Cosmology 3

Effective theories and structure formation

NExT summer school 2015

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An effective field theory for structure formation

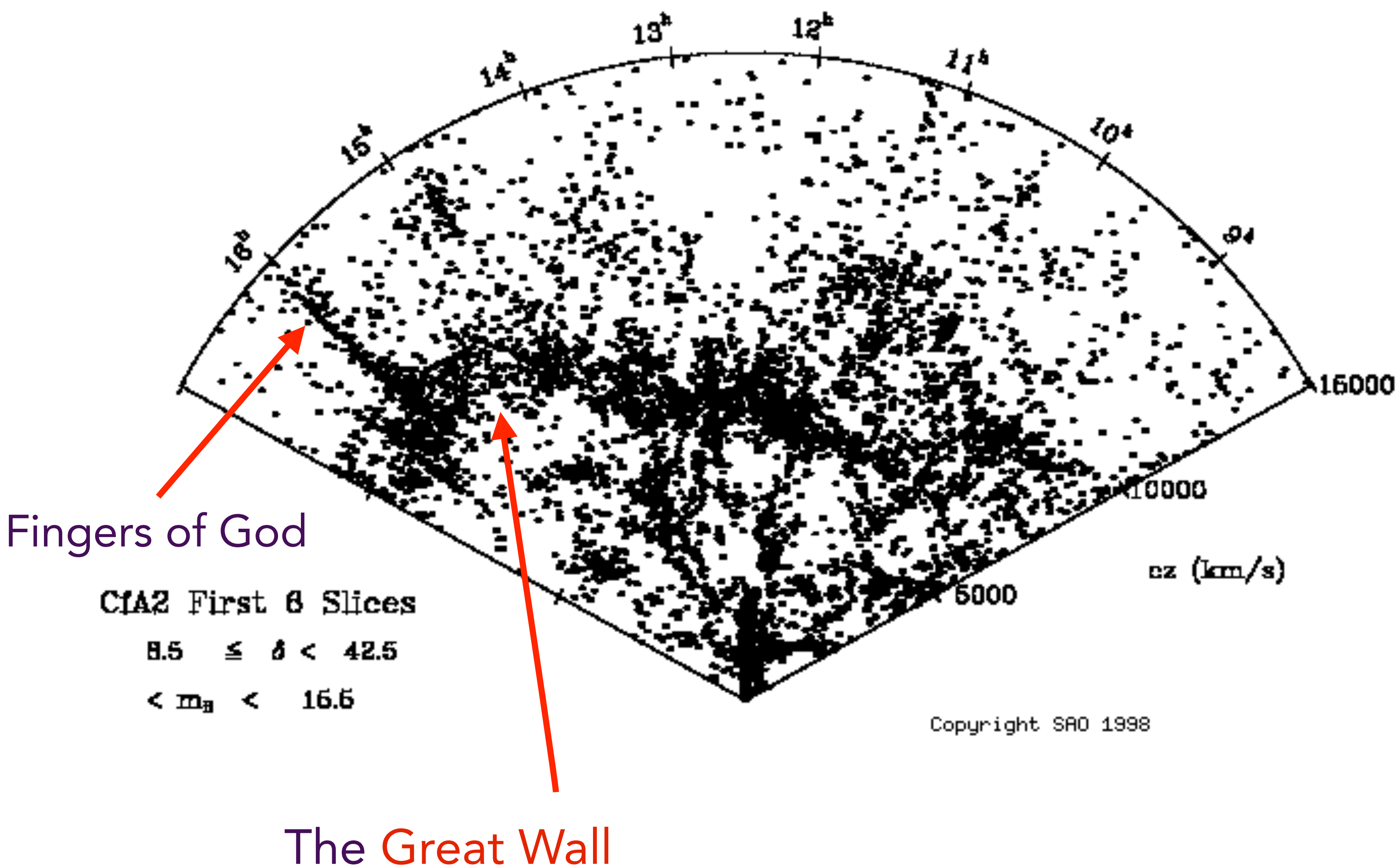
This time I'm going to talk about a slightly unusual application of EFT principles, to structure formation in the universe.

This is not something I'm involved with personally, so none of this is my own work (although you can blame me for the presentation)

- Cosmological non-linearities as an effective fluid, *Baumann et al.* **arXiv:1004.2488**
- The effective field theory of cosmological large scale structures, *Carrasco et al.* **arXiv:1206.2926**
- The effective field theory of large scale structures at two loops, *Carrasco et al.* **arXiv:1310.0464**
- Renormalized halo bias, *Assassi et al.* **arXiv:1402.5916**
- Effective theory of large-scale structure with primordial non-Gaussianity, *Assassi et al.* **arXiv:1505.06668**

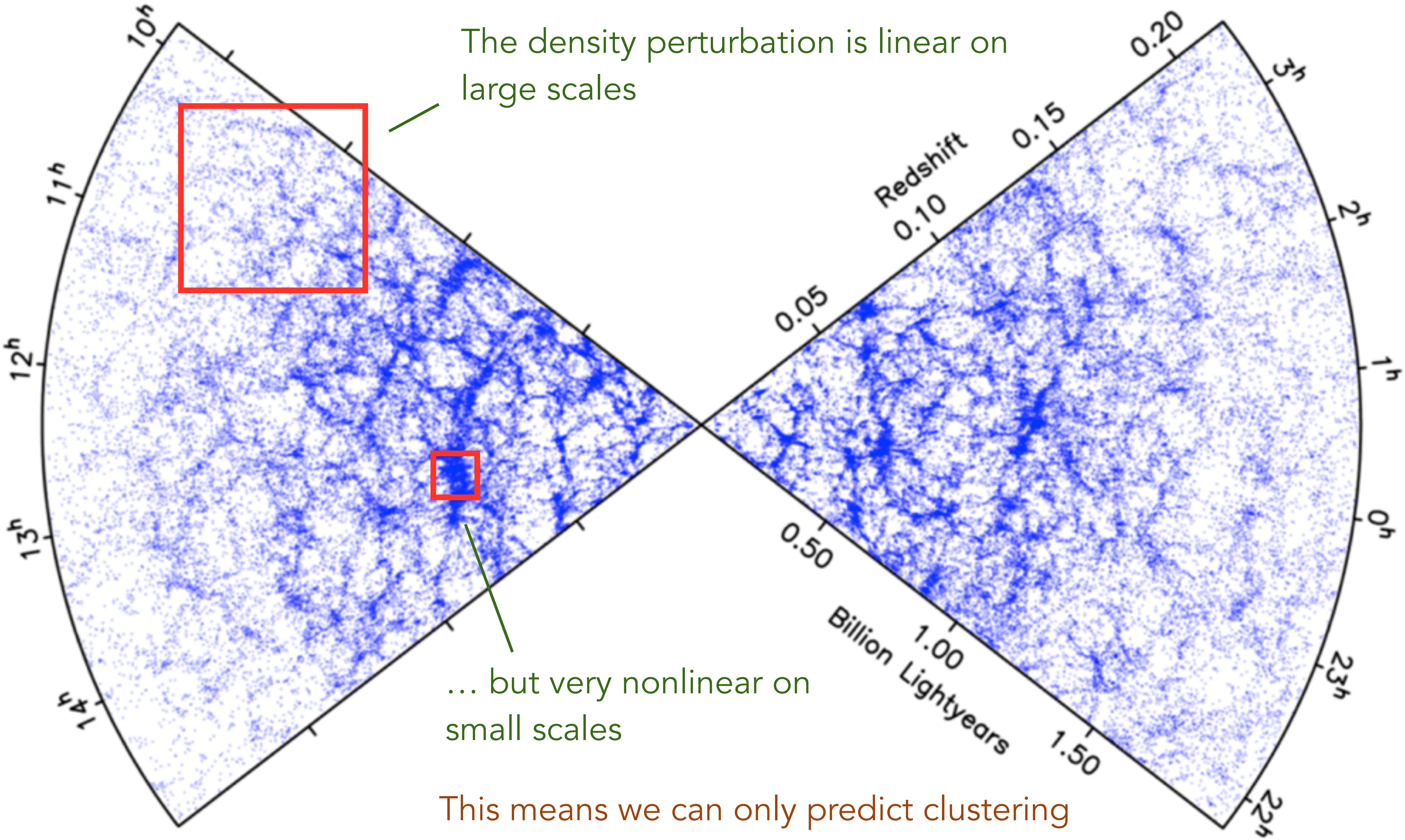
CfA2 (1985–1995)

To accurately determine the redshift of each galaxy, it was necessary to make painstaking measurements of the spectrum. This could require up to an hour of observing per galaxy.



CREDIT SAO, CfA2

2dF ("2 degree field," 1997–2002)



The density perturbation is linear on large scales

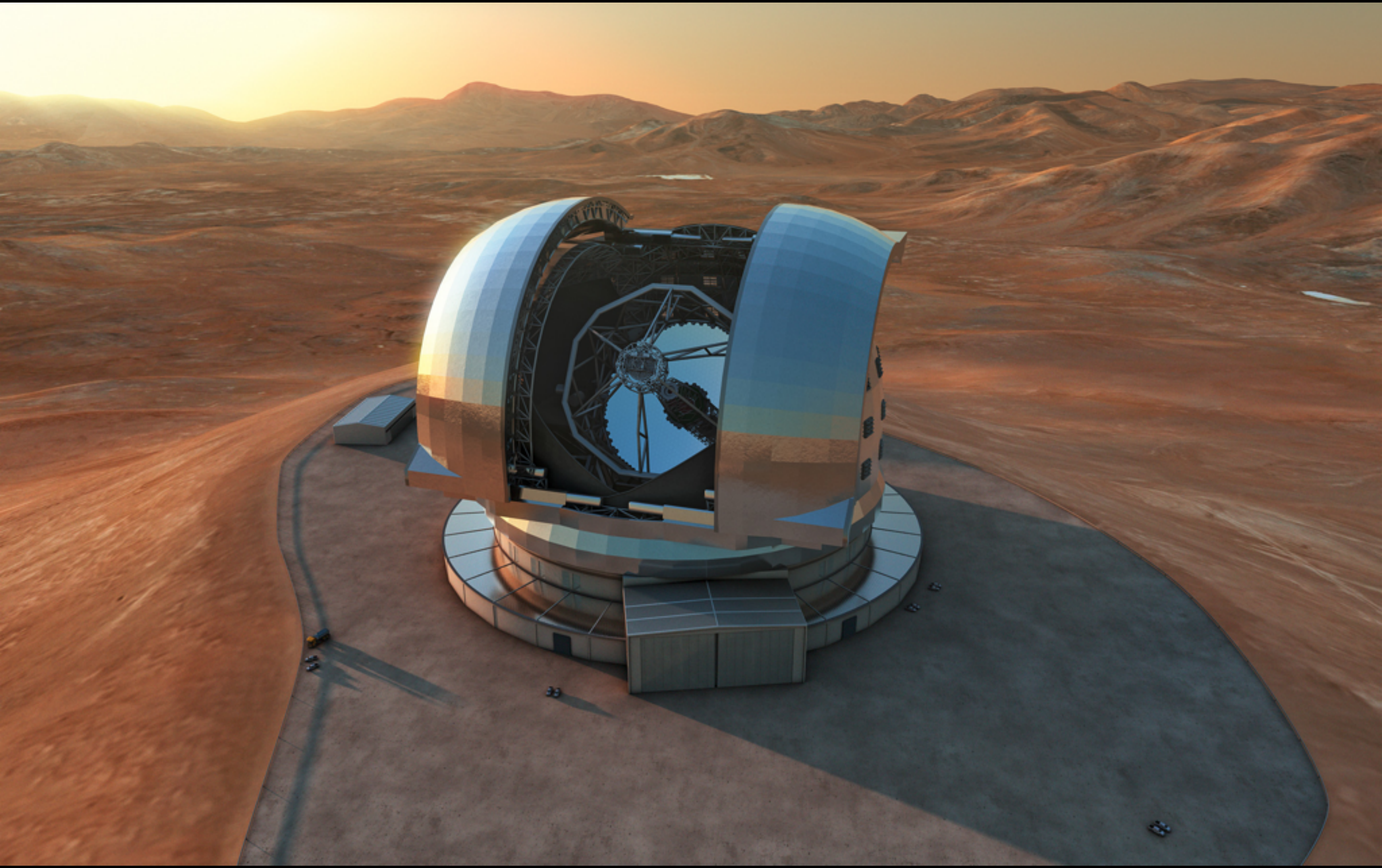
… but very nonlinear on small scales

This means we can only predict clustering properties on large scales.
It limits the primordial information we can extract

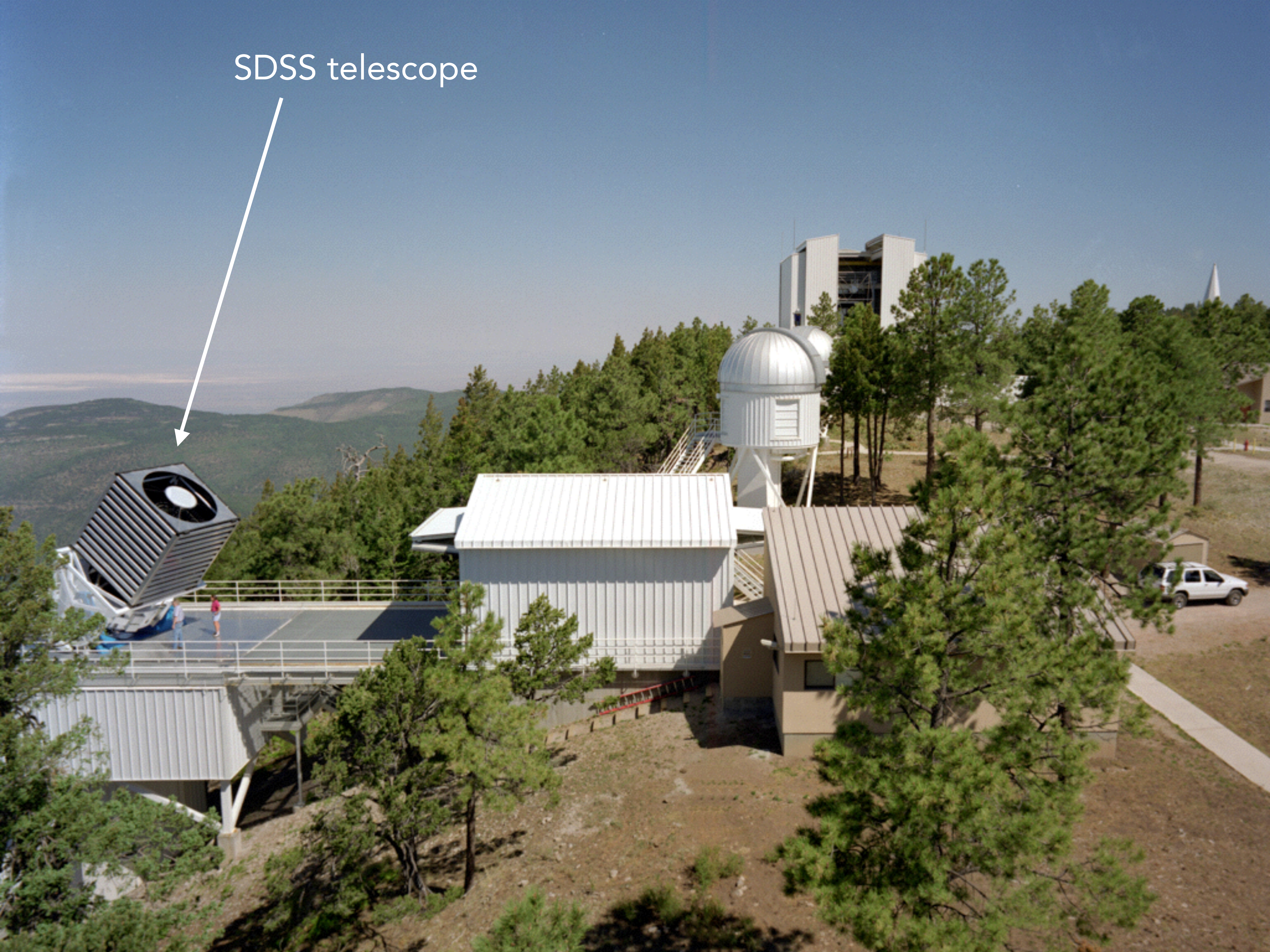
CREDIT 2dF team

Siding Spring Observatory

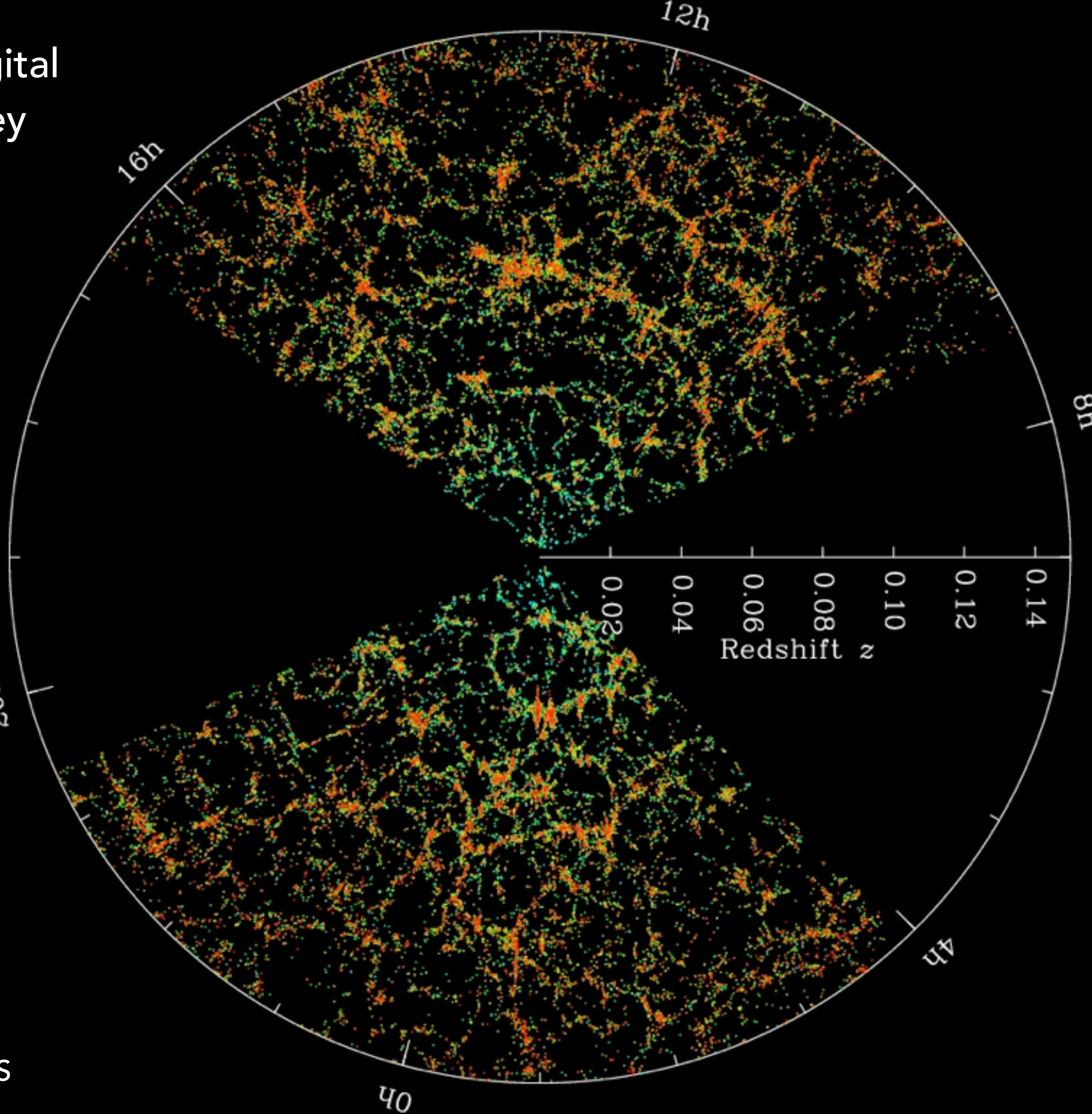
VLT at Cerro Paranal



Artist's impression of the European Extremely Large Telescope

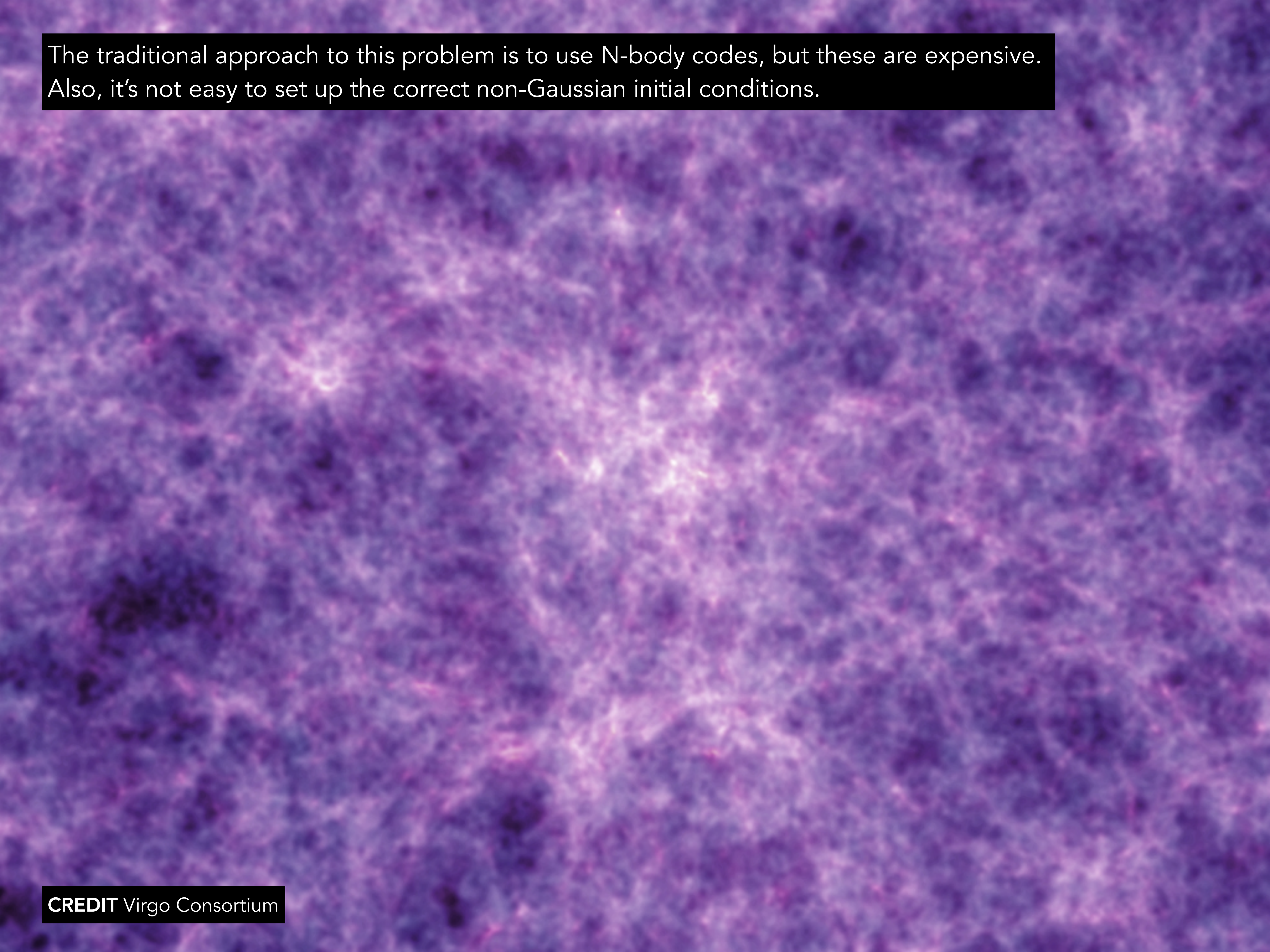

SDSS telescope

Sloan Digital Sky Survey



CREDIT SDSS

The traditional approach to this problem is to use N-body codes, but these are expensive. Also, it's not easy to set up the correct non-Gaussian initial conditions.



CREDIT Virgo Consortium

Cosmology: background

Our first job is to describe the spacetime

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + a(t)^2 \mathrm{d}\boldsymbol{x}^2 = -\mathrm{d}t^2 + a(t)^2 \left[\mathrm{d}x^2 + \mathrm{d}y^2 + \mathrm{d}z^2\right]$$

scale factor

inflation makes space flat

The evolution of *a*(*t*) is given by the Einstein equations

$$G_{ab} = 8\pi G T_{ab}$$

$$R_{ab} - \frac{1}{2} R g_{ab}$$

"Einstein tensor"

$$\frac{1}{M_{\mathrm{P}}^2}$$

Reduced Planck mass

$M_{\mathrm{P}} \approx 10^{18}\,\mathrm{GeV}$

energy–momentum tensor

stress–energy tensor

Cosmology: background

For the background, the only Einstein equation we need is

$$3H^2 M_{\rm P}^2 = \rho = \sum_i \rho_i \approx \rho_m + \rho_r + \rho_\Lambda$$

matter

dark energy

for independent species

radiation

divide through by $3H^2 M_{\rm P}^2$

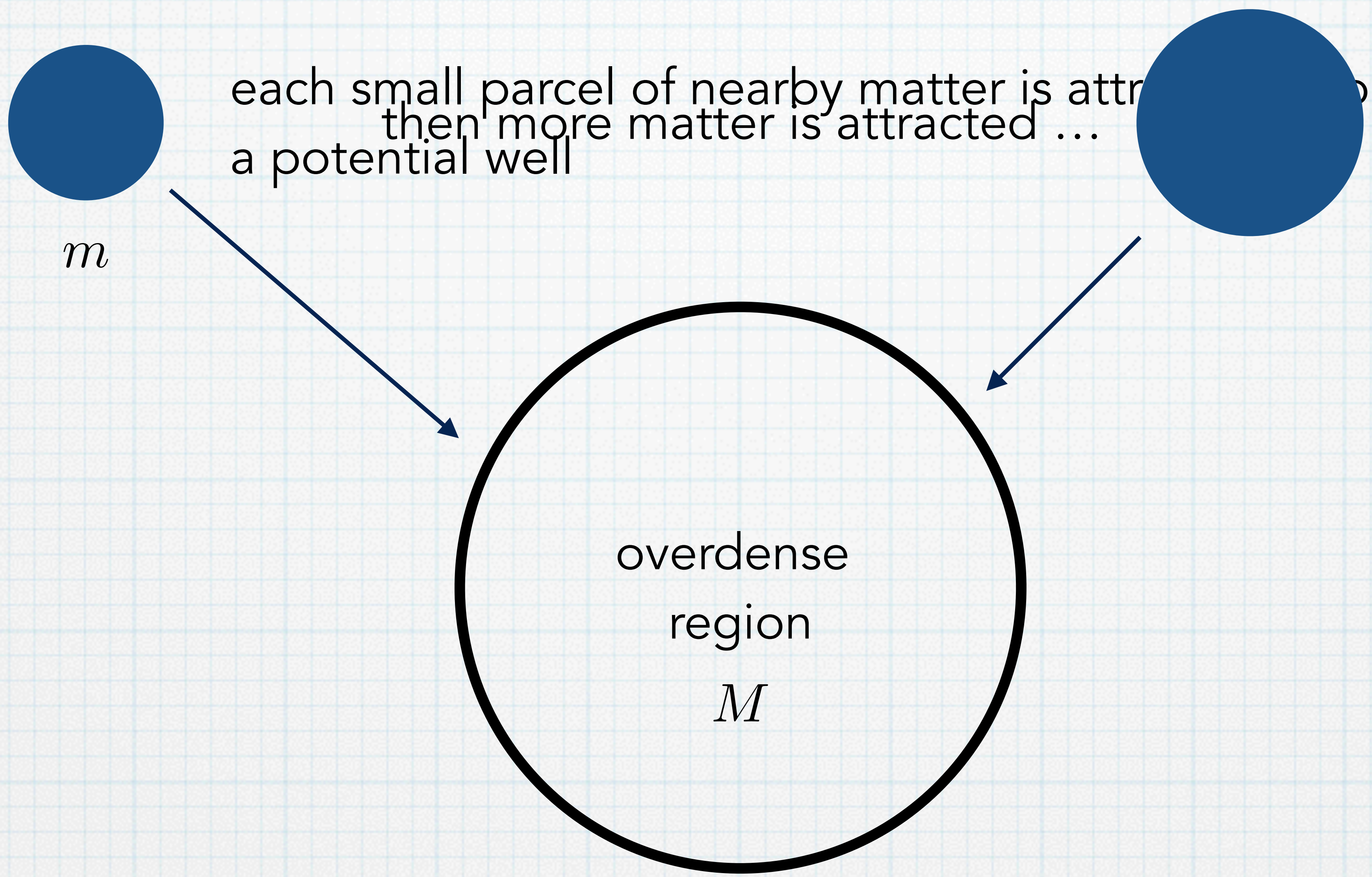
$$1 = \frac{\rho_m}{3H^2 M_{\rm P}^2} + \frac{\rho_r}{3H^2 M_{\rm P}^2} + \frac{\rho_\Lambda}{3H^2 M_{\rm P}^2}$$

$$= \Omega_m + \Omega_r + \Omega_\Lambda$$

we call these the “density parameters”
In a flat model they sum to 1

Structure forms by condensation

$\boldsymbol{F} = m\boldsymbol{a}$



$M \to M + m$

The condensation process is non-relativistic and the gravitational potential stays perturbatively small, $\Phi \approx 10^{-5}$

(in fact Φ is close to this value almost everywhere)

	acceleration	internal pressure	redshifting	gravity

$\boldsymbol{F} = m\boldsymbol{a}$	$\partial_t v^i + v^m \partial_m v^i + \frac{1}{p+\rho}\left(v^i \partial_t p + \frac{1}{a^2}\partial_i p\right) + 2Hv^i - \frac{1}{a^2}\partial_i \Phi = 0$
$M \to M + m$	$\partial_t \rho + \partial_m [v^m (p+\rho)] + 3(H + \dot{\Phi})(p+\rho) = 0$

change of mass	redshifting	relativistic volume modulation

↓

often we ignore this term,
called the "quasistatic" approximation

The condensation process is non-relativistic and the gravitational potential is perturbatively small, $\Phi \approx 10^{-5}$

nonlinear advection terms
cf. Navier–Stokes

viscosity — forces

$$\partial_t \boldsymbol{u} + \boldsymbol{u}\cdot\nabla\boldsymbol{u} - \nu\nabla^2\boldsymbol{u} = -\nabla w + \boldsymbol{g}$$

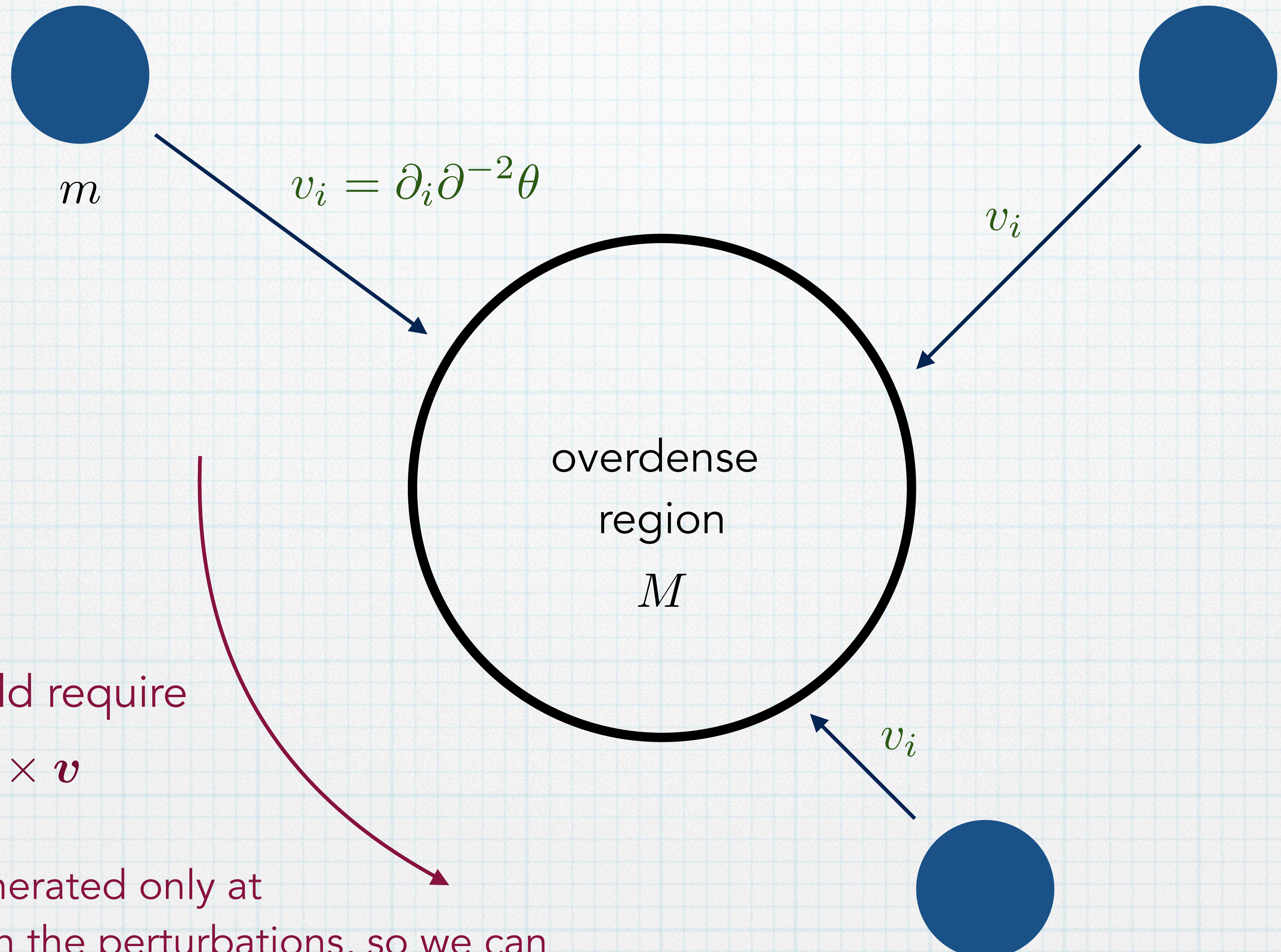
gravity

$\boldsymbol{F} = m\boldsymbol{a}$	$\partial_t v^i + v^m \partial_m v^i + \frac{1}{p+\rho}\left(v^i \partial_t p + \frac{1}{a^2}\partial_i p\right) + 2Hv^i - \frac{1}{a^2}\partial_i \Phi = 0$
$M \to M + m$	$\partial_t \rho + \partial_m[v^m(p+\rho)] + 3(H + \dot{\Phi})(p+\rho) = 0$

Poisson constraint $$-\frac{1}{a^2}\partial^2\Phi = \frac{\delta\rho}{2M_{\mathrm{P}}^2}$$

define $\delta = \delta\rho/\rho_0$ $$= \frac{3H^2}{2}\Omega_m \delta$$

if the inflow is effectively radial then we need only $\theta = \partial_i v^i$



rotation would require

$$\boldsymbol{\omega} = \nabla \times \boldsymbol{v}$$

rotation is generated only at higher order in the perturbations, so we can ignore it

$$\beta(\boldsymbol{q},\boldsymbol{r}) = \frac{\boldsymbol{q}\cdot\boldsymbol{r}}{2q^2r^2}(\boldsymbol{q}+\boldsymbol{r})^2$$

in Fourier space

$$\dot{\theta}_k + 2H\theta_k + \frac{3H^2}{2}\Omega_m\delta_k = -\int \frac{\mathrm{d}^3q\,\mathrm{d}^3r}{(2\pi)^3}\theta_q\theta_r\delta(\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{r})\beta(\boldsymbol{q},\boldsymbol{r})$$

$\boldsymbol{F} = m\boldsymbol{a}$

$$\dot{\theta} + \partial_m\partial^{-2}\theta\partial_m\theta + \partial_m\partial_n\partial^{-2}\theta\partial_m\partial_n\partial^{-2}\theta + 2H\theta + \frac{3H^2}{2}\Omega_m\delta = 0$$

$M \to M + m$

$$\dot{\delta} + \partial_m\partial^{-2}\theta\partial_m\delta + (1+\delta)\theta = 0$$

in Fourier space

$$\dot{\delta}_k + \theta_k = -\int \frac{\mathrm{d}^3q\,\mathrm{d}^3r}{(2\pi)^3}\theta_q\delta_r\delta(\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{r})\alpha(\boldsymbol{q},\boldsymbol{r})$$

$$\alpha(\boldsymbol{q},\boldsymbol{r}) = \frac{\boldsymbol{q}\cdot(\boldsymbol{q}+\boldsymbol{r})}{q^2}$$

in Fourier space

$$\dot{\theta}_k + 2H\theta_k + \frac{3H^2}{2}\Omega_m\delta_k = -\int \frac{\mathrm{d}^3q\,\mathrm{d}^3r}{(2\pi)^3}\theta_q\theta_r\delta(\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{r})\beta(\boldsymbol{q},\boldsymbol{r})$$

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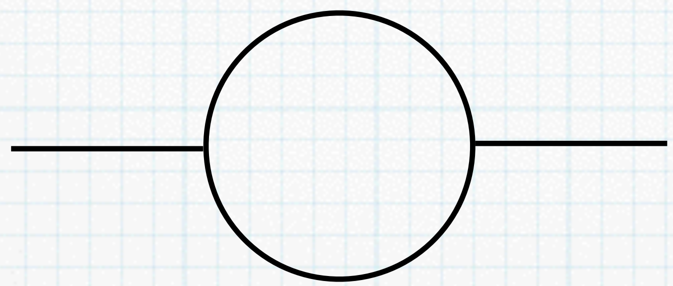
Given an initial state, whose statistical properties we can calculate using the rules from the last lecture, we can solve perturbatively. This is just the same as in field theory.

When we do so, we will encounter loop integrals.

Field theory

$$\int \frac{\mathrm{d}^4 q}{(2\pi)^4} \frac{1}{q^2 + m^2}$$

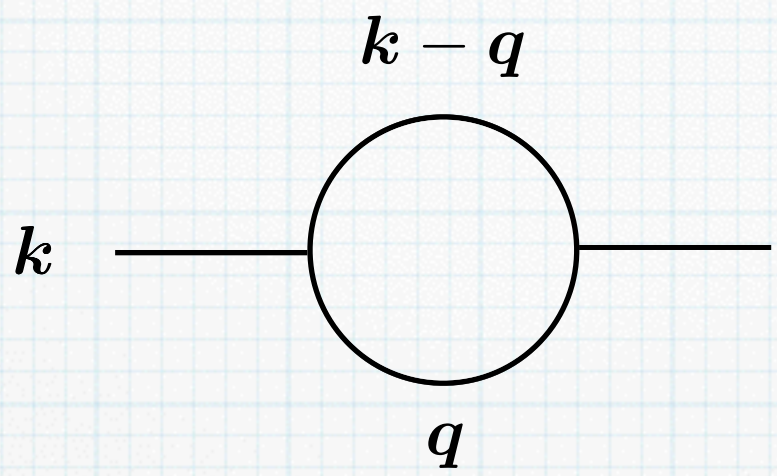


vacuum state has no excitations

loop averages over quantum fluctuations

Structure formation

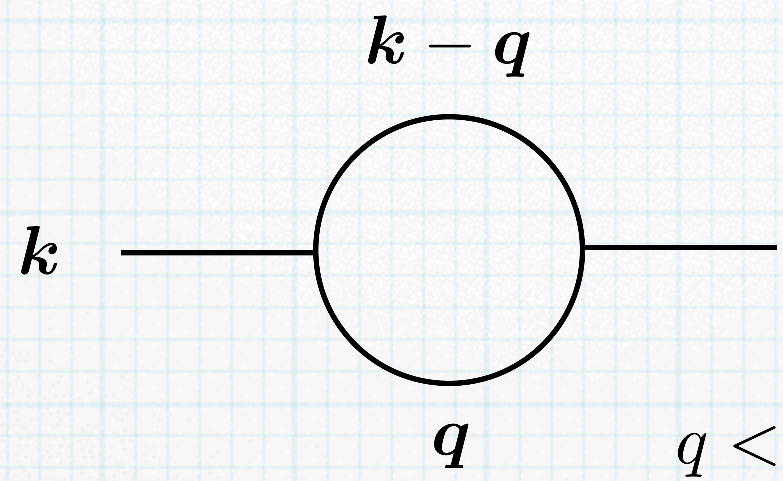
$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3} P(k)$$



state populated with excitations described statistically
loop averages the effect of these excitations

Structure formation

$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3} P(k)$$



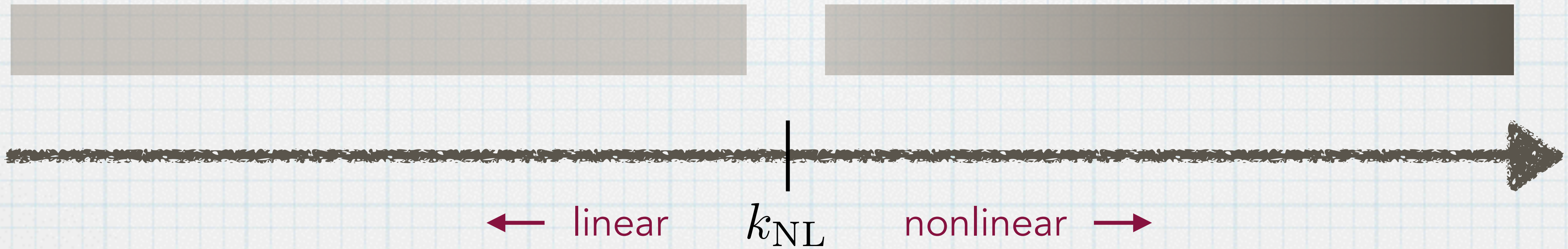
$q < k_{\mathrm{NL}}$

$q > k_{\mathrm{NL}}$ ultraviolet

The loop momentum $\boldsymbol{q}$ runs over all scales, including those we don't understand

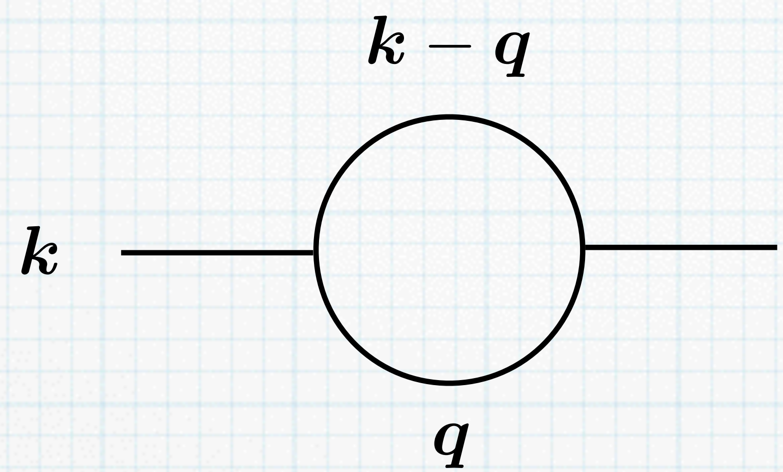
$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3}$$

perturbative rearrangement of initial conditions + unknown details of galaxy formation, gas dynamics, feedback …



Structure formation

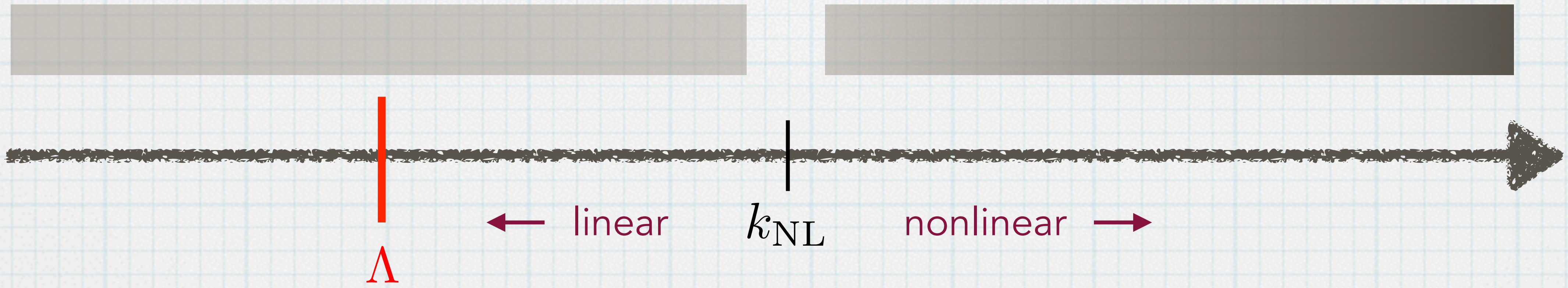
$$\int \frac{d^3q}{(2\pi)^3} P(k)$$



Our standard tool for separating averages over fluctuations we understand from those we don't is **effective field theory**

Cut off the loop integrals at some momentum Λ
The cut-off parametrizes their dependence on the unknown UV scales
Finally, renormalize to make predictions independent of Λ

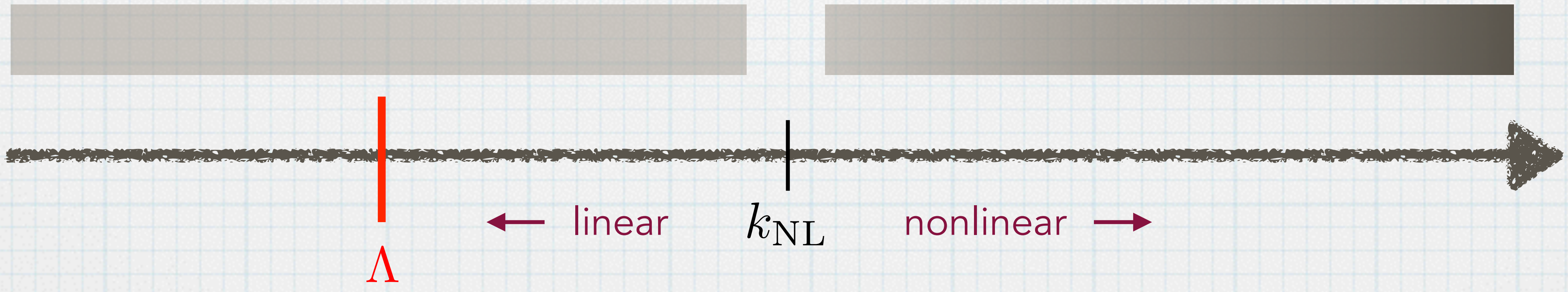
$$\int \frac{d^3q}{(2\pi)^3}$$ perturbative rearrangement of initial conditions + unknown details of galaxy formation, gas dynamics, feedback …



Structure formation

$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3} P(k)$$

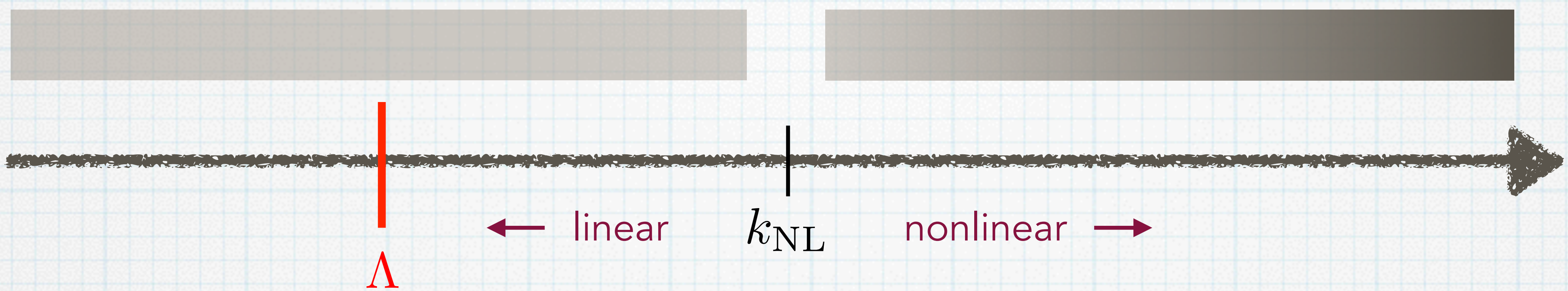
$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3}$$ perturbative rearrangement of initial conditions + unknown details of galaxy formation, gas dynamics, feedback …



Structure formation

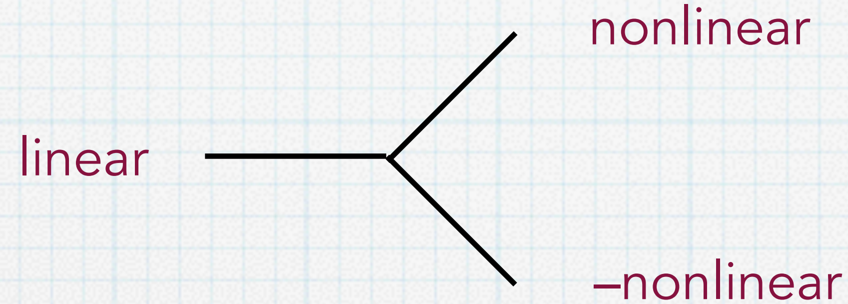
$$\int \frac{d^3q}{(2\pi)^3} P(k)$$

$$\int \frac{d^3q}{(2\pi)^3}$$ perturbative rearrangement of initial conditions + unknown details of galaxy formation, gas dynamics, feedback …



Forward process

linear → nonlinear

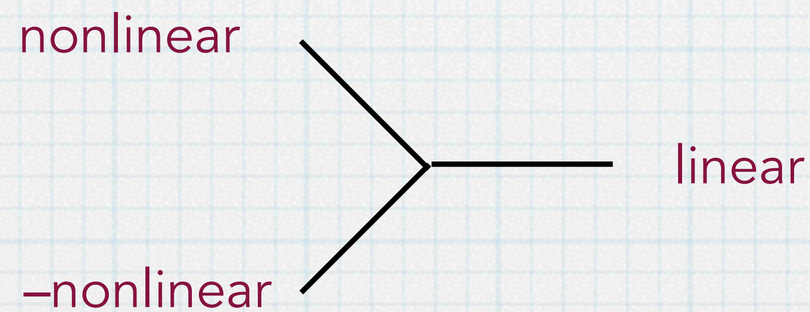


a linear mode splits into two nonlinear modes

its energy is lost from the linear regime → *dissipation*

Backward process

nonlinear → linear



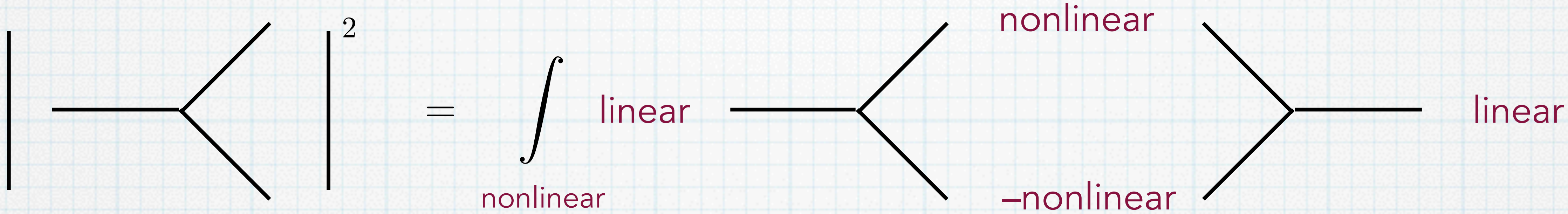
two nonlinear modes coalesce into a linear mode

energy is recovered from the nonlinear bath → *fluctuations*

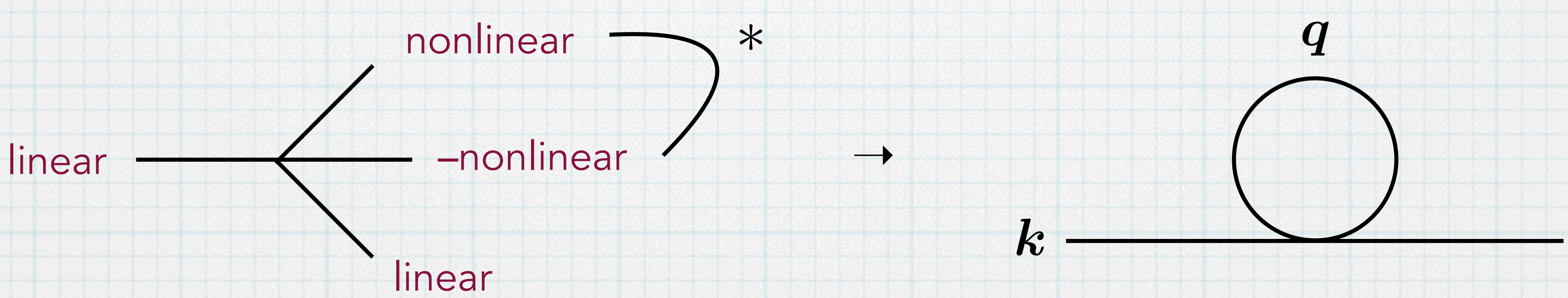
Structure formation

$$\int \frac{\mathrm{d}^3 q}{(2\pi)^3} P(k)$$

The loop is the average of these two processes

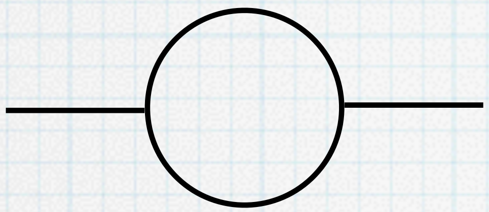


Other loops have a similar interpretation



The 22 contribution

At 1-loop these are the only possibilities. First, consider the 22 contribution



$$P_{22}(\boldsymbol{k}) = \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, P(q) P(\boldsymbol{k}-\boldsymbol{q}) \Big(\boldsymbol{k},\ \boldsymbol{q} \text{ and } z\text{-dependent piece} \Big)$$

Capture low-energy behaviour by expanding in $\boldsymbol{k}$

$$P(q) + k^2 \times \cdots \qquad A_0(q,z) + k^2 A_2(q,z) + \cdots$$

Result is *analytic in* k^2

$$= \int \frac{\mathrm{d}^3 q}{(2\pi)^3}\, P(q)^2 A_0(q,z) + k^2 \times \cdots$$

Inverse Fourier transform

$$\langle \delta(\boldsymbol{x}) \delta(\boldsymbol{x}+\boldsymbol{r}) \rangle \sim \delta(\boldsymbol{r}) + \partial_r^2 \delta(\boldsymbol{r}) + \cdots$$

The 22 contribution

Result is *analytic in k*^{2} $$= \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \, P(q)^2 A_0(q,z) + k^2 \times \cdots$$

Inverse Fourier transform $$\langle \delta(\boldsymbol{x}) \delta(\boldsymbol{x}+\boldsymbol{r}) \rangle \sim \delta(\boldsymbol{r}) + \partial_r^2 \delta(\boldsymbol{r}) + \cdots$$

The 22 contribution

Result is *analytic in* k^2

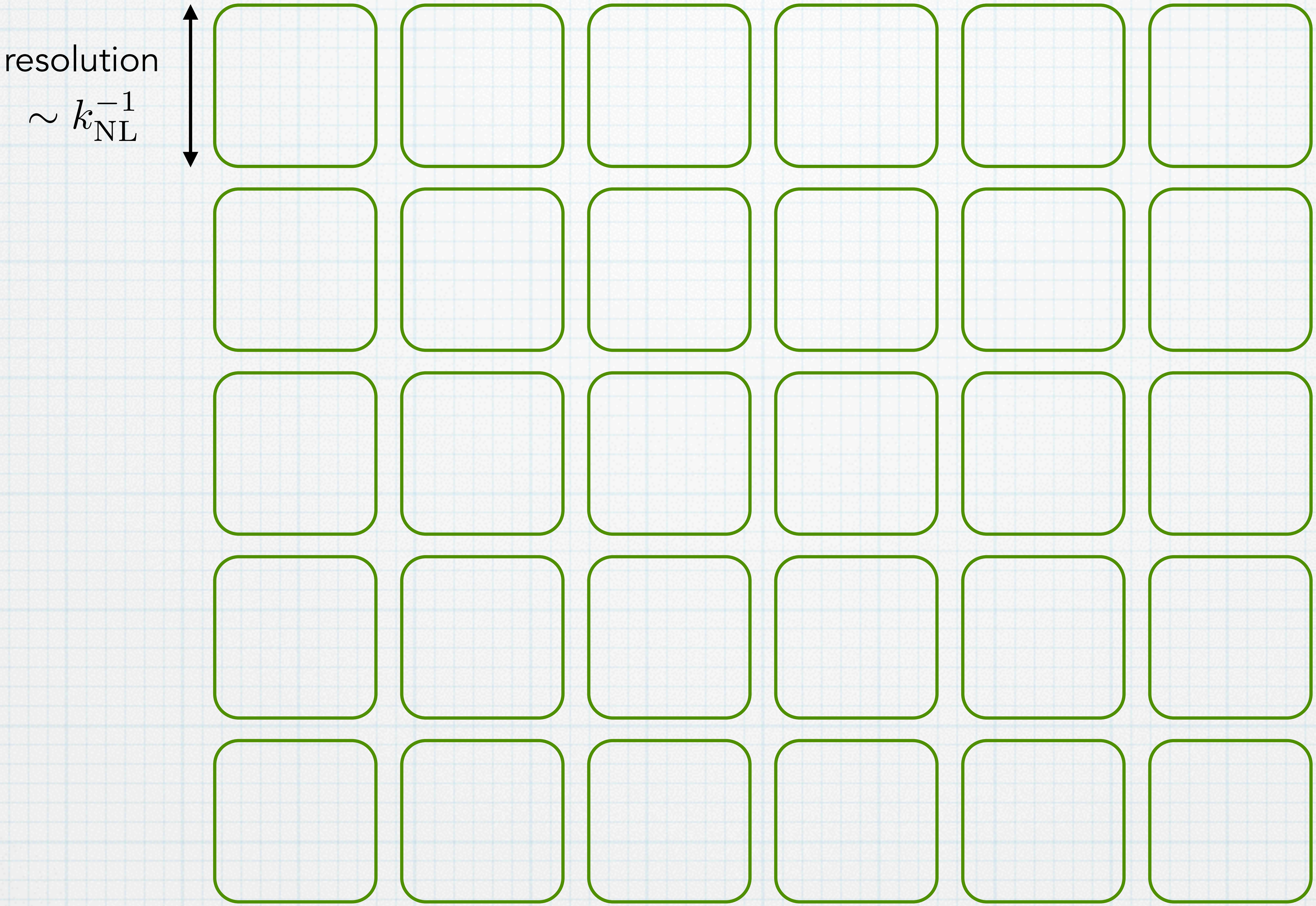
$$= \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \, P(q)^2 A_0(q, z) + k^2 \times \cdots$$

Inverse Fourier transform

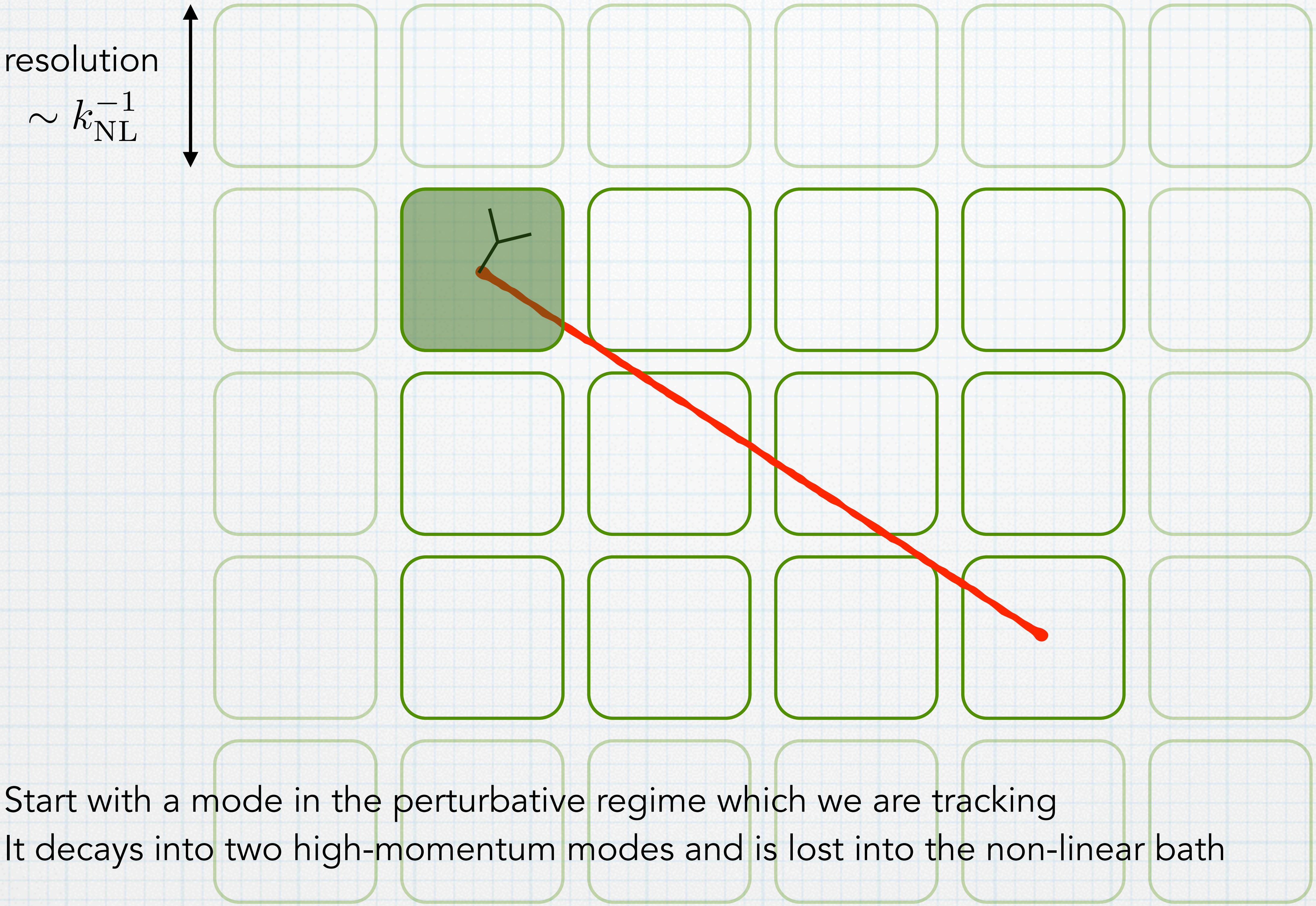
$$\langle \delta(\boldsymbol{x}) \delta(\boldsymbol{x} + \boldsymbol{r}) \rangle \sim \delta(\boldsymbol{r}) + \partial_r^2 \delta(\boldsymbol{r}) + \cdots$$

This is a standard argument in field theory — only non-analytic terms lead to long-range effects; everything else can be absorbed into a local counterterm.

Energy transfer to and from the non-linear bath happens below the resolution of the EFT.
It looks purely local viewed from larger scales.

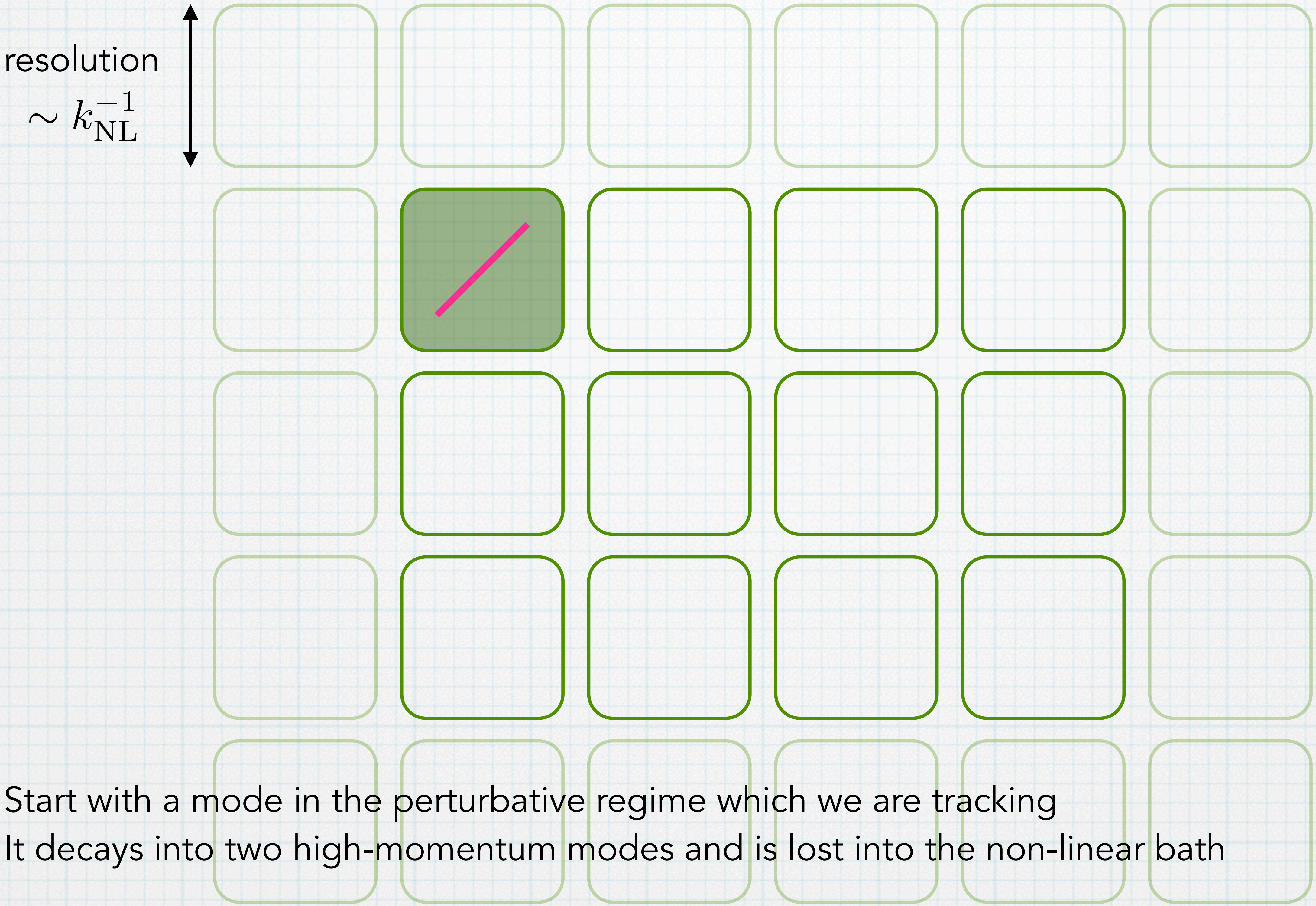


The effective theory doesn't resolve what happens on scales below the cutoff



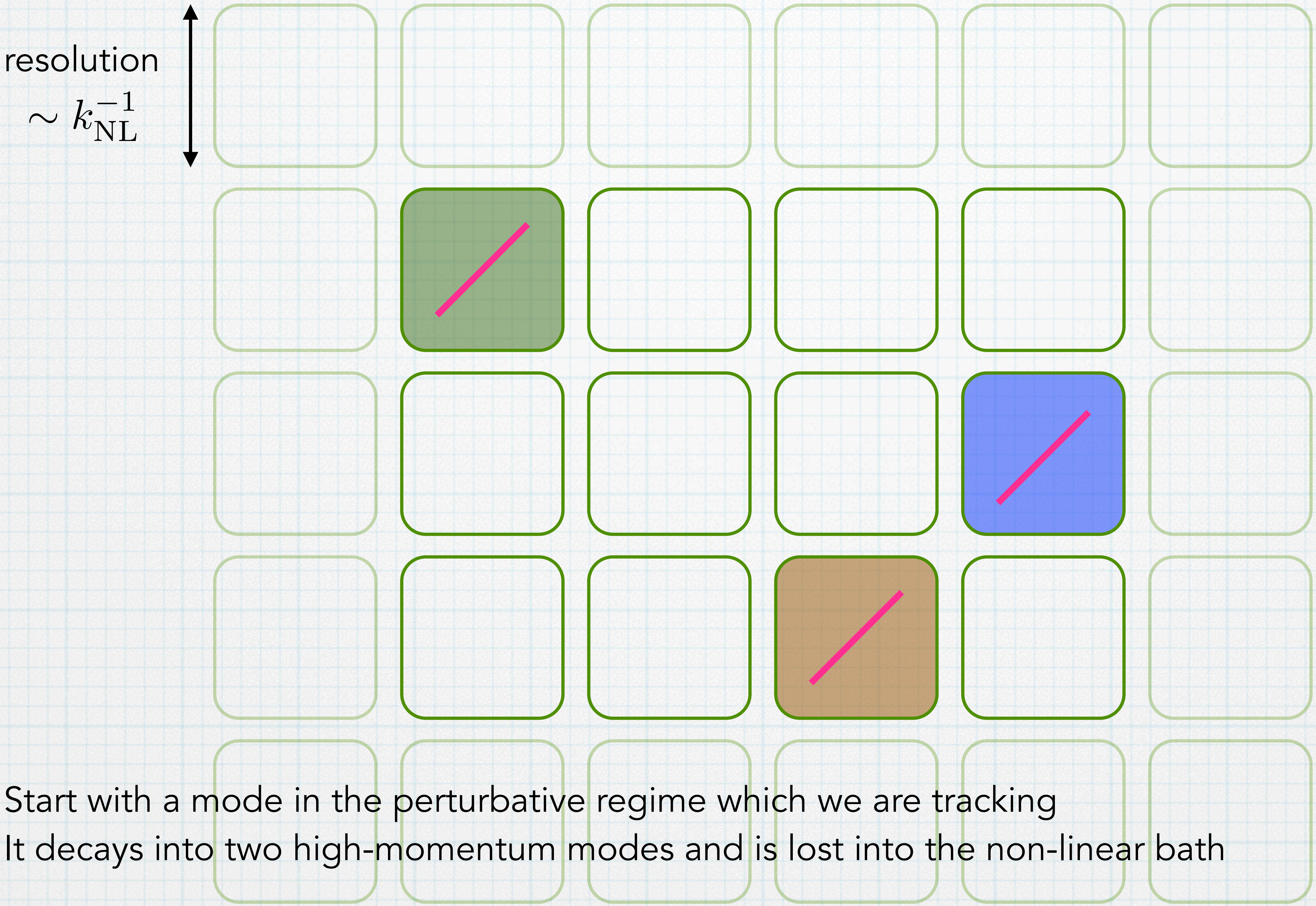
Start with a mode in the perturbative regime which we are tracking

It decays into two high-momentum modes and is lost into the non-linear bath



Start with a mode in the perturbative regime which we are tracking
It decays into two high-momentum modes and is lost into the non-linear bath

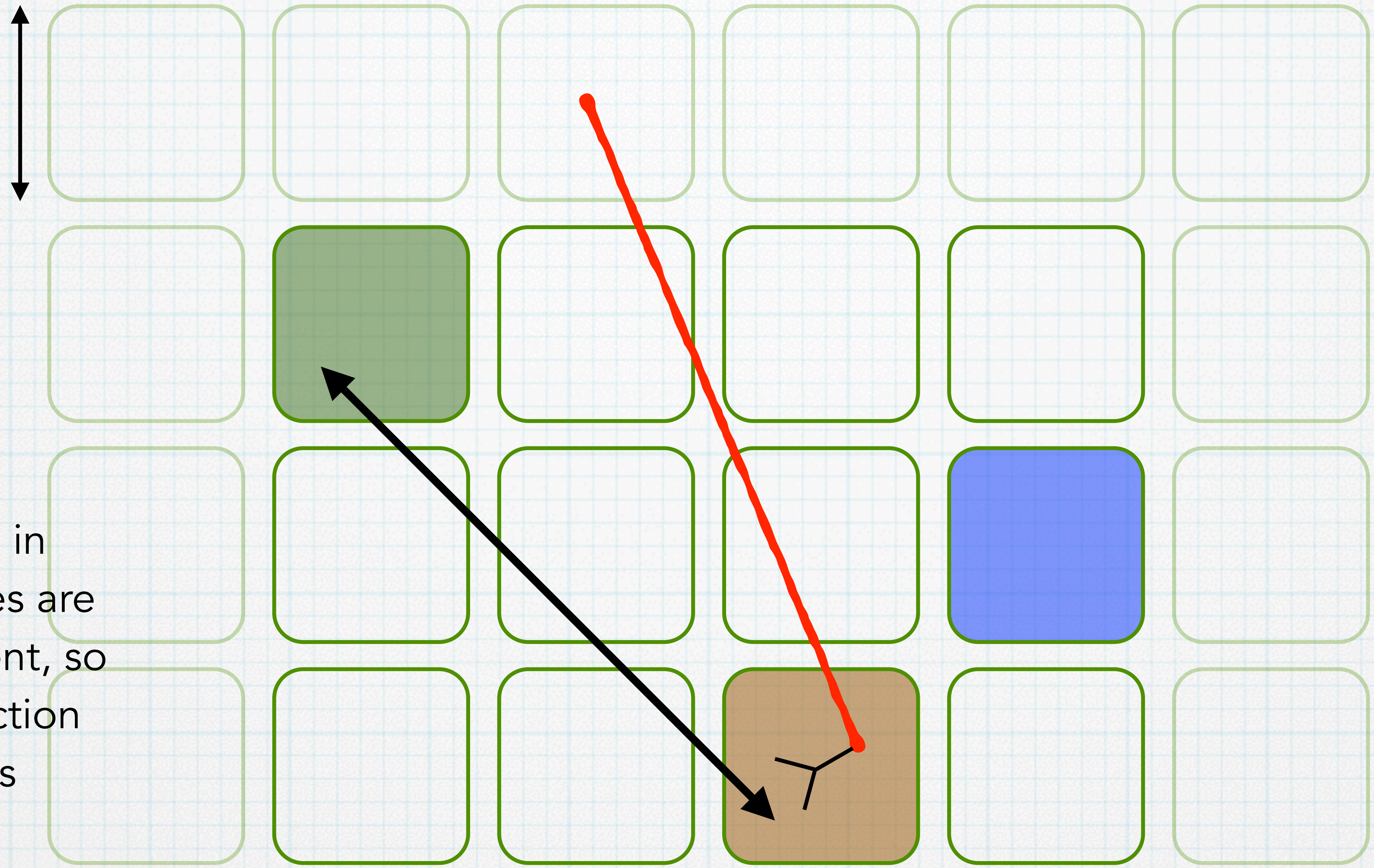
The high-momentum modes get scrambled while we are not looking



Start with a mode in the perturbative regime which we are tracking
It decays into two high-momentum modes and is lost into the non-linear bath

The high-momentum modes get scrambled while we are not looking resolution

$\sim k_{\rm NL}^{-1}$



Processes happening in these boxes are independent, so give δ-function correlations

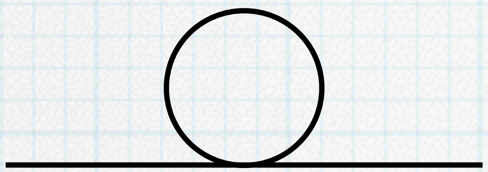
Start with a mode in the perturbative regime which we are tracking

It decays into two high-momentum modes and is lost into the non-linear bath

The high-momentum modes get scrambled while we are not looking

Then they get returned to the low-energy regime

The 13 contribution



$$P_{13}(\boldsymbol{k}) = P(k) \int \frac{\mathrm{d}^3 q}{(2\pi)^3} \, P(q) \Big(\boldsymbol{k}, \, \boldsymbol{q} \text{ and } z\text{-dependent piece} \Big)$$

nonanalyticity inherited from linear result

low-***k*** expansion will systematically modify the linear long-range behaviour

$$\sim k^2 \times \cdots$$

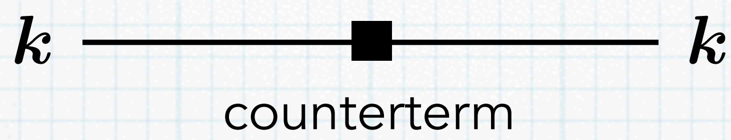
The leading behaviour is k^2. We could keep more terms in the low-energy expansion if we wished.

Its coefficient is $\sim \int^{\Lambda} \frac{\mathrm{d}^3 q}{(2\pi)^3} P(q) B_2(q, z)$

The cutoff dependence has to be renormalized

Removing the Λ dependence

To absorb the cutoff we need a counter term which gives k^2 behaviour at tree-level



$\boldsymbol{F} = m\boldsymbol{a}$	$\dot{\theta}_k + 2H\theta_k + \frac{3H^2}{2}\Omega_m\delta_k = -\int \frac{\mathrm{d}^3q\,\mathrm{d}^3r}{(2\pi)^3}\theta_q\theta_r\delta(\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{r})\beta(\boldsymbol{q},\boldsymbol{r})$ $+\ \text{counterterm} \qquad \partial^2\delta_k, \partial^2\theta_k$
$M \to M + m$	$\dot{\delta}_k + \theta_k = -\int \frac{\mathrm{d}^3q\,\mathrm{d}^3r}{(2\pi)^3}\theta_q\delta_r\delta(\boldsymbol{k}-\boldsymbol{q}-\boldsymbol{r})\alpha(\boldsymbol{q},\boldsymbol{r})$

Removing the Λ dependence

The other candidate counterterm at k^2 would be $\partial_i \partial_j \Phi$, but it is redundant

To interpret the counterterms, go back to the momentum equation in terms of ***v***

$$\partial_t v_i + \cdots - \frac{1}{a^2}\partial_i \Phi = A(t)\partial_i \delta + B(t)\partial^2 v_i + C(t)\partial_i \partial_j v^j$$

sound speed $\nabla p \sim c_s^2 \nabla \rho$

viscosity

Compare to Navier–Stokes $\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} - \nu \nabla^2 \boldsymbol{u} - \eta \nabla (\nabla \cdot \boldsymbol{u}) = -\nabla w + \boldsymbol{g}$

(of course, this is why these terms appear in the Navier–Stokes equation anyway, because it is a long-wavelength approximation to nonlinear small scale dynamics)

Removing the Λ dependence

The other candidate counterterm at k^2 would be $\partial_i \partial_j \Phi$, but it is redundant

To interpret the counterterms, go back to the momentum equation in terms of ***v***

$$\partial_t v_i + \cdots - \frac{1}{a^2}\partial_i \Phi = A(t)\partial_i \delta + B(t)\partial^2 v_i + C(t)\partial_i \partial_j v^j$$

A(*t*), *B*(*t*) and *C*(*t*) are unknown functions of time that must measured from observations

Only a single linear combination appears in the one-loop power spectrum

Removing the Λ dependence

The other candidate counterterm at k^2 would be $\partial_i \partial_j \Phi$, but it is redundant

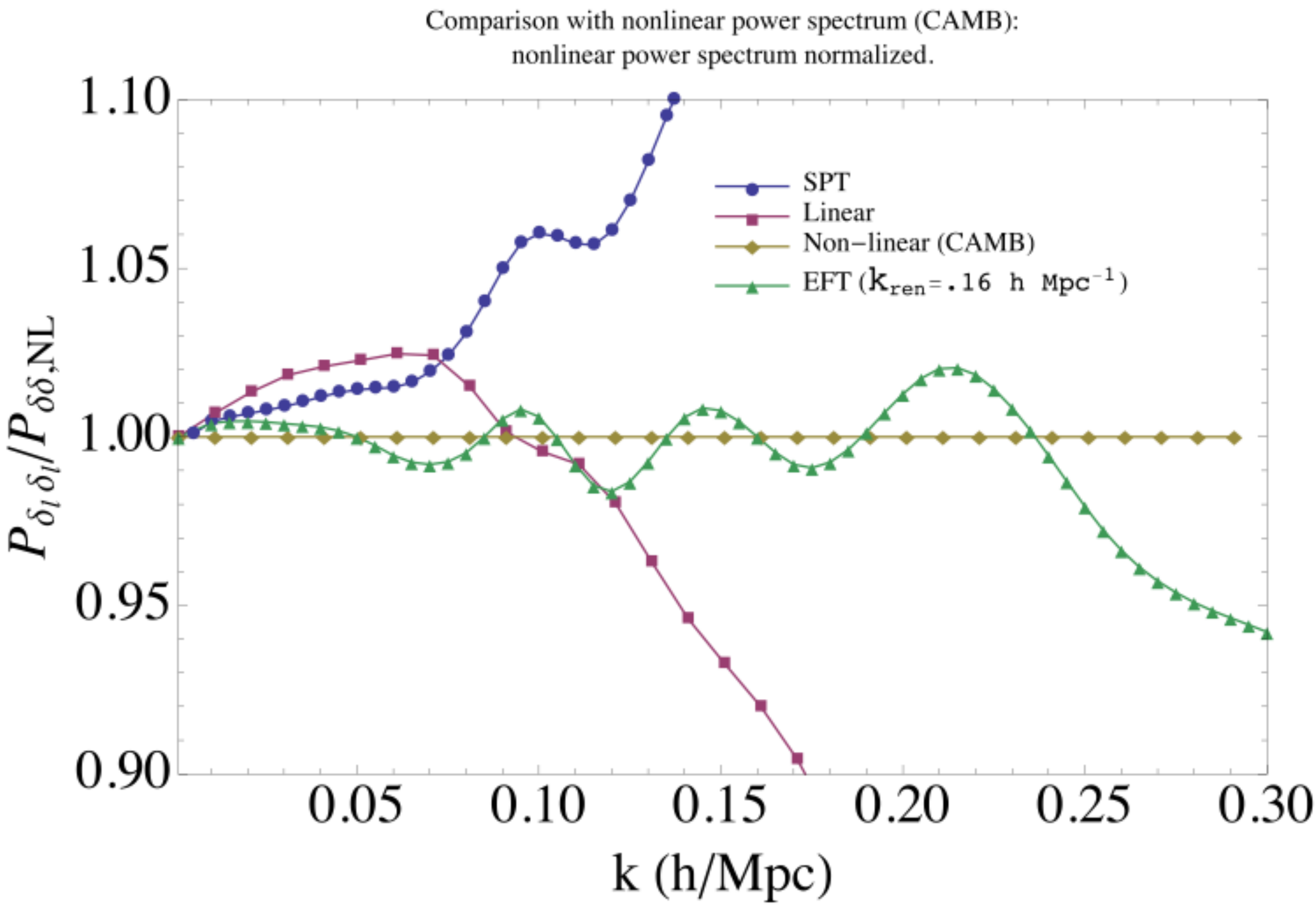
To interpret the counterterms, go back to the momentum equation in terms of **v**

$$\partial_t v_i + \cdots - \frac{1}{a^2}\partial_i \Phi = A(t)\partial_i \delta + B(t)\partial^2 v_i + C(t)\partial_i \partial_j v^j \ + \Theta$$

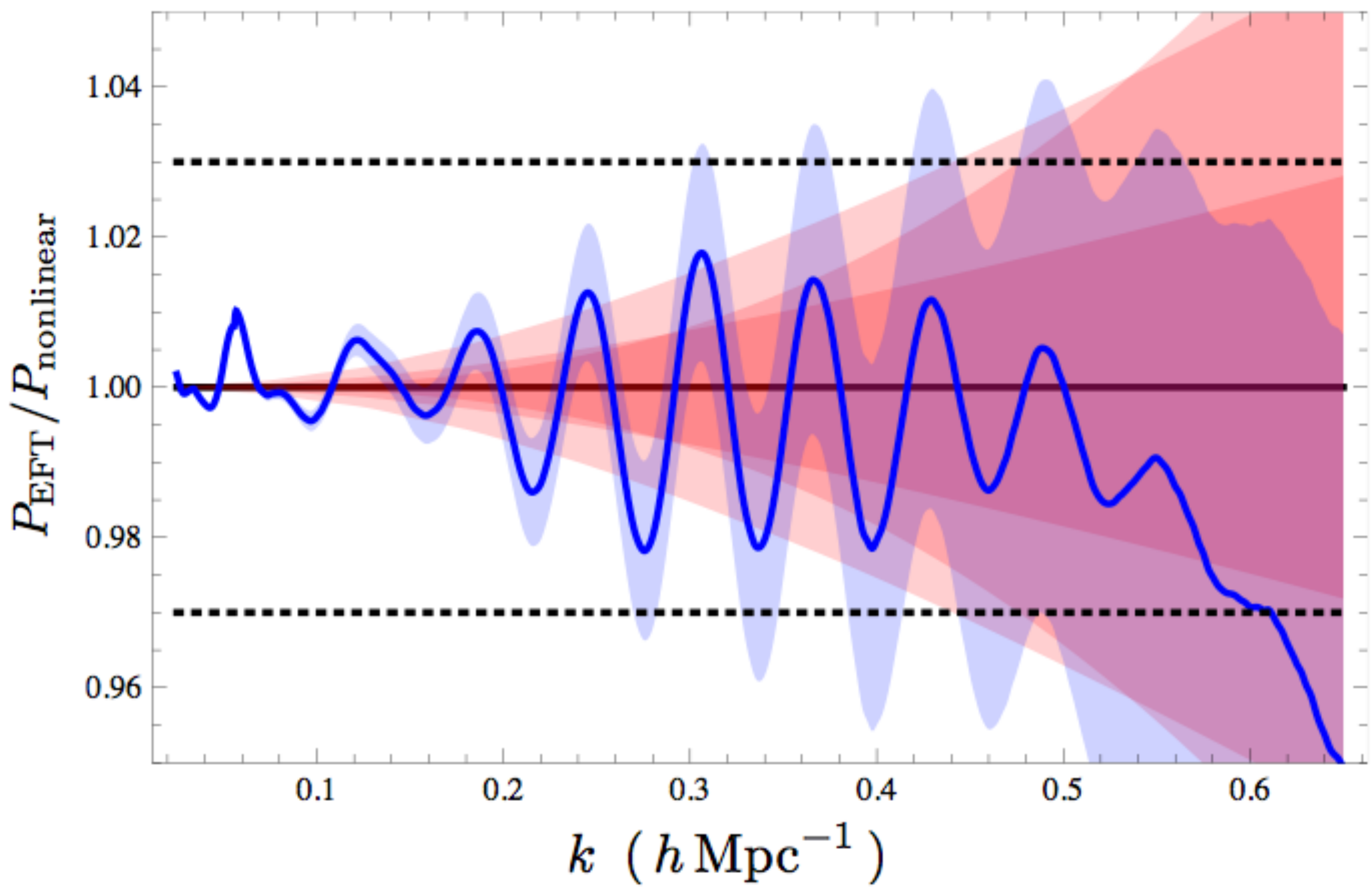
Take Θ to be a stochastic variable with (nearly-) δ-function correlations

$$\langle \Theta(\boldsymbol{x})\Theta(\boldsymbol{x}+\boldsymbol{r})\rangle \sim \delta(\boldsymbol{r}) + \partial_{\boldsymbol{r}}^2 \delta(\boldsymbol{r}) + \cdots$$

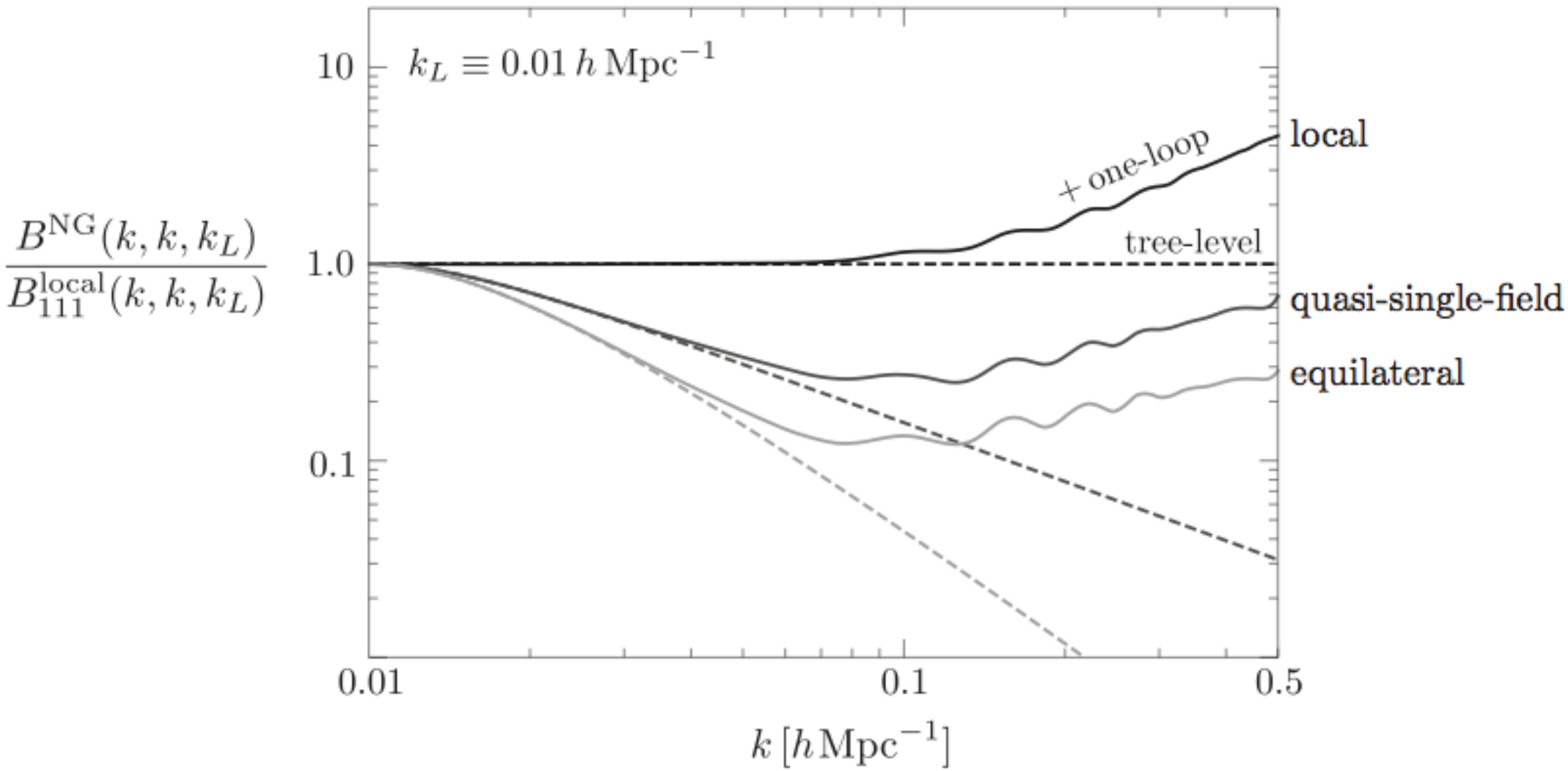
by adjusting the time-dependent coefficients of these δ-functions
we can absorb the δ-function divergences in P_{22}



from *Carrasco, Hertzberg & Senatore* **arXiv:1206.2926**



from *Carrasco, Foreman, Green & Senatore* **arXiv:1310.0464**



from *Assassi et al.* **arXiv:1505.06668**

Pros and cons

- Works well for dark matter where we need only a few counterterms to renormalize the power spectrum
- In reality the situation is more complex because of **bias**.
  The observed density fluctuation depends on composite operators
  δ, δ^2, δ^3 and others which need to be renormalized
  Assassi, Baumann et al. **arXiv:1402.5916**
- With too many counterterms the theory stops being predictive.
  This is the usual trade-off with an effective field theory.
- Recently generalized for non-Gaussian initial conditions, which would let us look at the primordial physics on Tuesday
  Assassi, Baumann et al. **arXiv:1505.06668**