

Cosmic-ray diffusive reacceleration - a critical look

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Basic idea is very simple...

- “Follow the energy” is always a good motto in Astrophysics.
- So-called “reacceleration” is popular in cosmic ray diffusive propagation models (eg GALPROP) mainly to fit the low energy B/C ratio (and expectations of a “Kolmogorov-like” turbulence spectrum which would also help explain the low HE anisotropy).
- But how much energy is contributed by the reacceleration to the cosmic rays in such models?
- Surprisingly does not seem to have been examined in the literature to date.

Standard picture

- Cosmic ray particles are produced in discrete sources (probably SNRs) in the Galaxy.
- They then wander through the interstellar medium interacting with the magnetic field.
- Turbulent magnetic field structures scatter the particles and lead to a random walk.
- Can be modelled as a diffusion process.
- If fields are not static this diffusion is in both physical space and momentum space.

Mathematical formulation....

Sources

Other stuff

$$\frac{\partial f}{\partial t} = Q + \nabla (D_{xx} \nabla f) + \frac{1}{4\pi p^2} \frac{\partial}{\partial p} \left(4\pi p^2 D_{pp} \frac{\partial f}{\partial p} \right) + \dots$$

Diffusion in space

Diffusion in momentum

plus boundary conditions (escape)!

Naive treatment of diffusion...

$$D_{xx} \approx \frac{1}{3} \lambda v = \frac{1}{3} \tau v^2 = \frac{1}{3} \frac{\lambda^2}{\tau} = \frac{1}{3} \frac{(\Delta x)^2}{\tau}$$

If scattering centres move with random velocity of order the Alfvén speed V_A , then at each scattering

$$\Delta p \approx \frac{V_A}{v} p \quad \text{and thus} \quad D_{pp} \approx \frac{1}{3} \frac{(\Delta p)^2}{\tau} = \frac{1}{3} \frac{p^2 V_A^2}{v^2 \tau}$$

so that

$$D_{pp} D_{xx} \approx \frac{1}{9} p^2 V_A^2$$

Much more elaborate quasi-linear theory
(Tsytovich, Skilling et al) gives

$$D_{xx}D_{pp} = p^2 V_A^2 \left\langle \frac{1 - \mu^2}{\nu_+ + \nu_-} \right\rangle \left\langle \frac{(1 - \mu^2)\nu_+\nu_-}{\nu_+ + \nu_-} \right\rangle$$

where ν_{\pm} are the scattering rates on forward and backward propagating Alfvén waves and the angle brackets denote averaging over an isotropic pitch-angle distribution

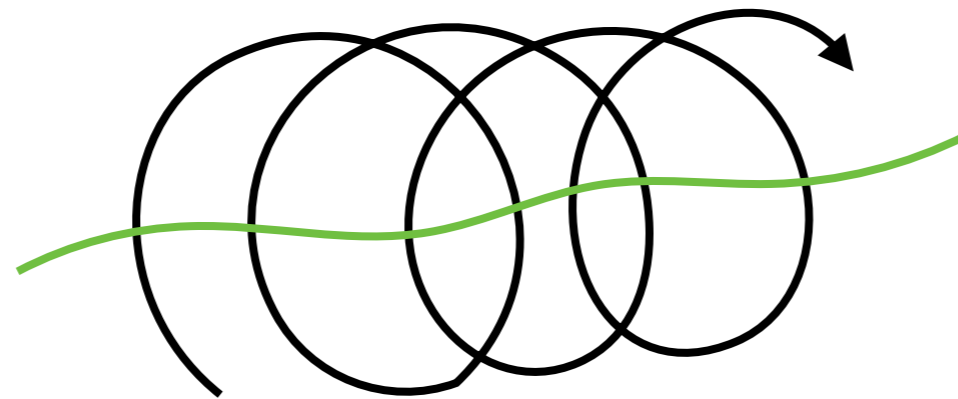
$$\langle \rangle = \int_{-1}^{+1} \frac{d\mu}{2}$$

In terms of the wave power spectrum

$$\nu_{\pm} \approx \frac{v}{r_g} \frac{k_{\text{res}} W_{\pm}(k_{\text{res}})}{B_0^2 / 2\mu_0}$$

where the gyro-resonant condition is

$$\mu r_g k_{\text{res}} \approx 1.$$



For a power-law wave spectrum

$$W(k) \propto k^{-a}$$

($a=5/3$ being a Kolmogorov spectrum) this gives

$$D_{xx}D_{pp} = p^2 V_A^2 \frac{1}{a(4-a)(4-a^2)}$$

and

$$D_{xx} \propto v \left(\frac{p}{e} \right)^{(2-a)} \propto v R^\delta$$

This is the conventional parametrisation of the spatial diffusion coefficient used in most diffusion models, velocity times a power-law in rigidity.

$$\delta = 2 - a$$

so that for a Kolmogorov spectrum

$$a = 5/3, \quad \delta = 1/3$$

In terms of the rigidity dependence δ

$$D_{xx}D_{pp} = p^2 V_A^2 \frac{1}{\delta(4 - \delta)(4 - \delta^2)} \geq \frac{1}{9} p^2 V_A^2$$

For convenience write

$$D_{pp} = v p^2 V_A^2 \frac{1}{D_{xx}}$$

The reacceleration power density can be shown to be given by the integral over the particle spectrum,

$$P_R = \int_0^\infty 4\pi p^2 f \frac{1}{p^2} \frac{\partial}{\partial p} (p^2 D_{pp} v) dp$$

subject to very mild regularity conditions on f
(Thornbury and Drury, arXiv:1404.2104)

which is

$$P_R = \int_0^\infty 4\pi p^2 f \left(\frac{\vartheta V_A^2 p v}{D_{xx}} \right) \left[4 + \frac{\partial \ln(v/D_{xx})}{\partial \ln p} \right] dp.$$

or if

$$D_{xx} = D_0 \left(\frac{v}{c} \right) \left(\frac{p}{mc} \right)^\delta$$

$$P_R = \vartheta(4 - \delta) \frac{V_A^2}{D_0} mc^2 \int 4\pi p^2 f \left(\frac{p}{mc} \right)^{1-\delta} dp$$

For a truncated power-law particle spectrum,

$$f(p) = f_0 \left(\frac{p}{mc} \right)^{-\gamma}, \quad p > p_{\min}$$

this gives

$$P_R = \vartheta(4 - \delta) \frac{V_A^2}{D_0} mc^2 (mc)^3 f_0 \frac{1}{\delta + \gamma - 4} \left(\frac{p_{\min}}{mc} \right)^{4 - \delta - \gamma}$$

which diverges at low energies

$$\delta + \gamma > 4$$

Reacceleration power is thus dominated by the low-energy part of the spectrum which is poorly known.

But can at least calculate the contribution from above about 1 GeV where the data are reliable

PDG gives

$$I(E) = 1.8 \times 10^4 \left(\frac{E}{1 \text{ GeV}} \right)^{-2.7} \text{ m}^{-2} \text{ s}^{-1} \text{ sr}^{-1} (\text{GeV})^{-1}$$

for the flux above 1 GeV

giving

$$P_R > \vartheta \frac{4 - \delta}{0.7 + \delta} \left(\frac{V_A}{30 \text{ km s}^{-1}} \right)^2 \left(\frac{D_0}{10^{28} \text{ cm}^2 \text{ s}^{-1}} \right)^{-1} 7 \times 10^{-16} \text{ eV cm}^{-3} \text{ s}^{-1}$$

For a cosmic ray residence time in the Galaxy of at least 10^7 yr and an energy density of order 1 eV cm^{-3} this shows that at least 20% of the energy comes from reacceleration!

Preliminary Galprop calculations by Andy Strong
confirm this

Parameters chosen to fit B/C with reacceleration.

Model z04LMS from Strong et al 2010 used as reference.

Alfvén speed

$$V_A = 30 \text{ km s}^{-1}$$

$$V_A = 40 \text{ km s}^{-1}$$

No reacceleration

Proton luminosity

$$7.8 \times 10^{40} \text{ erg s}^{-1}$$

$$8.7 \times 10^{40} \text{ erg s}^{-1}$$

$$6.0 \times 10^{40} \text{ erg s}^{-1}$$

Further calculations in progress, but clear already that if reacceleration is invoked at the levels commonly used to fit the B/C ratio, then a significant part of the total CR energy budget is contributed by reacceleration in the ISM and not by SNRs.

Is this plausible?

Not impossible if SNRs are the major drivers of ISM turbulence, but then has to compete with all other turbulence damping processes; this seems difficult.

We should not just blindly invoke reacceleration to fit B/C etc without considering the energy budget implications

Conclusions

- There is a remarkably simple integral formula for the reacceleration power density.
- Models that use reacceleration to fit B/C etc have substantial energy input from reacceleration.
- This is potentially difficult in view of the many competing damping mechanisms for ISM turbulence; **reacceleration is not a cost-free solution!**
- Alternatives (such as convection by winds) should be seriously considered.

Energy flow from SN to GCRs

