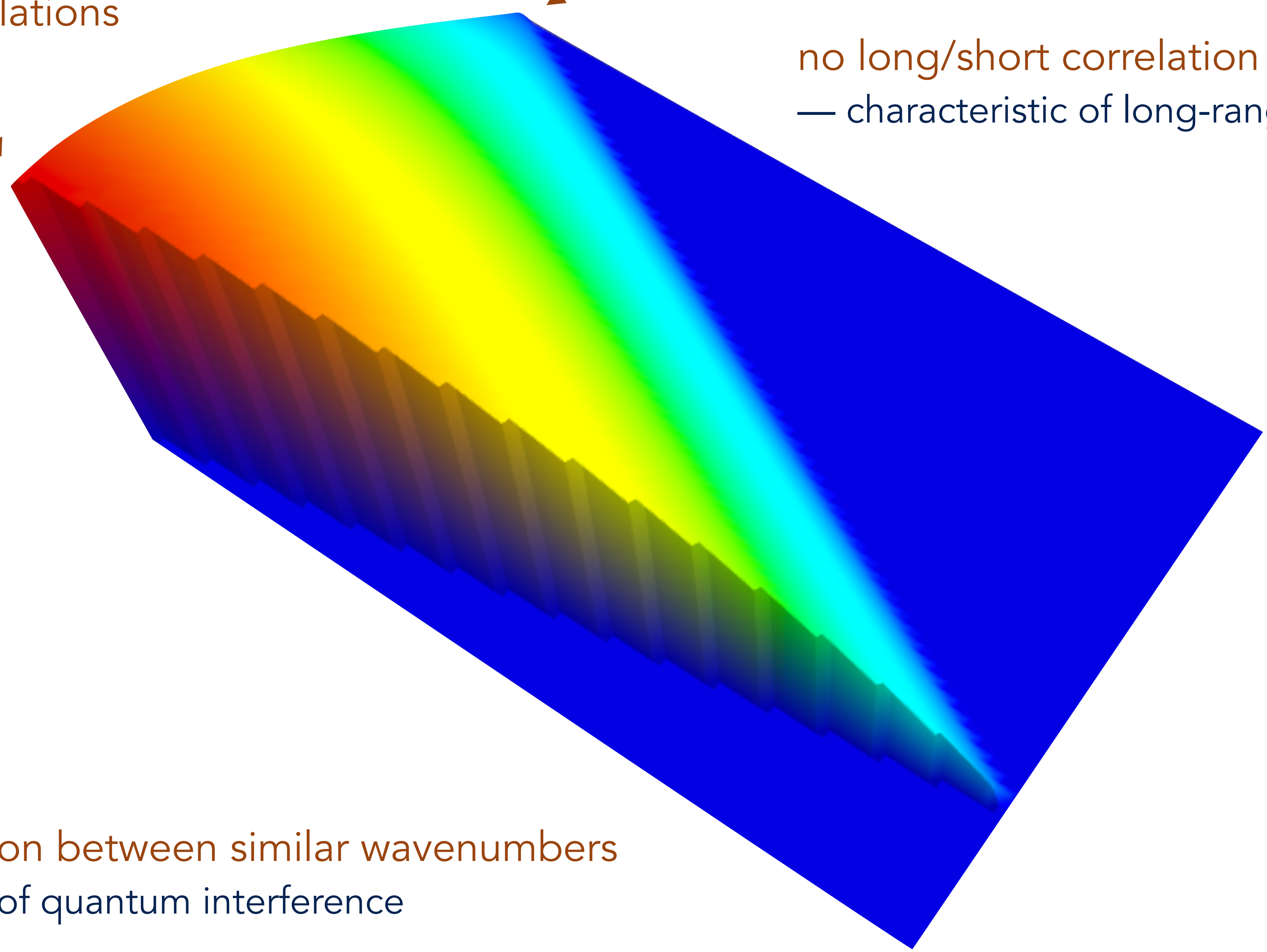
- **Flattened**. Indicates a near "resonance" between positive and negative energy modes.
  Favours a non-vacuum initial state
  A special case of like–like correlation

There is a lot of information in the bispectrum

equilateral template

near zero correlations

enhanced correlations

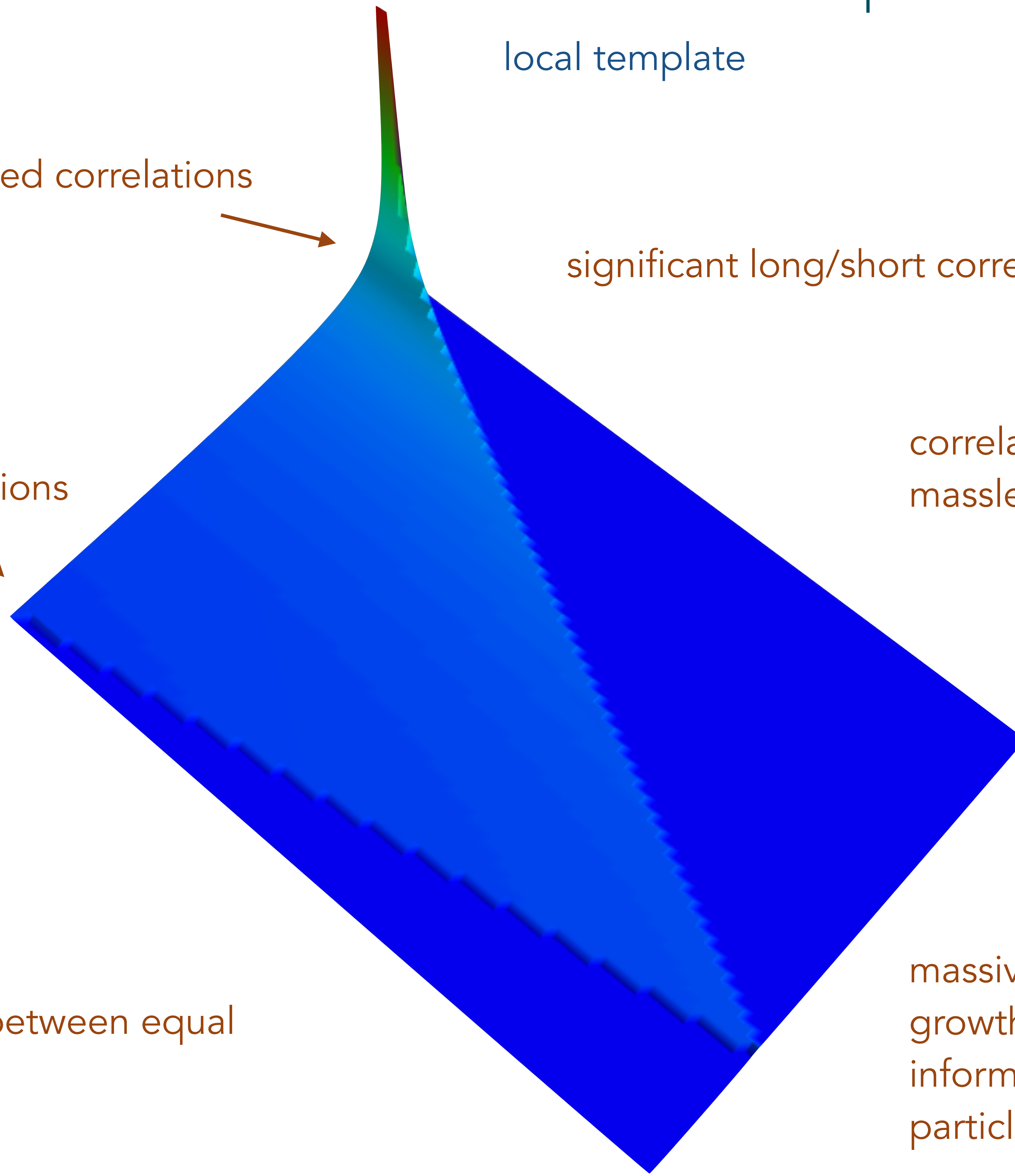


no long/short correlation
— characteristic of long-range forces

strong correlation between similar wavenumbers
— characteristic of quantum interference

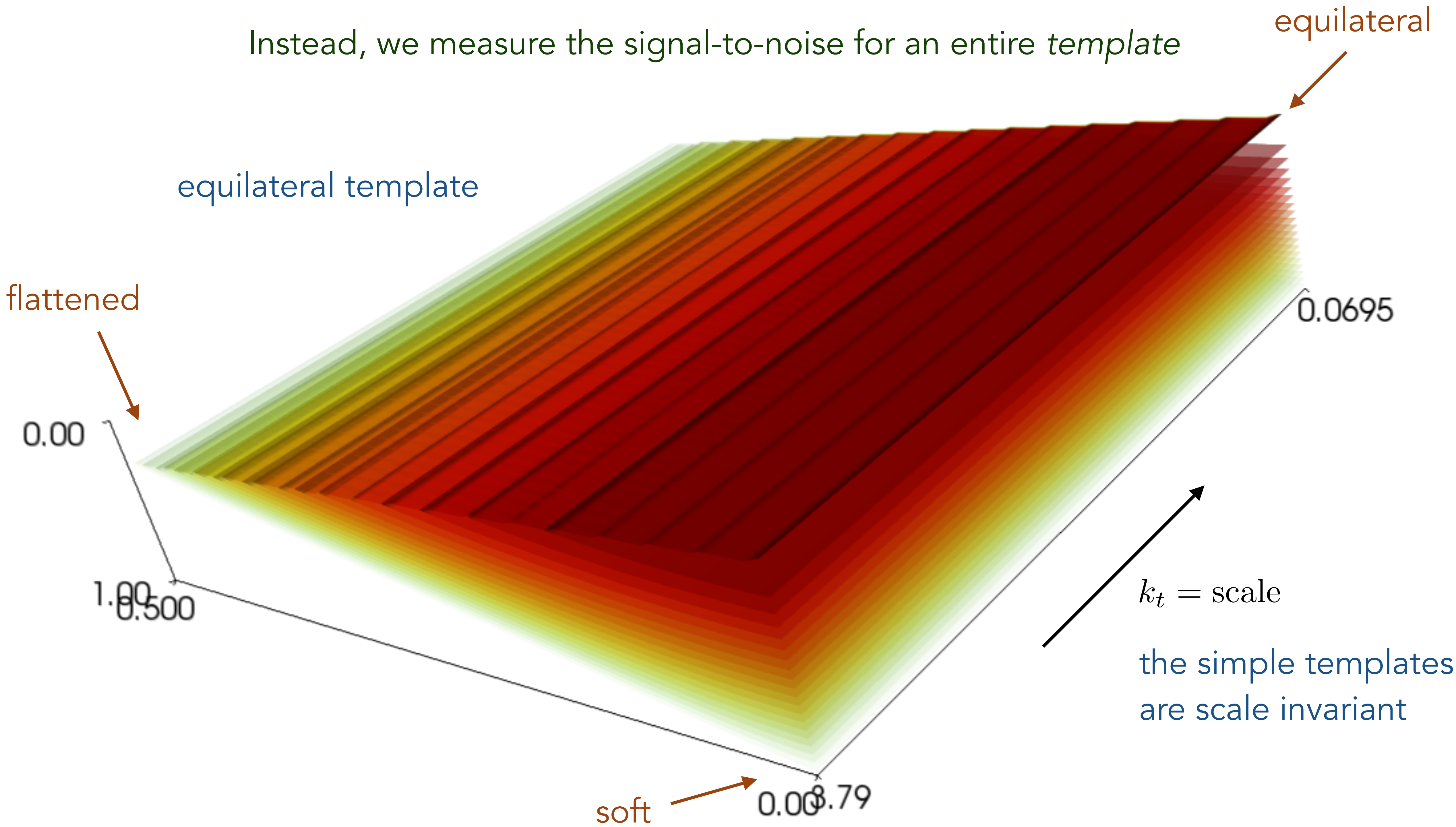
There is a lot of information in the bispectrum

There is a lot of information in the bispectrum



Unfortunately, there is very little signal-to-noise in any given configuration.
So we do not *measure* the bispectrum at a given k_1, k_2, k_3 or k_t, α, β

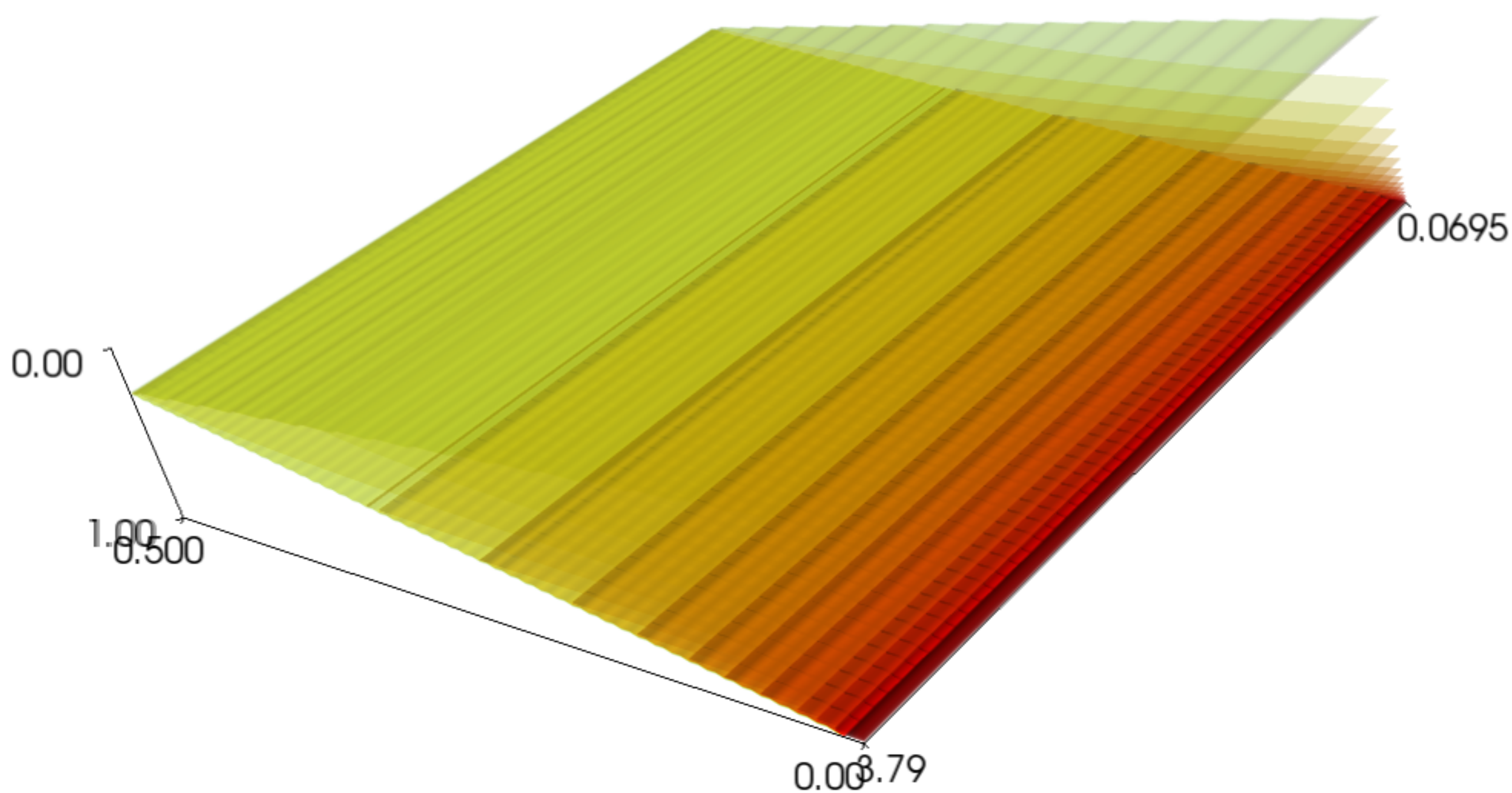
Instead, we measure the signal-to-noise for an entire *template*



$$\text{amplitude} = f_{\rm NL}^{\rm equi} = -4 \pm 43$$

(Planck2015 temperature+polarization)

local template

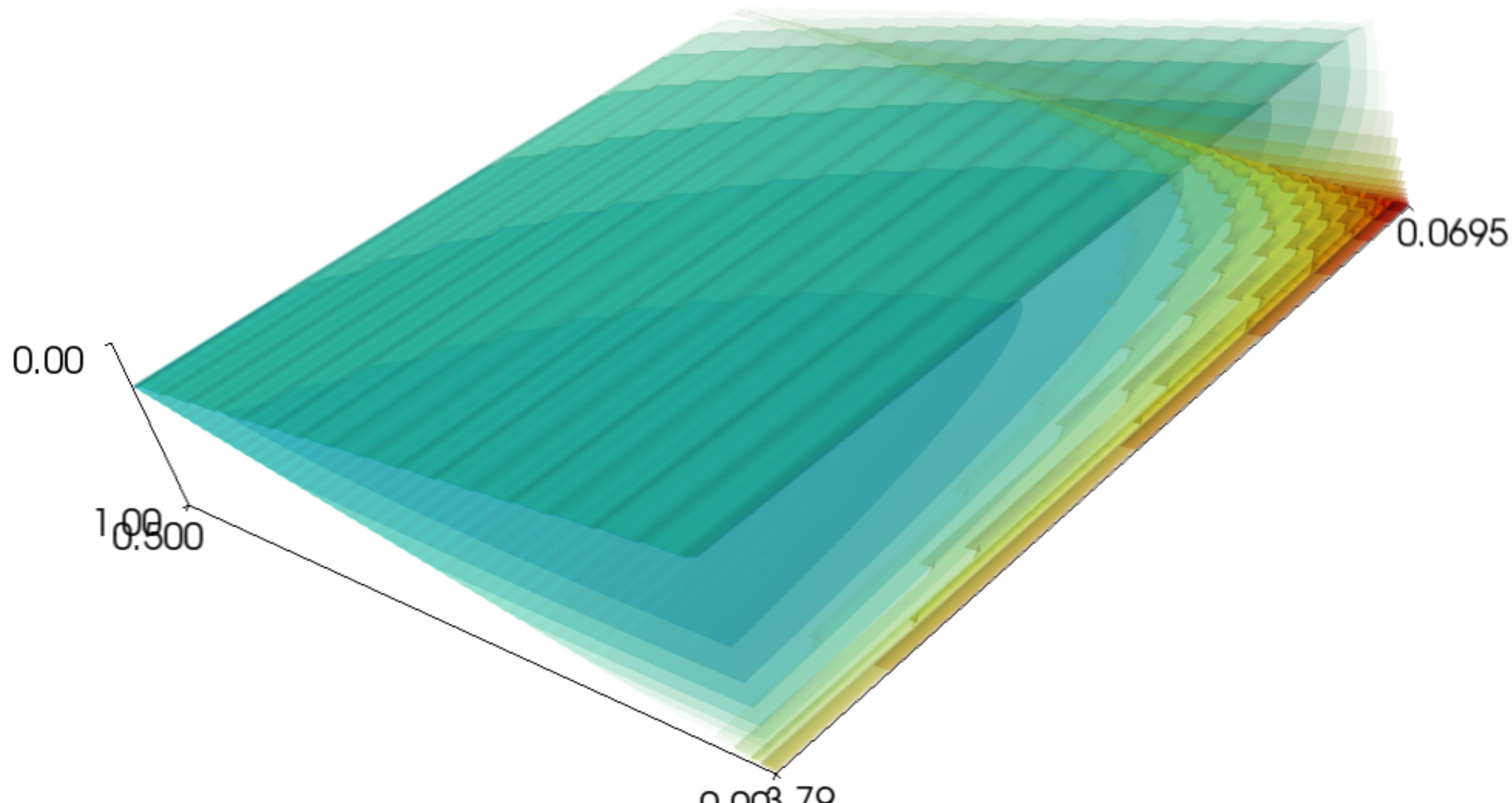


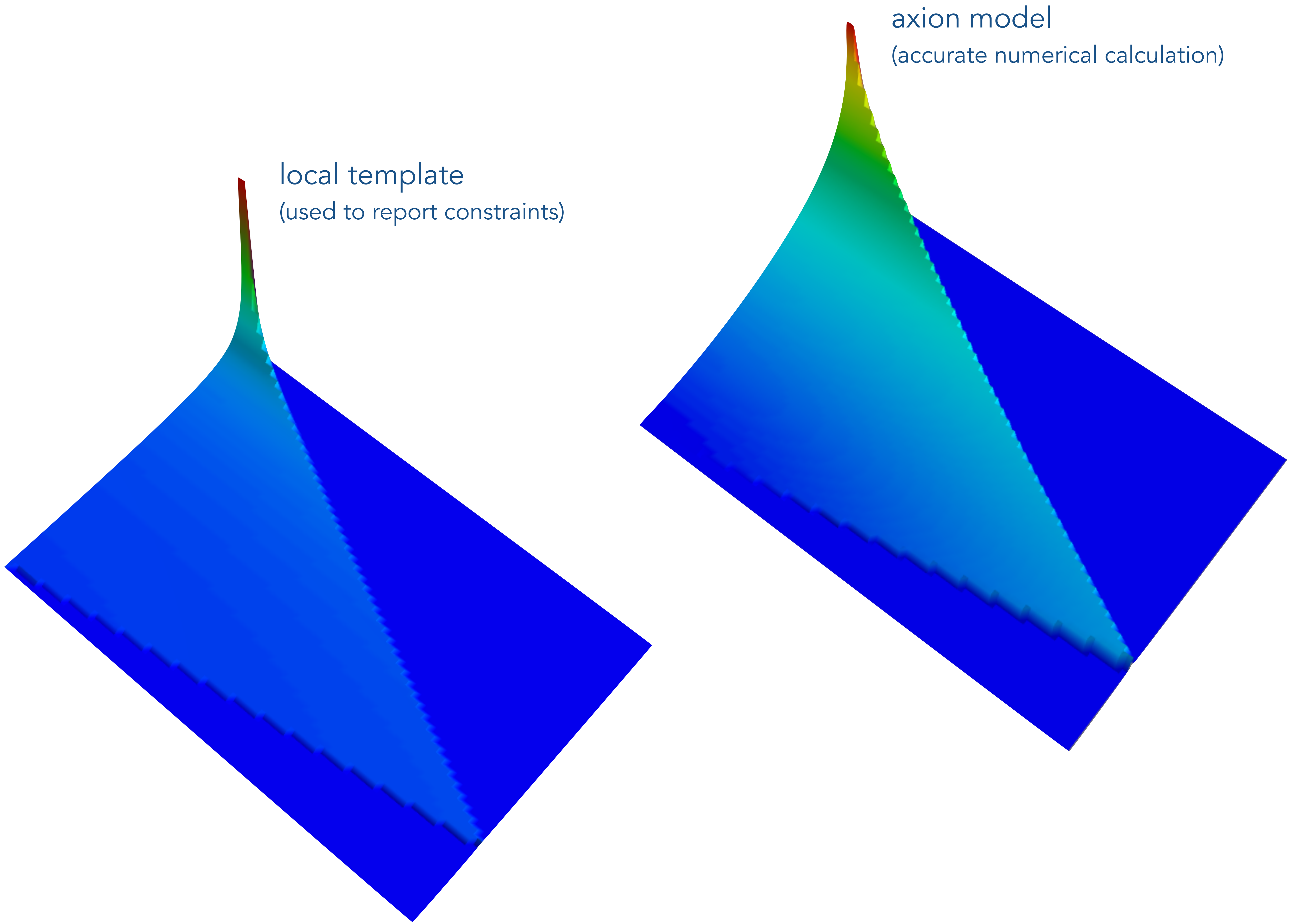
$$\text{amplitude} = f_{\rm NL}^{\rm local} = 0.8 \pm 5.0$$ (Planck2015 temperature+polarization)

Some models match the templates accurately, but others don't.
Numerical calculations are needed for more than just an estimate

axion + quadratic model with stronger scale dependence

$$V = \frac{1}{2}m^2\phi^2 + \Lambda^4 \cos\frac{2\pi\chi}{f}$$





local template
(used to report constraints)
axion model
(accurate numerical calculation)

Bispectrum codes (in chronological order)

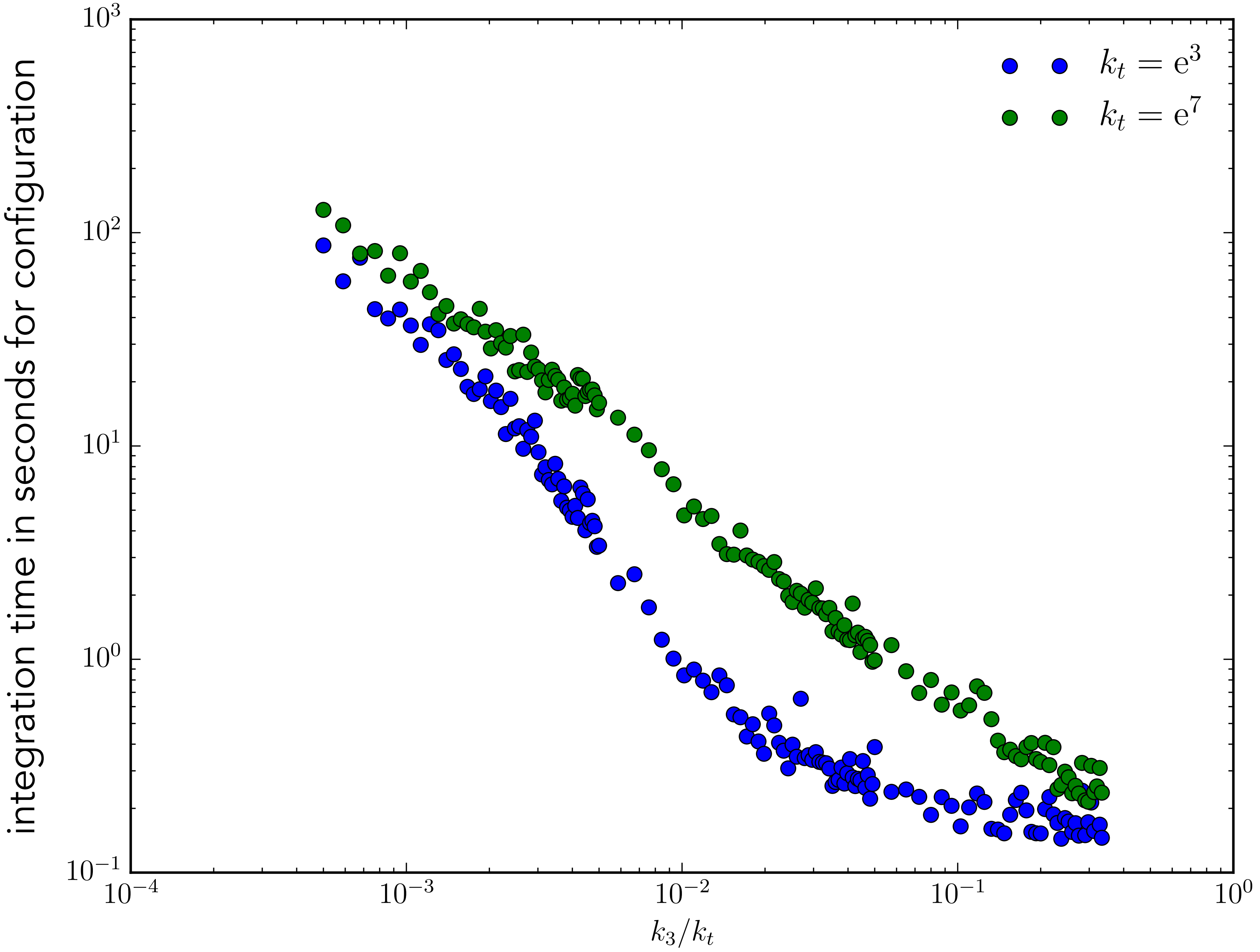
- BINGO, Hazra, Martin, Sreenath, Sriramkumar, **arXiv:1201.0926, 1410.0252** — FORTRAN)
  single-field only; https://sites.google.com/site/codecosmo/bingo

- Horner & Contaldi, **arXiv:1311.3224**
  single-field only (as far as I know); not publicly available

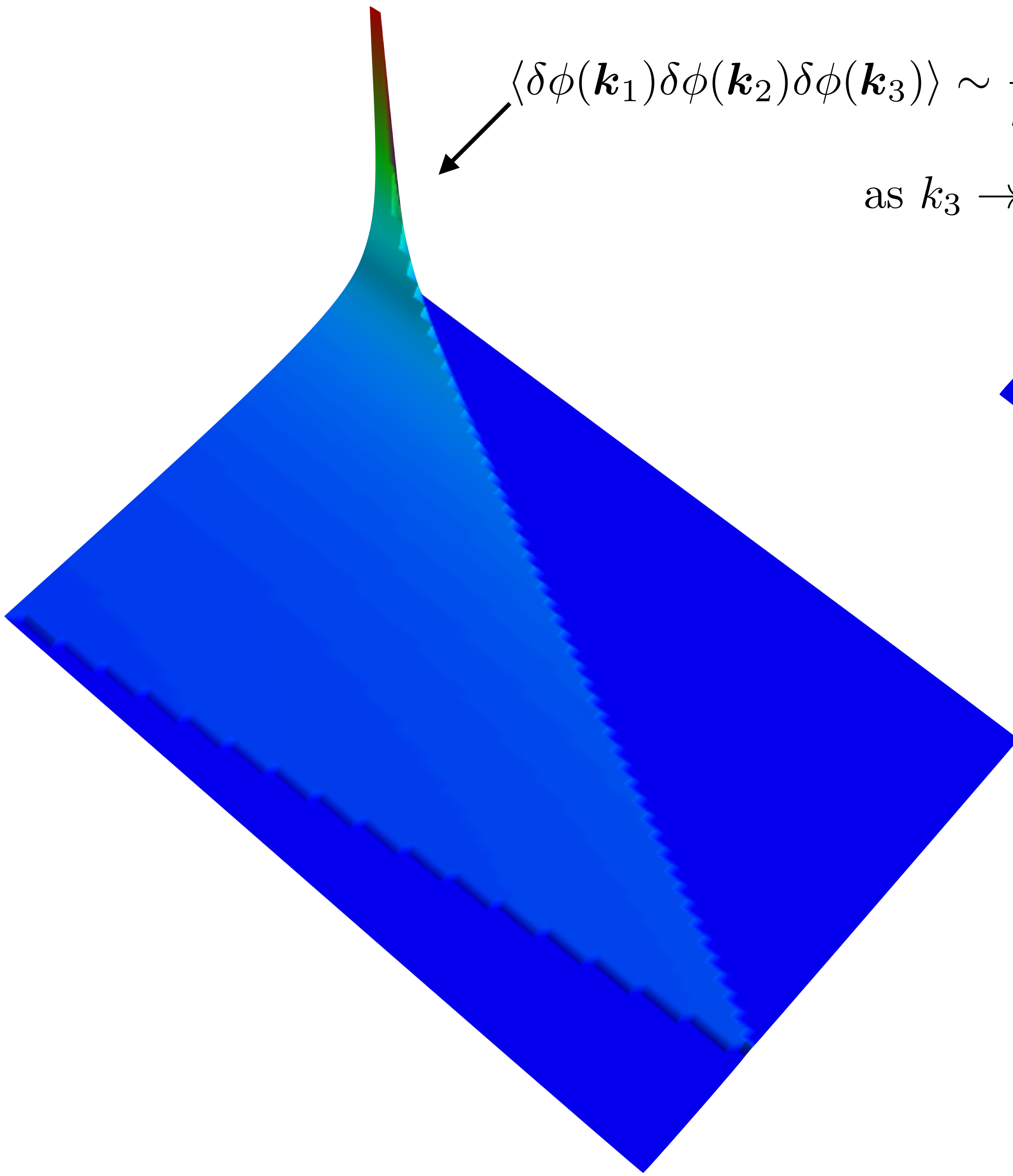
- Sussex & QMUL code in development (C++)
  for 3D iso-surface plot, used 173,502 configurations (likely more than is needed for constraints)
  average of 0.15 s/configuration = 7h 12m CPU time

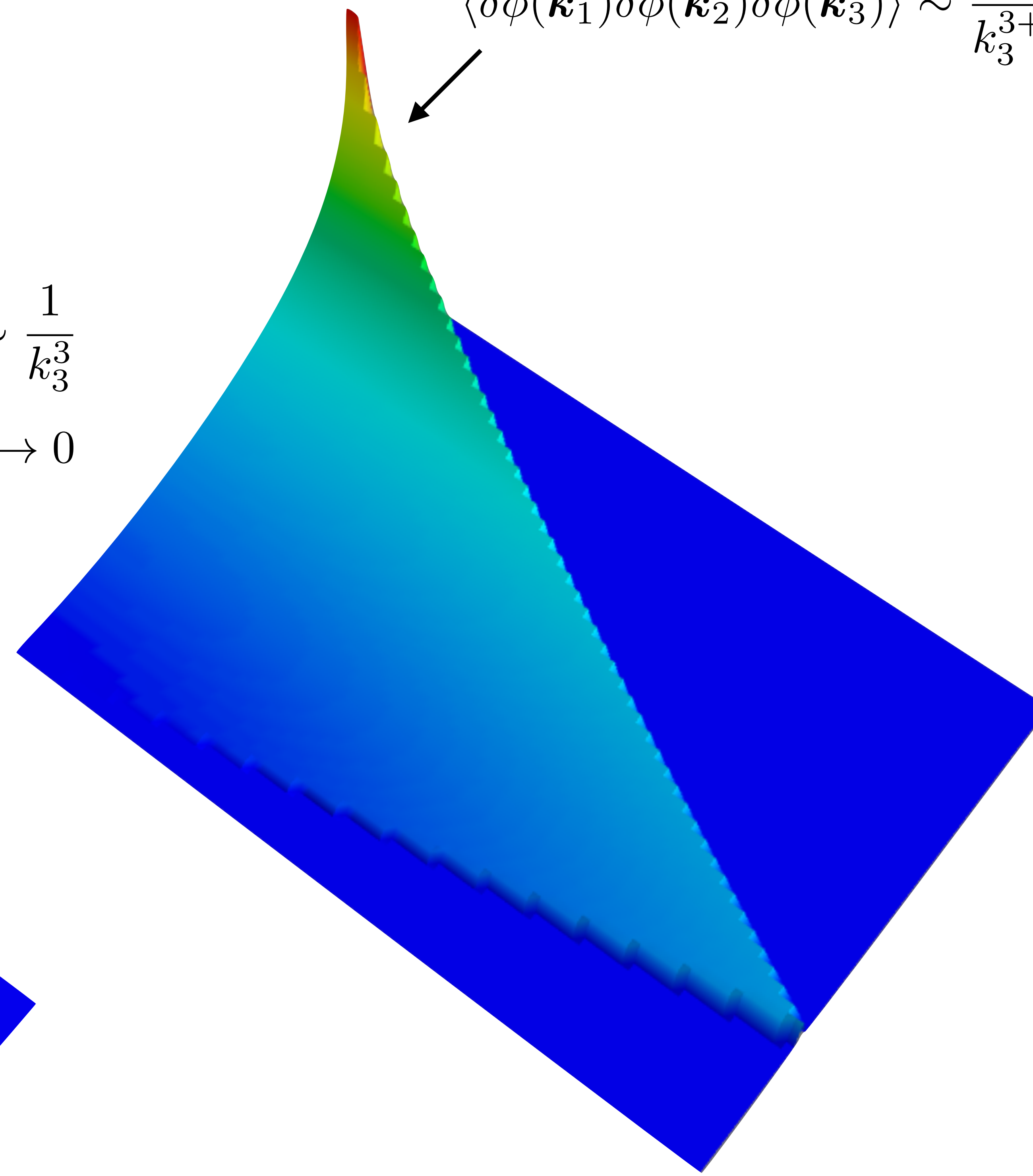
  however, that headline figure is a bit misleading

The squeezed limit is expensive to compute, but it's the easiest part of the bispectrum to see

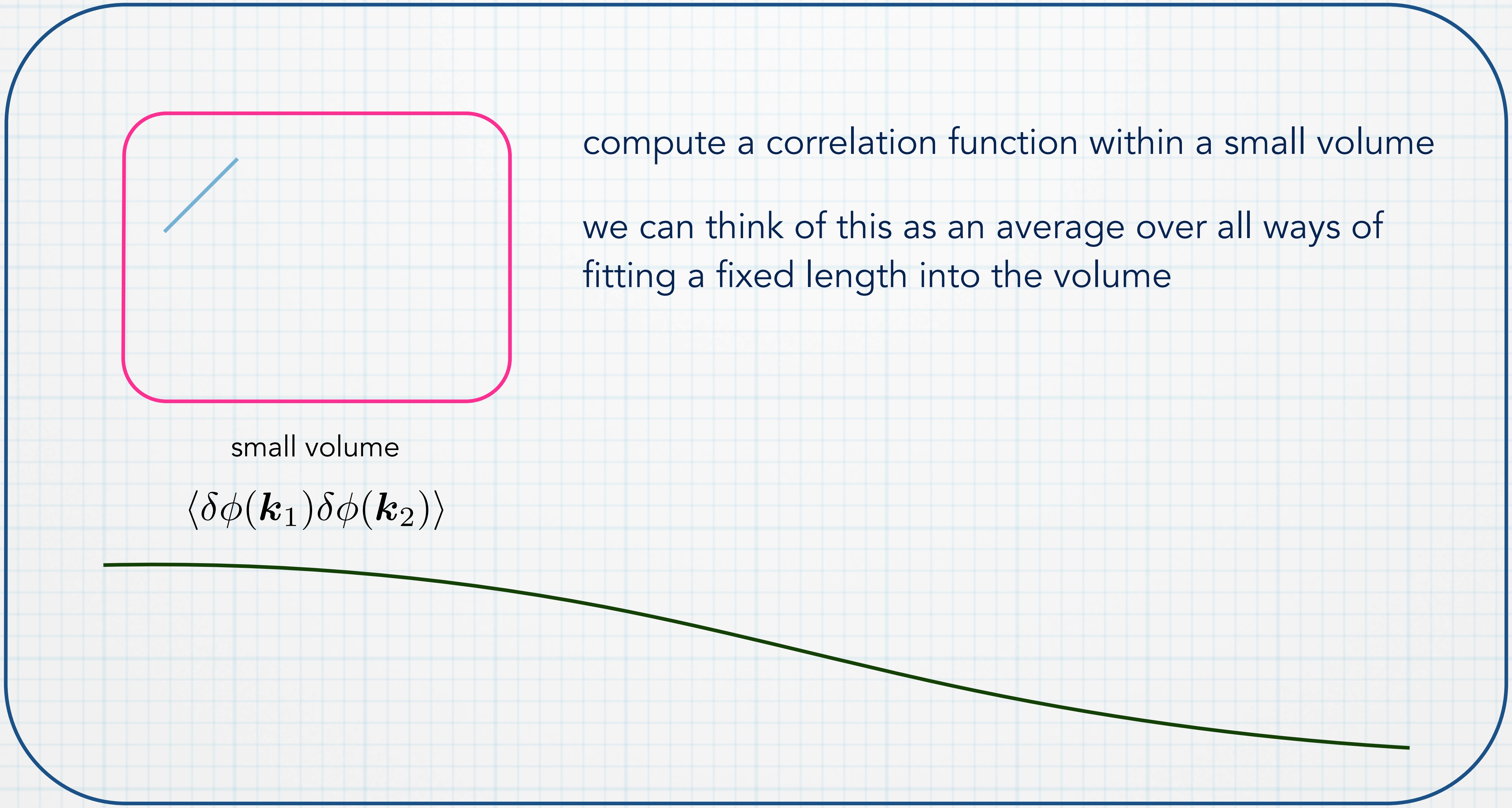


what does this spike mean?





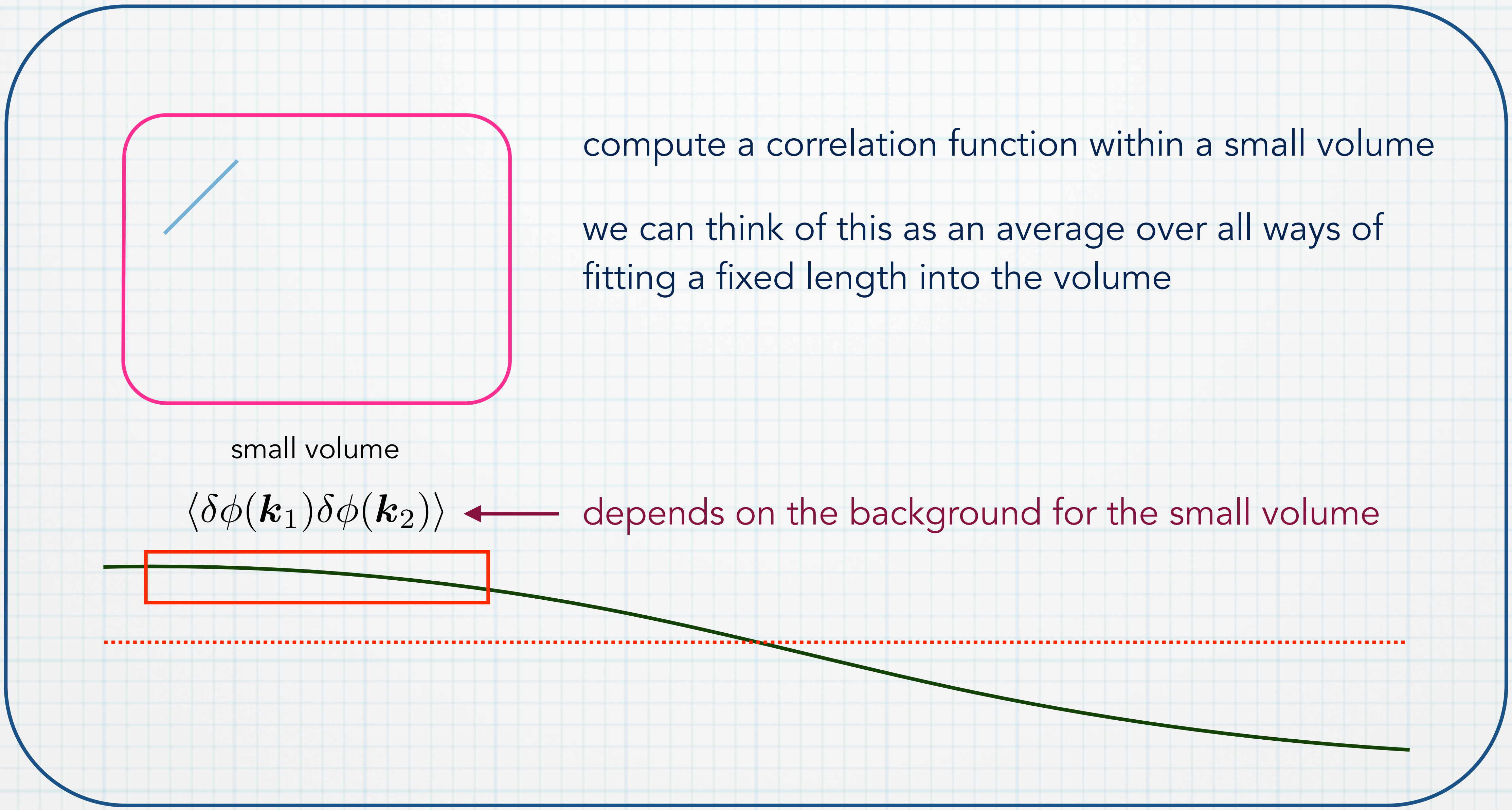
LARGE VOLUME



now suppose there are long wavelength modes crossing the large volume.

How does the correlation function depend on these long modes?

LARGE VOLUME



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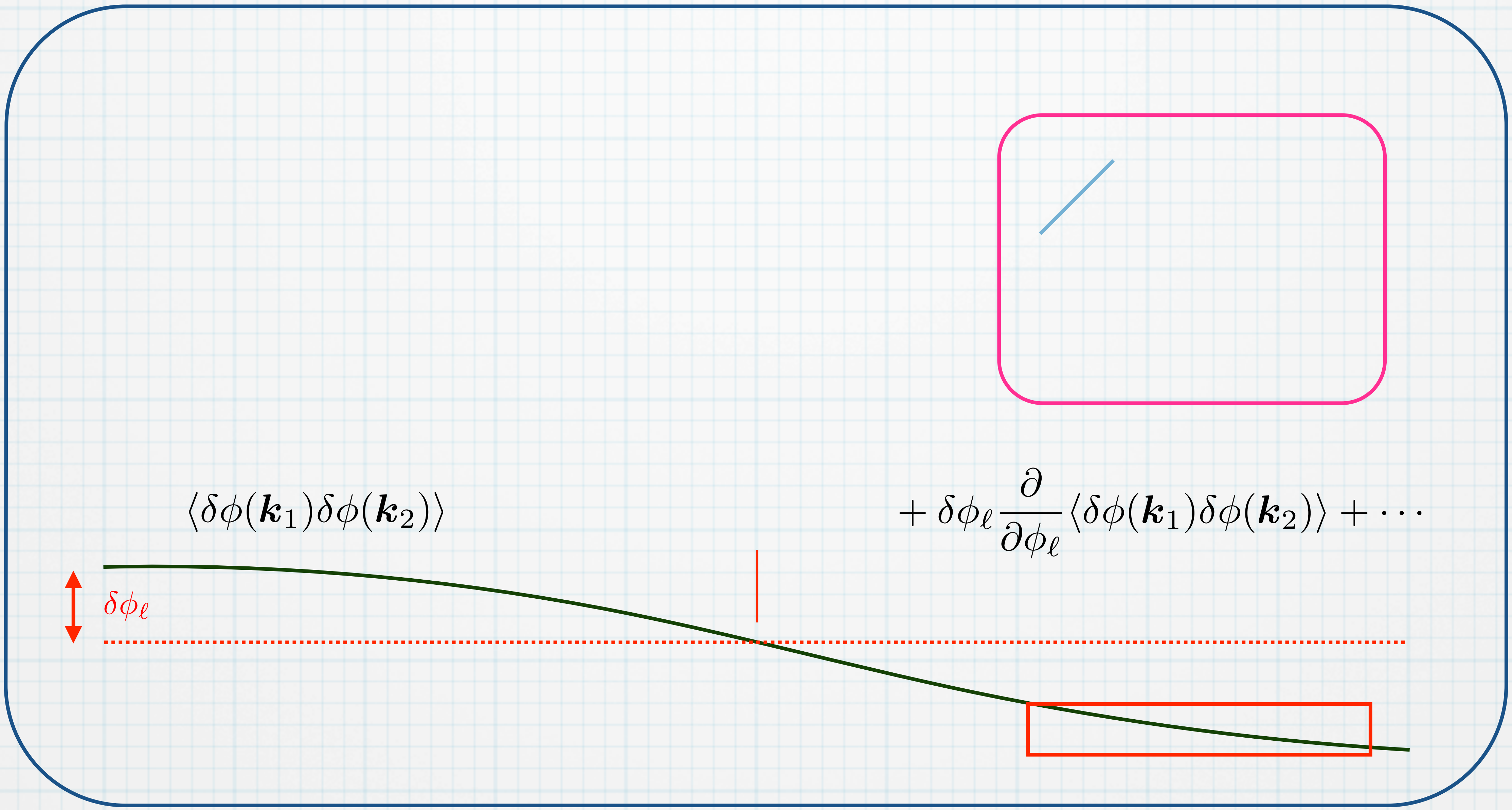


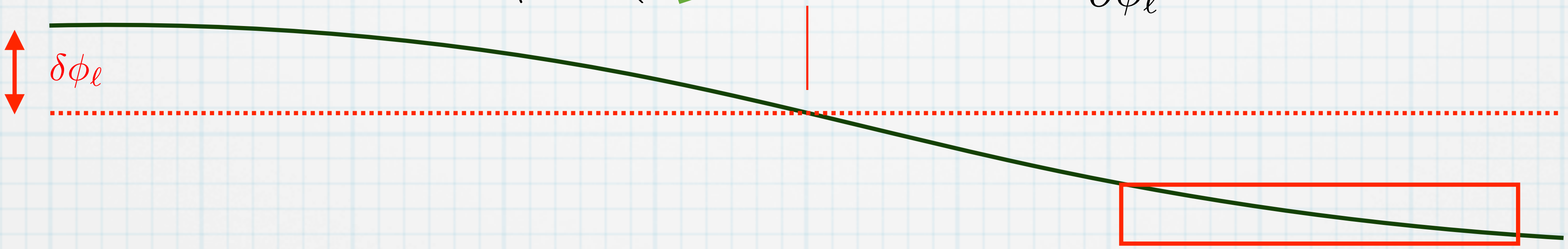
now suppose there are long wavelength modes crossing the large volume.

How does the correlation function depend on these long modes?

LARGE VOLUME

Finally, compute the correlation of the correlation in the small box with the long mode

$$\left\langle \delta\phi_\ell \left(\cancel{\langle \delta\phi(\boldsymbol{k}_1)\delta\phi(\boldsymbol{k}_2)\rangle} + \delta\phi_\ell \frac{\partial}{\partial\phi_\ell}\langle\delta\phi(\boldsymbol{k}_1)\delta\phi(\boldsymbol{k}_2)\rangle + \cdots \right)\right\rangle$$



now suppose there are long wavelength modes crossing the large volume.

How does the correlation function depend on these long modes?

in other words, roughly

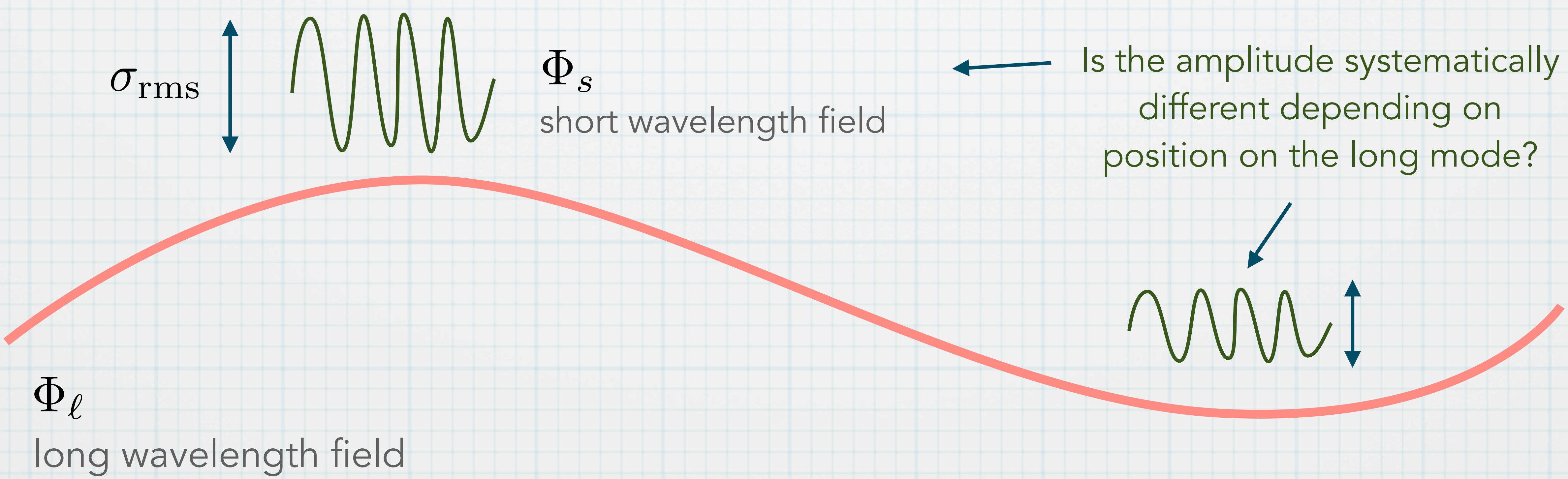
$$\langle \delta\phi(\boldsymbol{k}_1)\delta\phi(\boldsymbol{k}_2)\delta\phi(\boldsymbol{k}_3)\rangle \sim \langle \delta\phi(\boldsymbol{k}_3)\delta\phi(\boldsymbol{k}_3)\rangle \left\langle \frac{\partial}{\partial\phi_\ell}\langle \delta\phi(\boldsymbol{k}_1)\delta\phi(\boldsymbol{k}_2)\rangle \right\rangle$$

$k_3 \ll k_1, k_2$

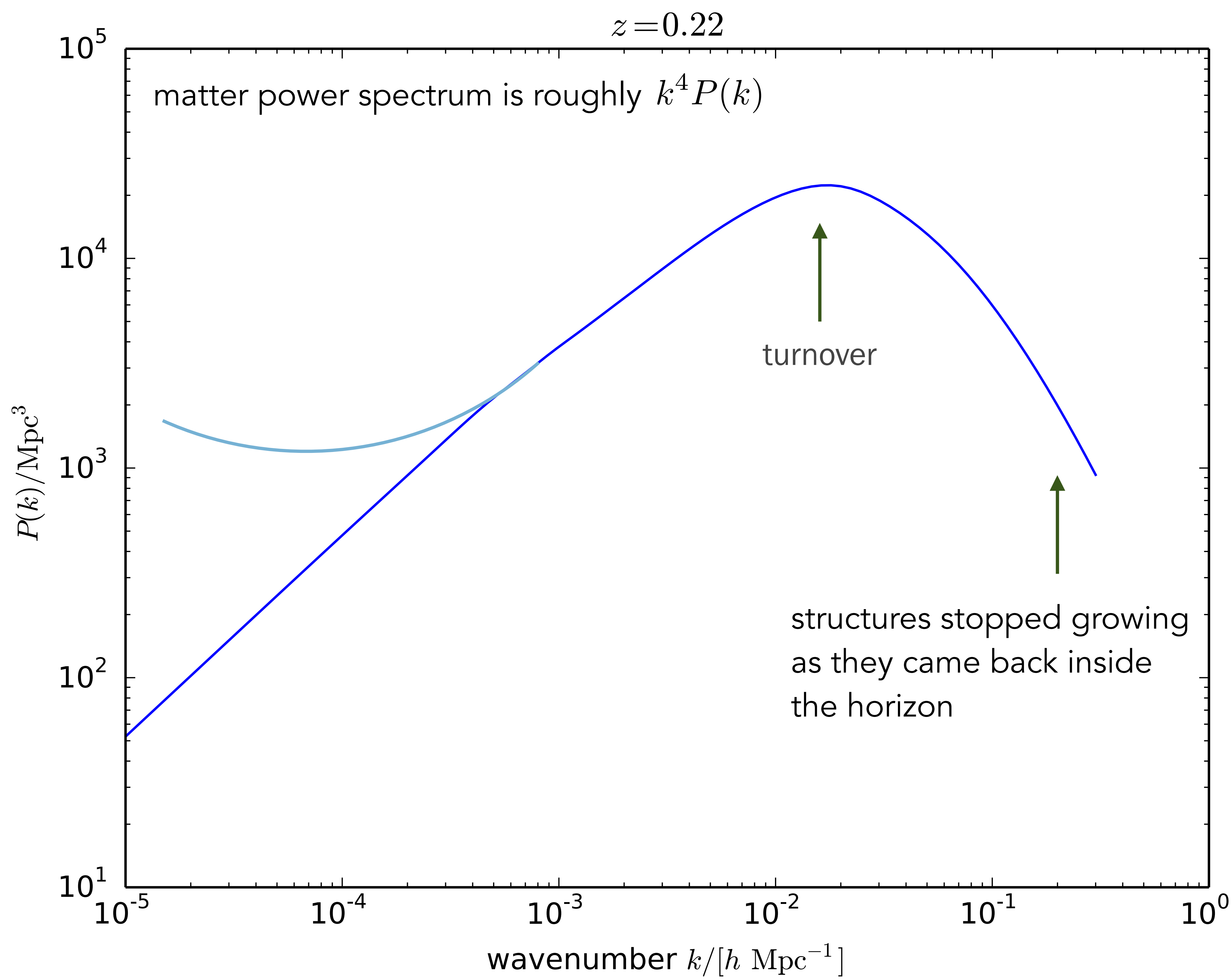
typical response of two-point function to a long-wavelength mode

so in the presence of a nontrivial bispectrum there is a correction to the power spectrum

$$\Delta\langle \delta\phi(\boldsymbol{k}_1)\delta\phi(\boldsymbol{k}_2)\rangle \sim \frac{B(k_1, k_2, k_\ell)}{P(k_\ell)} \sim \frac{k_\ell^{-3-\alpha}}{k_\ell^{-3+(n_s-1)}}$$



We can search for this by looking for an upturn in the clustering power on large scales



Galaxy survey measurements (from SDSS)