Higgs theory

Andrea Banfi

University of Sussex

The Higgs boson in the Standard Model

Outline

- The Brout-Englert-Higgs mechanism
- The Higgs boson in the Standard Model
- Stability of the Higgs potential
- Custodial symmetry

A historical prelude: Goldstone theorem

To understand the importance of the Brout-Englert-Higgs mechanism we need to go back to the Goldstone theorem

Our main characters:

- A conserved current $\partial_\mu J^\mu = 0$
- A set of scalar fields ϕ_a, on which the current can act $[J_\mu, \phi_a] = i t_{ab} \phi_b$

Example: $U(1)$ current and a complex scalar field ϕ : $[J_\mu, \phi] = i\phi$

If $\langle 0|\phi|0\rangle \neq 0$ (spontaneous symmetry breaking) the theory contains one massless "Goldstone" boson

Example of a Lagrangian leading to Goldstone bosons

$$\mathcal{L} = \partial_\mu \phi^* \partial^\mu \phi + \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

Note: the scalar field ϕ does not need to be an elementary field

Proof of the Goldstone theorem

The basic object entering the Goldstone theorem is the commutator of the current with the scalar field

$$\langle 0|[J_\mu(x),\phi(0)]|0\rangle = \int d^4p \left[e^{ip\cdot x}\sum_N \delta^4(p_N - p)\langle 0|J_\mu(0)|N\rangle\langle N|\phi(0)|0\rangle \right.$$
$$\left. - e^{-ip\cdot x}\sum_N \delta^4(p_N - p)\langle 0|\phi(0)|N\rangle\langle N|J_\mu(0)|0\rangle \right]$$

Exercise. Derive the above expression

Crucial point of Goldstone theorem is the Lorentz decomposition of the expectation value on the complete set of states $|N\rangle$

$$\sum_N \delta^4(p_N - p)\langle 0|J_\mu(0)|N\rangle\langle N|\phi(0)|0\rangle = ip_\mu\Theta(p^0)\rho(p^2)$$

$$\sum_N \delta^4(p_N - p)\langle 0|\phi(0)|N\rangle\langle N|J_\mu(0)|0\rangle = -ip_\mu\Theta(p^0)\rho(p^2)$$

Proof of the Goldstone theorem

Crucial point of Goldstone theorem is the Lorentz decomposition of the expectation value on the complete set of states $|N\rangle$

$$\sum_N \delta^4(p_N - p)\langle 0|J_\mu(0)|N\rangle\langle N|\phi(0)|0\rangle = ip_\mu \Theta(p^0)\rho(p^2)$$

$$\sum_N \delta^4(p_N - p)\langle 0|\phi(0)|N\rangle\langle N|J_\mu(0)|0\rangle = ip_\mu \Theta(p^0)\tilde{\rho}(p^2)$$

Exercise. Show that, because of causality, $\tilde{\rho}(p^2) = -\rho(p^2)$

Proof of the Goldstone theorem

The basic object entering the Goldstone theorem is the commutator of the current with the scalar field

$$\langle 0|[J_\mu(x), \phi(0)]|0\rangle = \partial_\mu \int_0^\infty dm^2 \Delta(x, m^2)\rho(m^2)$$

$$\Delta(x, m^2) \equiv \int d^4p\, \Theta(p^0)\delta(p^2 - m^2)\left[e^{ipx} - e^{-ipx}\right] \;\Rightarrow\; (\Box + m^2)\Delta(x, m^2) = 0$$

Exercise. Show that $\partial_\mu J^\mu = 0$ and $Q|0\rangle \neq 0 \;\Rightarrow\; \rho(m^2) = N\delta(m^2)$ with $N \neq 0$, i.e. there exists at least one massless state, a Goldstone boson

Problems with Goldstone theorem

There are as many massless Goldstone boson as the number of broken generators. However, no one has ever seen any. What is their fate?

Longitudinal waves in plasmas

Maxwell's equations in Lorenz gauge $k^\mu \tilde{A}^\mu(k) = 0$

$$k^2 \tilde{A}^\mu(k) = (\omega^2 - c^2 \vec{k}^2)\tilde{A}^\mu(k) = -\tilde{J}^\mu(k)$$

If $\tilde{J}^\mu(k) \propto \tilde{A}^\mu(k)$ we can induce a mass term for the electromagnetic field

In spite of a local conservation law, the corresponding gauge bosons are not massless

[Anderson '64]

Example. In the propagation of a plasma with velocity of sound c_s

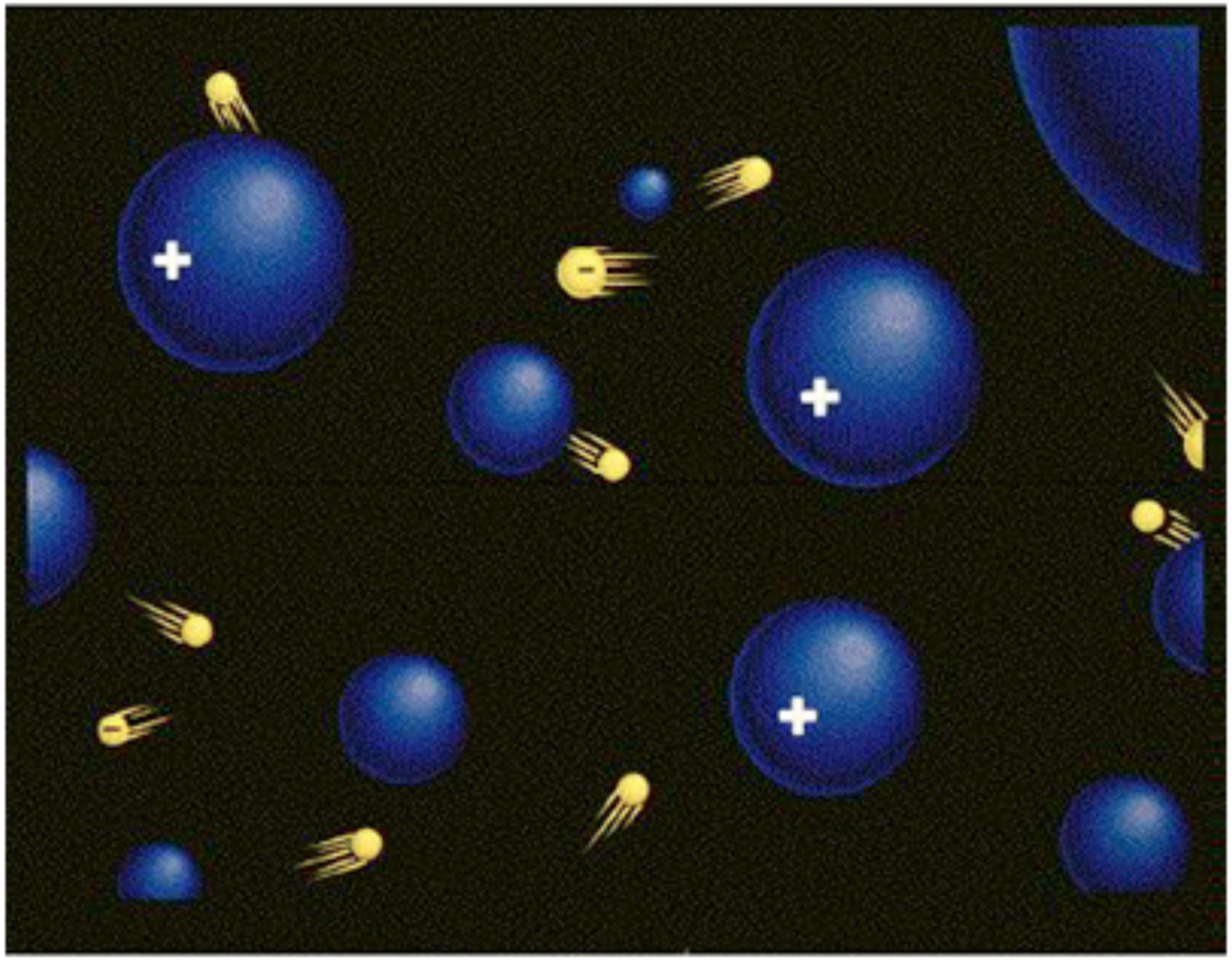
$$J^0(\omega, \vec{k}) = -\frac{\omega^2 - c^2\vec{k}^2}{\omega^2 - c_s^2\vec{k}^2}\omega_p^2\, A^0(\omega, \vec{k}) \qquad \vec{J}_\perp(\omega, k) = -\omega_p^2 \vec{A}_\perp(\omega, k)$$

$\omega_p = \sqrt{\dfrac{ne^2}{m}}$ is the plasma frequency of the medium

What have longitudinal waves in plasma have to do with Goldstone theorem?

Longitudinal waves in plasma

In a plasma, the ions are still, giving a uniform charge density $\rho_0 = n_e e$



The fluctuations of the electron density give a longitudinal wave, that propagates with the speed of sound of the plasma

Relation to Goldstone theorem

In a non-relativistic theory, there exists a special reference frame n^μ (e.g. the rest frame of the ions in a plasma)

$$p_\mu \rho(p^2) \to p_\mu \rho_1(p^2, n \cdot p) + n_\mu \rho_2(p^2, n \cdot p) + C_3 n_\mu \delta^4(p)$$

Imposing current conservation $\partial_\mu J^\mu = 0$

$$p_\mu \rho_1 + n_\mu \rho_2 \to p_\mu \delta(p^2) \rho_4 + \left[p^2 n_\mu - p_\mu (nk)\right] \rho_5$$

In non-relativistic theories, ρ_4 can be zero, thus avoiding the problem of Goldstone bosons.

Problem. In relativistic theories there seem to be no preferred reference frame, yet Goldstone bosons seem not to exist

Higgs' solution to Goldstone problem

BROKEN SYMMETRIES, MASSLESS PARTICLES AND GAUGE FIELDS

P. W. HIGGS
Tait Institute of Mathematical Physics, University of Edinburgh, Scotland

Received 27 July 1964

Recently a number of people have discussed the Goldstone theorem [1,2]: that any solution of a Lorentz-invariant theory which violates an internal symmetry operation of that theory must contain a massless scalar particle. Klein and Lee [3] showed that this theorem does not necessarily apply in non-relativistic theories and implied that their considerations would apply equally well to Lorentz-invariant field theories. Gilbert [4], however, gave a proof that the failure of the Goldstone theorem in the nonrelativistic case is of a type which cannot exist when Lorentz invariance is imposed on a theory. The purpose of this note is to show that Gilbert's argument fails for an important class of field theories, that in which the conserved currents are coupled to gauge fields.

Following the procedure used by Gilbert [4], let us consider a theory of two hermitian scalar fields

Higgs' solution to Goldstone problem

In order to quantise gauge theories, one needs to introduce a gauge-fixing condition, e.g. Coulomb gauge $n_\mu A^\mu = 0$

$$i\langle 0|[A_\mu(x), \phi_1(0)]|0\rangle|_{\mathrm{F.T.}} = p_\mu \rho_1 + \left[p^2 n_\mu - p_\mu (np)\right] \rho_2 + C_3 n_\mu \delta^4(p)$$

Combining with Maxwell's equations

$$\partial_\mu F^{\mu\nu} = J^\nu \qquad F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\Downarrow$$

$$i\langle 0|[J_\mu(x), \phi_1(0)]|0\rangle|_{\mathrm{F.T.}} = \left[p^2 n_\mu - p_\mu (np)\right] \rho(p^2, (np))$$

$$\Downarrow$$

In a broken gauge symmetry there are no massless Goldstone bosons

The Higgs model

VOLUME 13, NUMBER 16 PHYSICAL REVIEW LETTERS 19 OCTOBER 1964

BROKEN SYMMETRIES AND THE MASSES OF GAUGE BOSONS

Peter W. Higgs
Tait Institute of Mathematical Physics, University of Edinburgh, Edinburgh, Scotland
(Received 31 August 1964)

In a recent note[1] it was shown that the Goldstone theorem,[2] that Lorentz-covariant field theories in which spontaneous breakdown of symmetry under an internal Lie group occurs contain zero-mass particles, fails if and only if the conserved currents associated with the internal group are coupled to gauge fields. The purpose of the present note is to report that, as a consequence of this coupling, the spin-one quanta of some of the gauge fields acquire mass; the longitudinal degrees of freedom of these particles (which would be absent if their mass were zero) go over into the Goldstone bosons when the coupling tends to zero. This phenomenon is just the relativistic analog of the plasmon phenomenon to which Anderson[3] has drawn attention: that the scalar zero-mass excitations of a superconducting neutral Fermi gas become longitudinal plasmon modes of finite mass when the gas is charged.

The simplest theory which exhibits this behavior is a gauge-invariant version of a model used by Goldstone[2] himself: Two real[4] scalar fields φ_1, φ_2 and a real vector field A_μ interact through the Lagrangian density

$$L = -\tfrac{1}{2}(\nabla\varphi_1)^2 - \tfrac{1}{2}(\nabla\varphi_2)^2 - V(\varphi_1^2 + \varphi_2^2) - \tfrac{1}{4}F_{\mu\nu}F^{\mu\nu}, \qquad (1)$$

about the "vacuum" solution $\varphi_1(x) = 0$, $\varphi_2(x) = \varphi_0$:

$$\partial^\mu\{\partial_\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\} = 0, \qquad (2a)$$

$$\{\partial^2 - 4\varphi_0^2 V''(\varphi_0^2)\}(\Delta\varphi_2) = 0, \qquad (2b)$$

$$\partial_\nu F^{\mu\nu} = e\varphi_0\{\partial^\mu(\Delta\varphi_1) - e\varphi_0 A_\mu\}. \qquad (2c)$$

Equation (2b) describes waves whose quanta have (bare) mass $2\varphi_0\{V''(\varphi_0^2)\}^{1/2}$; Eqs. (2a) and (2c) may be transformed, by the introduction of new variables

$$B_\mu = A_\mu - (e\varphi_0)^{-1}\partial_\mu(\Delta\varphi_1),$$

$$G_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu = F_{\mu\nu}, \qquad (3)$$

into the form

$$\partial_\mu B^\mu = 0, \quad \partial_\nu G^{\mu\nu} + e^2\varphi_0^2 B^\mu = 0. \qquad (4)$$

Equation (4) describes vector waves whose quanta have (bare) mass $e\varphi_0$. In the absence of the gauge field coupling ($e = 0$) the situation is quite different: Equations (2a) and (2c) describe zero-mass scalar and vector bosons, respectively. In passing, we note that the right-hand side of (2c) is just the linear approximation to the conserved current: It is linear in the vector potential,

The Higgs model

Consider a pair of scalar field ϕ_1, ϕ_2 coupled to an electromagnetic field A_μ

$$\mathcal{L} = \frac{1}{2}(D_\mu\phi_1)(D^\mu\phi_1) + \frac{1}{2}(D_\mu\phi_2)(D^\mu\phi_2) - V\left(|\phi|^2\right) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}$$

$$D_\mu\phi_1 = \partial_\mu\phi_1 - eA_\mu\phi_2 \qquad D_\mu\phi_2 = \partial_\mu\phi_2 + eA_\mu\phi_1 \qquad |\phi|^2 = \frac{\phi_1^2 + \phi_2^2}{2}$$

The potential exhibits spontaneous symmetry breaking, e.g. $\langle 0|\phi_1|0\rangle = v \neq 0$, i.e. it has a minimum for

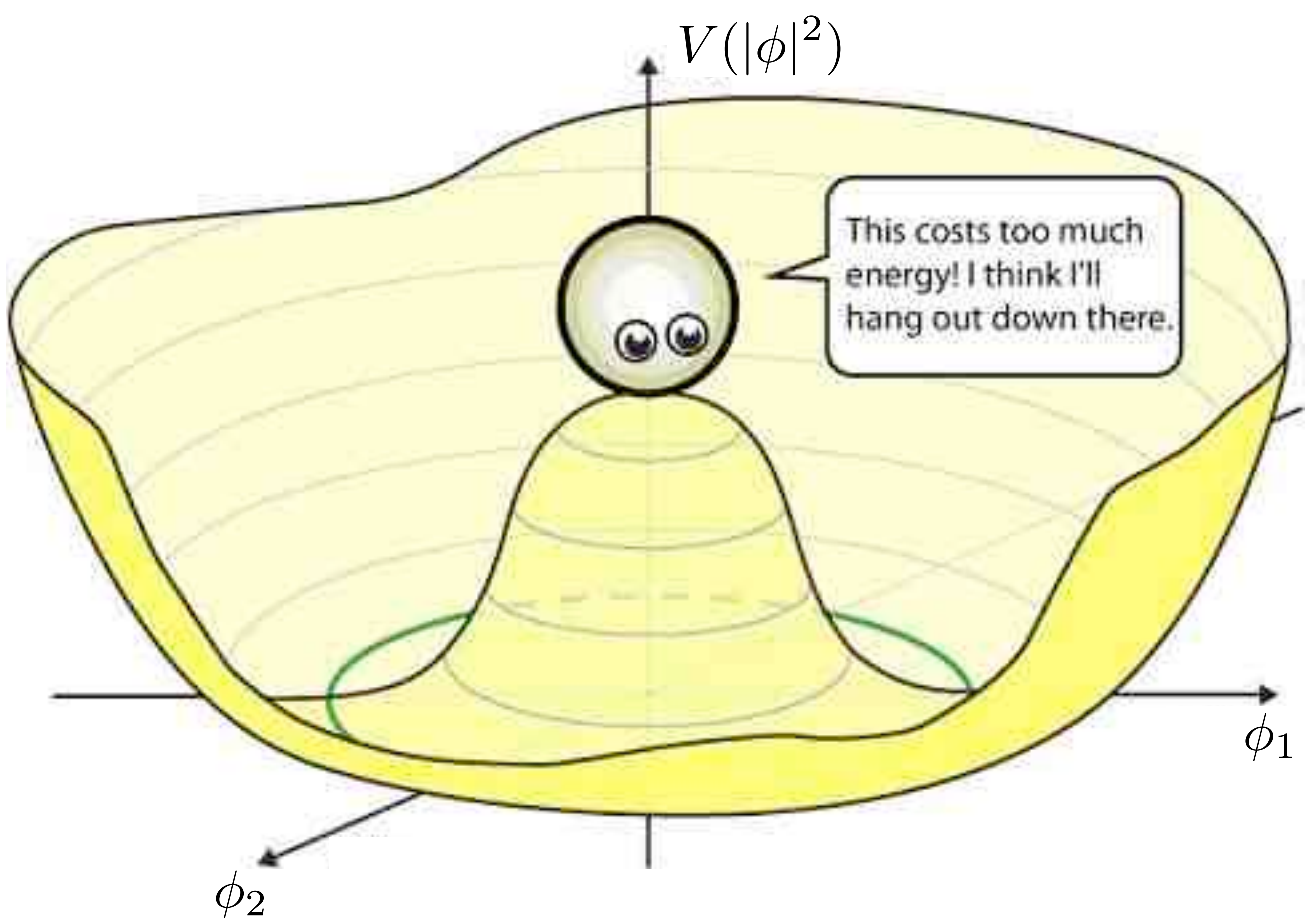
$$|\phi| = \frac{v}{\sqrt{2}}$$

Example. The potential

$$V(|\phi|^2) = -\mu^2|\phi|^2 + \frac{\lambda}{2}(|\phi|^2)^2$$

has a minimum for

$$|\phi| = \sqrt{\frac{\mu^2}{\lambda}}$$



Higgs model: mass for the photon

Expand all fields around the minimum

$$\phi_1 \simeq v + \eta_1 \qquad \phi_2 \simeq \eta_2 \qquad V(|\phi|^2) \simeq V_0 + \frac{V_0''}{2} v^2 \eta_1^2$$

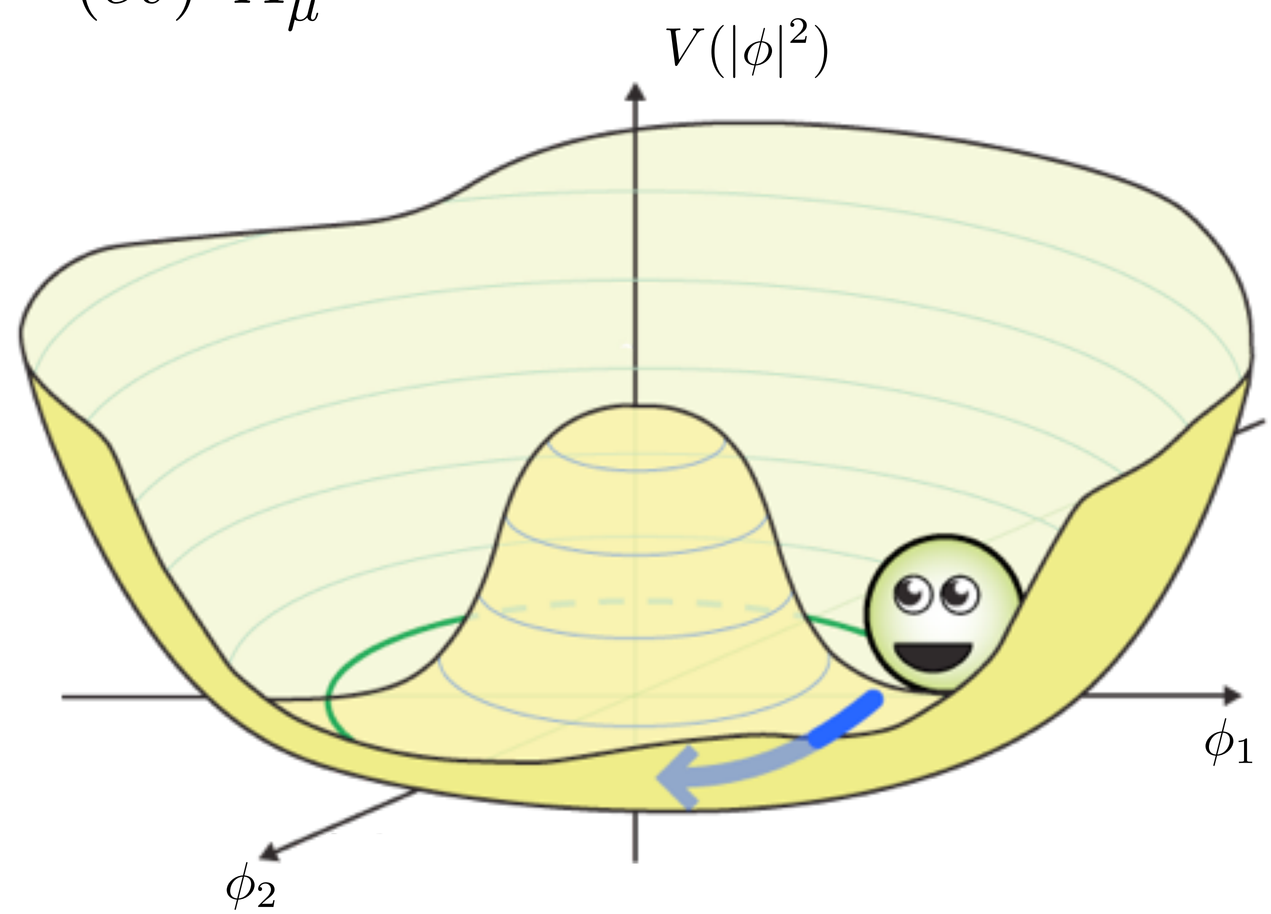
and linearise the equations of motions, assuming $A_\mu \sim \eta_1 \sim \eta_2 \ll v$

$$\partial^\mu(\partial_\mu \eta_1 + evA_\mu) = \partial^\mu A'_\mu = 0 \qquad A'_\mu = A_\mu + \partial_\mu(\eta_2/ev)$$

$$\partial^\mu F_{\mu\nu} = J_\nu \simeq -ev(\partial_\mu \eta_2 + evA_\mu) = -(ev)^2 A'_\mu$$

This is the Anderson condition, giving a mass $m_A = ev$ to the photon

Note. The fact that the photon gets a mass relies only on the fact that the scalar field gets a VEV



The calculation of Englert and Brout

BROKEN SYMMETRY AND THE MASS OF GAUGE VECTOR MESONS*

F. Englert and R. Brout
Faculté des Sciences, Université Libre de Bruxelles, Bruxelles, Belgium
(Received 26 June 1964)

It is of interest to inquire whether gauge vector mesons acquire mass through interaction[1]; by a gauge vector meson we mean a Yang-Mills field[2] associated with the extension of a Lie group from global to local symmetry. The importance of this problem resides in the possibility that strong-interaction physics originates from massive gauge fields related to a system of conserved currents.[3] In this note, we shall show that in certain cases vector mesons do indeed acquire mass when the vacuum is degenerate with respect to a compact Lie group.

Theories with degenerate vacuum (broken symmetry) have been the subject of intensive study since their inception by Nambu.[4-6] A characteristic feature of such theories is the possible existence of zero-mass bosons which tend to restore the symmetry.[7,8] We shall show that it is precisely these singularities which maintain the gauge invariance of the theory, despite the fact that the vector meson acquires mass.

We shall first treat the case where the original fields are a set of bosons φ_A which transform as a basis for a representation of a compact Lie group. This example should be considered as a rather general phenomenological model. As such, we shall not study the particular mechanism by which the symmetry is broken but simply assume that such a mechanism exists. A calculation performed in lowest order perturbation theory indicates that those vector mesons which are coupled to currents that "rotate" the original vacuum are the ones which acquire mass [see Eq. (6)].

We shall then examine a particular model based on chirality invariance which may have a more fundamental significance. Here we begin with a chirality-invariant Lagrangian and introduce both vector and pseudovector gauge fields, thereby guaranteeing invariance under both local phase and local γ_5-phase transformations. In this model the gauge fields themselves may break the γ_5 invariance leading to a mass for the original Fermi field. We shall show in this case that the pseudovector field acquires mass.

In the last paragraph we sketch a simple argument which renders these results reasonable.

(1) Lest the simplicity of the argument be shrouded in a cloud of indices, we first consider a one-parameter Abelian group, representing, for example, the phase transformation of a charged boson; we then present the generalization to an arbitrary compact Lie group.

The interaction between the φ and the A_μ fields is

$$H_{\text{int}} = ieA_\mu \varphi^* \overleftrightarrow{\partial}_\mu \varphi - e^2 \varphi^* \varphi A_\mu A_\mu, \tag{1}$$

where $\varphi = (\varphi_1 + i\varphi_2)/\sqrt{2}$. We shall break the symmetry by fixing $\langle\varphi\rangle \neq 0$ in the vacuum, with the phase chosen for convenience such that $\langle\varphi\rangle = \langle\varphi^*\rangle = \langle\varphi_1\rangle/\sqrt{2}$.

We shall assume that the application of the

Higgs model: massive scalar boson

Expand all fields around the minimum

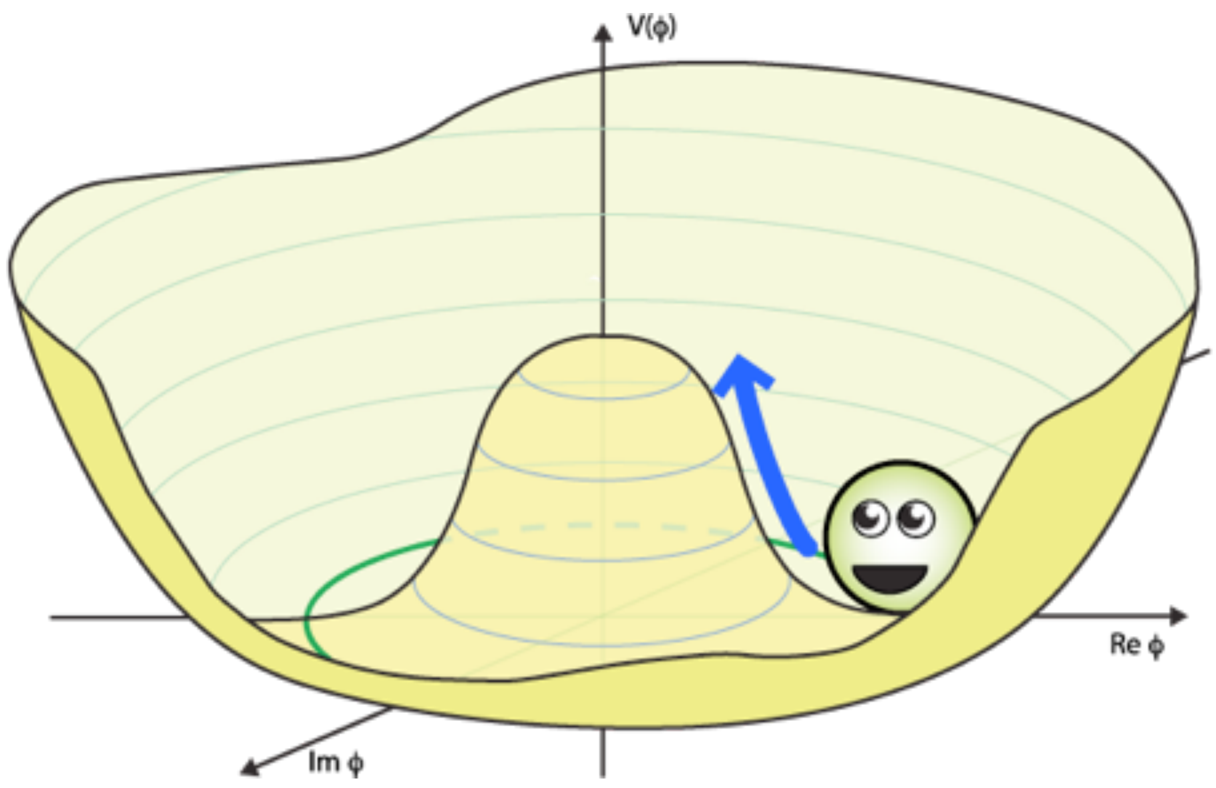
$$\phi_1 \simeq v + \eta_1 \qquad \phi_2 \simeq \eta_2 \qquad V(|\phi|^2) \simeq V_0 + \frac{V_0''}{2} v^2 \eta_1^2$$

and linearise the equations of motions, assuming $A_\mu \sim \eta_1 \sim \eta_2 \ll v$

$$(\Box + v^2 V_0'')\eta_1 = 0$$

This is the equation for a scalar ``Higgs'' boson of mass $m_H = v\sqrt{V_0''}$

Note. The fact that a boson is found relies on the reliability of the linear approximation, i.e. quantum fluctuations leave η_1 around the minimum



The Higgs field in the Standard Model

The Standard Model for EW interactions has $SU(2) \times U(1)$ gauge symmetry

$$\mathcal{L}_{\text{gauge}} = -\frac{1}{4}\text{Tr}(F_{\mu\nu}F^{\mu\nu}) - \frac{1}{4}B_{\mu\nu}B^{\mu\nu}$$

$$F^i_{\mu\nu} = \partial_\mu W^i_\nu - \partial_\nu W^i_\mu - g_2\epsilon^{ijk}W^j_\mu W^k_\nu$$

$$B^i_{\mu\nu} = \partial_\mu B_\nu - \partial_\nu B_\mu$$

To give a mass to the vector bosons we introduce a complex Higgs doublet

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$$

with a Lagrangian

$$\mathcal{L}_H = (D^\mu\Phi)^*(D_\mu\Phi) - V(\Phi^*\Phi)$$

$$D_\mu = \mathbf{I}\left(\partial_\mu + i\frac{g_1}{2}B_\mu\right) + ig_2\frac{\vec{\sigma}}{2}\cdot\vec{W}_\mu$$

The BEH mechanism in the Standard Model

Impose spontaneous symmetry breaking

$$\Phi = \frac{1}{\sqrt{2}} \begin{pmatrix} \phi_3 + i\phi_4 \\ \phi_1 + i\phi_2 \end{pmatrix} \qquad \langle 0|\phi_1|0\rangle = v \neq 0 \quad \Rightarrow \quad \langle 0|\Phi^\dagger \Phi|0\rangle = \frac{v^2}{2}$$

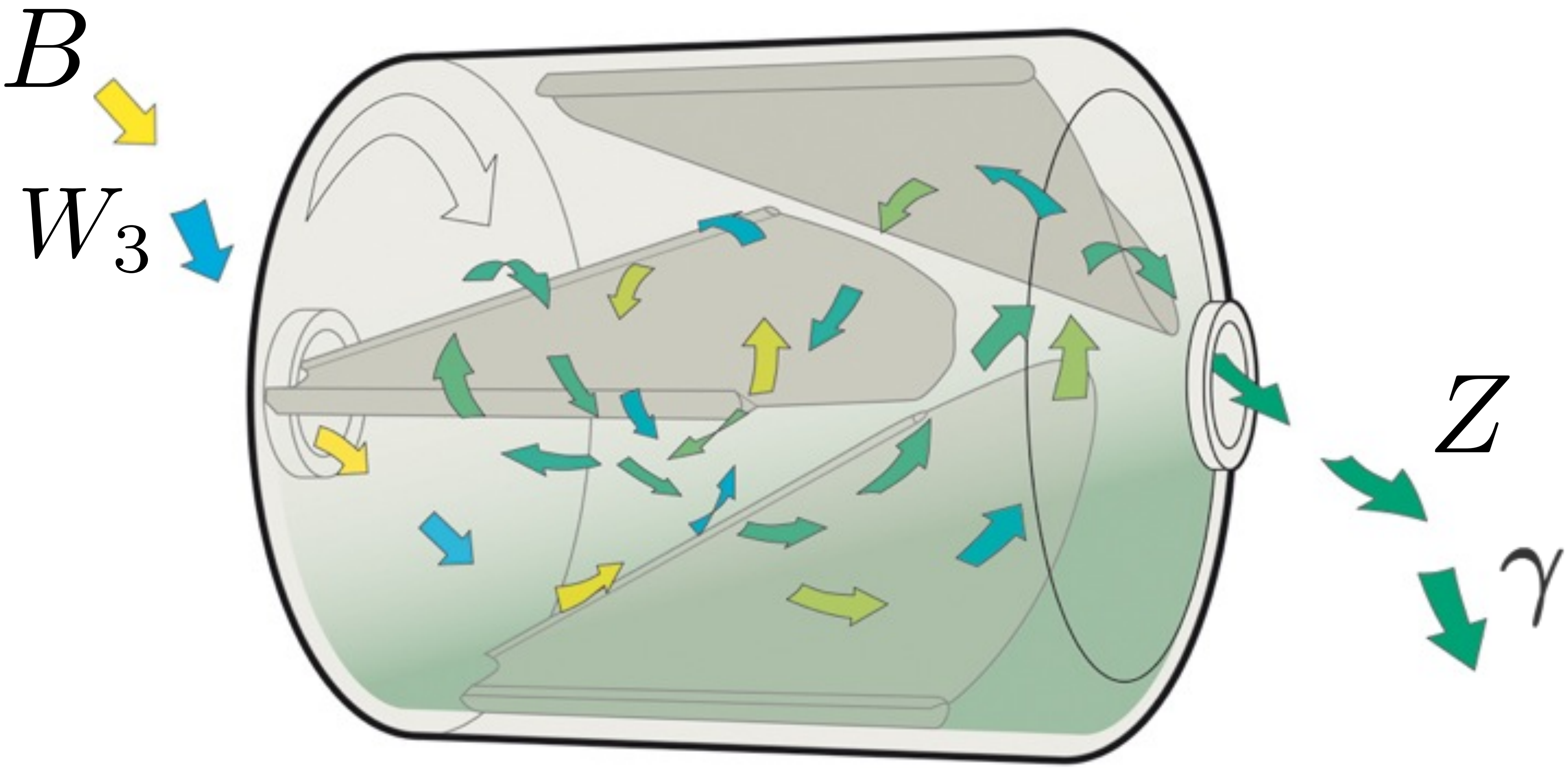
This is enough to give mass to the vector bosons through the coupling with the Higgs field, without any information on the potential

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp i W_\mu^2)$$

$$(D^\mu \Phi)^* (D_\mu \Phi) \to \left(\frac{v g_2}{2}\right)^2 W_\mu^+ W_-^\mu$$
$$+ \frac{v^2}{8} (W_\mu^3 \; B_\mu) \begin{pmatrix} g_2^2 & -g_1 g_2 \\ -g_1 g_2 & g_1^2 \end{pmatrix} \begin{pmatrix} W_3^\mu \\ B^\mu \end{pmatrix}$$

Vector boson mass eigenstates

Diagonalising the mass matrix one obtains the masses of the eigenstates, the W and Z bosons and the photon

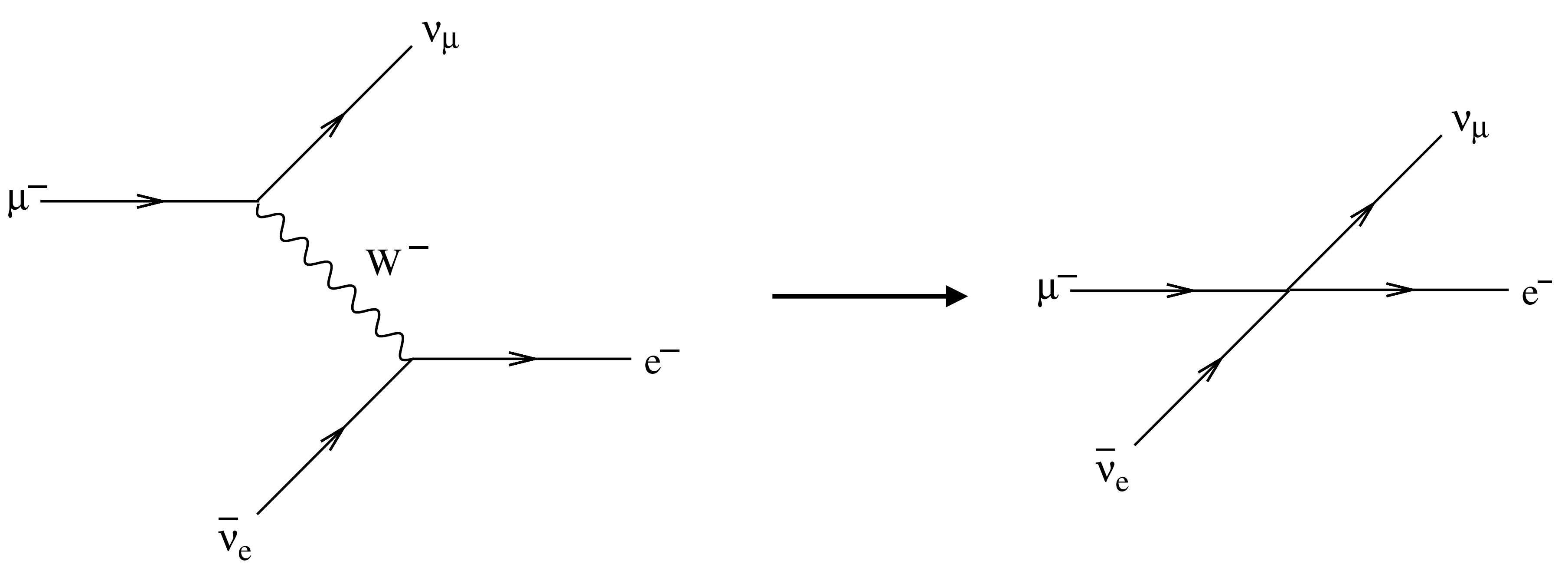


$$M_W = \frac{v}{2} g_2 \qquad M_Z = \frac{v}{2}\sqrt{g_1^2 + g_2^2} \qquad M_\gamma = 0$$

The VEV of the Higgs field

To measure the VEV of the Higgs field we need the coupling g_2 , which we obtain from weak decay, involving left particles only

$$L = \begin{pmatrix} \nu \\ \psi_L \end{pmatrix} \qquad \mathcal{L}_{\rm ew} = i\bar{L}\not{D}L \qquad D_\mu = \mathbf{I}\left(\partial_\mu + i\frac{g_1}{2}B_\mu Y_{\rm w}\right) + ig_2\frac{\vec{\sigma}}{2}\cdot\vec{W}_\mu$$



$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2}$$

$$M_W = \frac{v}{2}g_2 \qquad \Rightarrow \qquad v = (\sqrt{2}G_F)^{-1/2} \simeq 246\,\text{GeV}$$

The Higgs potential in the Standard Model

The Higgs potential in the Standard Model is the simplest compatible with spontaneous symmetry breaking and renormalisability

$$V(\Phi) = -\mu^2 \Phi^* \Phi + \lambda (\Phi^* \Phi)^2 \qquad v = \sqrt{\frac{\mu^2}{\lambda}}$$

Expanding around the minimum of the potential, as follows

$$\Phi = \frac{1}{\sqrt{2}} \left(\begin{array}{c} \sqrt{2}\phi^+ \\ v + H + i a^0 \end{array} \right)$$

we find a single neutral Higgs boson H with a mass $m_H = v\sqrt{2\lambda}$

Recently LHC has discovered a boson compatible with the Higgs of the Standard Model, with a mass $m_H \simeq 125\,\mathrm{GeV}$

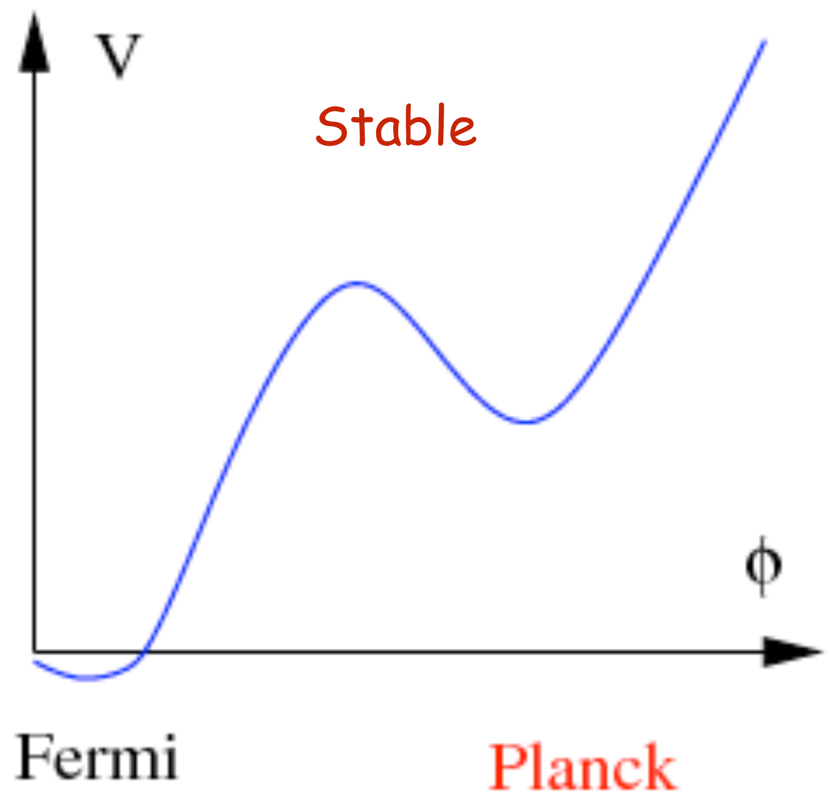
This is enough to fix all the parameters of the Higgs potential

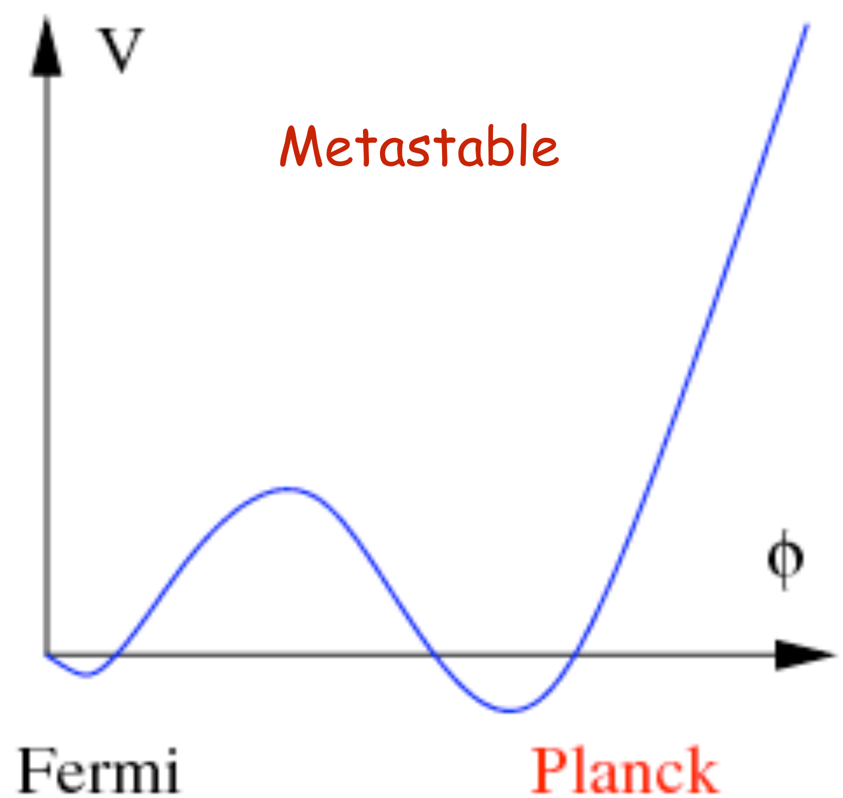
$$\mu \simeq 88.8\,\mathrm{GeV} \qquad \lambda \simeq 0.13$$

Stability of the Higgs potential

In order to have spontaneous symmetry breaking, we need to have $\lambda > 0$

But the value of λ can change with the value of ϕ $\Rightarrow$ problem with the stability of the electroweak vacuum





To compute the running of λ we need to introduce the interaction of the Higgs boson with fermions, which are Yukawa interactions

Fermion masses

To compute the running of λ we need to introduce the interaction of the Higgs boson with fermions, which are Yukawa interactions

$$\mathcal{L}_{\mathrm{Yukawa}} = -\hat{h}_d^{ij}\bar{Q}_{L,i}\Phi d_{R,j} - \hat{h}_u^{ij}\bar{Q}_{L,i}\tilde{\Phi} u_{R,j} - \hat{h}_L^{ij}\bar{L}_i\Phi e_{R,j} + \mathrm{h.c.}$$

In the Standard Model $\tilde{\Phi} = i\sigma_2\Phi^*$ (otherwise 2HDM)

At the minimum of the Higgs potential

$$\Phi = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 \\ v \end{pmatrix} \qquad \tilde{\Phi} = \frac{1}{\sqrt{2}}\begin{pmatrix} v \\ 0 \end{pmatrix}$$

after diagonalisation of the Yukawa couplings each fermion acquires a mass

$$m_f = h_f \frac{v}{\sqrt{2}}$$

Higgs interactions

Expanding $\mathcal{L}_{\rm gauge}, \mathcal{L}_{\rm Yukawa}$ and $V(\Phi)$ around the minimum of the Higgs potential we find all interactions of SM particles with the Higgs

$$\mathcal{L} = -g_{Hff} \bar{f} f H + \frac{g_{HHH}}{6} H^3 + \frac{g_{HHHH}}{24} H^4 + \delta_V V_\mu V^\mu \left(g_{HVV} H + \frac{g_{HHVV}}{2} H^2 \right)$$

In the SM all these couplings are constrained and proportional to the masses of fermions and to the masses of boson squared

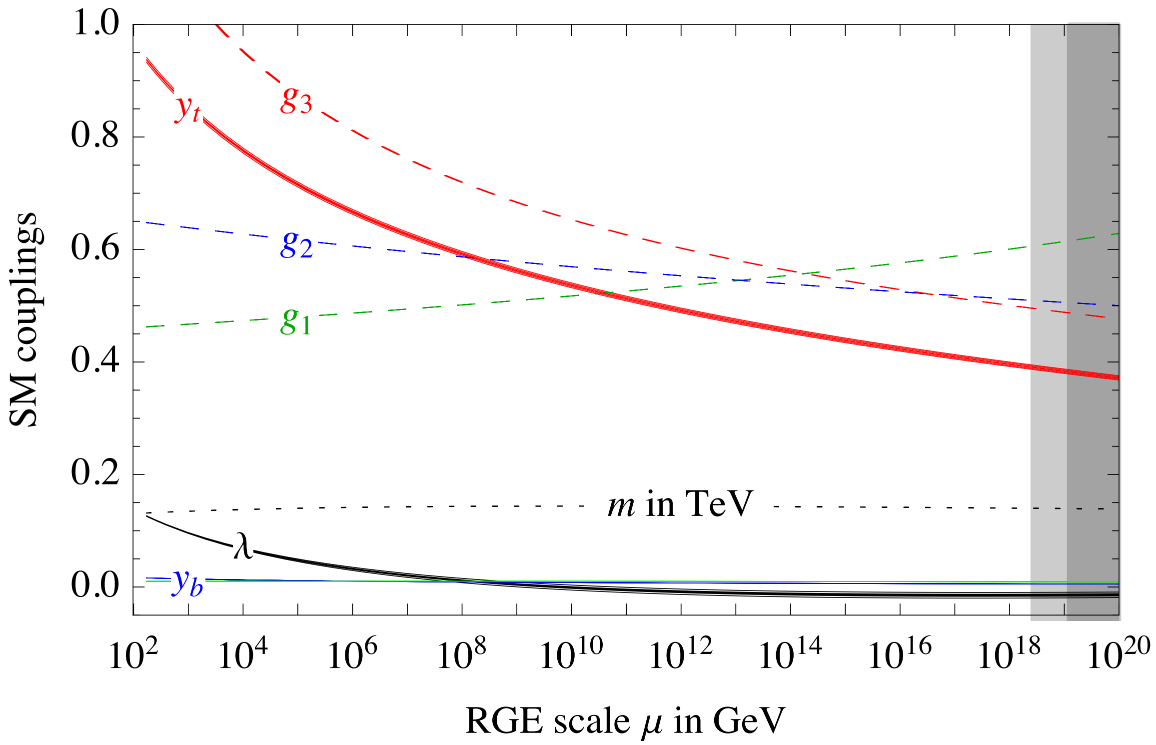
$$g_{Hff} = \frac{m_f}{v} \qquad g_{HVV} = \frac{2m_V^2}{v} \qquad g_{HHVV} = \frac{2m_V^2}{v^2} \qquad \delta_W = 1 \quad \delta_Z = 1/2$$

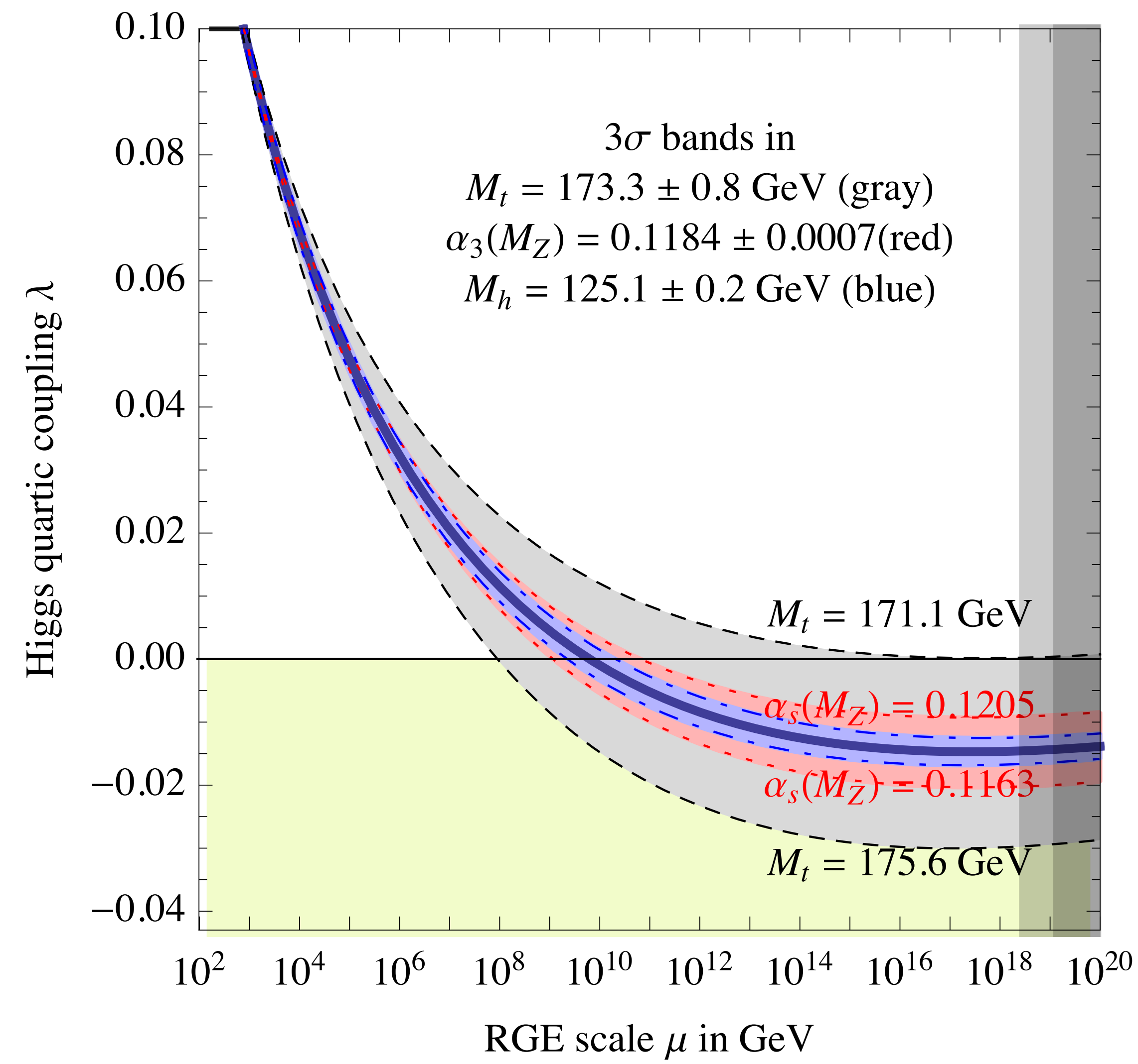
$$g_{HHH} = \frac{3m_H^2}{v} \qquad g_{HHHH} = \frac{3m_H^2}{v^2}$$

This is enough to compute the radiative corrections to the Higgs potential

RG flow of the SM couplings

The running with energy of the SM couplings has been computed up to three loops





All SM couplings stay small until the Planck scale $\Rightarrow$ no problem with perturbativity

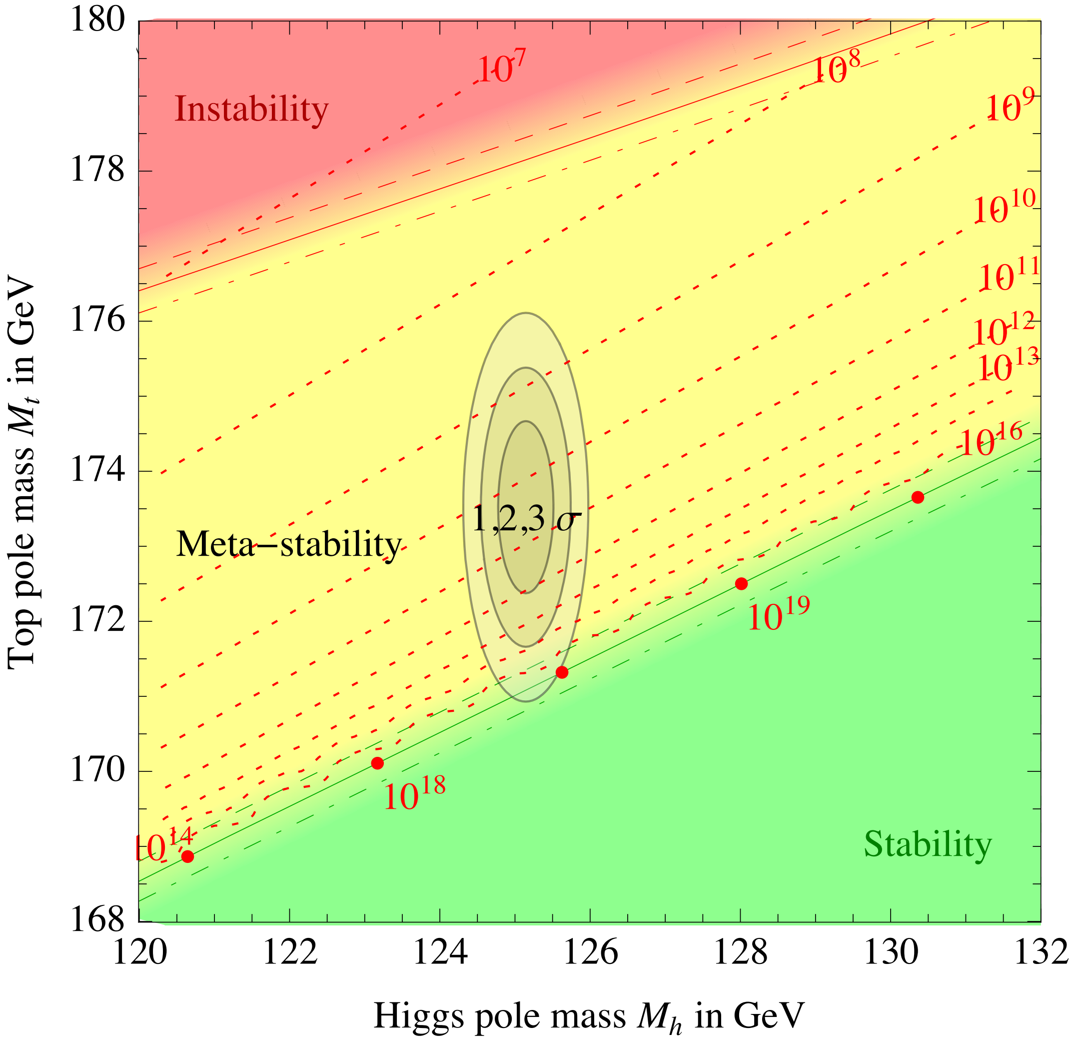
The Higgs potential becomes potentially unstable at around $\Lambda_I = 10^{10} - 10^{12}\,\text{GeV}$

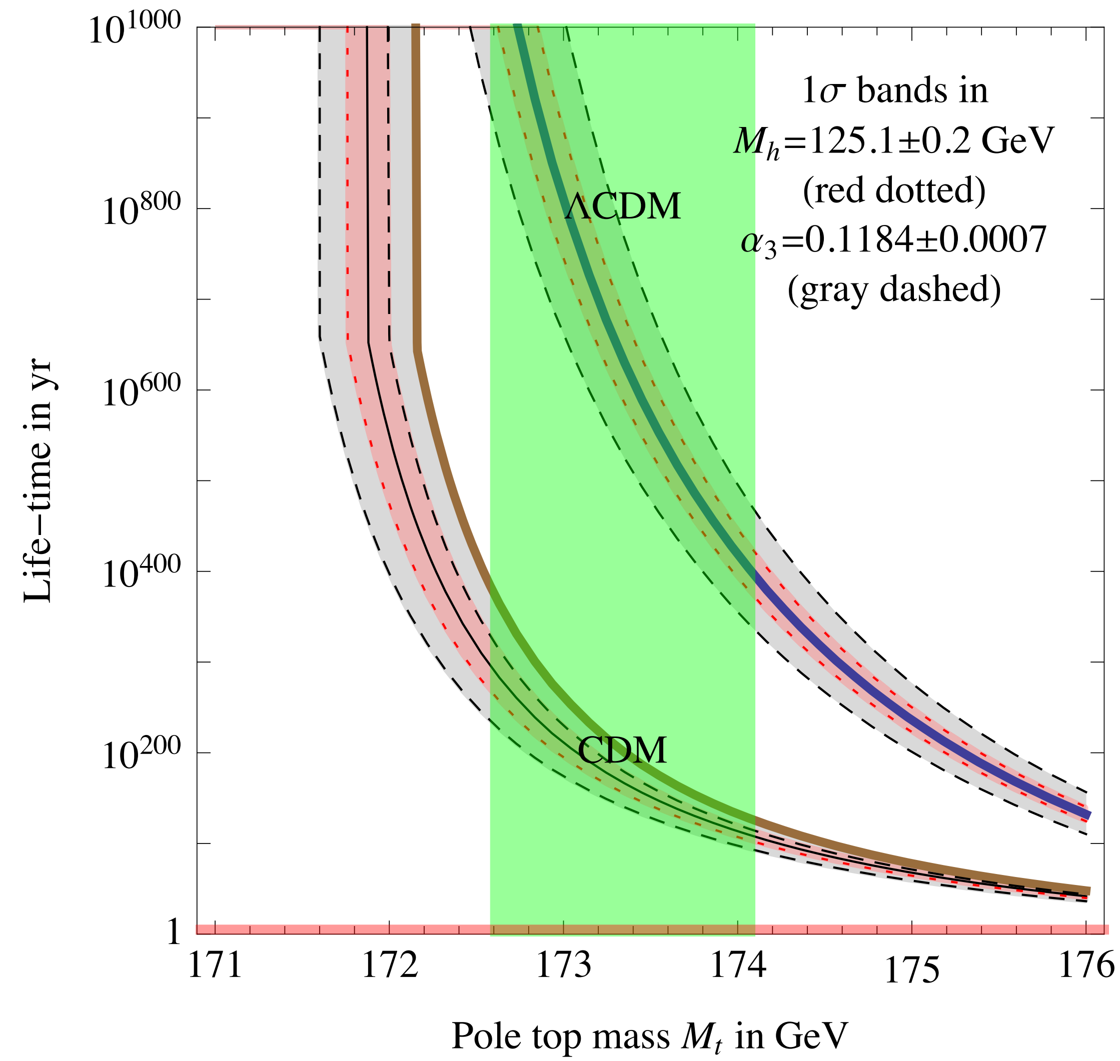
Stability of the EW vacuum

To study the stability of the EW vacuum one looks at the Higgs effective potential for $H \gg v$

$$V_{\text{eff}}(H) \simeq \frac{\lambda_{\text{eff}}(H)}{4} H^4$$

$$\lambda_{\text{eff}} - \lambda \simeq \frac{1}{(4\pi)^2}\left[3\lambda^2\left(4\ln\lambda - 6 + 3\ln 3\right) - 3y_t^2\left(\ln\frac{y_t^2}{2} - \frac{3}{2}\right)\right.$$
$$\left.\frac{3}{8}g_2^2\left(\ln\frac{g_2^2}{4} - \frac{5}{6}\right) + \frac{3}{16}(g_1^2+g_2^2)^2\left(\ln\frac{g_1^2+g_2^2}{4} - \frac{5}{6}\right)\right]$$





Custodial symmetry

Rearrange the Higgs field as a 2x2 complex matrix

$$\Phi = \frac{1}{\sqrt{2}} \left(\begin{array}{cc} \phi_0^* & \phi_+ \\ -\phi_- & \phi_0 \end{array} \right) \qquad V(\Phi) = -\mu^2 \mathrm{Tr}(\Phi^\dagger \Phi) + \lambda [\mathrm{Tr}(\Phi^\dagger \Phi)]^2$$

The Higgs potential is invariant under $SU(2)_L \times SU(2)_R$

$$SU(2)_L : \Phi \to U_L \Phi \qquad SU(2)_R : \Phi \to \Phi\, U_R^\dagger$$

The kinetic term

$$\mathrm{Tr}(D_\mu \Phi)^\dagger D^\mu \Phi \qquad D_\mu = \partial_\mu + i\frac{g_2}{2}\vec{\sigma} \cdot \vec{W}_\mu - i\frac{g_1}{2} B_\mu \Phi \sigma_3$$

is invariant under $SU(2)_L \times SU(2)_R$ only if $g' \to 0$

$$\mathcal{L}_{\mathrm{inv}} = \mathrm{Tr}(D_\mu \Phi)^\dagger D^\mu \Phi|_{g_1 = 0}$$

Custodial symmetry

After spontaneous symmetry breaking $SU(2)_L \times SU(2)_R$ is broken

$$\langle 0|\Phi|0\rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} v & 0 \\ 0 & v \end{pmatrix} \quad \Rightarrow \quad U_L \langle 0|\Phi|0\rangle\, U_R^\dagger \neq \langle 0|\Phi|0\rangle$$

However $U_L \langle 0|\Phi|0\rangle U_L^\dagger = \langle 0|\Phi|0\rangle$, leaving a residual $SU(2)_{L+R}$ symmetry, called ``custodial'' symmetry

In the limit $g_1 \to 0$, W^+, W^-, Z form a triplet under $SU(2)_{L+R}$ $\Rightarrow$ $M_W = M_Z$

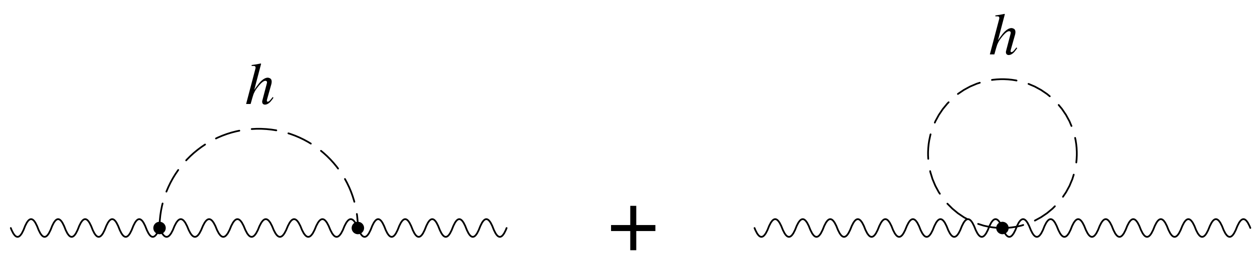
For $g_1 \neq 0$, one defines the ρ parameter as

$$\frac{M_W^2}{M_Z^2} = \frac{g_2^2}{g_1^2 + g_2^2} = \cos^2\theta_W \quad \Rightarrow \quad \rho \equiv \frac{M_W^2}{M_Z^2 \cos^2\theta_W} = 1 \text{ (tree level)}$$

Experimentally, $\rho_{\text{exp}} = 1.0050 \pm 0.0010$, suggesting that custodial symmetry is broken very mildly [ALEPH, DELPHI, L3, OPAL, SLD Phys. Rep 427 (2006) 257]

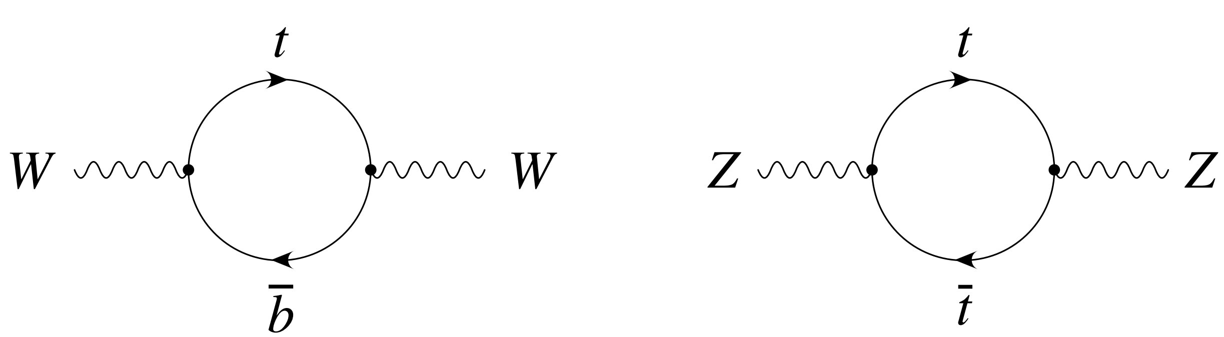
Breaking of custodial symmetry in the SM

Higgs loop corrections to W and Z masses



$$\Delta\rho|_{g'\neq 0} \simeq -\frac{11 G_F M_Z^2 \sin^2\theta_W}{24\sqrt{2}\pi^2} \ln\frac{m_H^2}{m_Z^2}$$

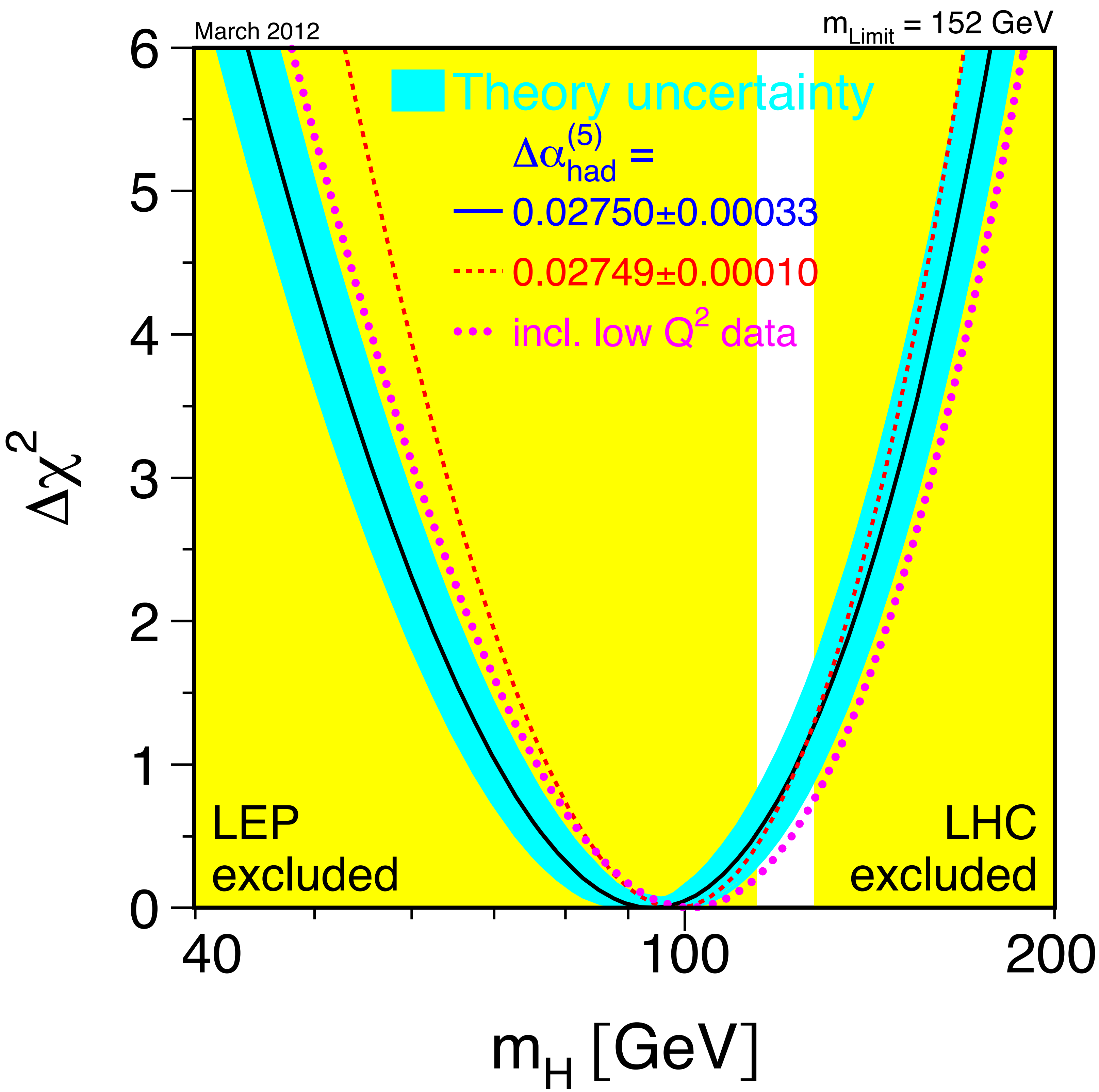
Yukawa interactions break custodial symmetry unless $h_u = h_d$



$$\Delta\rho|_{h_u\neq h_d} \simeq +\frac{3G_F}{8\pi^2\sqrt{2}}\left(m_t^2 + m_b^2 - 2\frac{m_t^2 m_b^2}{m_t^2 - m_b^2}\ln\frac{m_t^2}{m_b^2}\right)$$

Get an idea of SM m_H from EW precision data

Breaking of custodial symmetry in the SM



Learning outcomes

In this lecture we have learnt

- The BEH Higgs gives a mass to vector bosons via spontaneous symmetry breaking
- The form of the potential gives rise (or not) to an incomplete multiplet of scalar “Higgs” bosons
- The Standard Model contains only one neutral Higgs boson
- The observed mass of the SM Higgs boson is compatible with a metastable Higgs vacuum
- The SM Higgs potential possesses a custodial symmetry which, from experimental data, is broken very mildly