Perpendicular diffusion of energetic particles in noisy reduced MHD turbulence[1]

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ABSTRACT

The noisy reduced magneto-hydrodynamic (NRMHD) turbulence model was recently formulated and used to compute field line random walk (FLRW) diffusion coefficient, κ_{FL}. Using the same model, we investigate the diffusion of energetic particles across the mean magnetic field by calculating parallel and perpendicular mean free paths, $\lambda_{\parallel}$ & $\lambda_{\perp}$. We used the Non-Linear Guiding Centre (NLGC) theory, the Unified Non-Linear Transport (UNLT) theory, and test-particle simulations. It is shown that both theories provide very different results and only UNLT agrees with simulations.

Introduction

Different processes in space and astrophysics can be solved by understanding the diffusion of energetic particles. Such particles scatter upon interacting with the solar wind plasma in the solar system for example (Fig. 1). Spatial diffusion is mainly caused by turbulent magnetic fields δB. On top of that we also find a mean magnetic field B_0 which breaks the symmetry of the system. Therefore, we have to distinguish between diffusion of particles along ($\kappa_{\parallel}$ or $\lambda_{\parallel}$) and across ($\kappa_{\perp}$ or $\lambda_{\perp}$) the mean magnetic field.

$$\vec{B} = B_0 \hat{e}_z + \delta\vec{B}$$

Fig. 1: Solar wind arena
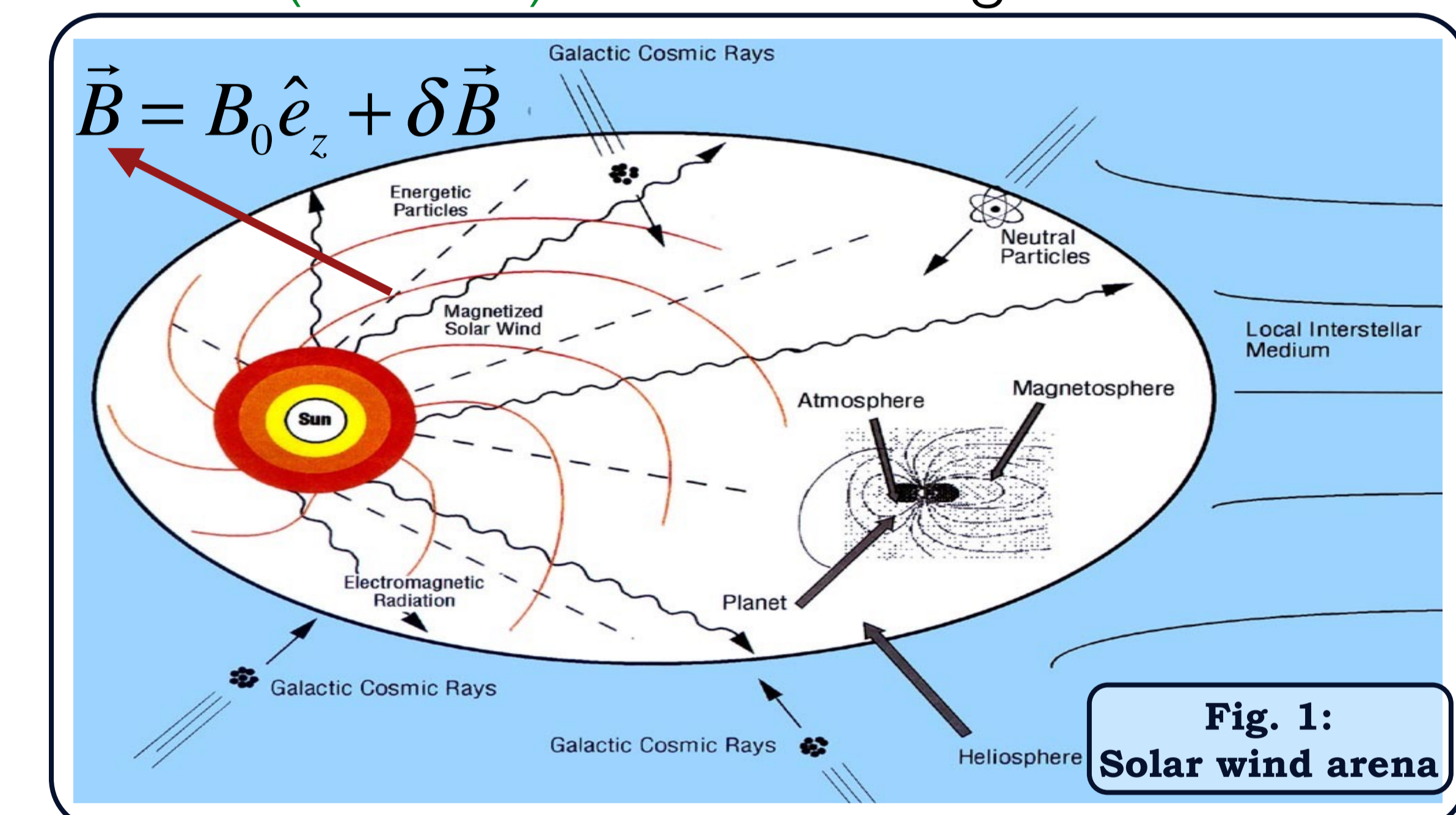
Analytical theories (NLGC vs. UNLT)

It is difficult to describe perpendicular diffusion due to non-linear effects. Couple of theories were proposed to approximate the perpendicular diffusion coefficient. The most promising are NLGC[2] and UNLT[3] theories. Combining the equation of motion

$$\tilde{v}_x = a v_z \frac{\delta B_x}{B_0} \qquad (1)$$

with TGK[4,5,6] formulation

$$\kappa_{xx} = \int_0^\infty dt \ \langle \tilde{v}_x(t)\tilde{v}_x(0)\rangle; \qquad (2)$$

and approximating 4th order correlation by a product of two 2nd order and using exponential and isotropic model for the parallel velocity correlation function, Matthaeus et al. (2003) derived the following nonlinear integral for NLGC Th.

$$\kappa_{\perp} = \frac{a^2 v^2}{3B^2} \int d^3k \frac{P_{xx}(\vec{k})}{\kappa_{\parallel}k_{\parallel}^2 + \kappa_{\perp}k_{\perp}^2 + v/\lambda_{\parallel}}.$$

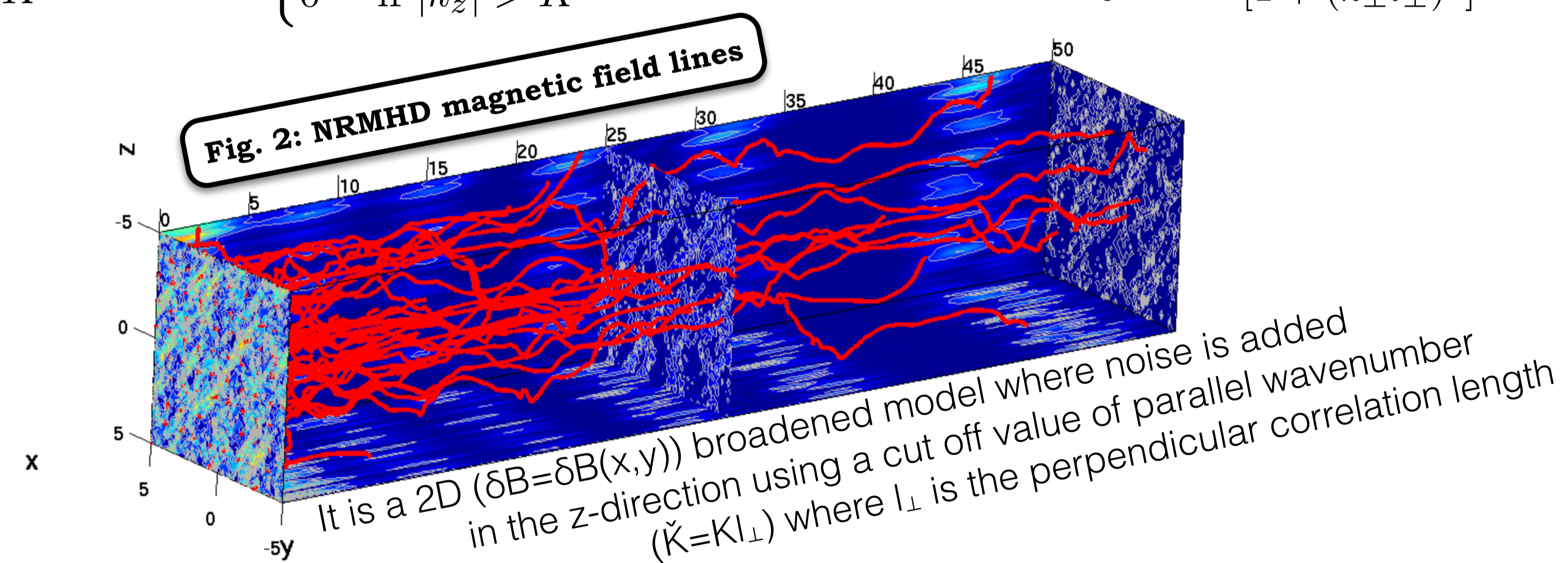$P_{xx}(\mathbf{k})$: magnetic correlation tensor; $k_{\parallel}$ & $k_{\perp}$: wavenumbers

Two problems were discovered for the NLGC theory. First it provides diffusion for slab turbulence in contradiction with theorem on reduced dimensionality[7]. Second, FLRW limit cannot be derived for arbitrary turbulence. Shalchi (2010) improved NLGC and developed the UNLT theory which is still based on Eqs. (1) and (2) but 4th order correlations are computed directly by using a pitch-angle dependent particle distribution function f (μ, x, y, z, t) with the pitch-angle cosine μ. Hence the problems of NLGC were solved and the following nonlinear integral was derived

$$\kappa_{\perp} = \frac{a^2 v^2}{3B^2} \int d^3k \frac{P_{xx}(\vec{k})}{(vk_{\parallel})^2/(3\kappa_{\perp}k_{\perp}^2) + 4\kappa_{\perp}k_{\perp}^2/3 + v/\lambda_{\parallel}} \ .$$
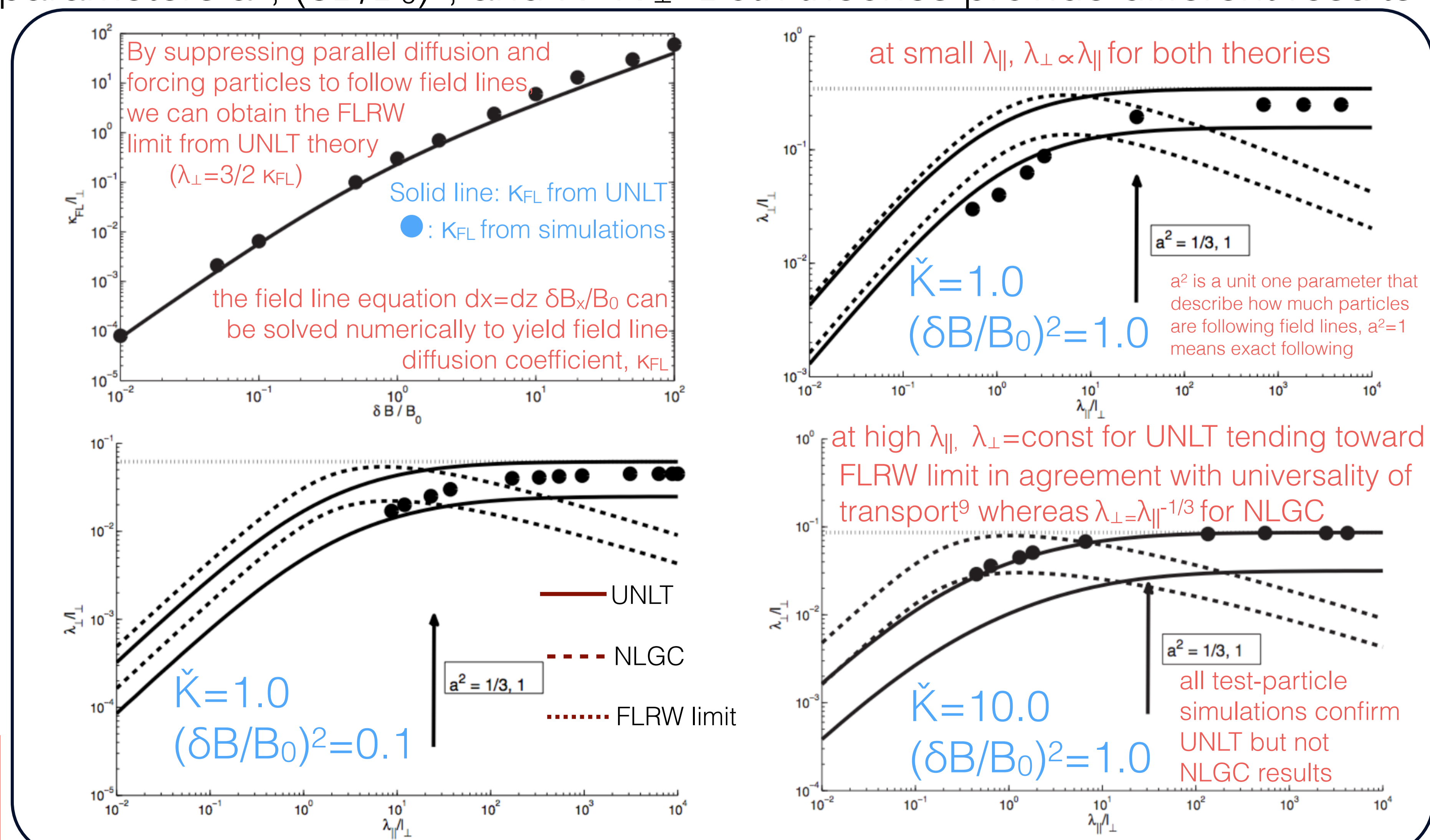
NRMHD turbulence model[8]

It is a highly anisotropic model of magnetic field lines appropriate for low frequency turbulence when superimposed with a strong mean field. Parallel wavenumbers are cut off at a value K (noisy part). The correlation tensor is

$$P_{xx}(\vec{k}) = \frac{1}{4\pi K}k_y^2 A(k_x,k_y)\begin{cases}1 & \text{if } |k_z| \le K \\ 0 & \text{if } |k_z| > K\end{cases} \text{ with } A(k_x,k_y) = A(k_{\perp}) = \frac{8}{9}l_{\perp}\delta B^2 \frac{1}{[1+(k_{\perp}l_{\perp})^2]^{7/3}}$$

Fig. 2: NRMHD magnetic field lines

It is a 2D ($\delta B = \delta B(x,y)$) broadened model where noise is added in the z-direction using a cut off value of parallel wavenumber ($\check{K} = Kl_{\perp}$) where $l_{\perp}$ is the perpendicular correlation length

FLRW and particle diffusion results

Parallel and perpendicular diffusion coefficients were obtained using NLGC, UNLT, and test-particle simulations using different values of parameters a^2, $(\delta B/B_0)^2$, and $\check{K} = Kl_{\perp}$. Both theories provide different results.

By suppressing parallel diffusion and forcing particles to follow field lines we can obtain the FLRW limit from UNLT theory ($\lambda_{\perp} = 3/2 \ \kappa_{FL}$)

Solid line: κ_{FL} from UNLT
●: κ_{FL} from simulations

the field line equation dx=dz $\delta B_x/B_0$ can be solved numerically to yield field line diffusion coefficient, κ_{FL}

at small $\lambda_{\parallel}$, $\lambda_{\perp} \propto \lambda_{\parallel}$ for both theories

$\check{K}=1.0$
$(\delta B/B_0)^2=1.0$

a^2 is a unit one parameter that describe how much particles are following field lines, $a^2=1$ means exact following

$a^2 = 1/3, 1$

$\check{K}=1.0$
$(\delta B/B_0)^2=0.1$

$a^2 = 1/3, 1$

— UNLT
- - - NLGC
····· FLRW limit

at high $\lambda_{\parallel}$, $\lambda_{\perp}$=const for UNLT tending toward FLRW limit in agreement with universality of transport[9] whereas $\lambda_{\perp} = \lambda_{\parallel}^{1/3}$ for NLGC

$\check{K}=10.0$
$(\delta B/B_0)^2=1.0$

$a^2 = 1/3, 1$

all test-particle simulations confirm UNLT but not NLGC results

References
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