A lateral distribution function for the radio emission of air showers

Anna Nelles

for the LOFAR Key Science Project Cosmic Rays

PoS(ICRC2015)376
LOFAR - LOw Frequency ARRay

- large distributed radio telescope in Europe
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- per station/location ~ 200 antennas
- two types of antennas 10 - 90 MHz and 110 - 250 MHz
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Radio and lateral distribution functions

Distance to shower axis [m]

Particle density [$m^{-2}$]

Radio pulse power [$J/m^2$]

typical detection threshold

10^0 10^1 10^2 10^3 10^4 10^5 10^6
0 50 100 150 200 250 300 350 400 450

10^{-19} 10^{-18} 10^{-17} 10^{-16} 10^{-15}
Radio and lateral distribution functions

“Radio LDF”

Particle density $[m^{-2}]$

Radio pulse power $[J/m^2]\)

Distance to shower axis $[m]$
Radio and lateral distribution functions

- Radio emission is not only a function of the distance to the shower axis
- “Features” in LDF are real
Radio and lateral distribution functions

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- Two-dimensional approach is needed
Radio and lateral distribution functions

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- Characteristic bean-shaped form

See also:
C. Glaser et al. (Auger Coll.), this session
The function
The function

- **Observation:**
  - Shape is always the same in shower plane defined by shower axis (\(\mathbf{v}\)) and magnetic field (\(\mathbf{B}\))
  - “bean shape” is function of zenith angle and height of shower maximum
  - signal falls off exponentially
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\[
P(x', y') = A_+ \cdot \exp \left( -\frac{[(x' - X_+)^2 + (y' - Y_+)^2]}{\sigma^2_+} \right) \\
- A_- \cdot \exp \left( -\frac{[(x' - X_-)^2 + (y' - Y_-)^2]}{\sigma^2_-} \right)
\]
The function

- Eight free parameters too many for stable fit
- Physical free parameters
  - **Shower direction** (two parameters, determined from arrival timing)
  - **Shower axis position** (two parameters)
  - **Height of shower maximum** (one parameter)
  - **Energy** (one parameter)

= four parameters for function
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- In Monte Carlo study, fit can be reduced to four free parameters

\[
P(x', y') = A_+ \cdot \exp\left(\frac{-(x' - X_c)^2 + (y' - Y_c)^2}{\sigma_+^2}\right)
\]

\[
-C_0 \cdot A_+ \cdot \exp\left(\frac{-(x' - (X_c + x_-))^2 + ((y' - Y_c))^2}{(C_1 \cdot e^{C_2 \cdot \sigma_+})^2}\right)
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\[ P(x', y') = A_+ \cdot \exp \left( -\frac{[\left( x' - X_c \right) + (y' - Y_c) \right)]}{\sigma_+^2} \right) \]

\[ -C_0 \cdot A_+ \cdot \exp \left( -\frac{[\left( x' - (X_c + x_-) \right) + (y' - Y_c) \right)]}{\left( C_1 \cdot e^{C_2 \cdot \sigma_+} \right)^2} \right) \]
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- Remaining experiment dependent parameters function of arrival direction

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P(x', y') = A_+ \cdot \exp \left( -\frac{((x' - X_c)^2 + (y' - Y_c)^2)}{\sigma^2 +}\right)
\]

\[
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\]

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Reconstructing shower axis

- Shower axis determines location in array
- Center (NOT location at strongest signal) corresponds to shower axis
- Offset only in $\mathbf{v} \times \mathbf{B}$ direction
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Reconstructing shower axis

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- Center (NOT location at strongest signal) corresponds to shower axis
- Offset only in $\text{vxB}$ direction
- Resolution of 15 meters for LOFAR with respect to scintillator array

Position in $\text{vxB}$ direction

- $\text{mean } = 25 \text{ m}$
- $\text{mean } = 0 \text{ m}$
Reconstruction of the Energy

- Radio signal is coherent
  = amplitude scales with number of particles
  = scales with energy

*see many papers before this*
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- e.g. scaling parameter $A_+$ of function shows clear correlation with energy

- Resolution of ~30%, dominated by resolution of particle array at LOFAR

Fit parameter corrected for magnetic field angle
now calibrated, see Hörandel et al. PoS(ICRC2015)662
Reconstruction of $X_{\text{max}}$
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- Size of footprint is determined by distance to the emission point since radio signals are not attenuated in the atmosphere.

- Correlation with distance to $X_{\text{max}}$

\[
D(X_{\text{max}}) [\text{g/cm}^2] = \frac{X_{\text{atm}} [\text{g/cm}^2]}{\cos(\theta)} - X_{\text{max}} [\text{g/cm}^2]
\]
Reconstruction of $X_{\text{max}}$

- At LOFAR:
  No independent detector to measure the height of the shower maximum
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- Comparison to full Monte-Carlo simulations possible (see S. Buitink, *PoS(ICRC2015)368* or *PoS(ICRC2015)369*)
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Conclusions

General radio lateral distribution function

- Based on CoREAS simulations:
- Four free parameters: energy, $X_{\text{max}}$, shower axis position
- Shape parameters dependent on magnetic field and height above sea-level
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General radio lateral distribution function
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Function is easy to use and suitable for all radio arrays
• Shape is found at all locations
• Best correlating quantity to energy and constants a function of location of experiment
• Corrections for magnetic field independent of shape
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General radio lateral distribution function
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  • Four free parameters: energy, \( X_{\text{max}} \), shower axis position
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Application shown for LOFAR air shower data
• ~ 15 meter resolution of axis position, no bias
• ~ 30 % energy resolution, limited by comparison to particle array
• ~ 35 g/cm\(^2\) \( X_{\text{max}} \) resolution against full Monte Carlo simulations, no independent detector at LOFAR