Cosmology 1

Primordial fluctuations

NExT summer school 2015

Tuesday 9 June

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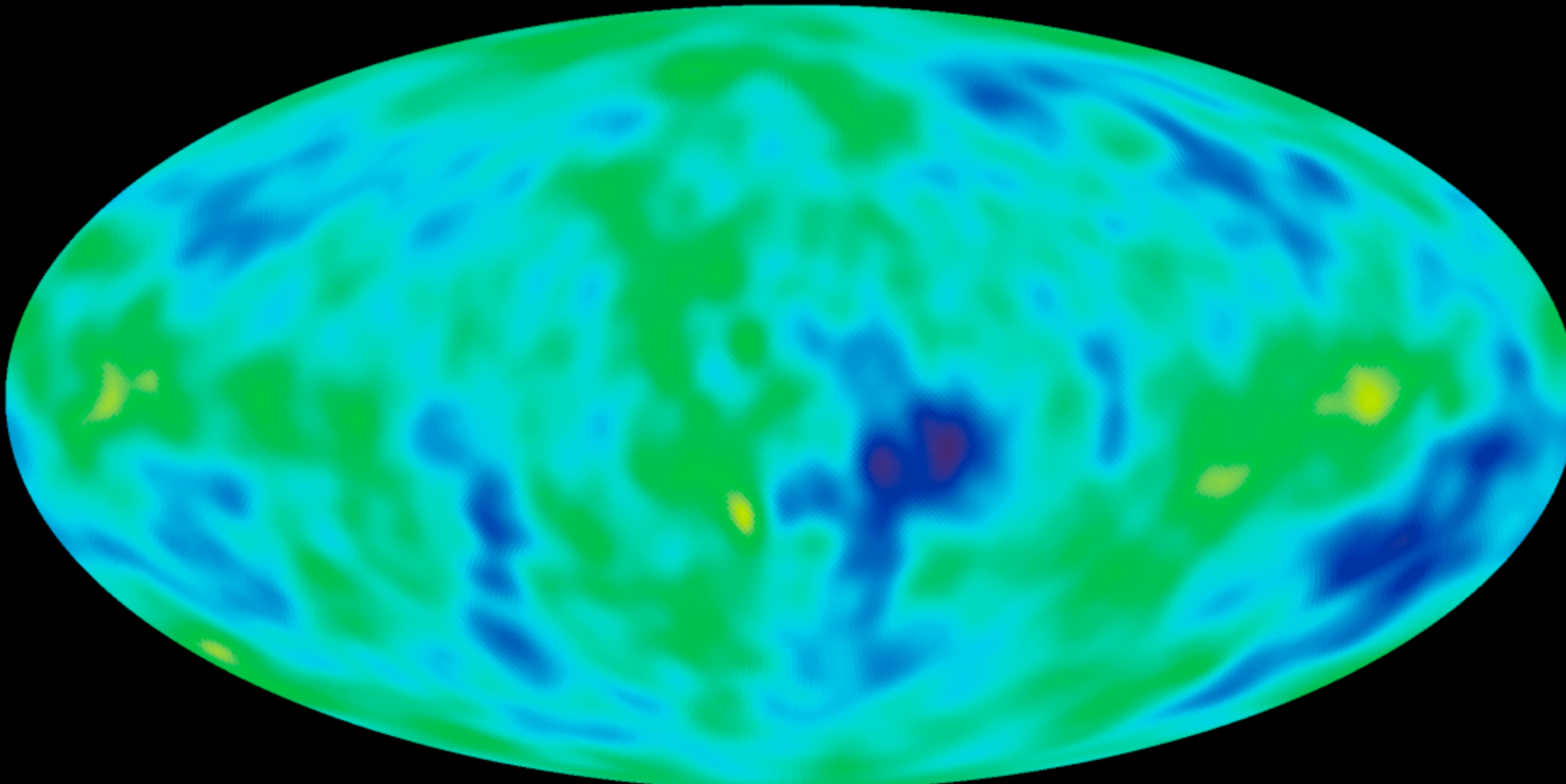
Today	Primordial perturbations and inflation
Tomorrow	Effective field theories applied to structure formation Dark energy

How we used to live

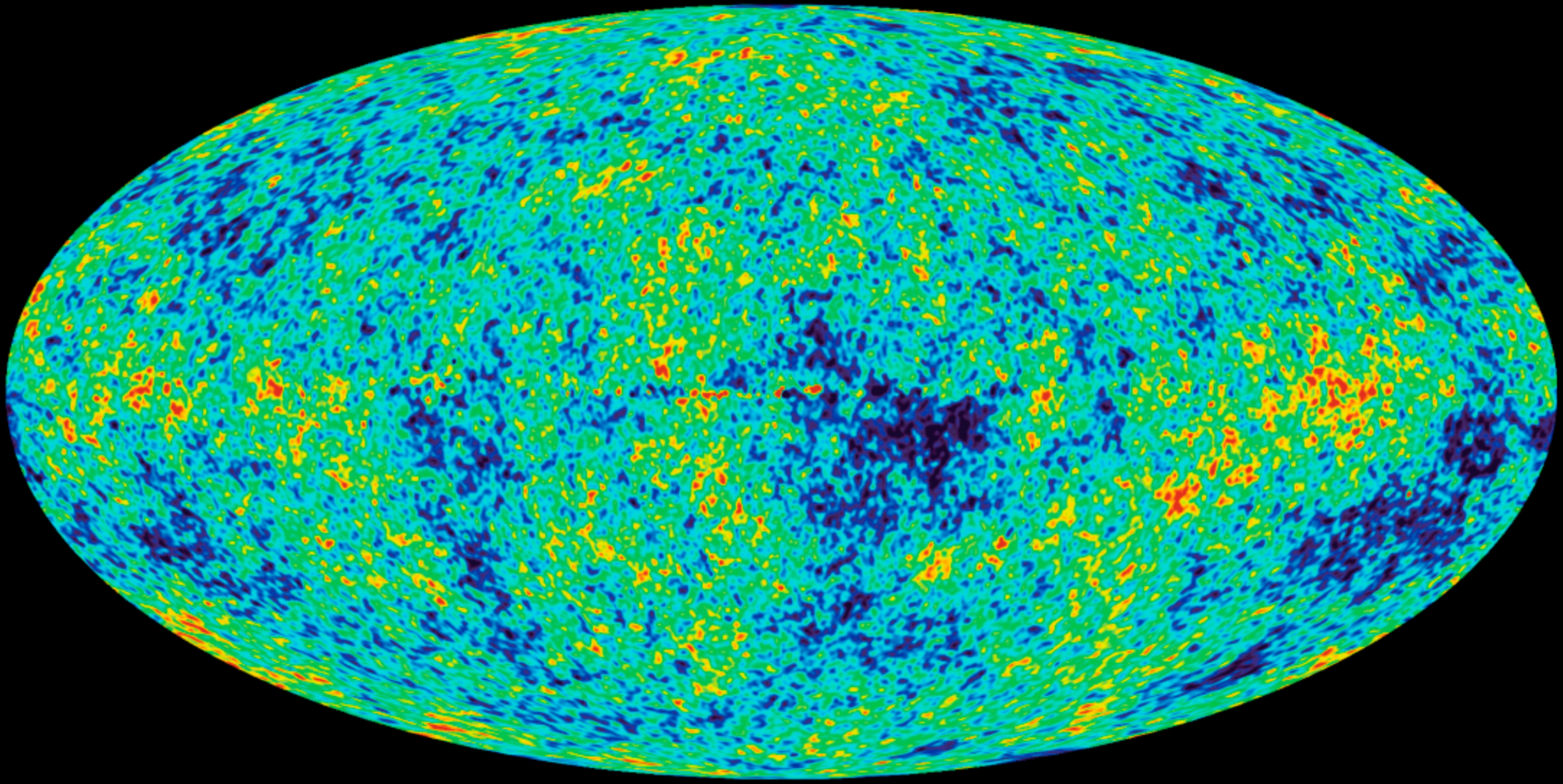
Over the last decade, the rapid arrival of data has made cosmology a major growth area

How we used to live

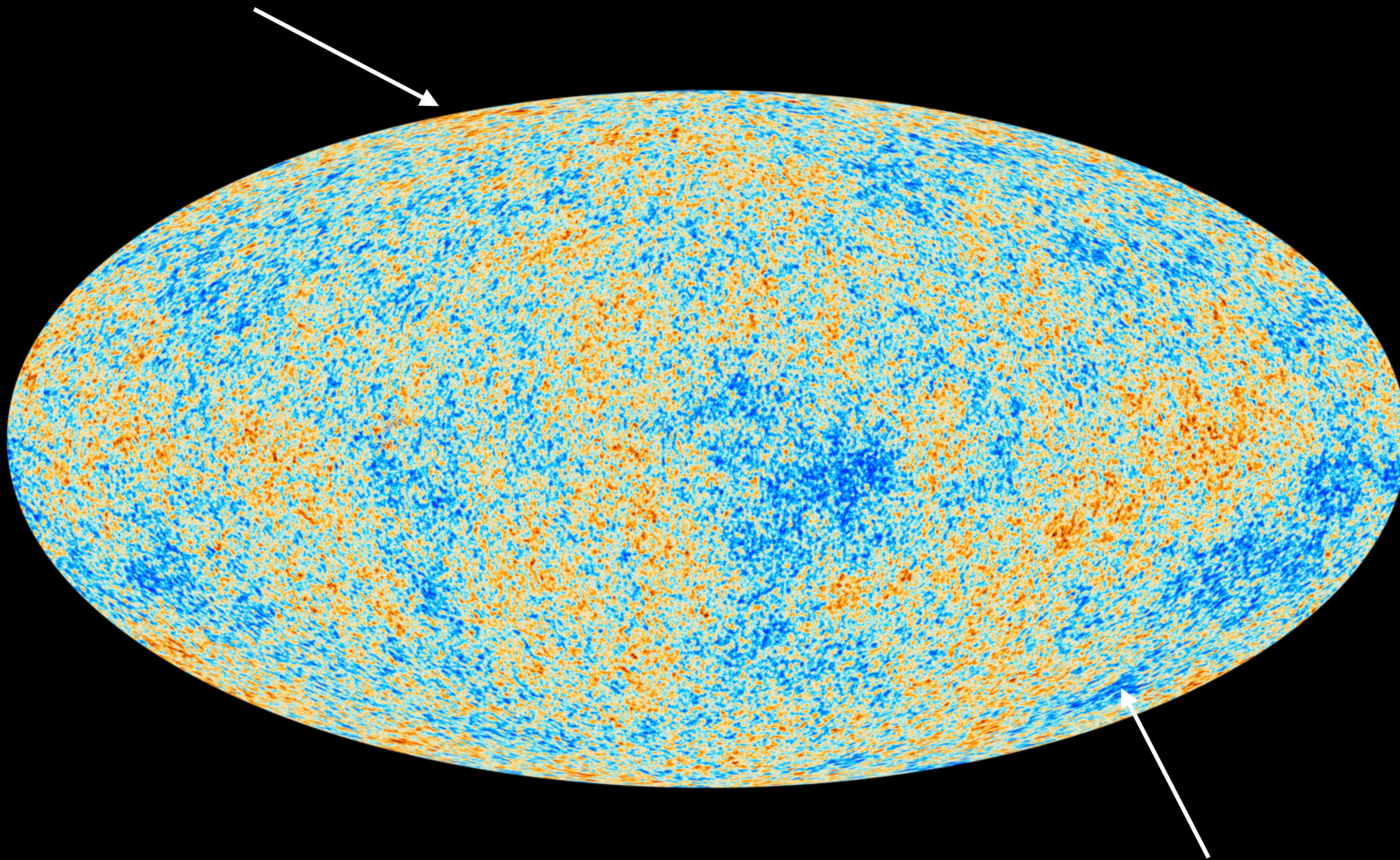
1964/5	measurement of 3K microwave background	↑ little data available
~1980	inflation	
1992	COBE measures CMB anisotropies at 1 part in 10^5	data begins to accumulate ↓
1999	measurement of cosmological acceleration	
2001	launch of NASA **WMAP** satellite	CMB era "precision cosmology" ↓
2003	WMAP first-year data support Standard Model	
2009	launch of ESA **Planck** satellite	
2013	Planck first-year data	galaxy survey era ↓
~2020	ESA **Euclid** launch	



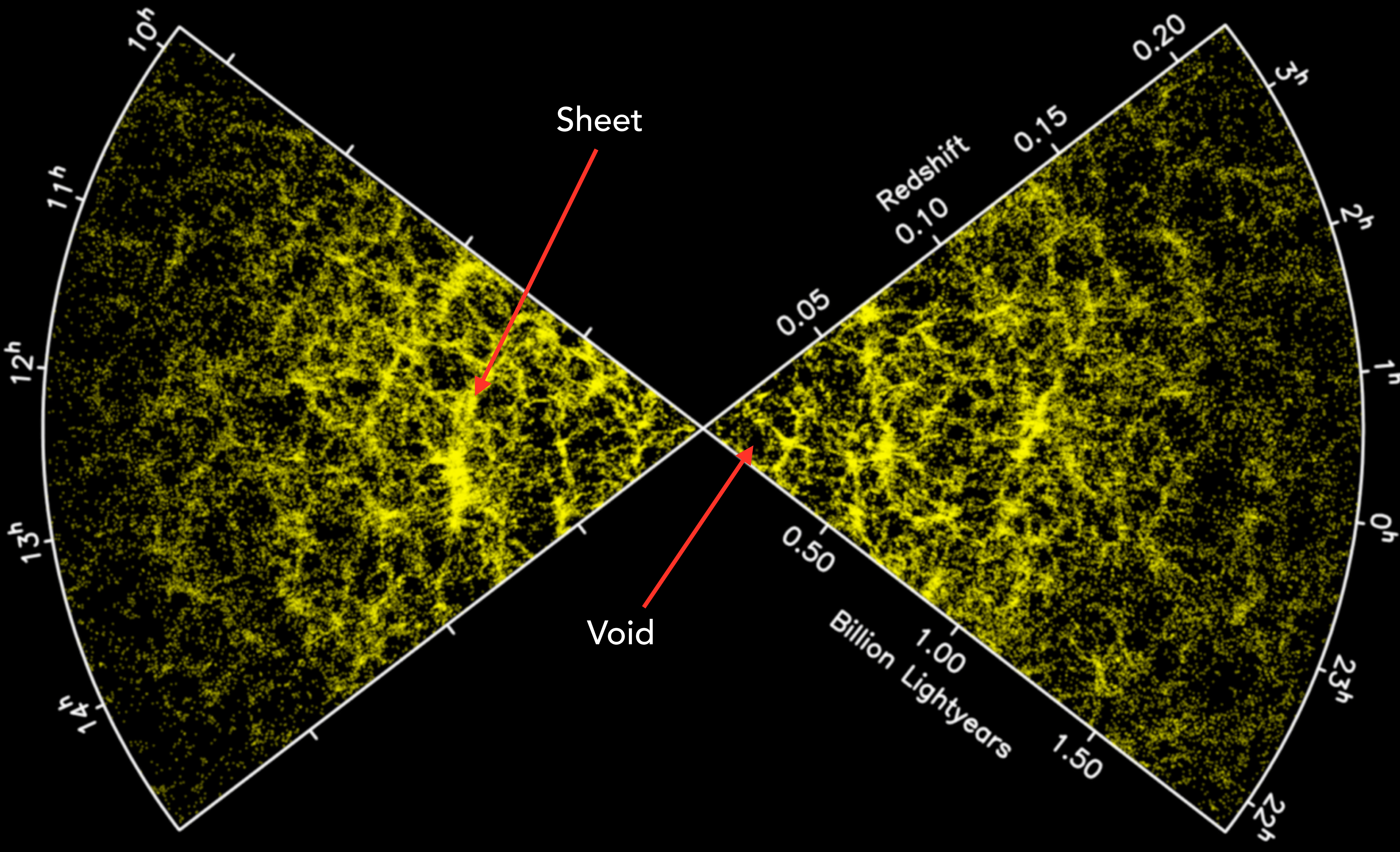
COBE CMB map, circa 1995



WMAP CMB map, circa 2007



Planck CMB map, 2013



2dFGRS galaxy map, circa 2003

High energy physics from cosmology

- **What produces these fluctuations?**
  If the origin is microphysical, can we use it to constrain the details of particle physics in the same way we do at a collider?

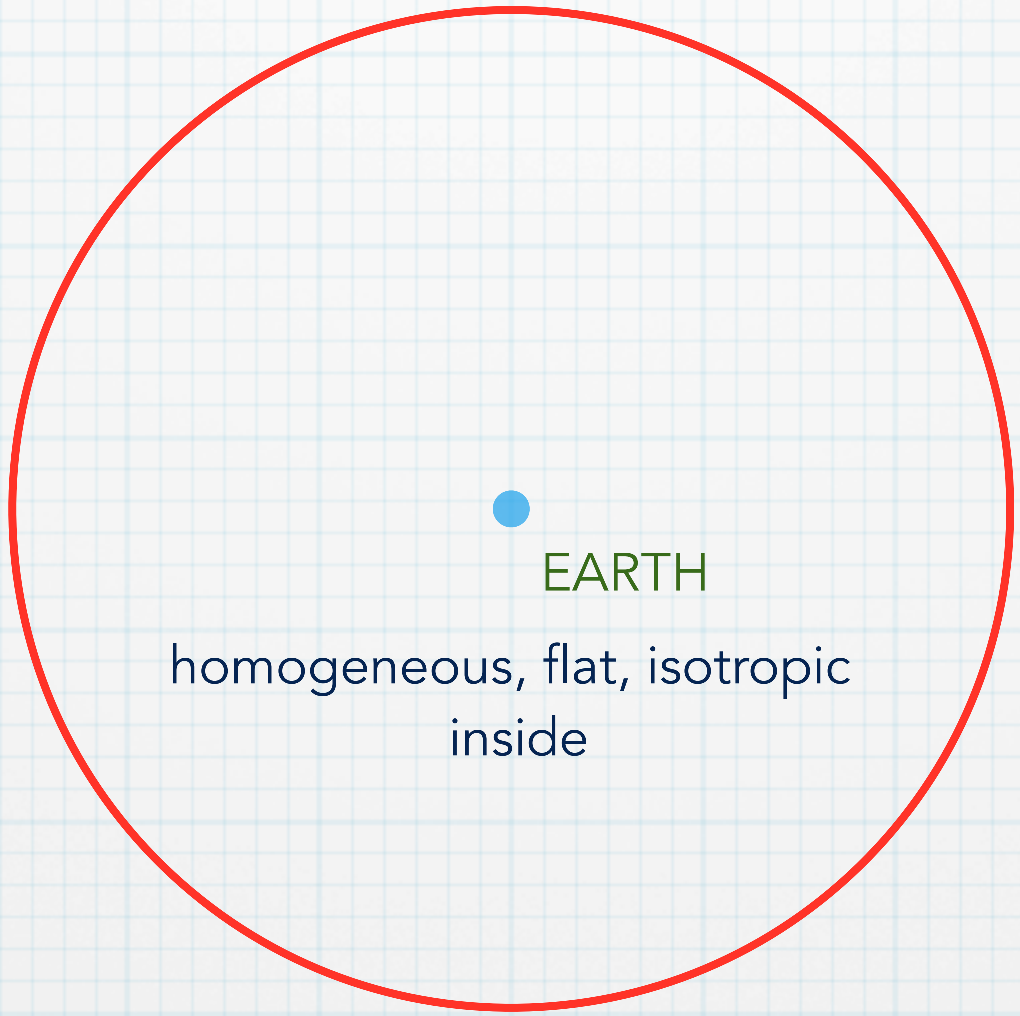
- **What does a theory of the early universe look like, and what tools do we need to produce concrete predictions?**

- **What do we get from the CMB, and what do we get from galaxies?**
  For example, do the CMB and galaxy surveys probe much the same physics, or are they complementary?

The universe presumably extends to very large distances in all directions

TO INFINITY AND BEYOND

no evidence for inhomogeneity outside



Only a small, finite portion is visible to us — the part inside the *cosmological horizon*

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TO INFINITY AND BEYOND

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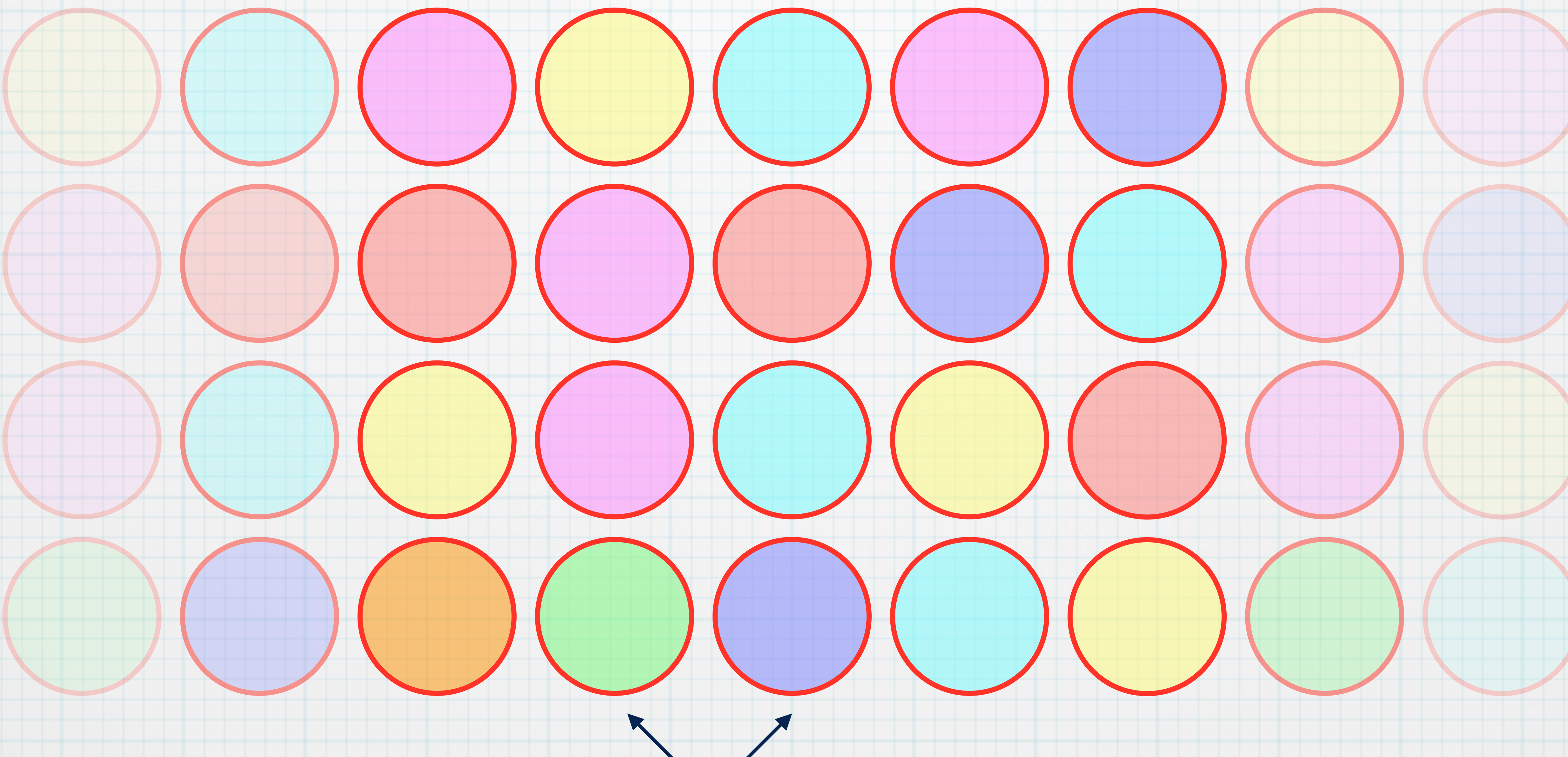
TO INFINITY AND BEYOND

We do not know what the early universe was like, but it is reasonable to imagine that it was quite disordered



Physical conditions vary between patches

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TO INFINITY AND BEYOND

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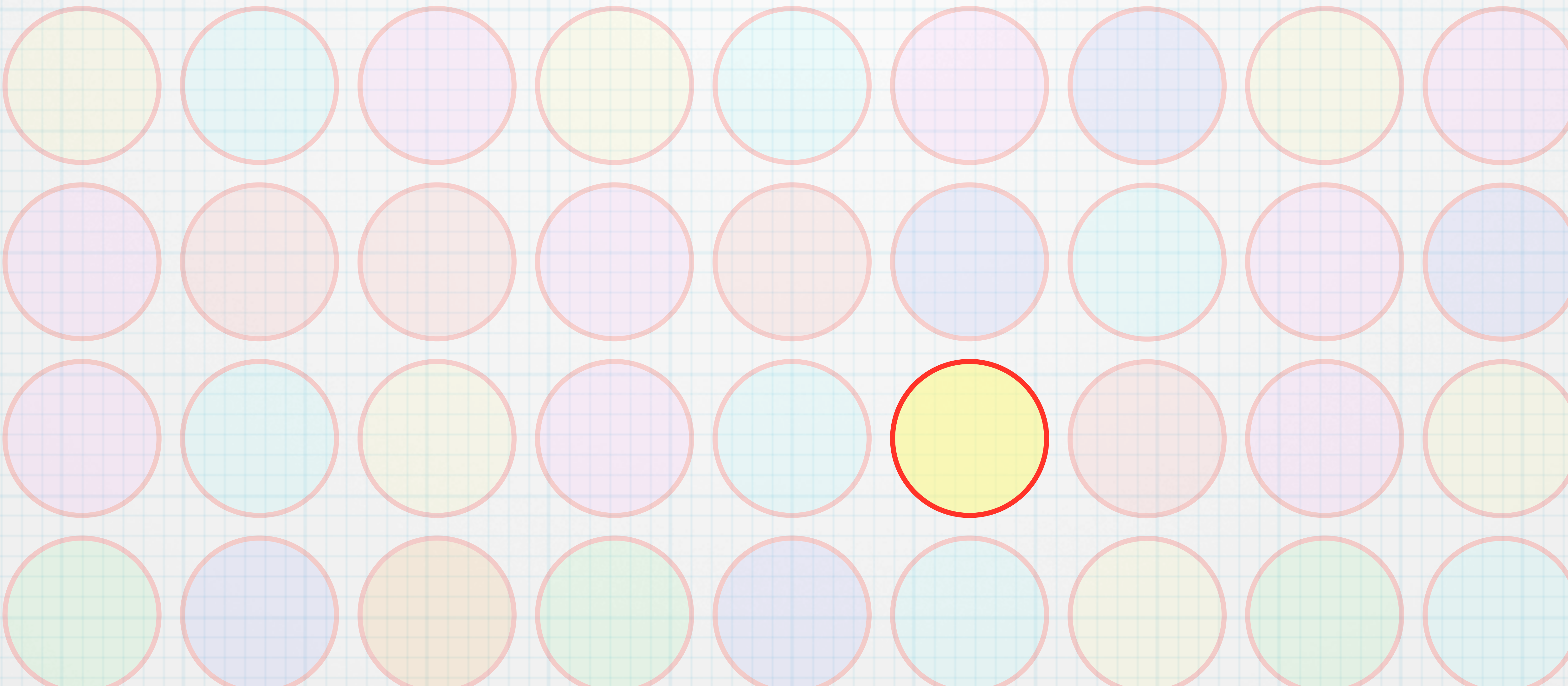

It can happen that one patch
grows very quickly

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TO INFINITY AND BEYOND

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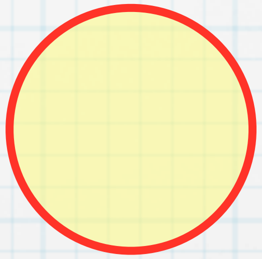

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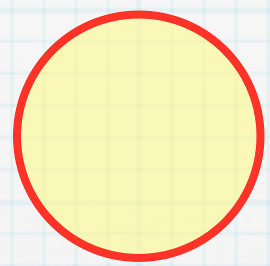

It can happen that one patch grows very quickly

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TO INFINITY AND BEYOND

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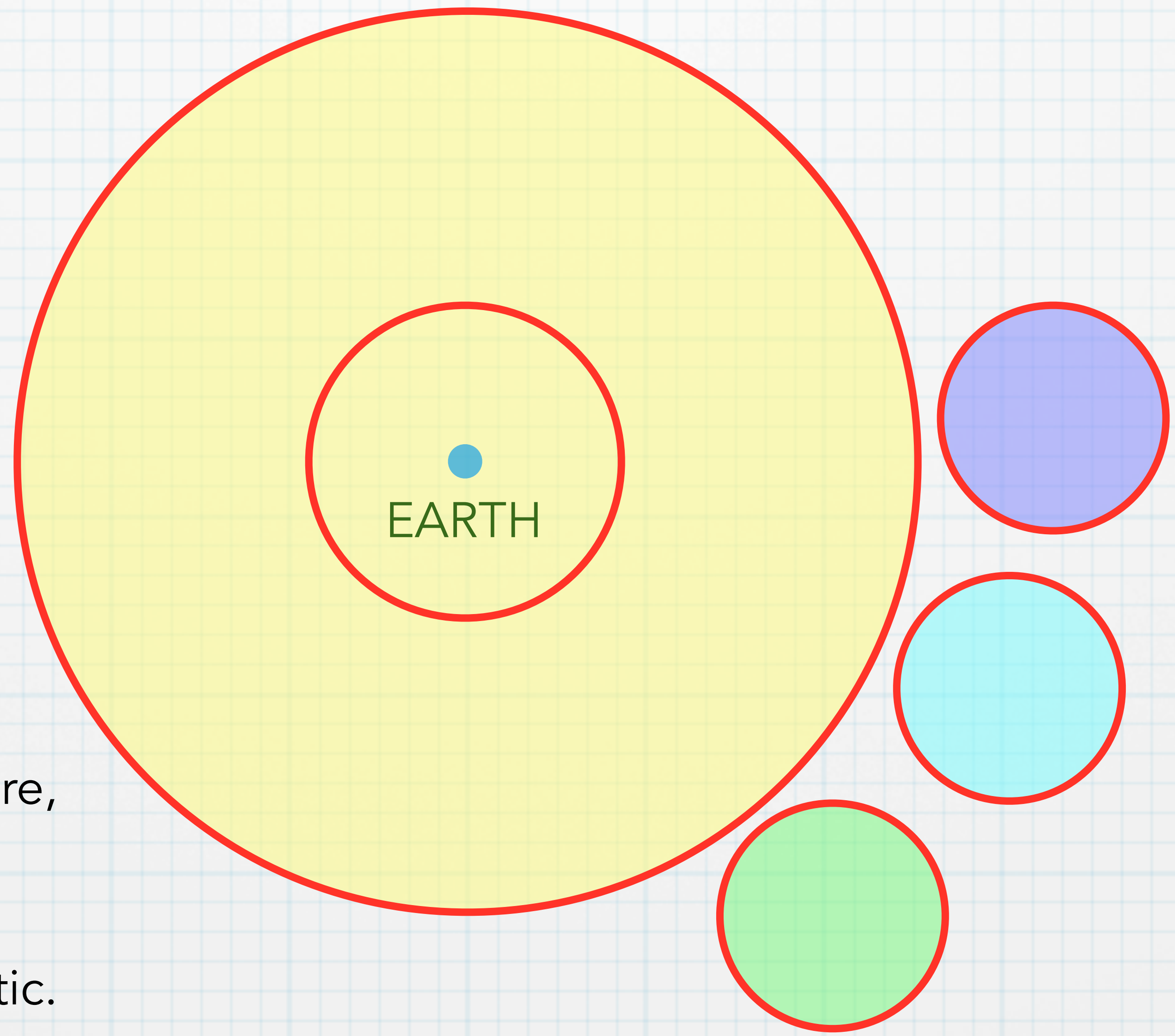
Our horizon volume would be a small region inside this inflated patch



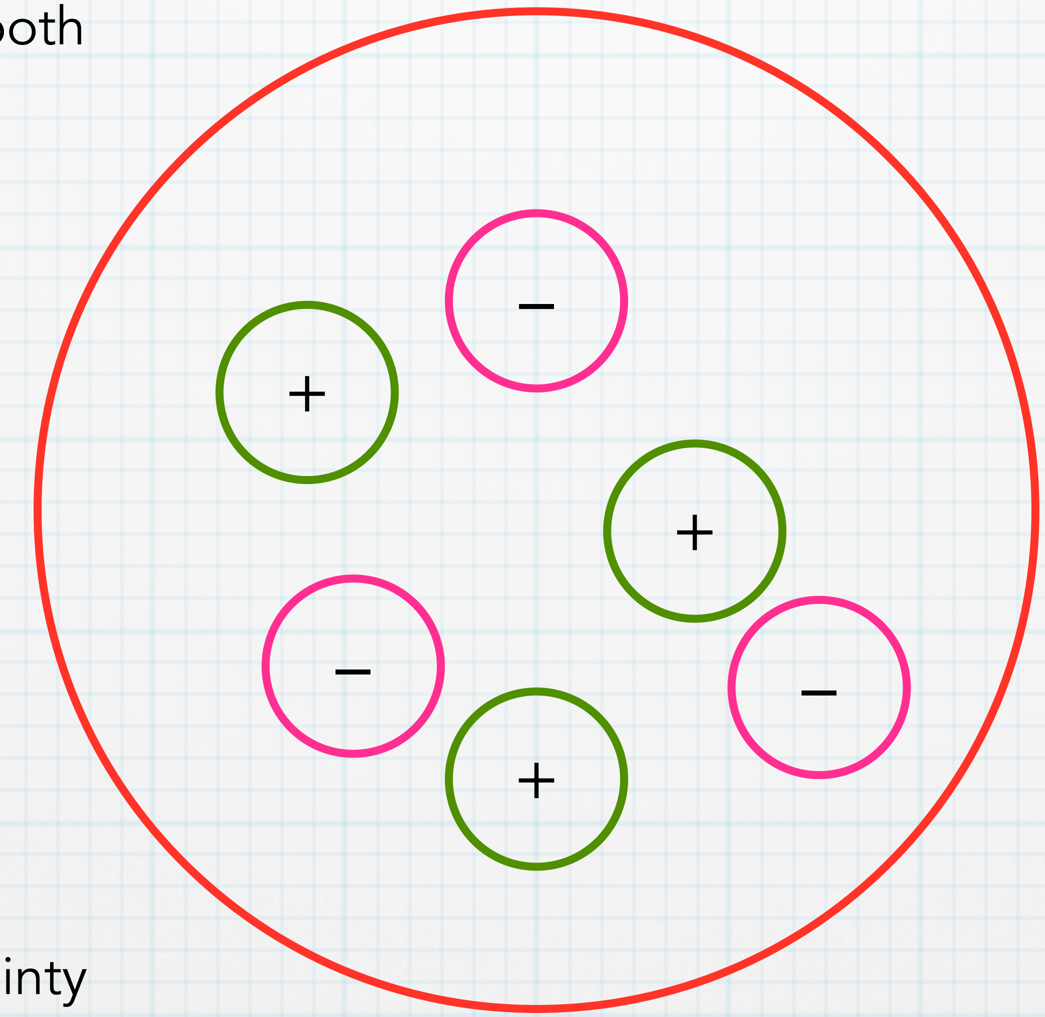
The adjacent patches are still there, but we can't see them. Presumably we don't live there because conditions are too chaotic.

It can happen that one patch grows very quickly

The universe presumably extends to very large distances in all directions

TO INFINITY AND BEYOND

This makes a large flat universe,
but one which is too smooth
and featureless

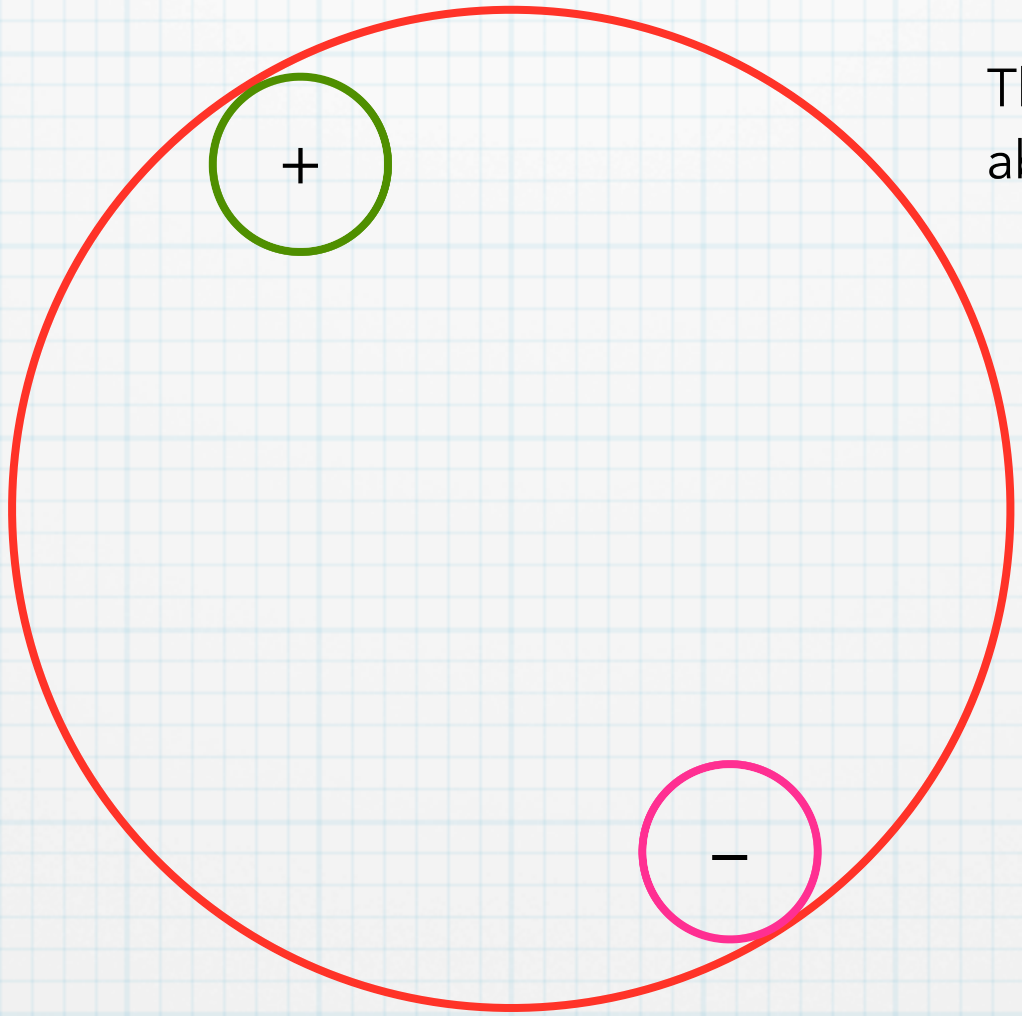


According to the uncertainty principle, the universe doesn't mind if we temporarily borrow a small amount of energy — as long as we give it back

The universe presumably extends to very large distances in all directions

TO INFINITY AND BEYOND

This fluctuation is just about able to equalize



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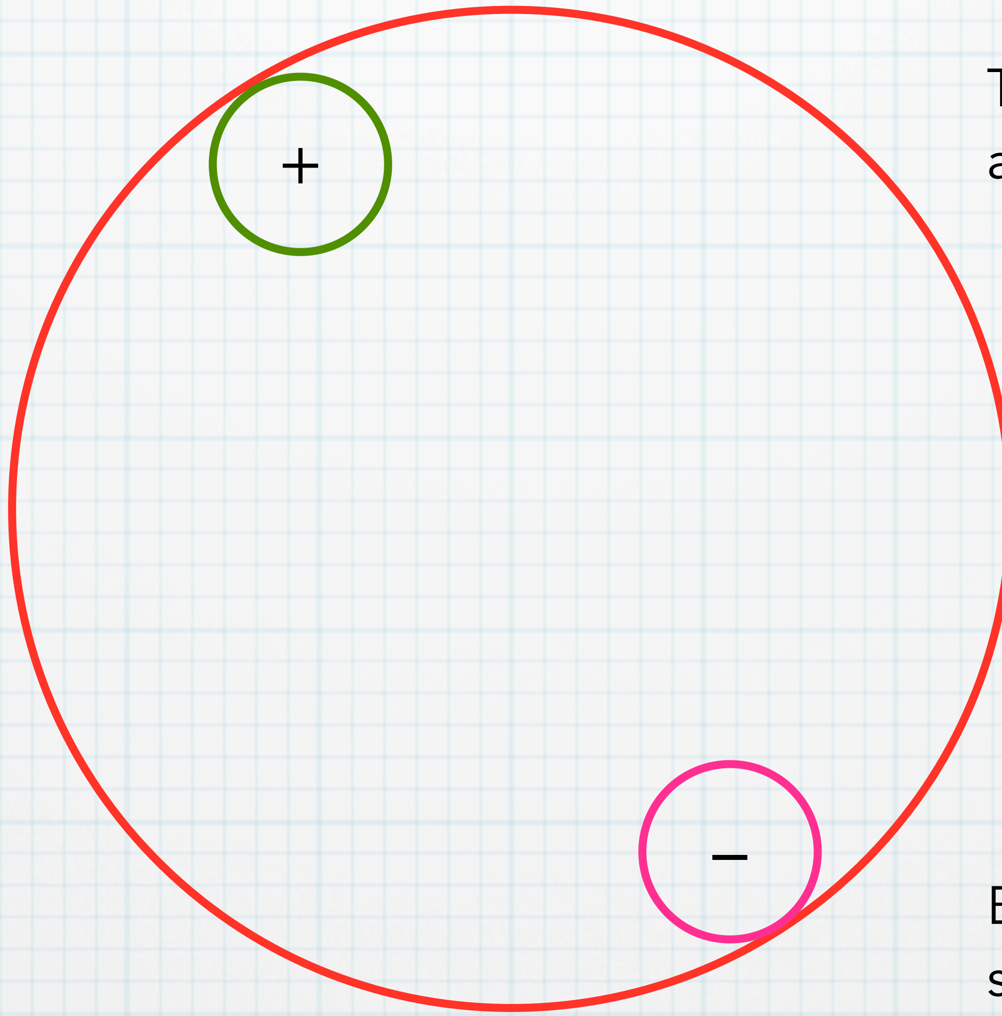
TO INFINITY AND BEYOND



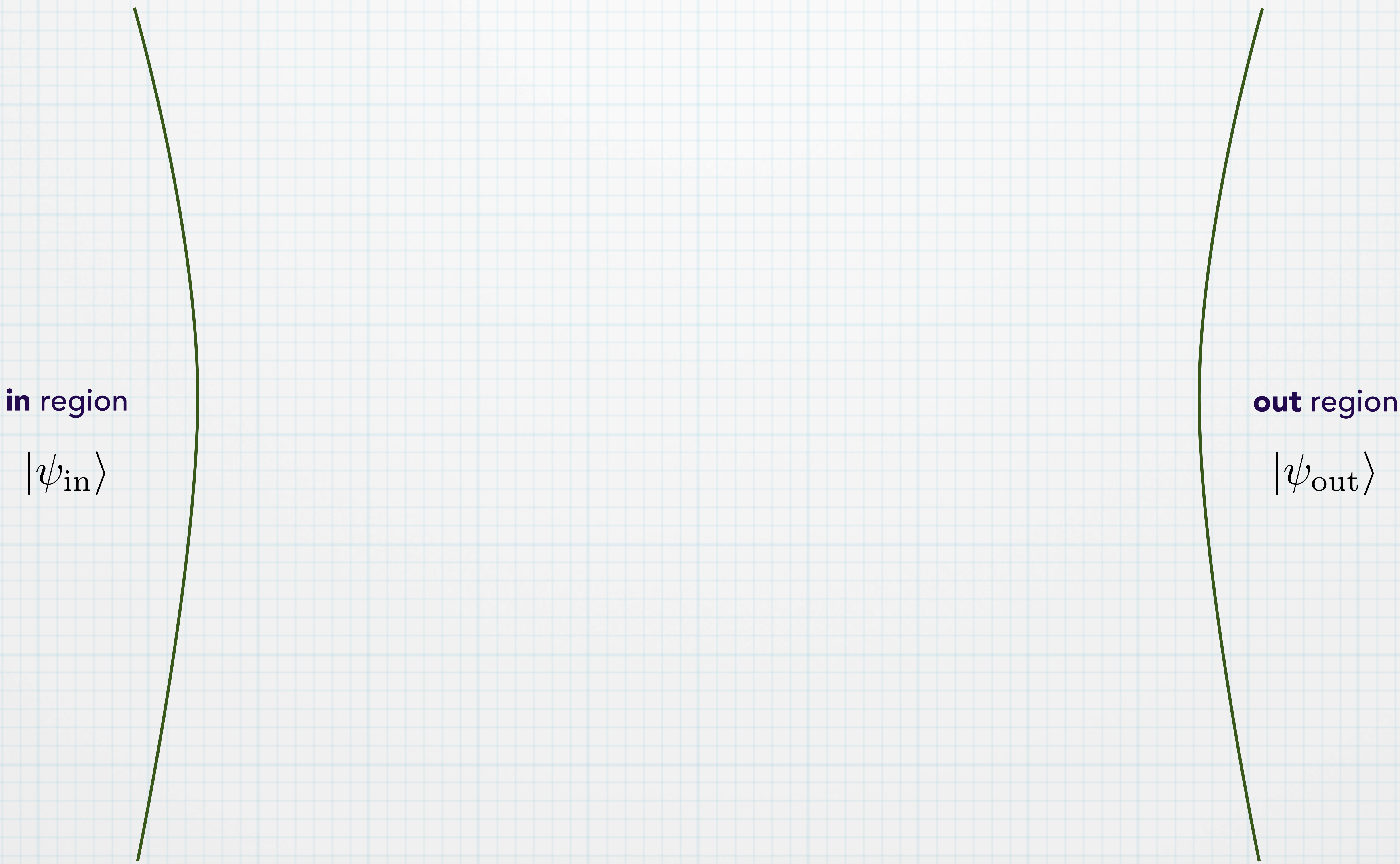
This fluctuation is just about able to equalize

But during inflation, physical scales are being stretched

The fluctuation gets stretched beyond the horizon.

It becomes fixed.

As particle physicists you would normally try to compute the properties of such fluctuations by identifying fairly long-lived, weakly-interacting states



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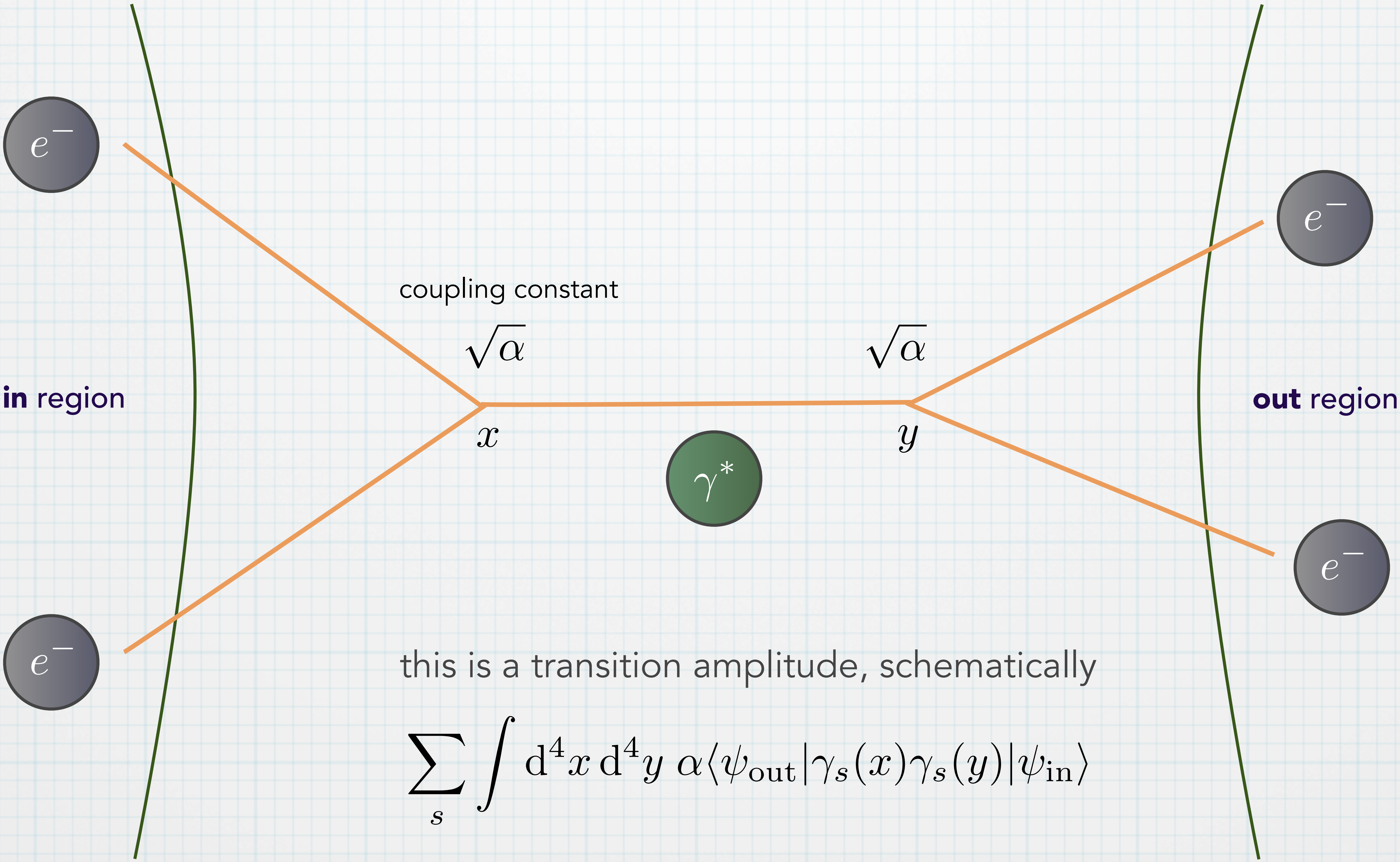
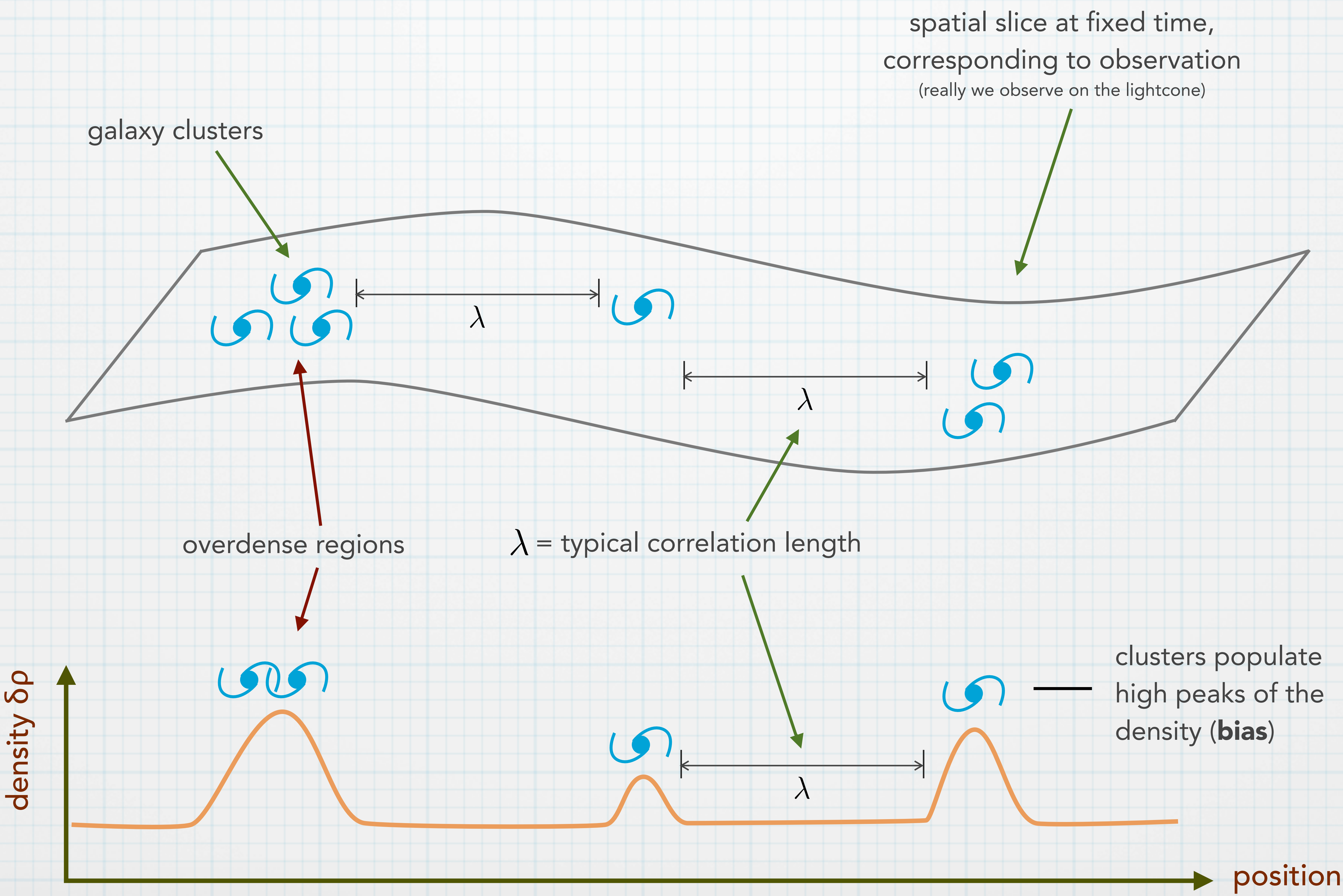


this is a transition amplitude, schematically

$$\sum_s \int \mathrm{d}^4x\, \mathrm{d}^4y\; \alpha \langle \psi_{\mathrm{out}} | \gamma_s(x) \gamma_s(y) | \psi_{\mathrm{in}} \rangle$$

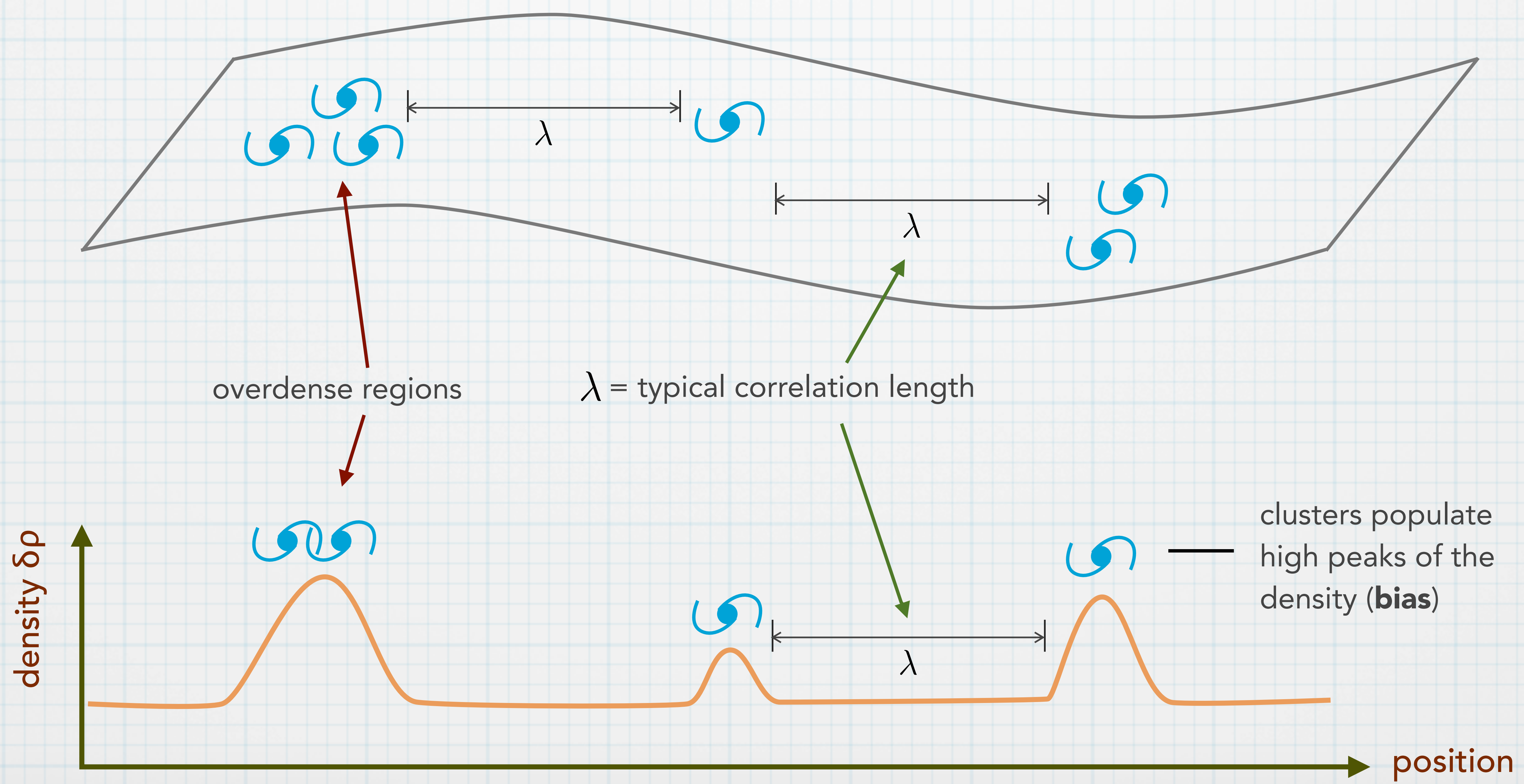
In cosmology, our "experiments" are not usually scattering events

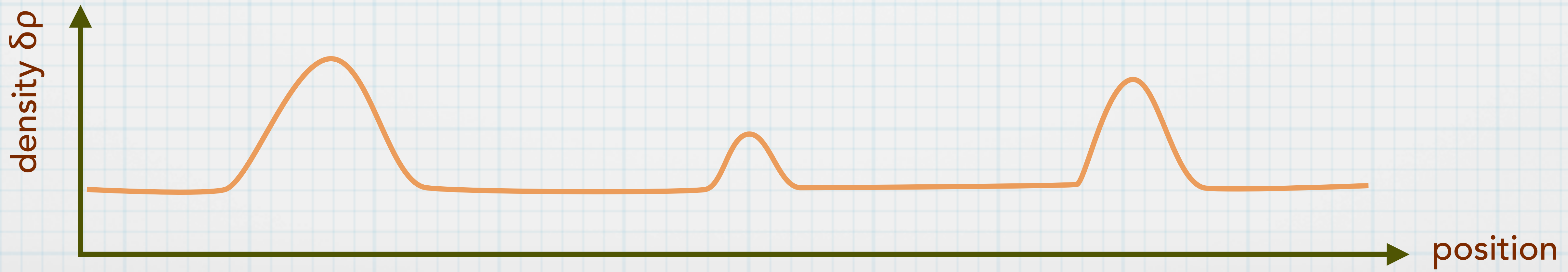


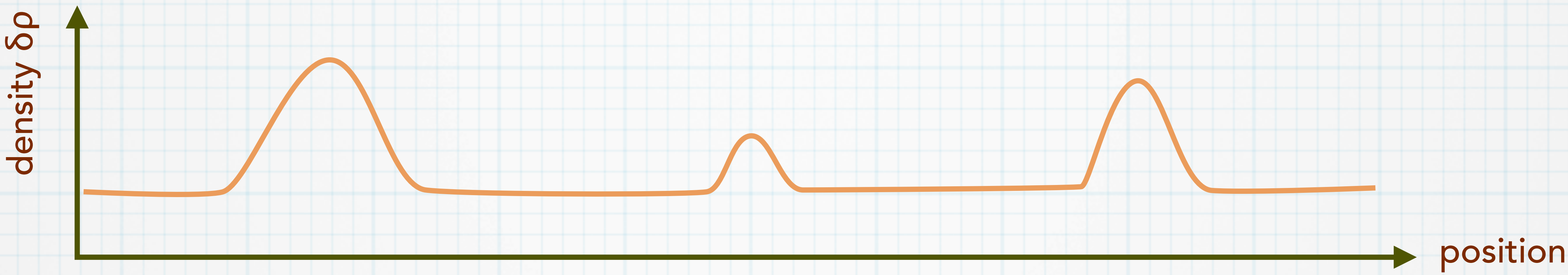
In cosmology, our "experiments" are not usually scattering events

(they are closer to condensed matter observations than particle physics)

we are interested in the **clustering properties** (eg. correlation length) of some tracer for the density field (eg. **galaxies**) at a nearly-fixed time of observation




density δρ
position



If we can find a suitable tracer, then clustering can be measured by the **equal-time correlation functions**, such as

$$\langle \delta\rho(\boldsymbol{x})\delta\rho(\boldsymbol{x}+\boldsymbol{r})\rangle \sim \frac{1}{\mathrm{Volume}}\int \mathrm{d}^3x\;\delta\rho(\boldsymbol{x})\delta\rho(\boldsymbol{x}+\boldsymbol{r})$$

(spatial average over a single realization)

"statistically homogeneous"—independent of x

ergodic

$$\langle \delta\rho(\boldsymbol{x})\delta\rho(\boldsymbol{x}+\boldsymbol{r})\rangle \sim \langle \psi|\delta\rho(\boldsymbol{x})\delta\rho(\boldsymbol{x}+\boldsymbol{r})|\psi\rangle$$

(ensemble average over many realizations)

How do I compute correlation functions?

If your focus is particle physics, cosmology is most likely to
to be of interest as an extra source of constraints — eg. Higgs inflation

In that case, you want to calculate as many observables as we have data for

CMB

Spectrum
amplitude and scale-dependence

Bispectrum
amplitudes for templates

Galaxy surveys

Spectrum
amplitude and scale-dependence

squeezed limit of bispectrum
from scale-dependent bias

Bispectrum
nothing yet but see next lecture

The spectrum

two-point function → $$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\rangle_\tau = (2\pi)^3\delta(\boldsymbol{k}_1 + \boldsymbol{k}_2)P_\tau(k_1)$$

curvature perturbation

evaluated at equal times

depends on Fourier convention

spectrum

doesn't matter if we write k_1 or k_2

The spectrum

$$\langle \zeta(\boldsymbol{k}_1)\zeta(\boldsymbol{k}_2)\rangle_\tau = (2\pi)^3\delta(\boldsymbol{k}_1+\boldsymbol{k}_2)P_\tau(k_1)$$

curvature perturbation

this is a perturbation in the volume of space $\mathrm{d}s^2 = a(\tau)^2\big[-\mathrm{d}\tau^2 + \mathrm{d}\boldsymbol{x}^2\big]$

then $\zeta(\boldsymbol{x}) = \delta \ln a(\boldsymbol{x},\tau)$

locally-defined scale factor

rarified, so cooler $\zeta > 0$

a bit larger than average

$\zeta < 0$ denser, so hotter

a bit smaller than average

The spectrum

rarified, so cooler

$\zeta > 0$

a bit larger than average

$\zeta < 0$

denser, so hotter

a bit smaller than average

The spectrum

rarified, so cooler

$\zeta > 0$

a bit larger than average

deeper potential well, so larger redshift when climbing out

$\zeta < 0$

denser, so hotter

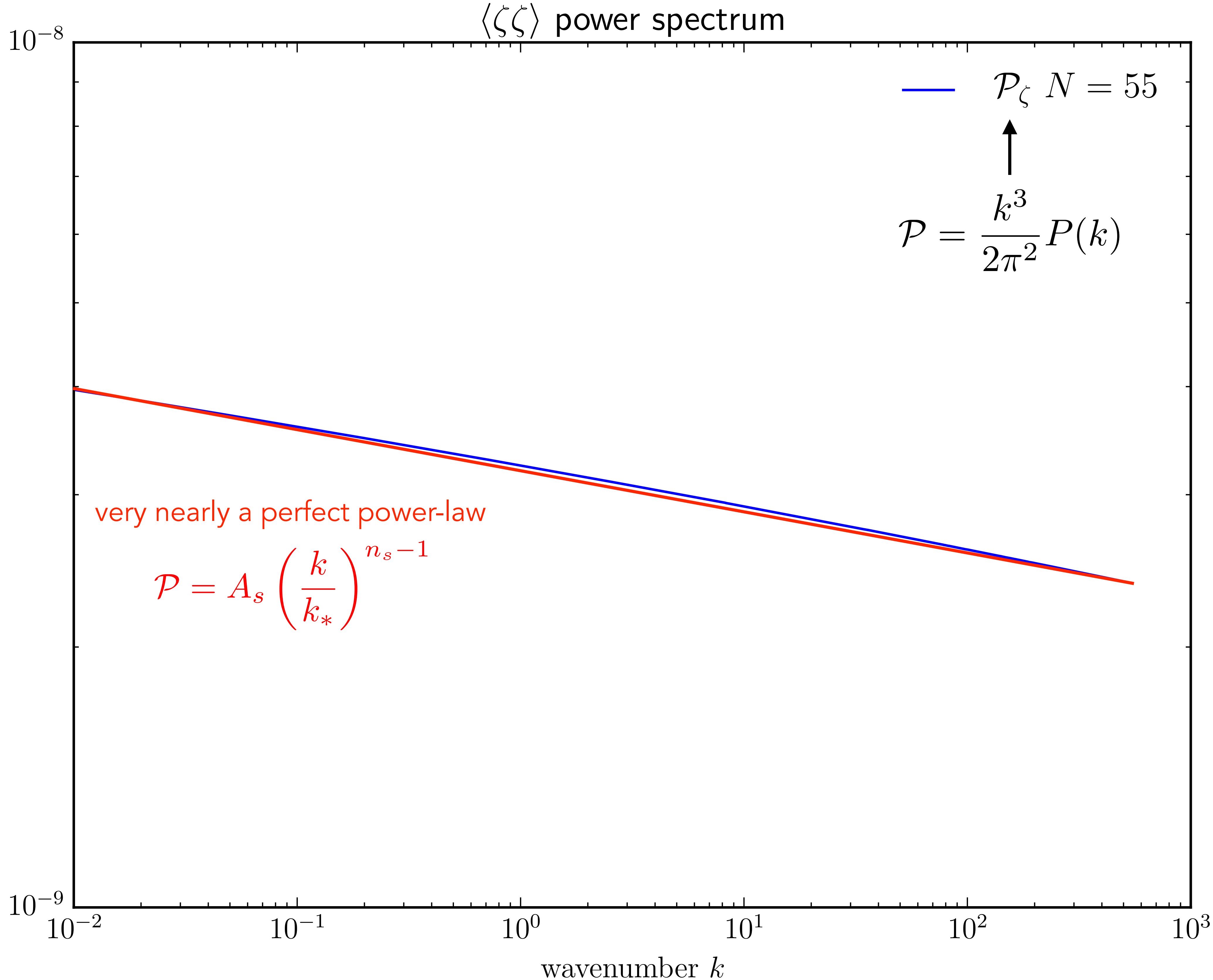
a bit smaller than average

shallower potential well, so smaller redshift when climbing out

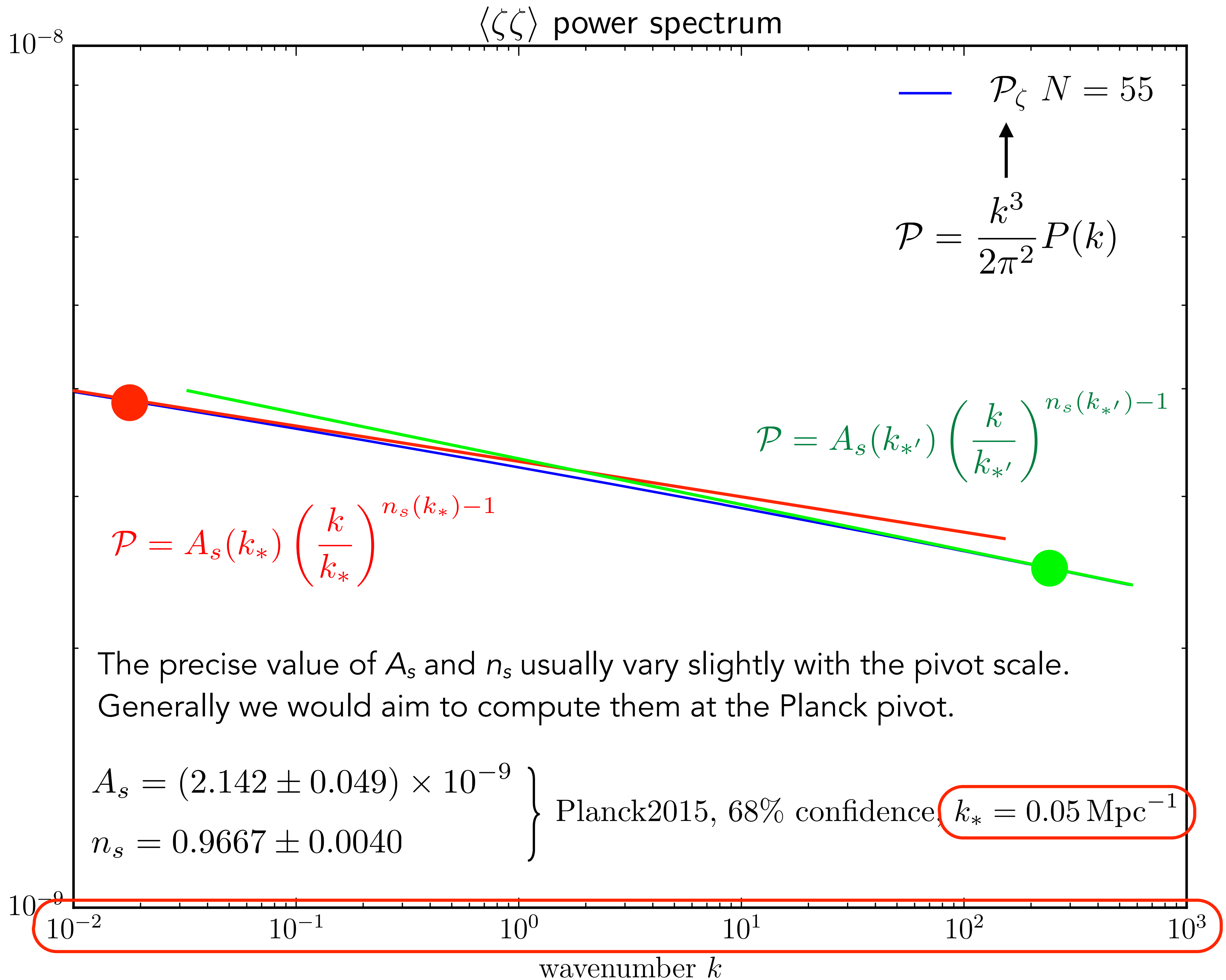
Another way to think of the effect is in terms of the gravitational potential $\Phi = -3\zeta/5$

("Sachs–Wolfe effect")

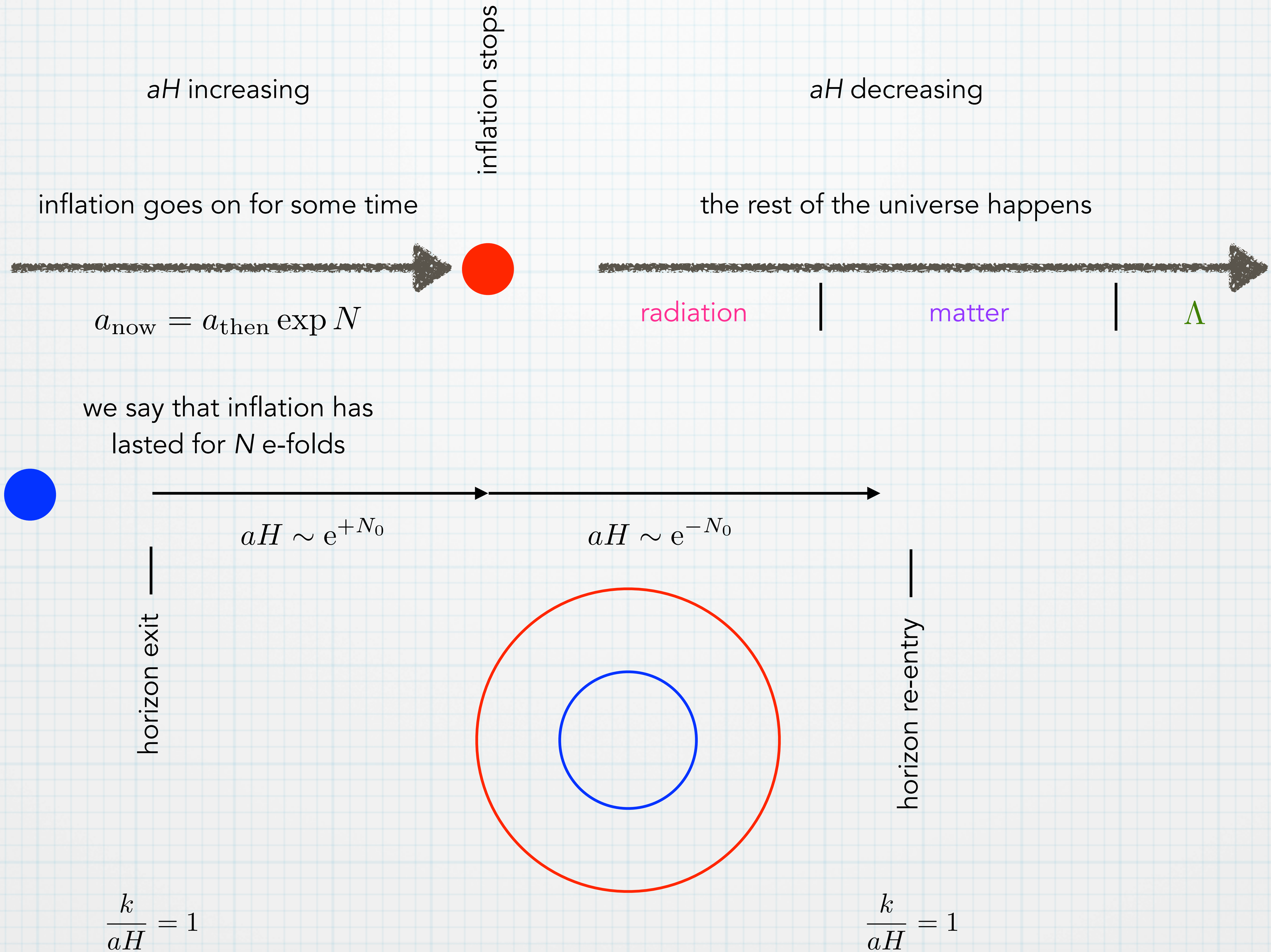
Power spectra are often (but not always) fairly featureless



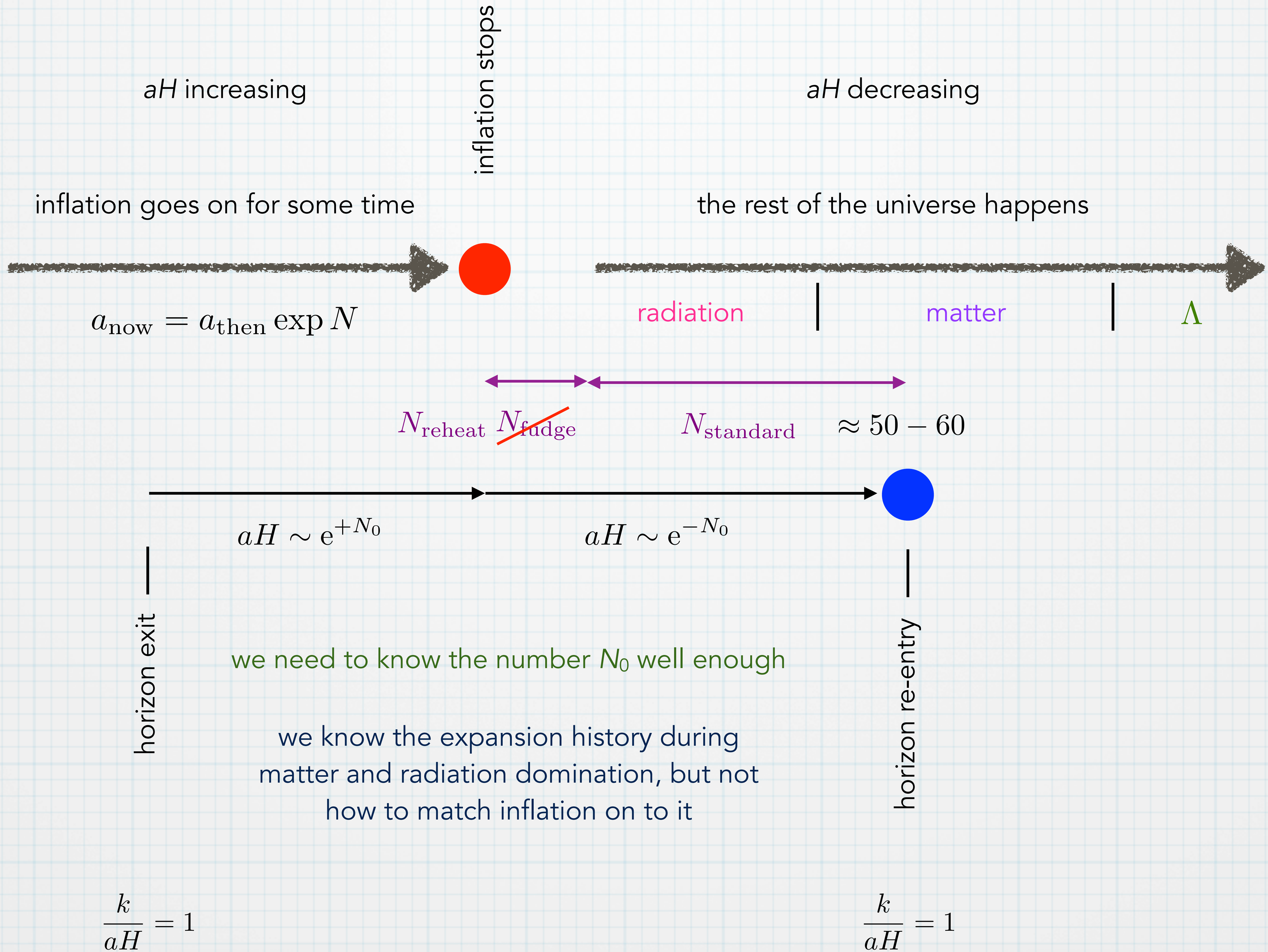
Power spectra are often (but not always) fairly featureless



notice there is a units mismatch



inflation stops
aH increasing
aH decreasing
inflation goes on for some time
the rest of the universe happens
$a_{\text{now}} = a_{\text{then}} \exp N$
radiation
matter
Λ
we say that inflation has lasted for N e-folds
$aH \sim \mathrm{e}^{+N_0}$
$aH \sim \mathrm{e}^{-N_0}$
horizon exit
horizon re-entry
$\frac{k}{aH} = 1$
$\frac{k}{aH} = 1$



inflation stops
aH increasing
aH decreasing
inflation goes on for some time
the rest of the universe happens
$a_{\rm now} = a_{\rm then} \exp N$
radiation
matter
Λ
$N_{\rm reheat}$ $N_{\rm fudge}$
$N_{\rm standard}$
$\approx 50 - 60$
$aH \sim {\rm e}^{+N_0}$
$aH \sim {\rm e}^{-N_0}$
horizon exit
horizon re-entry
we need to know the number N_0 well enough
we know the expansion history during
matter and radiation domination, but not
how to match inflation on to it
$\frac{k}{aH} = 1$
$\frac{k}{aH} = 1$

How to compute the spectrum

The action for small fluctuations can be written

$$M_{\alpha\beta} = \mathcal{D}_{(\alpha} V_{\beta)} - R_{\alpha\lambda\mu\beta}\dot{\phi}^{\lambda}\dot{\phi}^{\mu} - \frac{1}{a^3 M_{\mathrm{P}}}\mathcal{D}_t\left(a^3 \frac{\dot{\phi}_\alpha \dot{\phi}_\beta}{H}\right)$$

$$S = \frac{1}{2}\int \mathrm{d}^3x\,\mathrm{d}t\,a^3\left(G_{\alpha\beta}\mathcal{D}_t\delta\phi^\alpha\mathcal{D}_t\delta\phi^\beta + \left[G_{\alpha\beta}\frac{\partial^2}{a^2} - M_{\alpha\beta}\right]\delta\phi^\alpha\delta\phi^\beta\right)$$

$$\mathcal{D}_t X^\alpha = \dot{X}^\alpha + \Gamma^\alpha_{\beta\gamma}\dot{\phi}^\beta X^\gamma$$

field-space metric, inherited from parent theory

$$S = \frac{1}{2}\int \mathrm{d}^3x\,\mathrm{d}t\,\sqrt{-g}\left(M_{\mathrm{P}}^2 R - g_{\mu\nu}G_{\alpha\beta}\partial^\mu\delta\phi^\alpha\partial^\nu\delta\phi^\beta - 2V(\phi)\right)$$

eg. Kähler metric, or generated from nontrivial coupling to gravity

$$\delta\pi_\alpha = \frac{\delta p_\alpha}{Ha^3} = \frac{1}{H}\mathcal{D}_t\delta\phi_\alpha = \mathcal{D}_N\delta\phi_\alpha = -\frac{\mathrm{i}}{H}[\delta\phi_\alpha, \mathcal{H}]$$

$$\mathcal{D}_N\delta\pi_\alpha = -\frac{\mathrm{i}}{H}[\delta\pi_\alpha, \mathcal{H}] + (\epsilon - 3)\delta\pi_\alpha \longleftarrow \quad \epsilon = -\frac{\dot{H}}{H^2}$$

How to compute the spectrum

At tree-level we can use these equations of motion inside correlation functions

$$\mathcal{D}_N \langle X^a X^b \rangle = \langle (\mathcal{D}_N X^a) X^b \rangle + \langle X^a (\mathcal{D}_N X^b) \rangle$$

$$X^a = \begin{pmatrix} \delta\phi^\alpha \\ \delta\pi^\alpha \end{pmatrix}$$

$$= u_{ac} \langle X^c X^b \rangle + u_{bc} \langle X^a X^c \rangle$$

$$u_{ab} = \begin{pmatrix} 0 & \delta^\alpha_\beta \\ -\delta^\alpha_\beta \dfrac{k^2}{a^2 H^2} - \dfrac{M^\alpha{}_\beta}{H^2} & (\epsilon - 3)\delta^\alpha_\beta \end{pmatrix}$$

How to compute the spectrum

- These equations can be integrated with a suitable initial condition
  I haven't told you how to get that — see Dias et al. **arXiv:1502.03125**
- Doesn't involve any approximation beyond tree-level and our ability to compute the initial condition sufficiently accurately
  Initial condition requires slow-roll approximation, but not afterwards
- Computational cost is peanuts
  0.00507s per *k*-mode on my laptop — easily fast enough to include in a parameter-estimation Monte Carlo.
  Analytic estimates aren't the best way to compare to data.
- Freely available codes exist

Spectrum codes (in chronological order)

FieldInf (Ringeval, Martin — FORTRAN)

http://theory.physics.unige.ch/~ringeval/fieldinf.html

ModeCode, MultiModeCode (Easter, Frazer, Peiris, Price, Xu — FORTRAN)

http://modecode.org | only trivial field-space metric

Sussex & QMUL code (Dias, Frazer, DS — Mathematica)

http://transportmethod.com