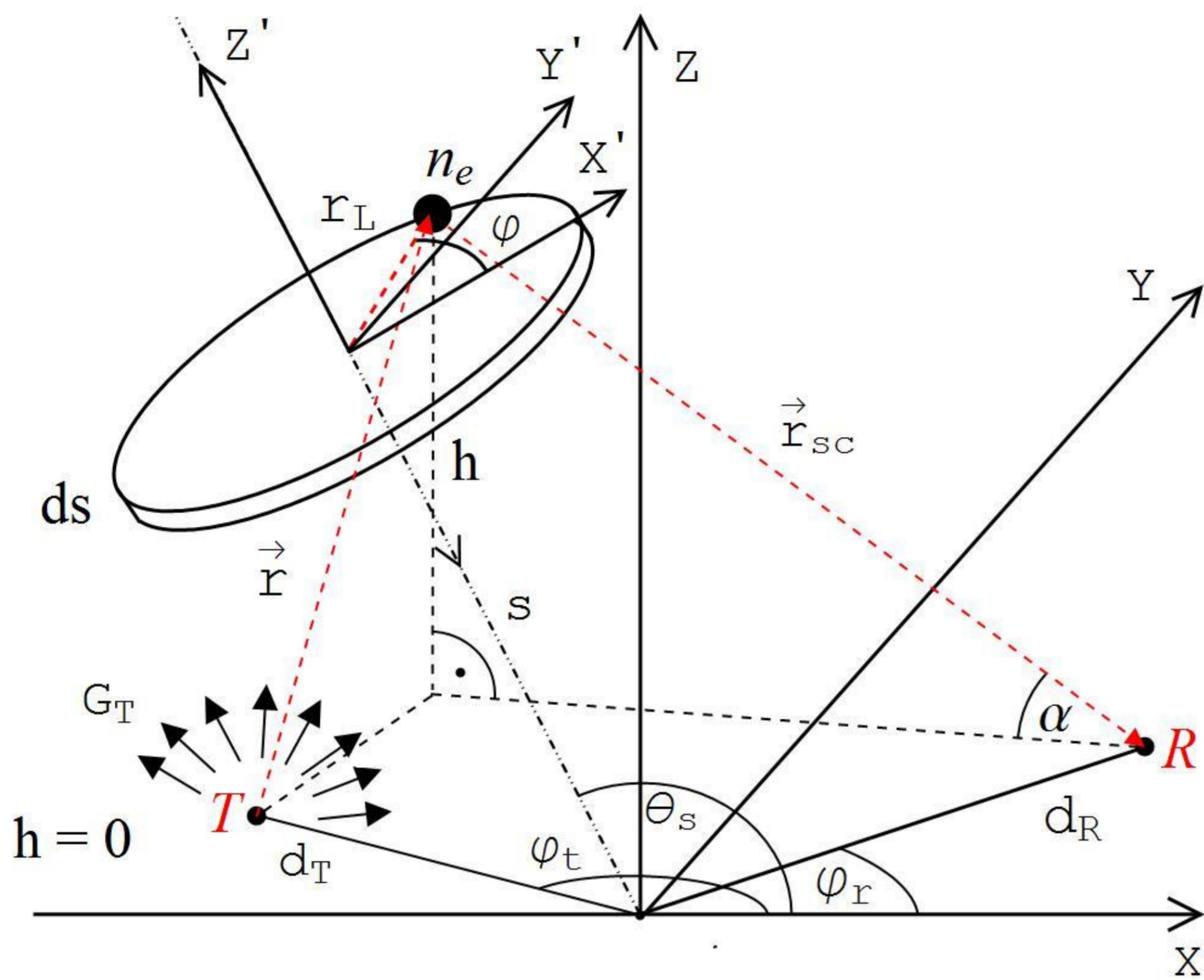


Calculation of the radar signal

A radio transmitter (**T**) irradiates a short-lived, disk-like, non-moving plasma left behind the shower front. The radio signal is scattered by free electrons in the ionization trail and subsequently received by the antenna (**R**). The geometry of the radar system is described by the distances from the shower core to the transmitter (d_T) and to the receiver (d_R), and by the angle θ_s .



The electric field of the radar echo at the receiver at time t :

$$U_E(t) = \frac{\omega}{v_c} \iiint_V U_E(t, s, r_L, \varphi) r_L dr_L d\varphi ds,$$

$$U_E(t, s, r_L, \varphi) \propto n_e e^{i\omega t} e^{-i \int_r n k dr} e^{-i \int_{r_{sc}} n k_{sc} dr_{sc}} \frac{\sqrt{\Delta\Omega_{sc}}}{|r|} \sqrt{P_T} \frac{d\sigma_T}{d\Omega},$$

where n_e is the plasma density, n is the refractive index of the air, $\omega = 2\pi\nu$ is the radian frequency of the emitted radio wave, P_T is the transmitted power, v_c is the collision frequency of an electron with neutral molecules, $d\sigma_T/d\Omega$ is the Thomson cross-section, and $\Delta\Omega_{sc}$ is the solid angle of the receiver as seen from the point of scattering. The factor ω/v_c is introduced to take the molecular quenching into account.

Air shower plasma

- plasma lifetime $\sim 15 - 40$ ns \Rightarrow length of the ionization trail $\sim 10 - 40$ m
- diameter of the disk containing 95% of the plasma electrons > 200 m
- plasma at rest, ionization (shower) front moves with the speed of light \Rightarrow frequency upshift of the scattered signal (**Doppler effect**)

Scattering off the plasma

Radio scattering properties of the plasma depend on the local characteristic plasma frequency ω_p and the electron collision frequency ν_c :

- overdense region: $\omega_p > \omega \gg \nu_c \Rightarrow$ the radio wave is reflected from the plasma surface, **large radar cross-section**
- underdense region: $\omega_p < \omega$ or $\omega < \nu_c \Rightarrow$ scattering on individual electrons within the plasma, **the radar reflection is heavily reduced by molecular quenching** (electron collisions with neutral molecules \Rightarrow reduction of the electron acceleration in the radio wave field \Rightarrow decrease in the power re-radiated by the electron)

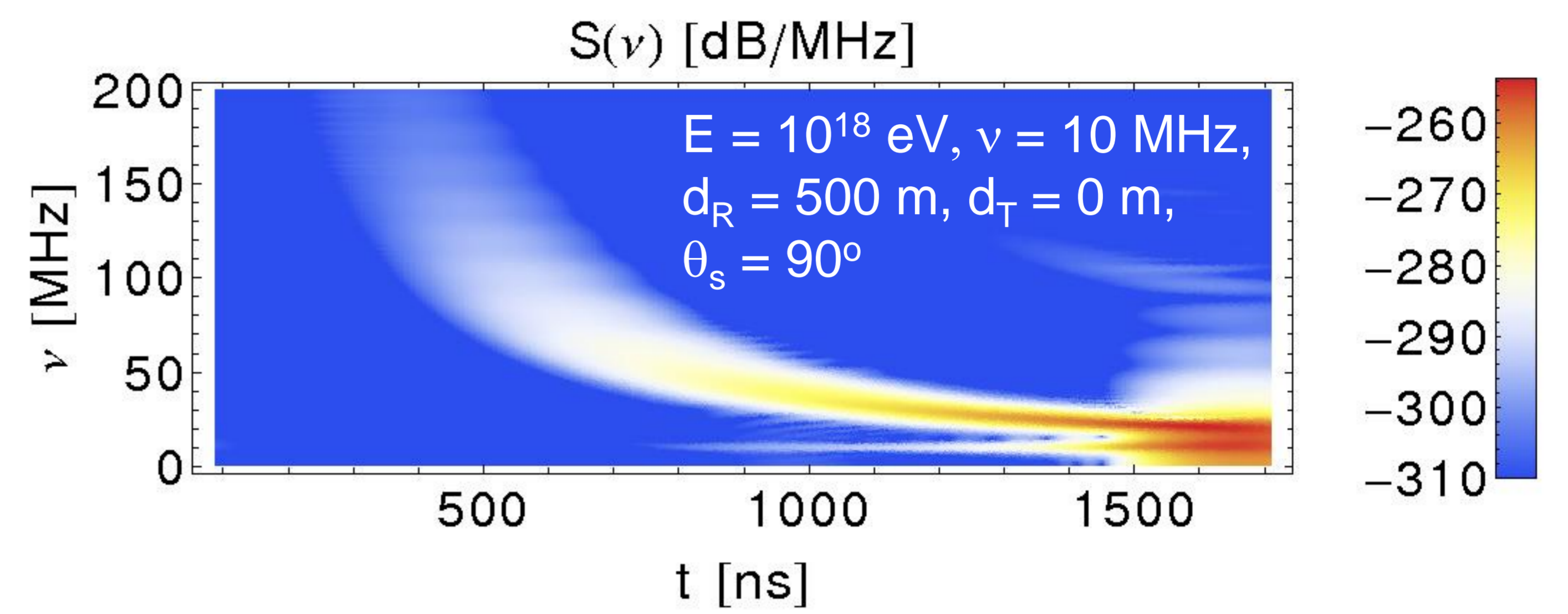
collision frequency $\nu_c \sim$ THz \Rightarrow treating the plasma as underdense

The maximum received power of the radar echo $P_{R,max}$ vs E

The dependence of the maximum received power $P_{R,max}$ on the shower energy E shows a universal scaling similar to that in coherent scattering:

$$P_{R,max} \sim E^{2.3}$$

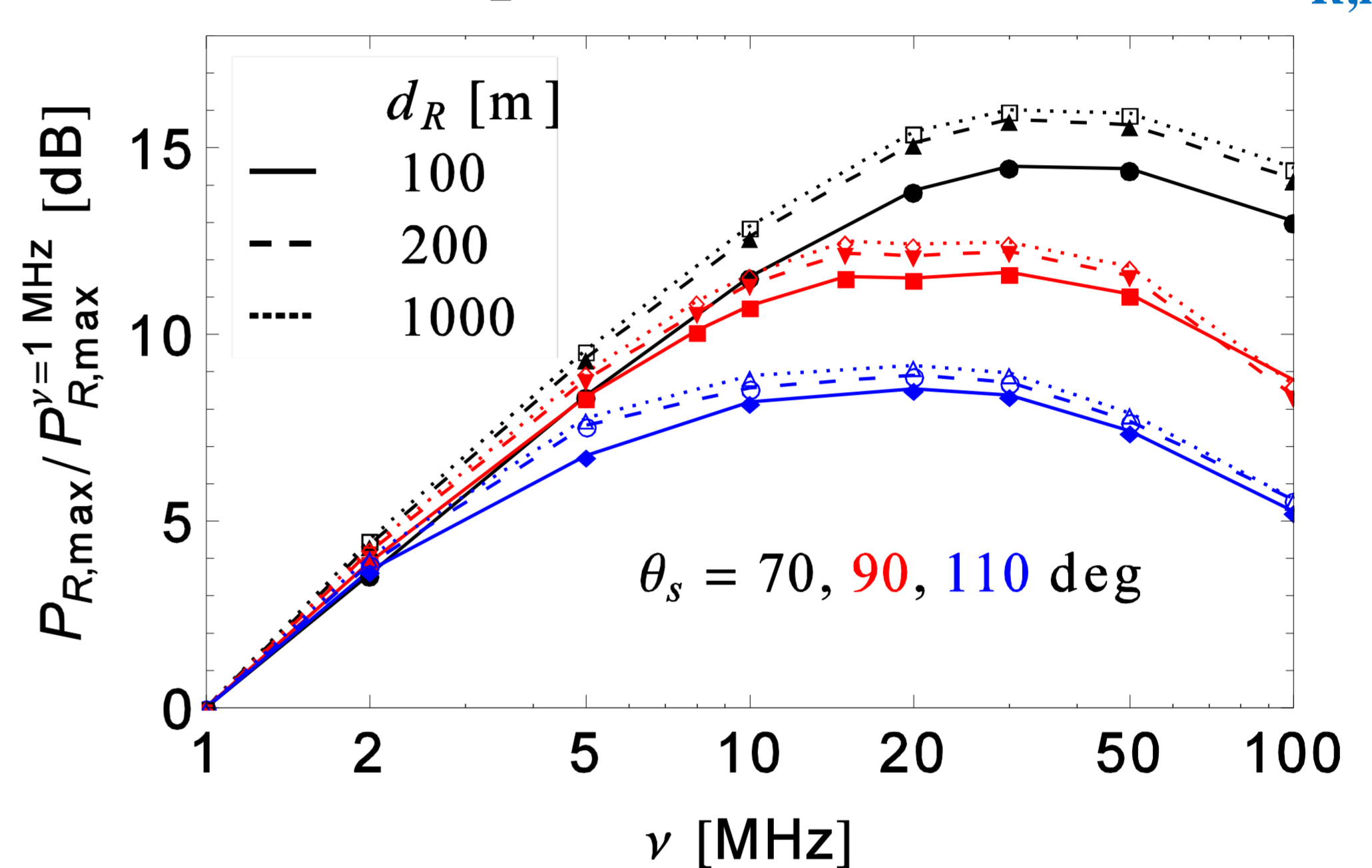
The typical spectrogram of the radar echo



The frequency of the radar echo decreases with time. The typical signal consists of two parts:

- a short signal upshifted to high-frequency with low amplitudes
- a long signal with frequency upshifts of only a few and larger amplitudes.

The maximum received power of the radar echo $P_{R,max}$ vs ν

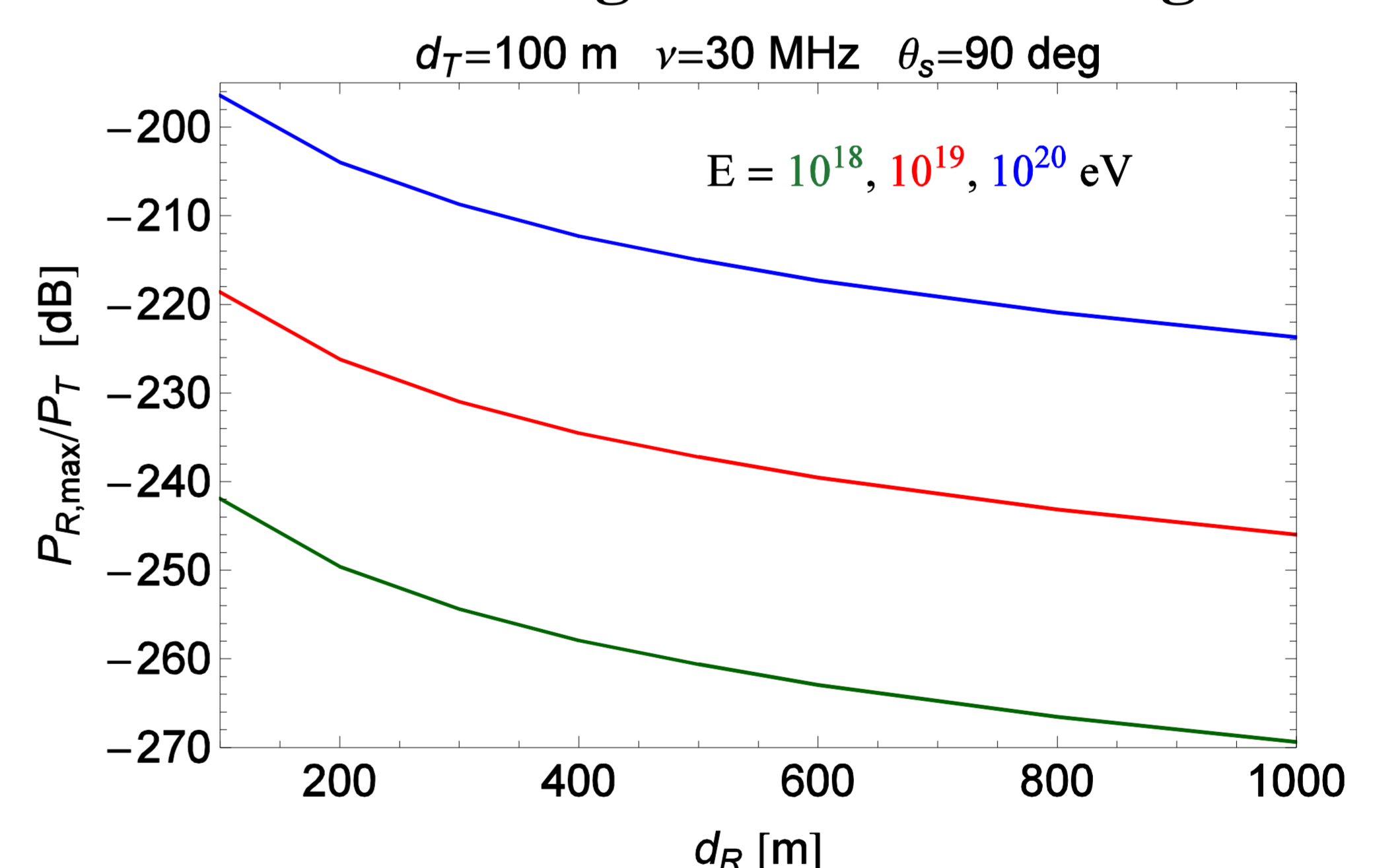


($P_{R,max}$ scaled to the respective maximum power received at the frequency of 1 MHz)

The strongest signal is expected at the radar frequencies around 30 MHz because of

- destructive interference across the shower disk at higher frequencies
- molecular quenching at lower frequencies.

Maximum strength of the radar signal



Ratio of the maximum received ($P_{R,max}$) to the emitted power (P_T) as a function of the shower core-receiver distance d_R for different shower energies E . The values of the power ratio $P_{R,max}/P_T$ represent the strongest signal that can be attained in the MHz frequency range for d_T and d_R larger than 100 m.

The received power is more than 20 orders of magnitude lower than the emitted one!!!

Conclusion

- The plasma produced by air showers has to be treated always as underdense and the reflected radar signal is reduced by several orders of magnitude by molecular quenching.
- Even with very optimistic assumptions, the weakness of the radar signal makes the radar technique impractical for air shower detection.