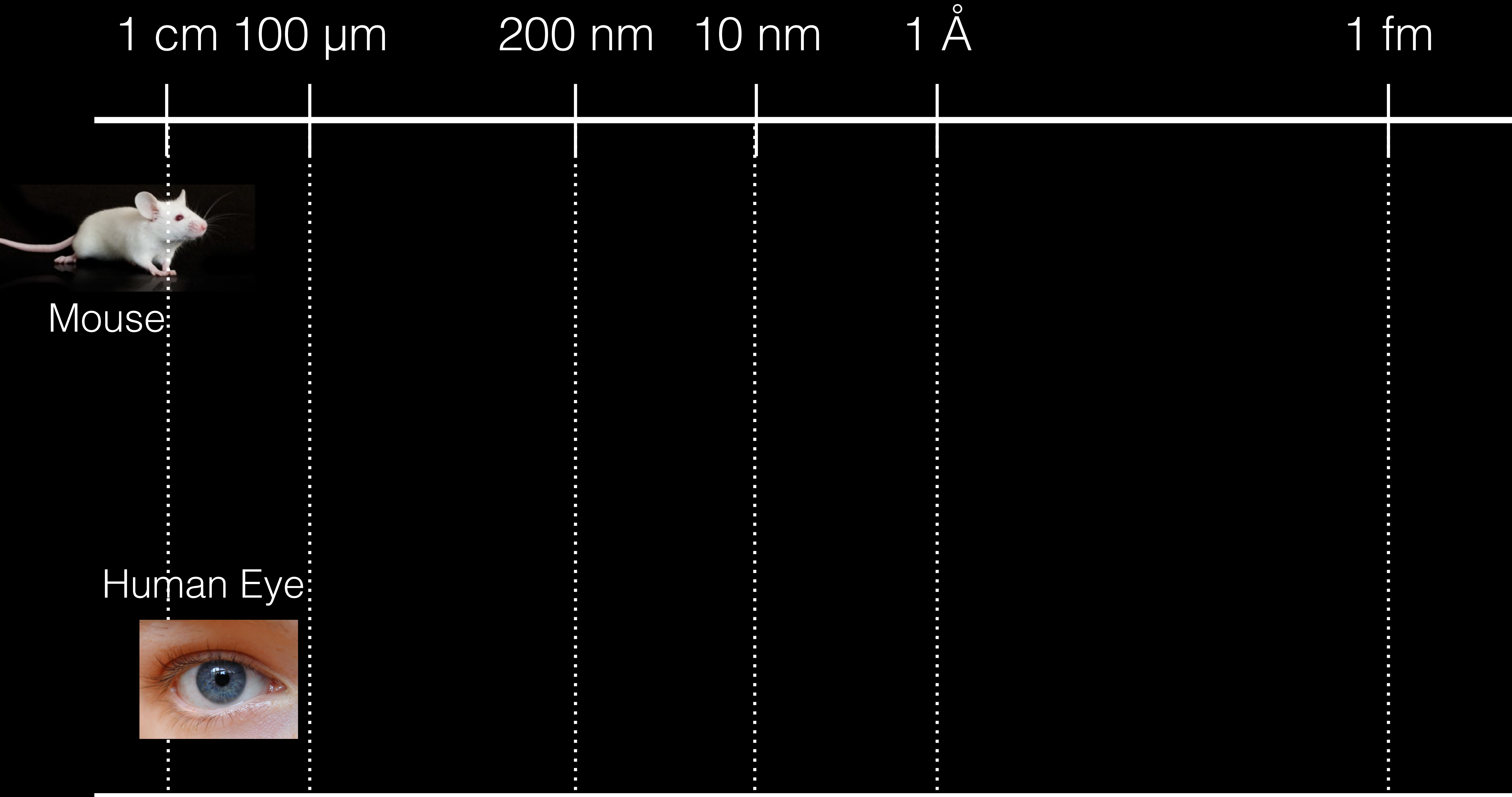
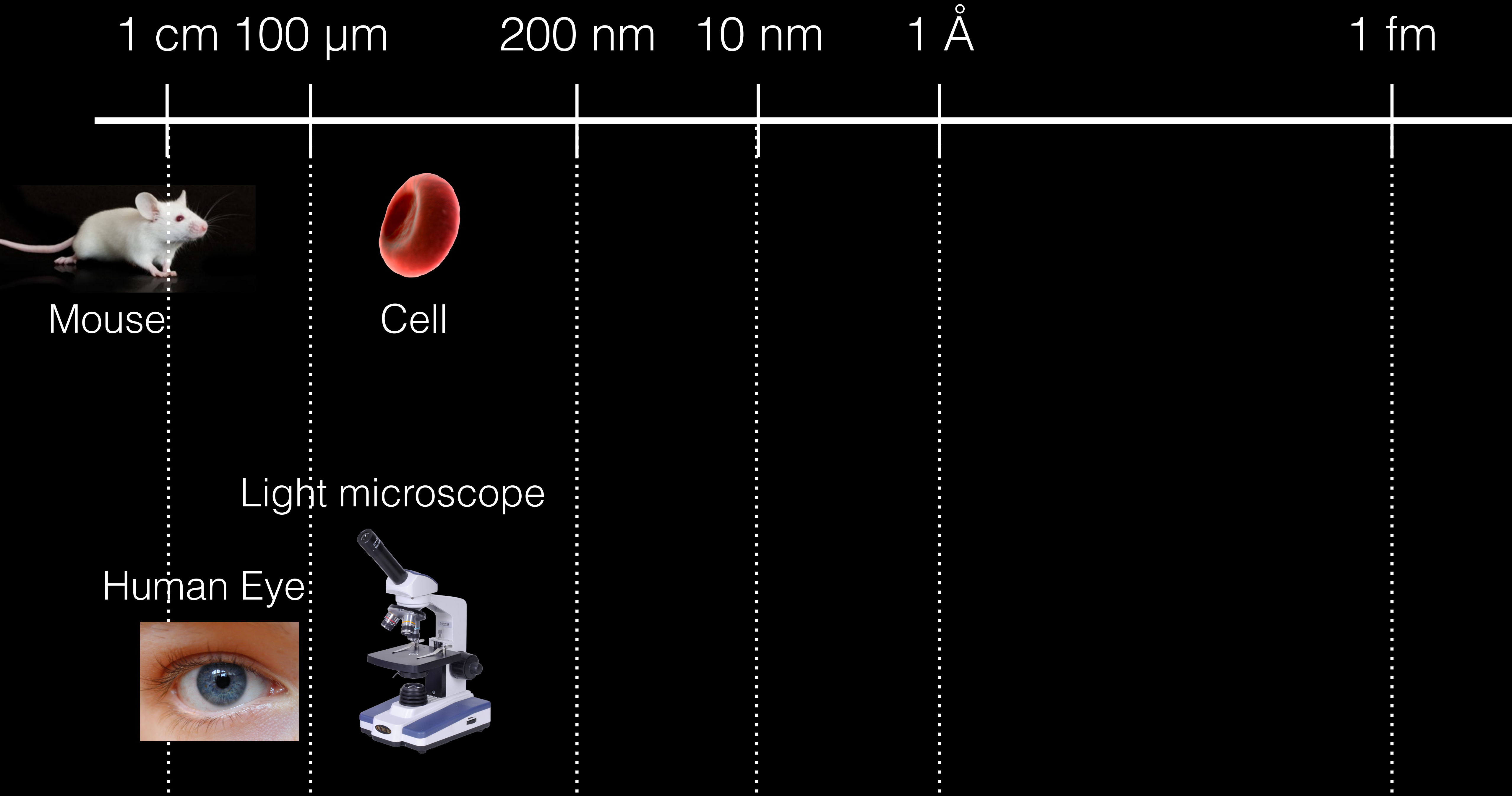
Quantum statistics measurements using 2-, 3-, and 4-pion Bose-Einstein correlations

Dhevan Gangadharan (LBNL)
on behalf of the ALICE collaboration
CERN PH seminar, Oct. 14th 2014

Measuring Spatial Structures

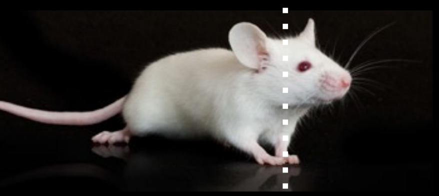


Measuring Spatial Structures

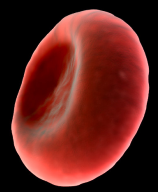


Measuring Spatial Structures

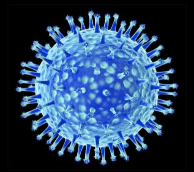
1 cm | 100 µm | 200 nm | 10 nm | 1 Å | 1 fm



Mouse



Cell

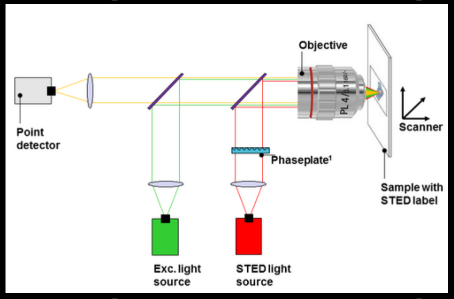


Virus

Light microscope

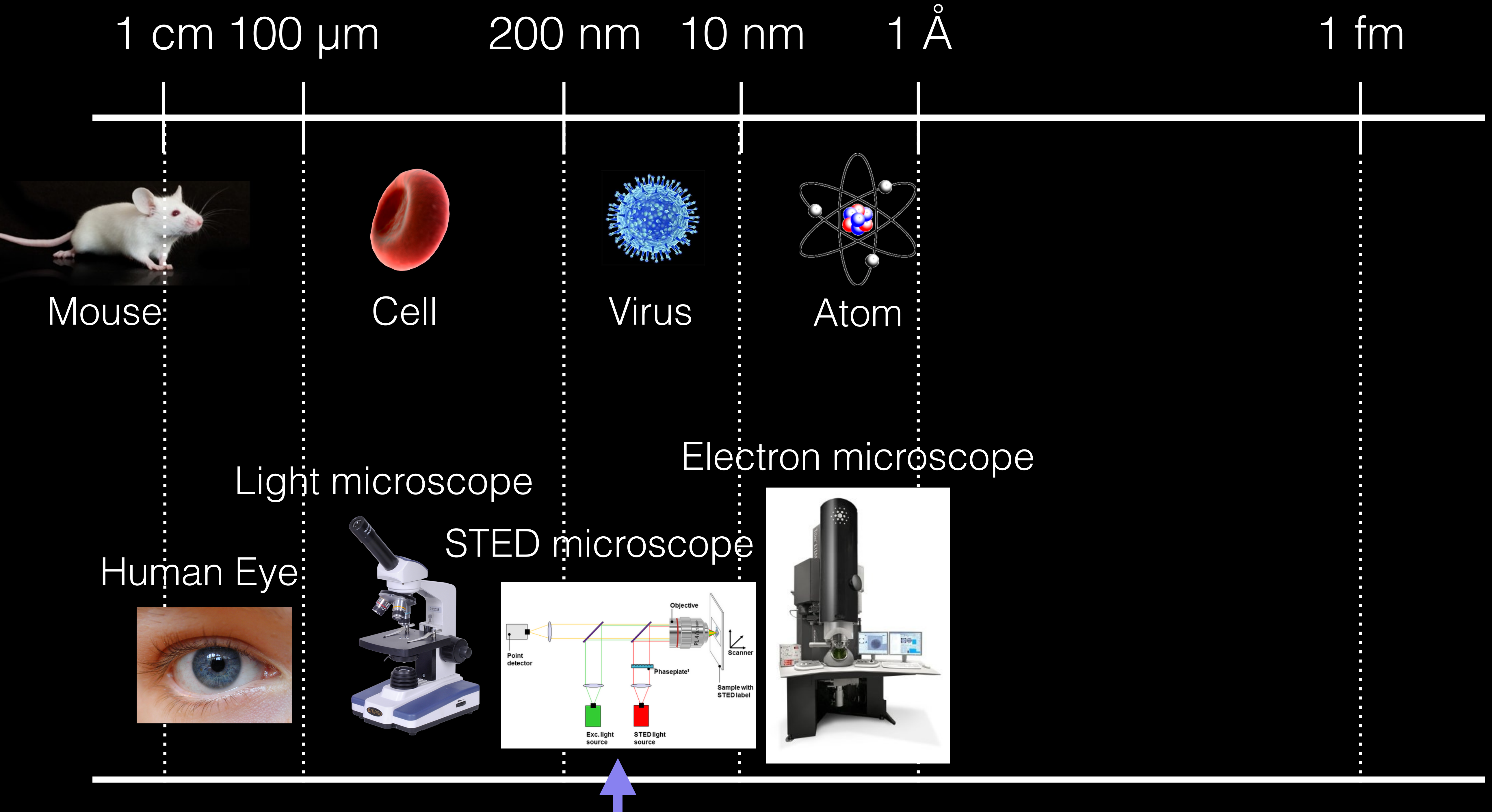
STED microscope

Human Eye



2014 Nobel Prize in Chemistry!

Measuring Spatial Structures



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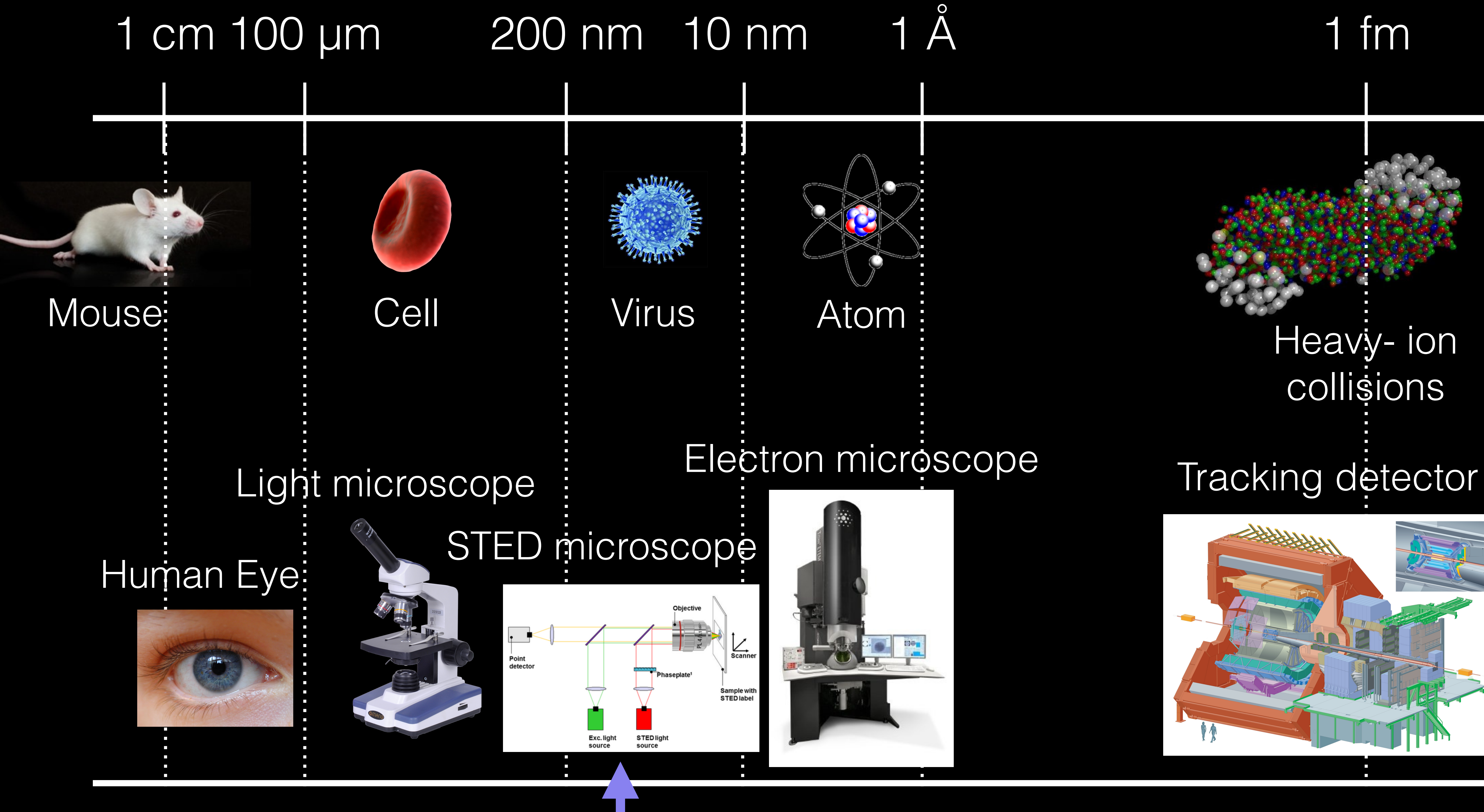


2014 Nobel Prize in Chemistry!

Exploiting Quantum Statistics (QS) to measure the source size

- The last stage of particle interactions is freeze-out
- At freeze-out in high-energy particle collisions, the characteristic separation of particles is femtoscopic ($\Delta x \sim 10^{-15}$ m).

$$\Delta x \Delta p \gg 2\pi\hbar$$

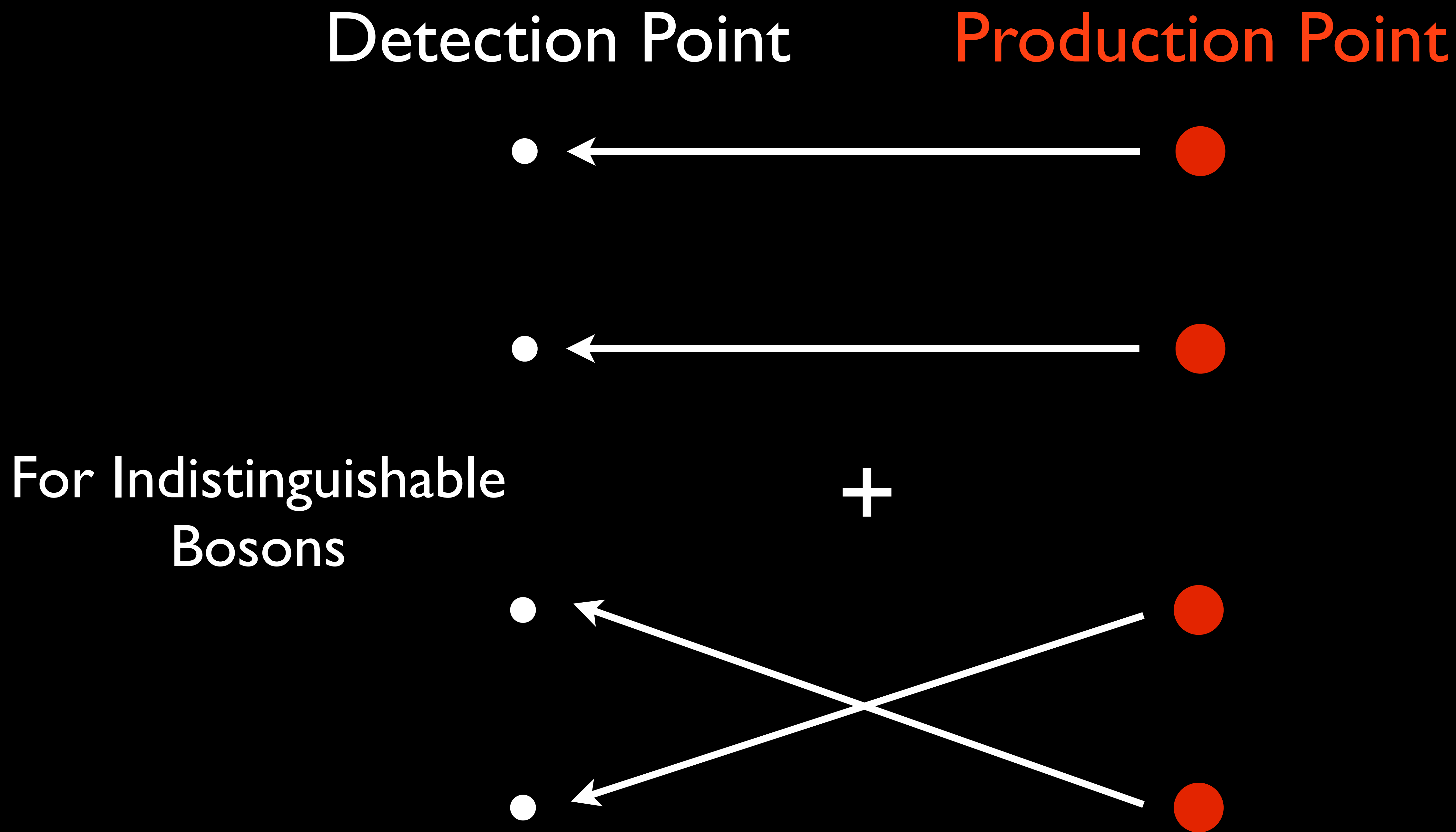
Classical:
no observable quantum phenomena

$$\Delta x \Delta p \sim 2\pi\hbar$$

Non Classical:
Bose-Einstein / Fermi-Dirac correlations

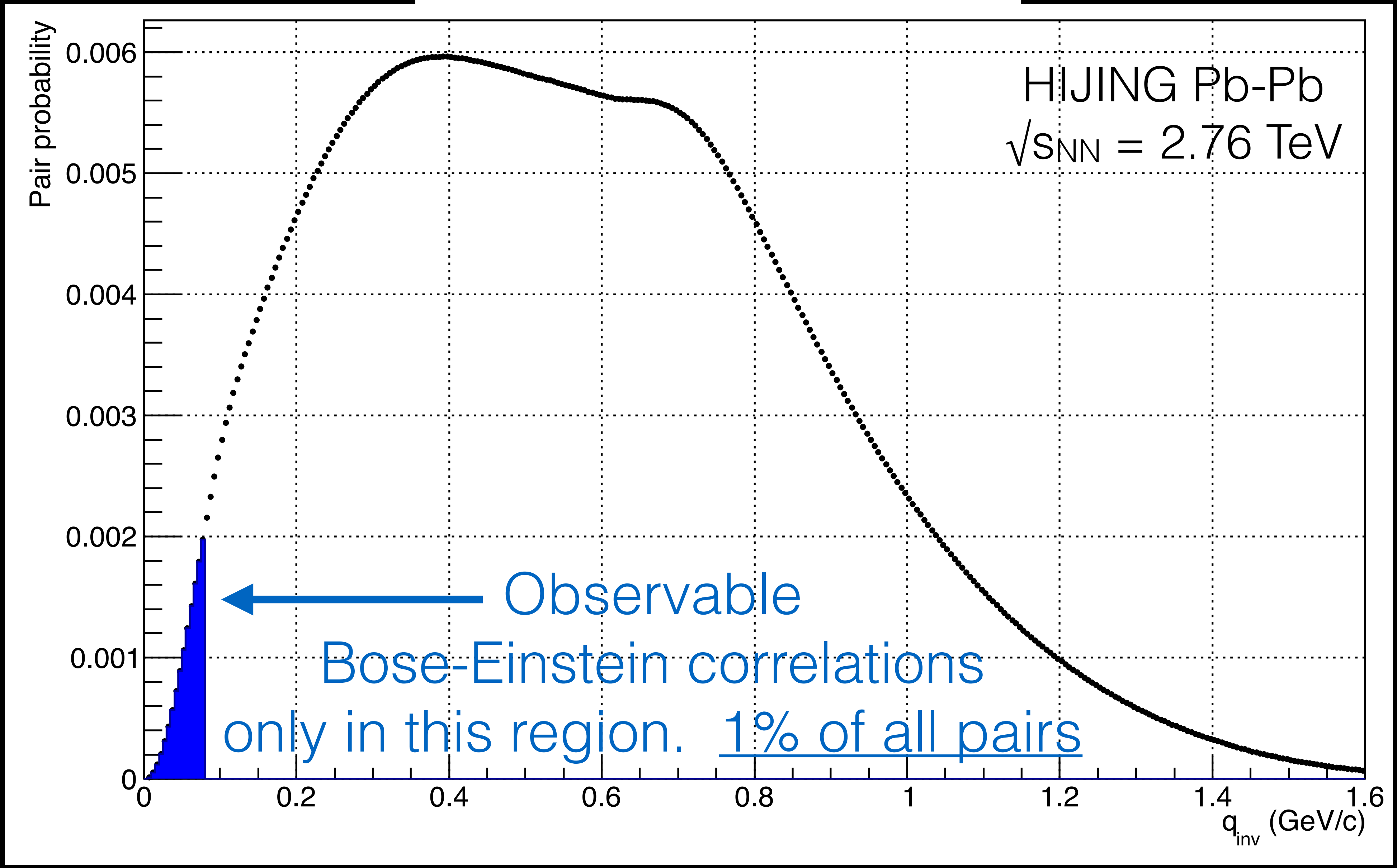
- Bose-Einstein correlations will be visible for $\Delta p <\sim 0.5$ GeV/c. Relative momentum correlations are sensitive to the relative separation at freeze out.

Exploiting Quantum Statistics (QS) to measure the source size



Bose-Einstein correlations are in a very narrow region of phase-space

$$\Delta x \Delta p \leq 2\pi\hbar$$



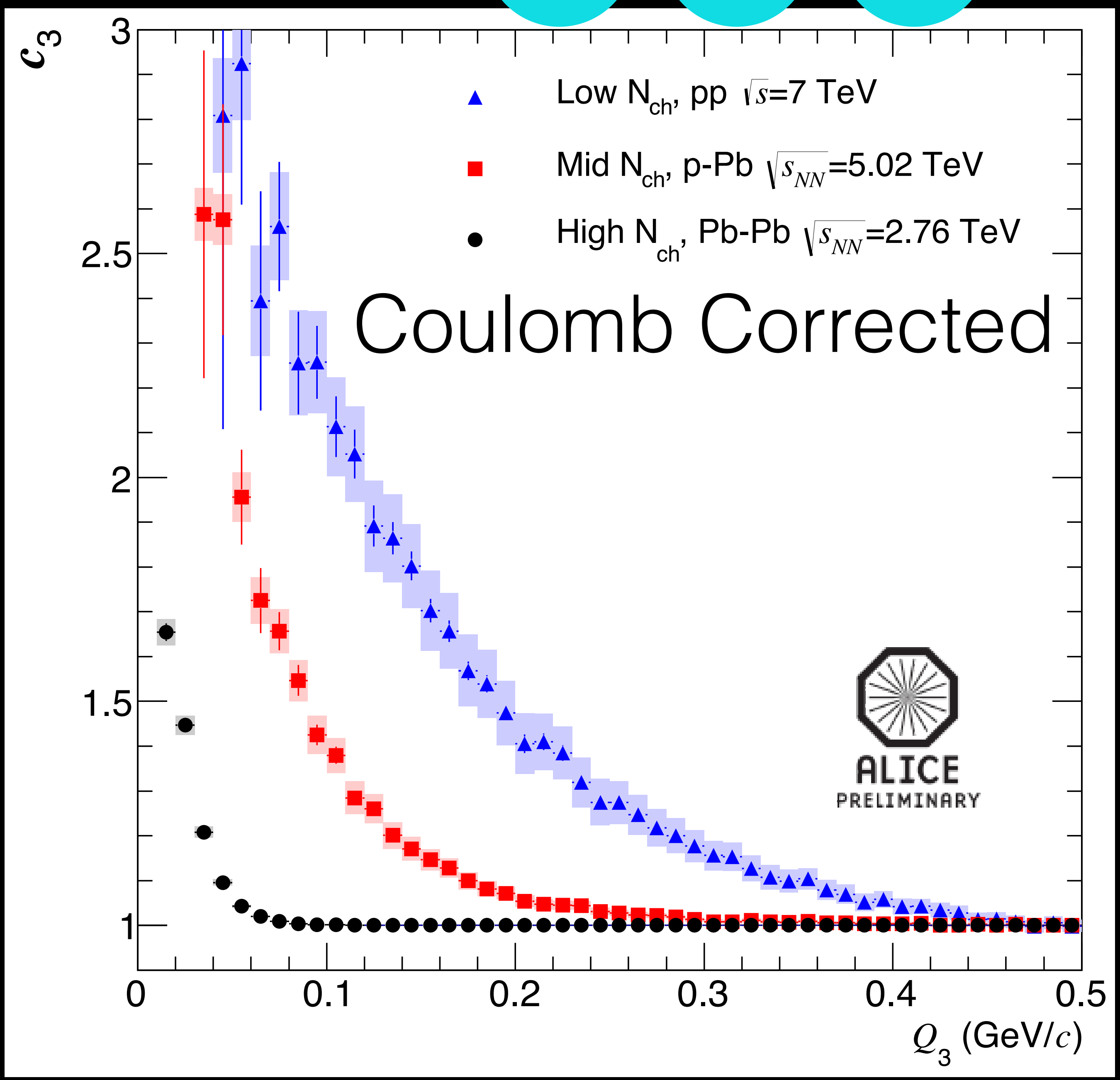
$$q_{\mathrm{inv}} = \sqrt{(\vec{p}_1 - \vec{p}_2)^2 - (E_1 - E_2)^2}$$

HIJING
Wang & Gyulassy
PRD **44** 3501

Femtoscopy (10^{-15})

The study of particle correlations at low relative momentum

3-pion Correlation: π^+ π^+ π^+



= Triplet relative momentum

Low Q region is dominated by Quantum Statistics (QS) and Coulomb correlations.

Clean region of study

AKA:

Bose-Einstein Correlations
Quantum Statistics Correlations
"HBT" Correlations

2 Uses of Femtoscopy

The last stage of particle interactions is “freeze-out”

Use 1

Measure:
Space-time structure at freeze-out
(e.g. Radius)

Sensitive to dynamics
of the collision.
(e.g. Hydrodynamics or not?)

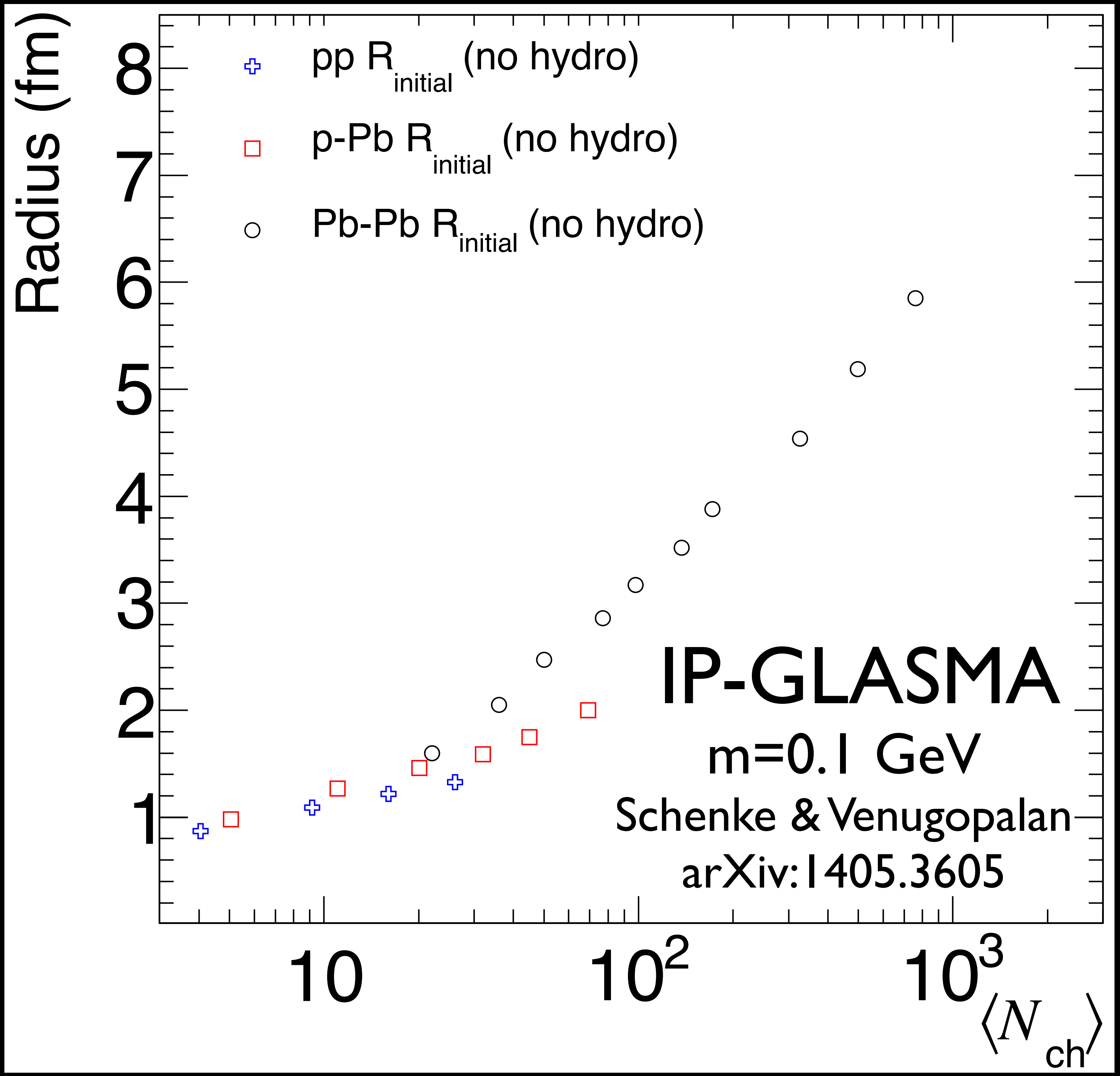
Use 2

Measure:
Quantum coherence
of particles at freeze-out.

Very sensitive to dynamics of the collision.

Use 1

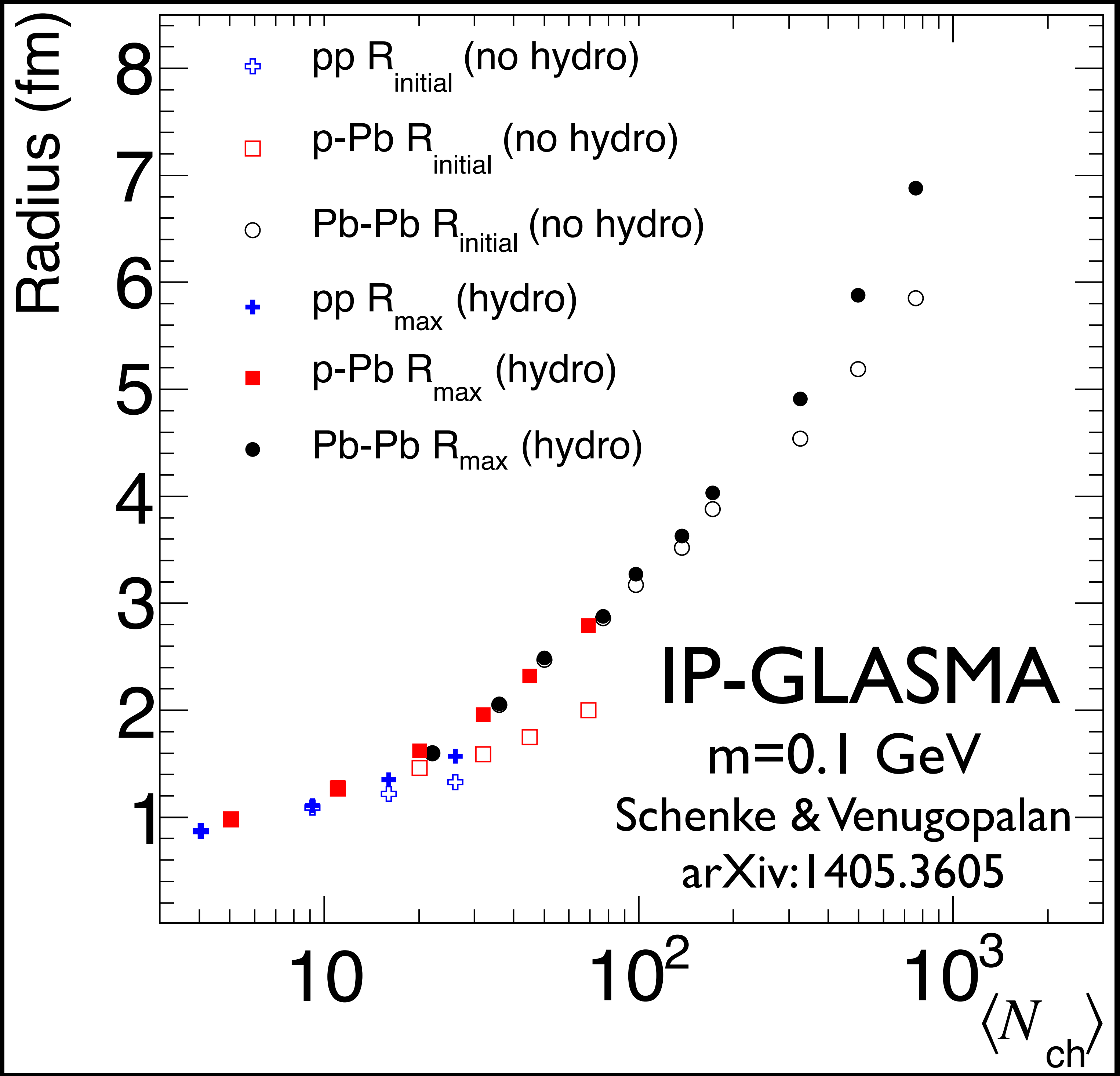
Why is the Source Radius Important



IP-GLASMA initial conditions alone (a model with only gluon fields).
→ Similar freeze-out radius in p-Pb as compared to pp.

Use 1

Why is the Source Radius Important



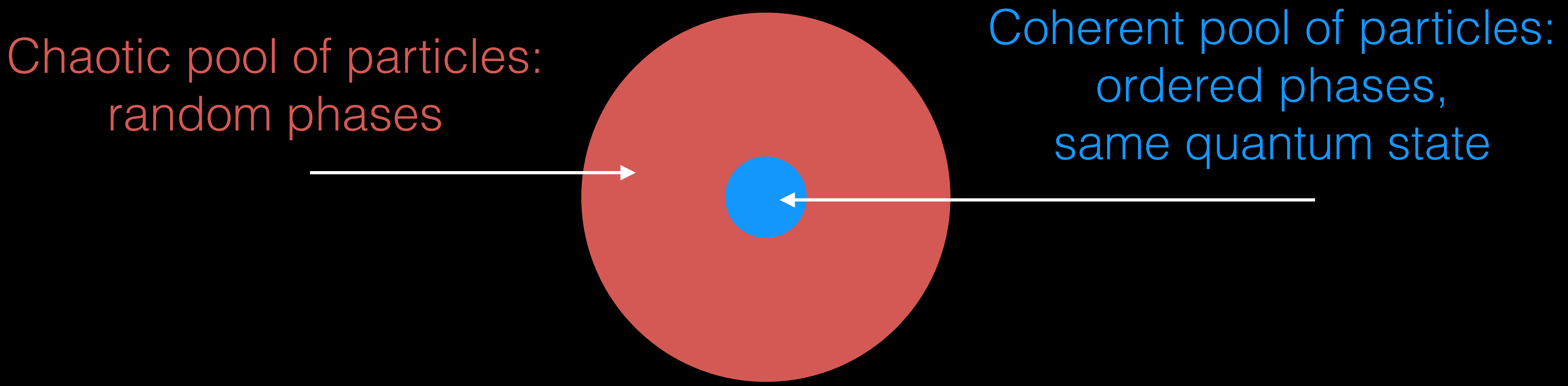
IP-GLASMA initial conditions alone (a model with only gluon fields).
→ Similar freeze-out radius in p-Pb as compared to pp.

Hydrodynamic expansion
→ Larger freeze-out radius.
p-Pb more comparable to Pb-Pb

There are other hydrodynamic predictions as well:
Bozek and Broniowski,
Phys. Lett. B 720, 250 (2013)

Use 2

Measuring the Coherent Fraction of Pions

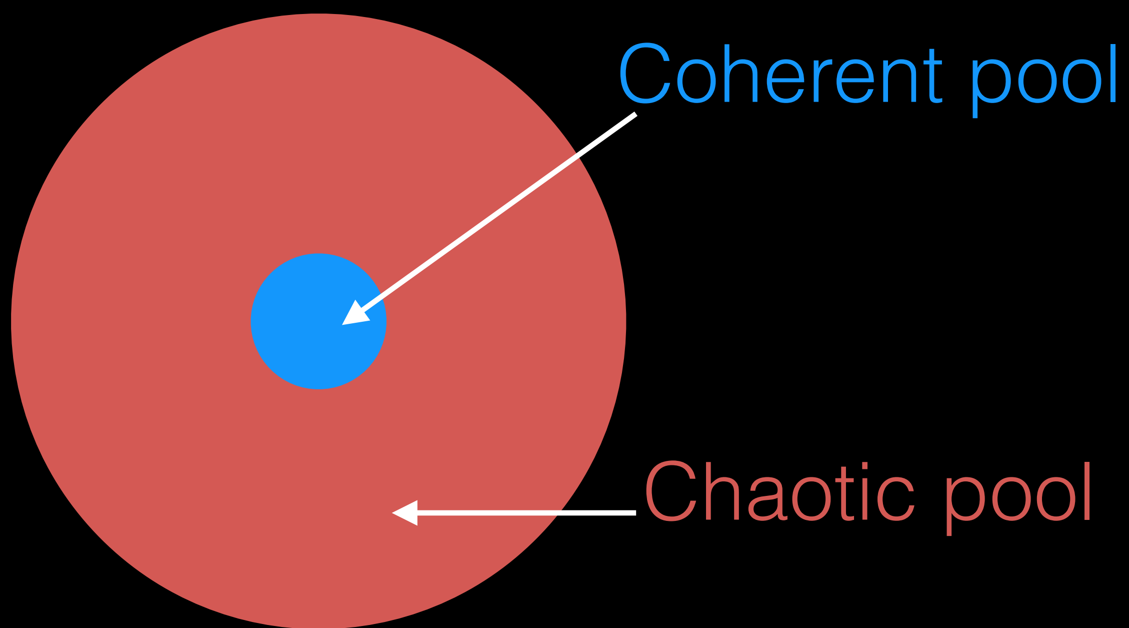


Pion condensation, Disoriented Chiral Condensates, +…..
may create a coherent pool of pions.

For coherence to survive in the final state,
the chaotic pool must not interact with the coherent pool.
Existence of such coherence would imply 2 disjunct sources!

Use 2

2-pion Bose-Einstein Contributions



<u>2-pions</u>

$\pi\ \pi$

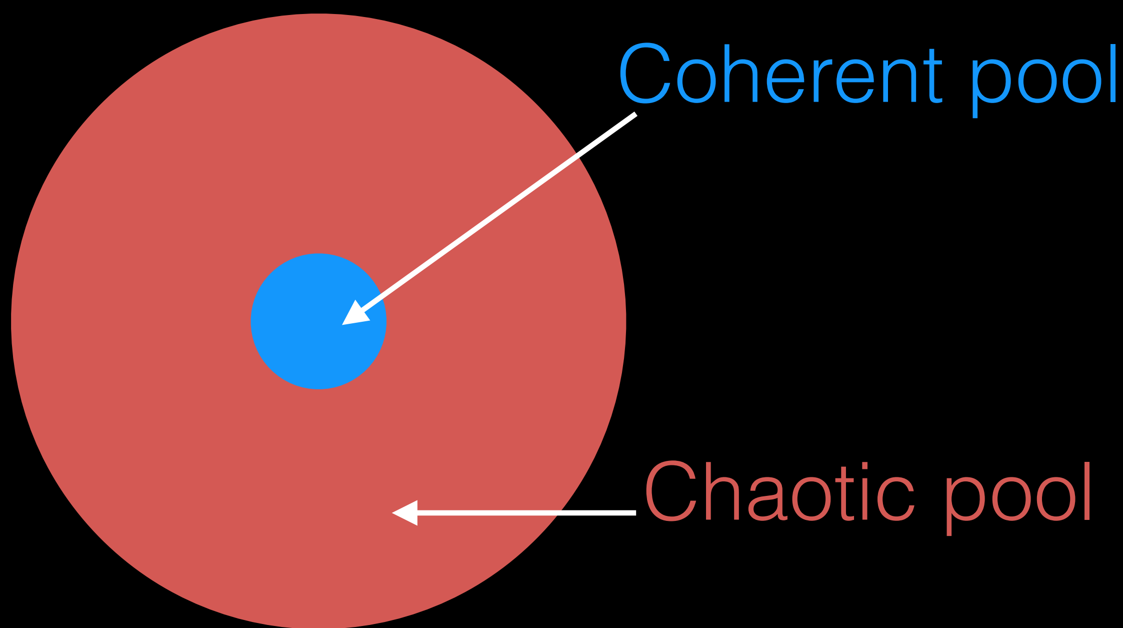
$\pi\ \pi$

$\cancel{\pi\ \pi}$

1 suppressed combination

Use
2

3-pion Bose-Einstein Contributions



3-pions

$\pi\ \pi\ \pi$

$\pi\ \pi\ \pi$

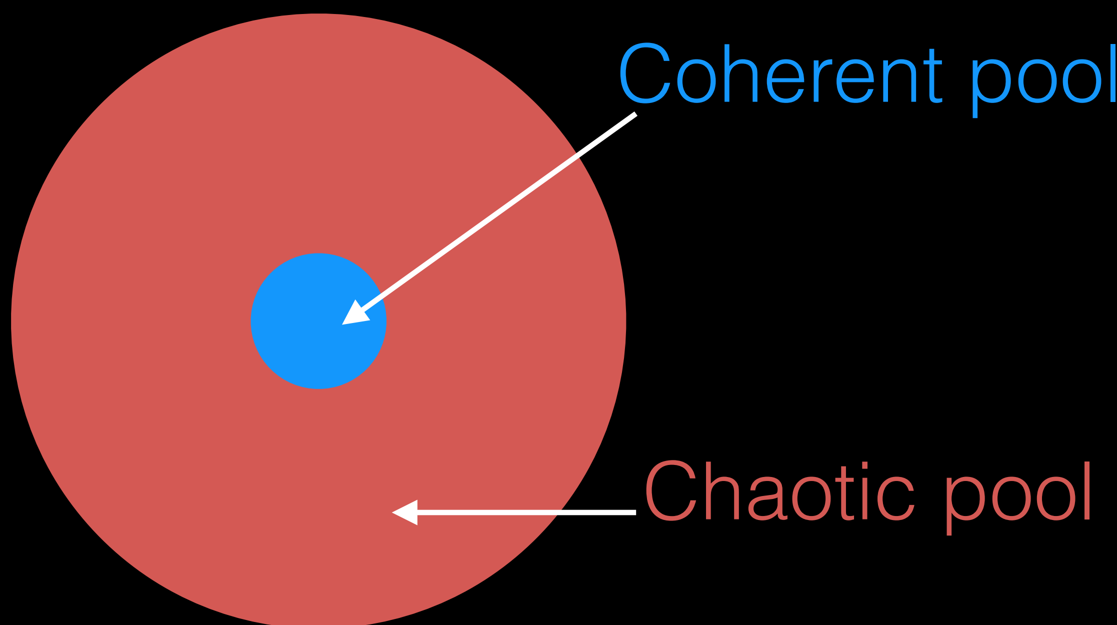
$\pi\ \cancel{\pi}\ \pi$

$\pi\ \cancel{\pi}\ \pi$

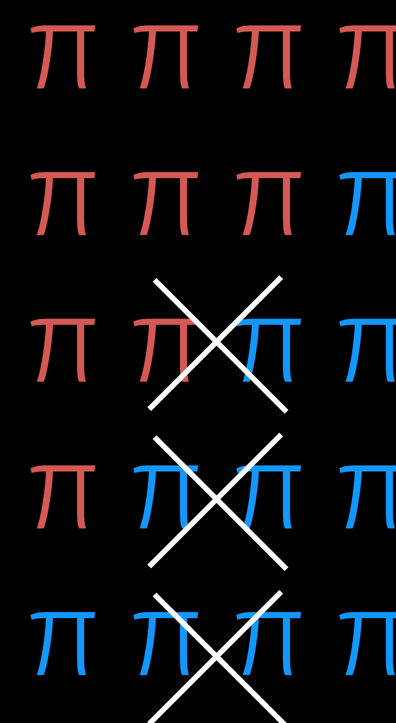
2 suppressed combinations

Use 2

4-pion Bose-Einstein Contributions



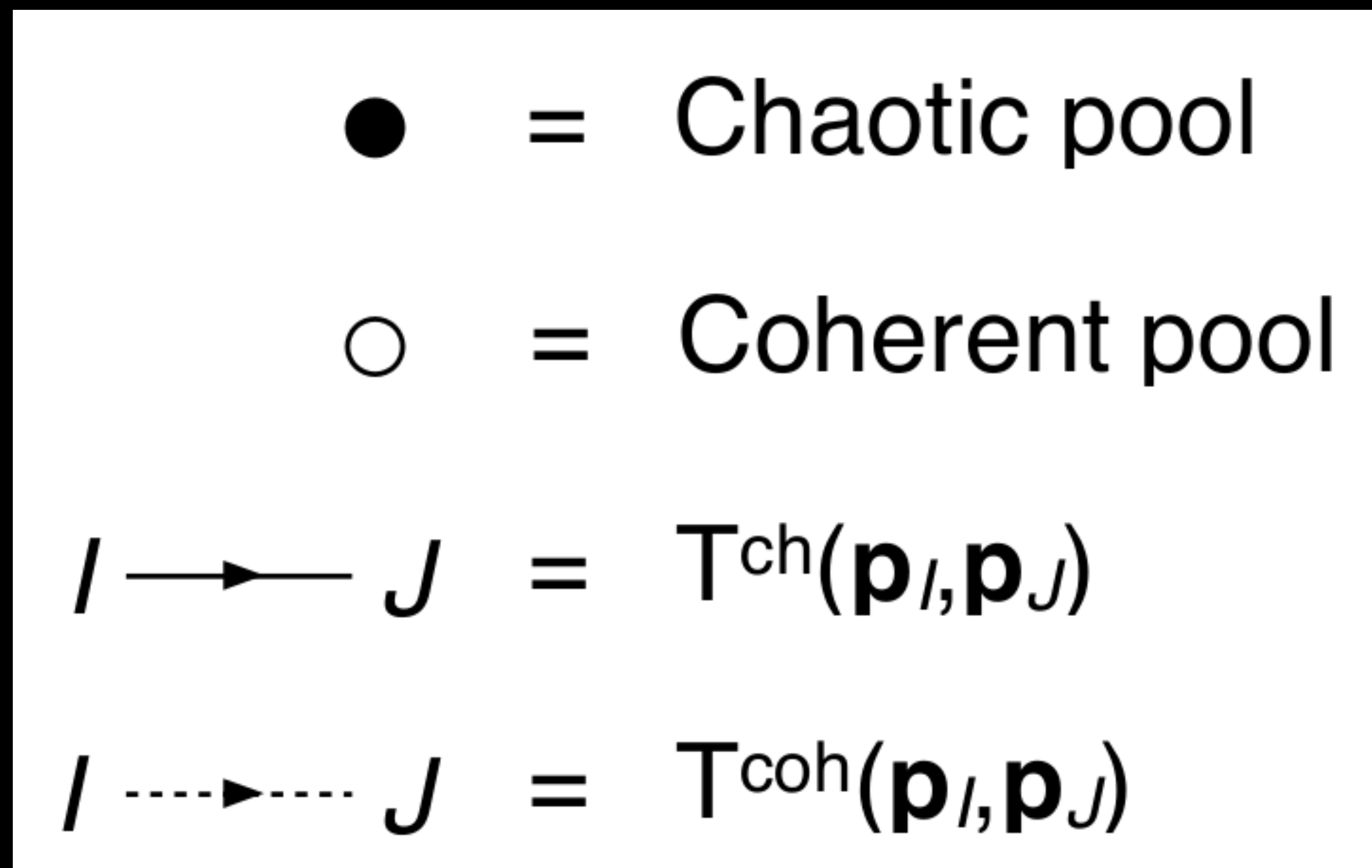
4-pions



3 suppressed combinations

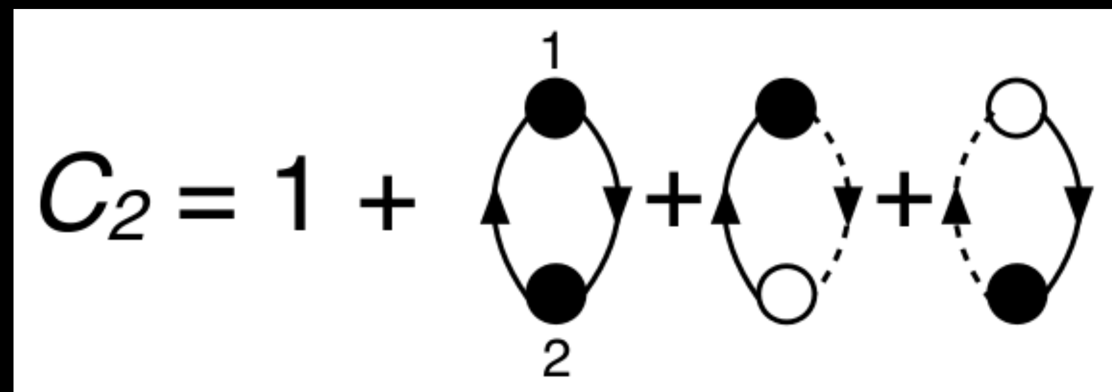
Resolution of coherence increases with the number of pions used.

Pair Exchange Amplitude
— Building Blocks of Bose-Einstein Correlations



T_{IJ} is the <u>pair exchange amplitude</u>:
Fourier Transform of source space-time distribution.
It is the building-block of all orders of Bose-Einstein correlations.

2-boson Symmetrization



Diagrams derived from
T. Csorgo
Heavy Ion Physics ***15*** *1-80*

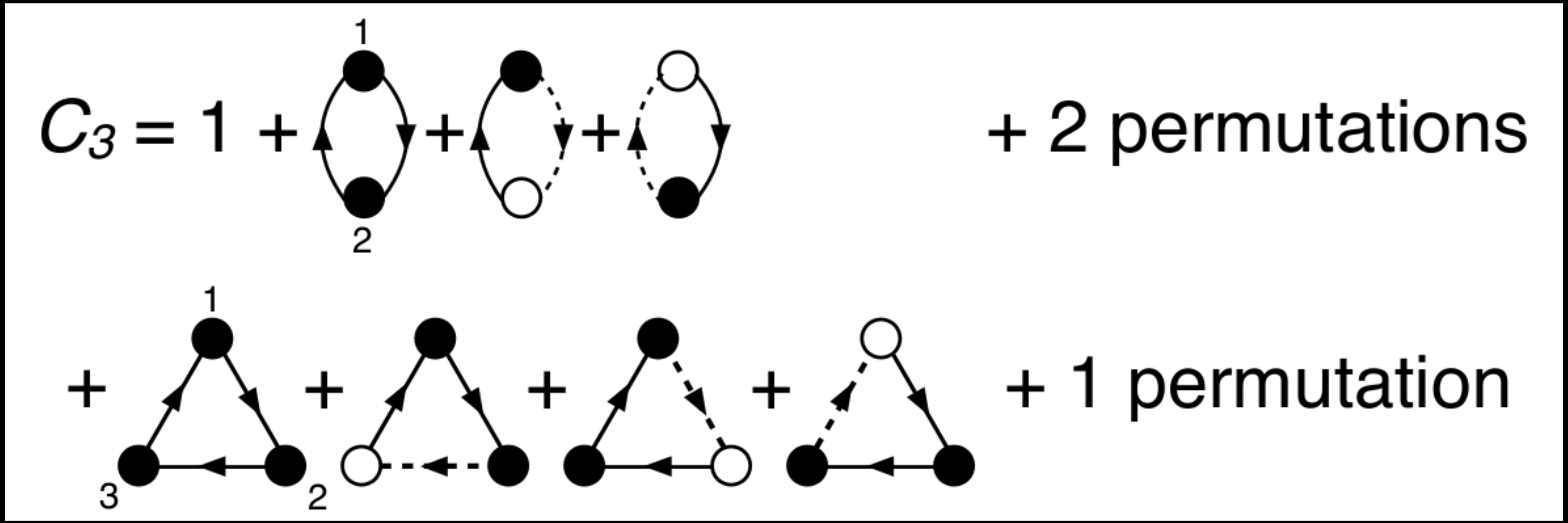
$$
\begin{aligned}
C_2(1,2) &= 1 + (1-G)^2 (T_{12}^{\mathrm{ch}})^2 \\
&+ 2G(1-G) T_{12}^{\mathrm{ch}} T_{12}^{\mathrm{coh}} \cos(\phi_{12}^{\mathrm{ch-coh}})
\end{aligned}
$$

phase of chaotic-coherent interference.

coherent fraction of pions

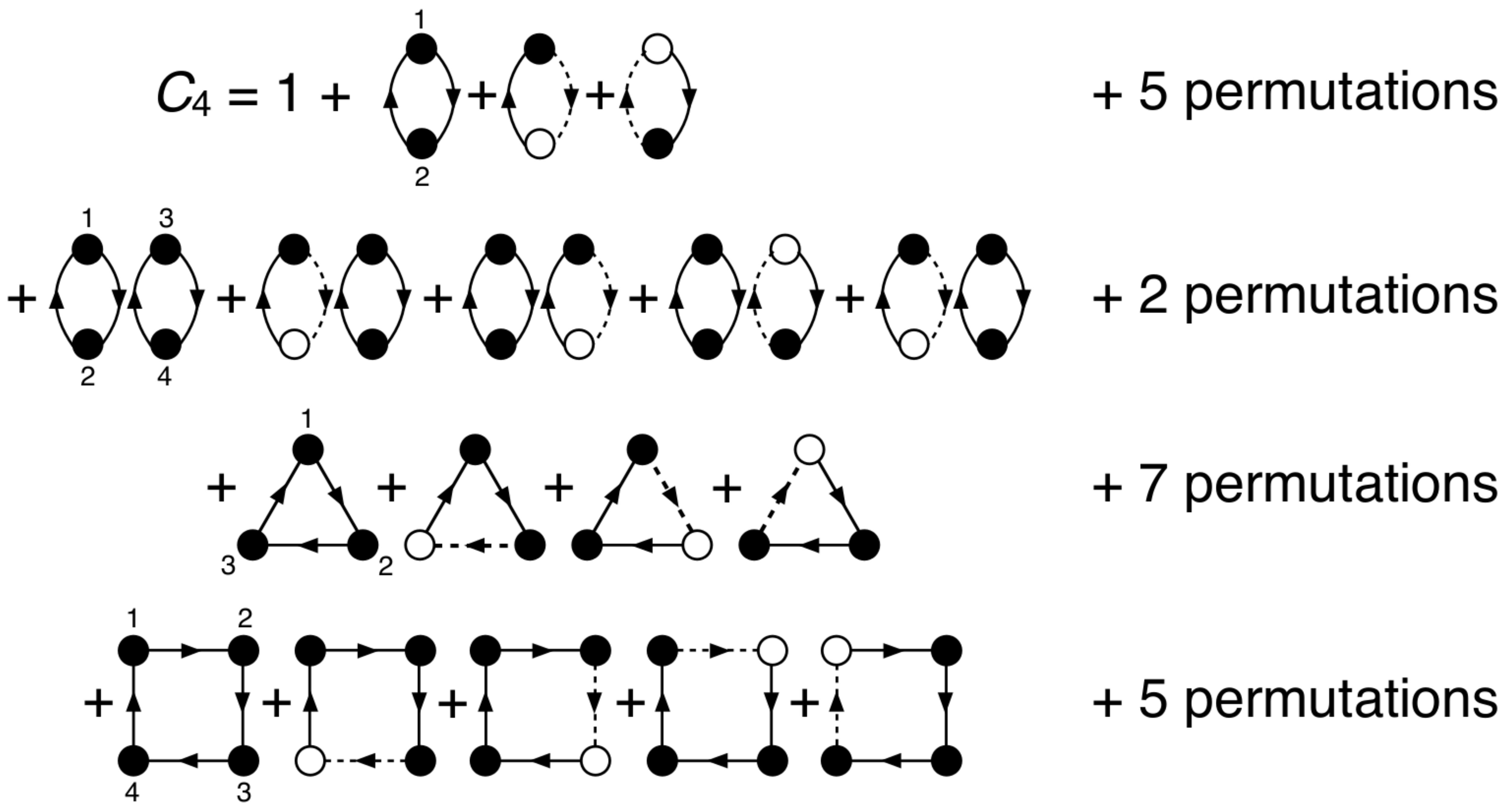
Equations derived from
I. Andreev et al.
Int. J. Mod. Phys. A ***8*** *4577*

3-boson Symmetrization



Diagrams derived from
T. Csorgo
Heavy Ion Physics ***15*** *1-80*

4-boson Symmetrization



Diagrams derived from
T. Csorgo
Heavy Ion Physics ***15*** *1-80*

Standard Correlation Functions

$$C_n = \frac{N_n(\mathbf{p_1}, \mathbf{p_2}, ..., \mathbf{p_n})}{N_1(\mathbf{p_1})N_1(\mathbf{p_2})...N_1(\mathbf{p_n})}$$

p = momentum

<u>Projection Variables</u>

$$q_{ij} = \sqrt{-(p_i - p_j)_\mu (p_i - p_j)^\mu}$$

$$k_T = |\vec{p}_{\mathrm{T}1} + \vec{p}_{\mathrm{T}2}|/2$$

$$Q_3 = \sqrt{q_{12}^2 + q_{13}^2 + q_{23}^2}$$

$$K_{T,3} = |\vec{p}_{\mathrm{T}1} + \vec{p}_{\mathrm{T}2} + \vec{p}_{\mathrm{T}3}|/3$$

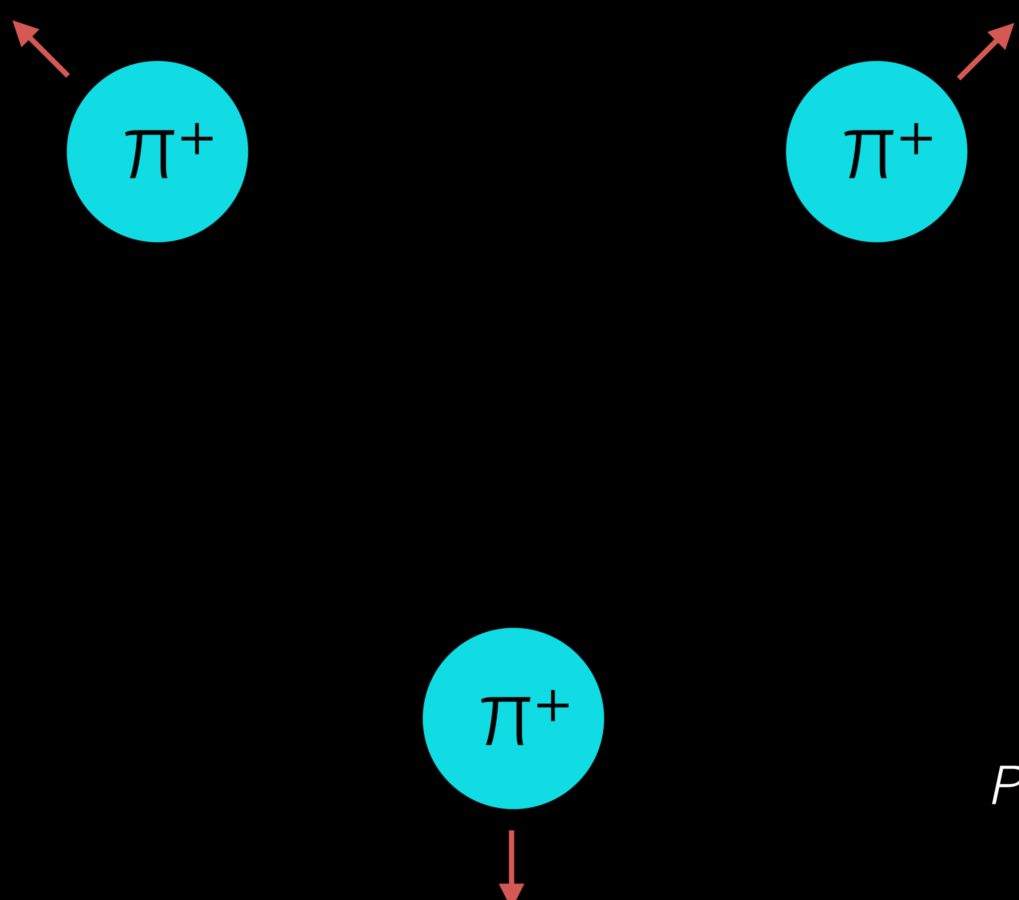
$$Q_4 = \sqrt{q_{12}^2 + q_{13}^2 + q_{14}^2 + q_{23}^2 + q_{24}^2 + q_{34}^2}$$

$$K_{T,4} = |\vec{p}_{\mathrm{T}1} + \vec{p}_{\mathrm{T}2} + \vec{p}_{\mathrm{T}3} + \vec{p}_{\mathrm{T}4}|/4$$

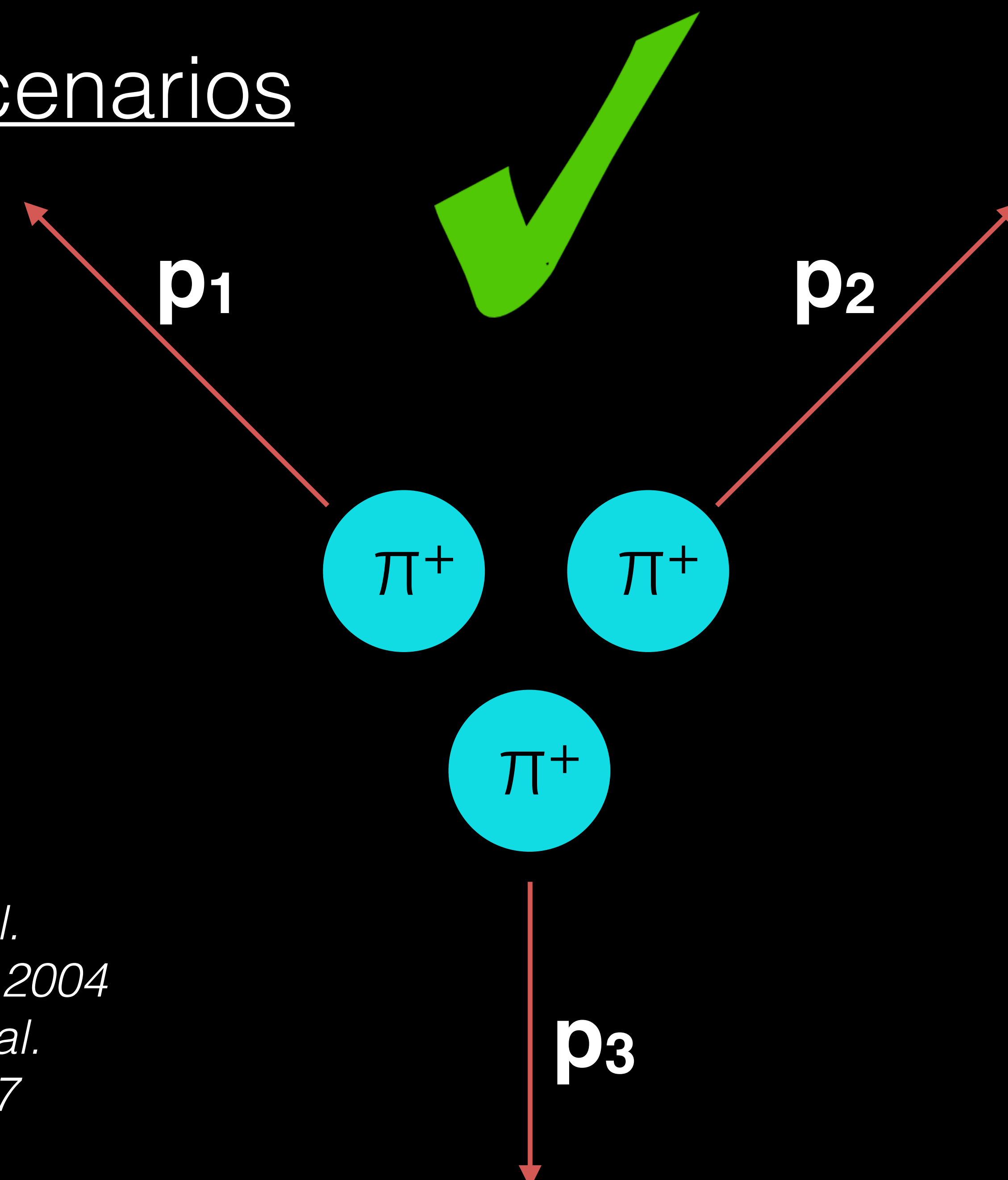
Multi-Pion Coulomb Interaction

Multi-body Coulomb wave-functions are not known exactly. However, asymptotic solutions exist which are applicable to high-energy collisions.

Asymptotic Scenarios

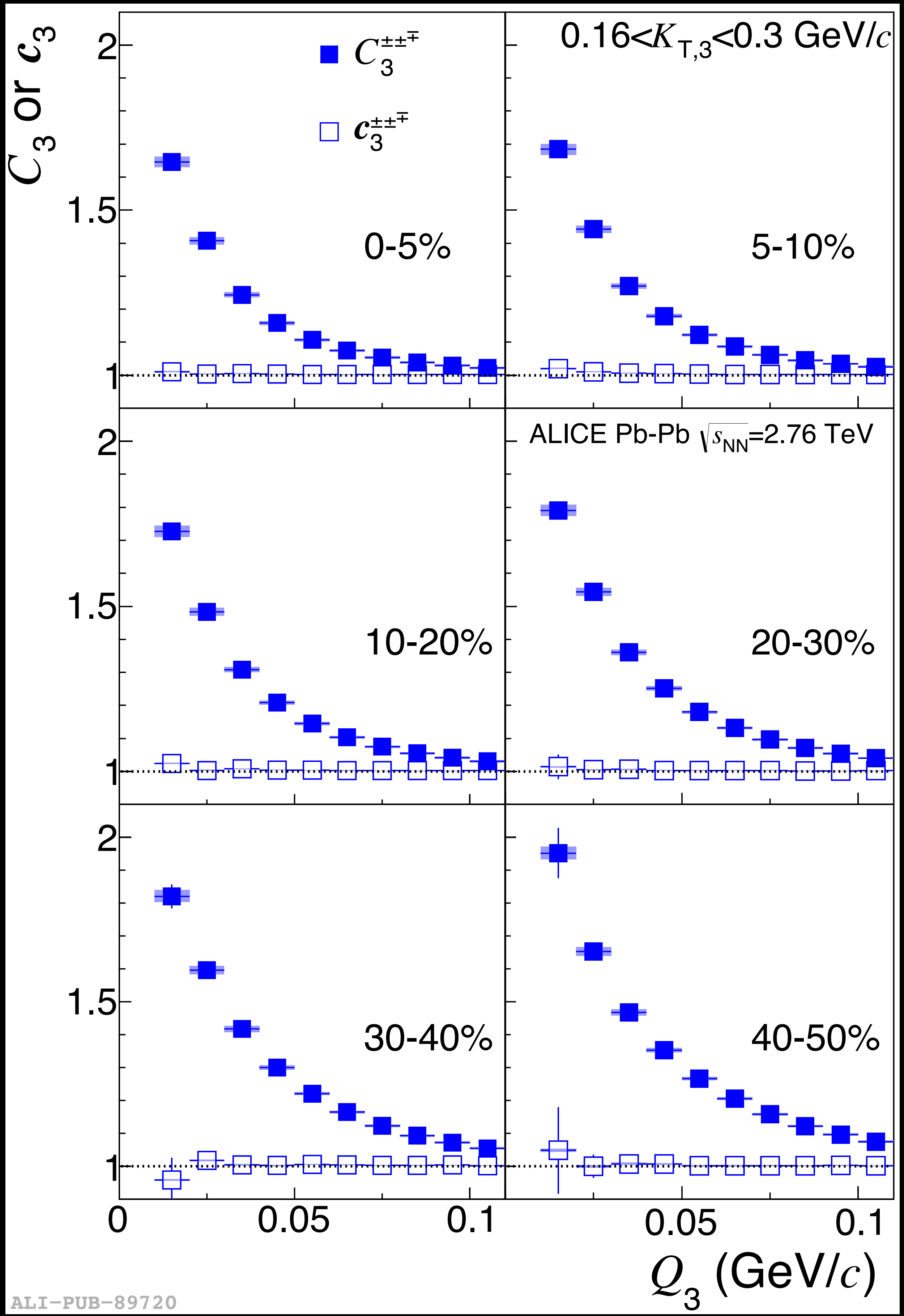


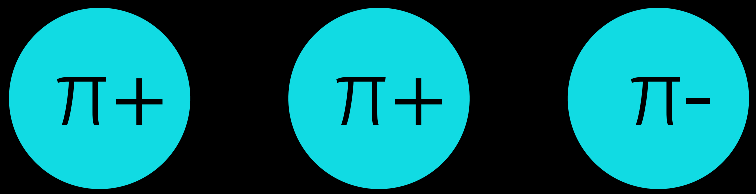
or



E.O. Alt et al.
*Phys. Rev. A **47** 2004*
T. Csorgo et al.
*PLB **458** 407*

Check that 3-body Coulomb corrections work





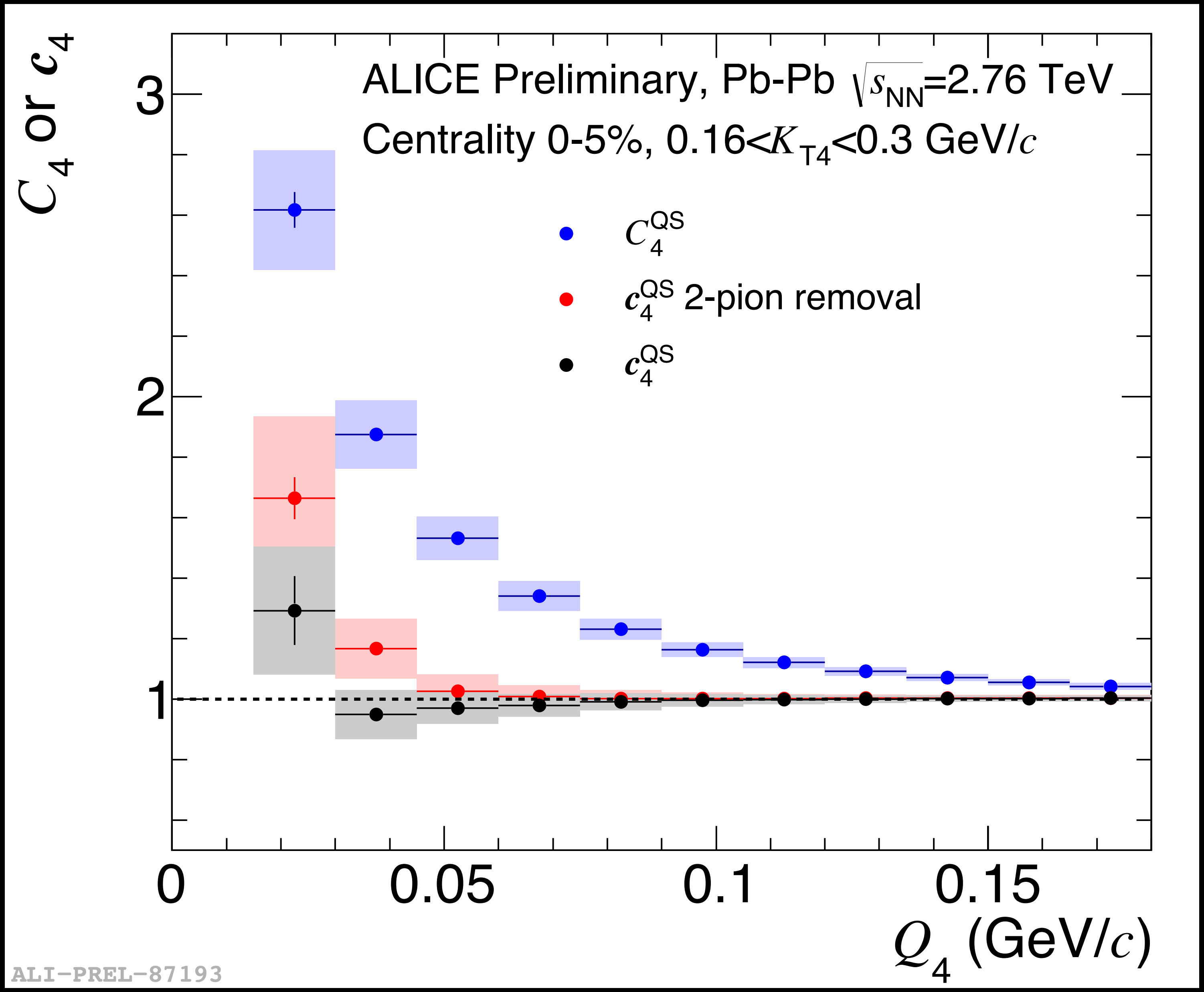
The cumulant (hollow points) are Coulomb corrected.

Consistency with unity demonstrates success of 3-body Coulomb ansatz

ALICE
PRC 89 024911 (2014)

New: 4-pion Coulomb Check

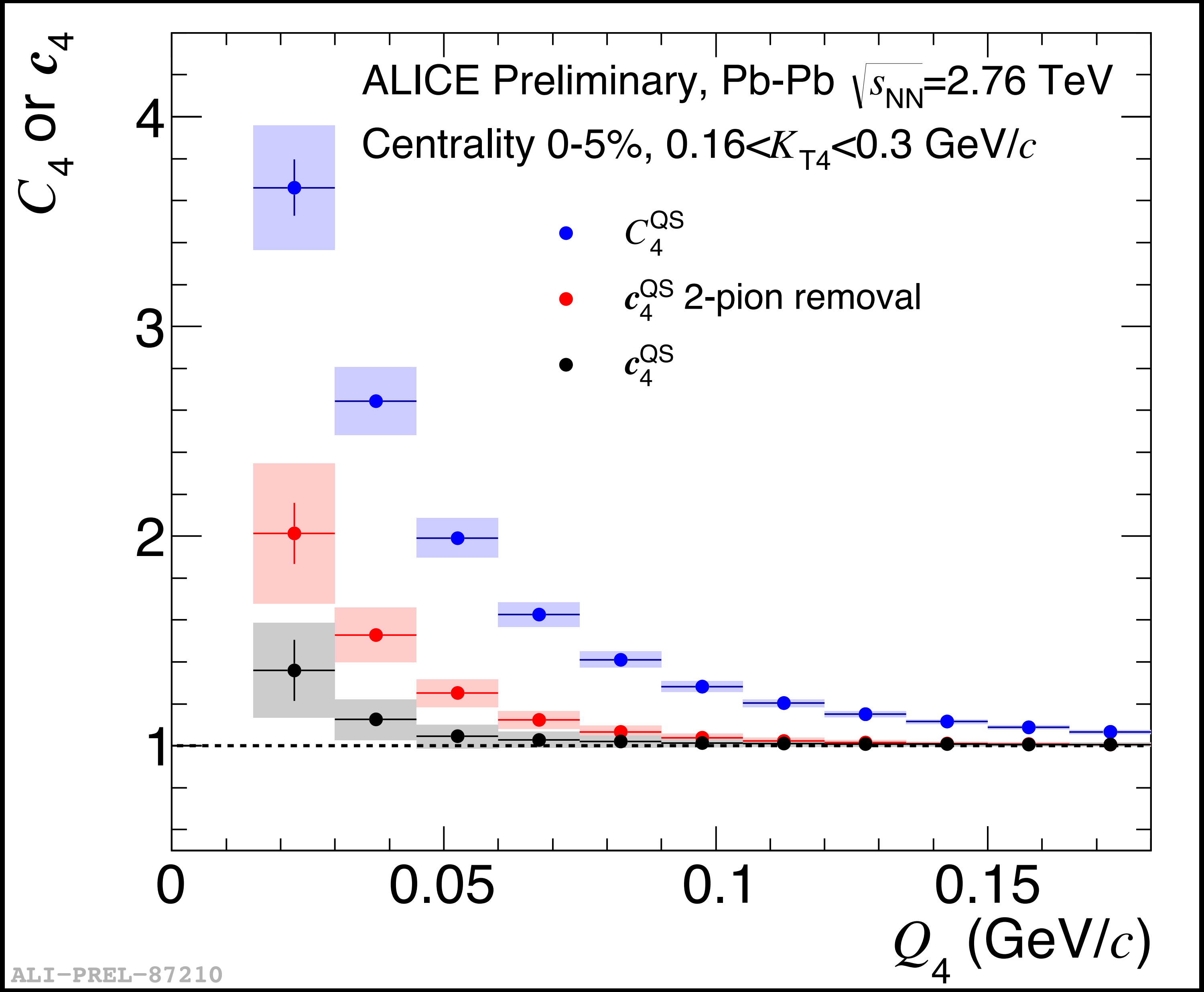
π+ π+ π- π-



- - - + + correlation well understood. Cumulant (black) near unity.

New: 4-pion Coulomb Check

π+ π+ π+ π-



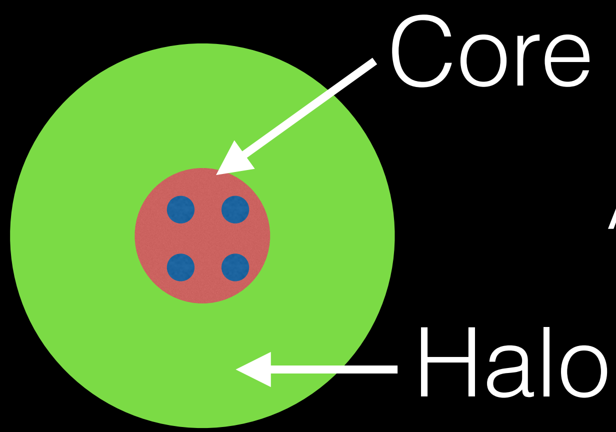
- - - - + correlation mostly understood. Cumulant (black) near unity.
- Ongoing studies in pp and p-Pb suggest that the reside is not Coulomb related.

Dilution from non-Femtoscopic Pairs

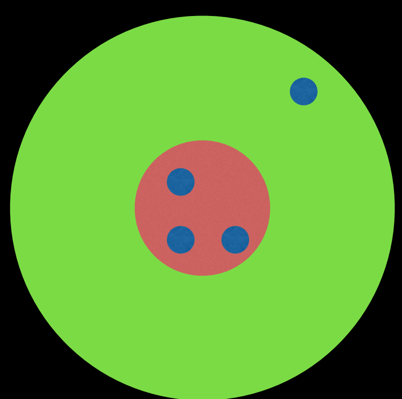
For pairs with relative separation $\gtrsim$ 50 fm
there is no observable Bose-Einstein correlation.

4-pion possibilities in the Core/Halo picture

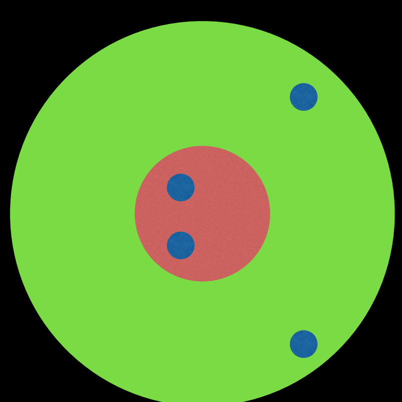
T. Csorgo et al.
Z. Phys. C ***71*** *491*



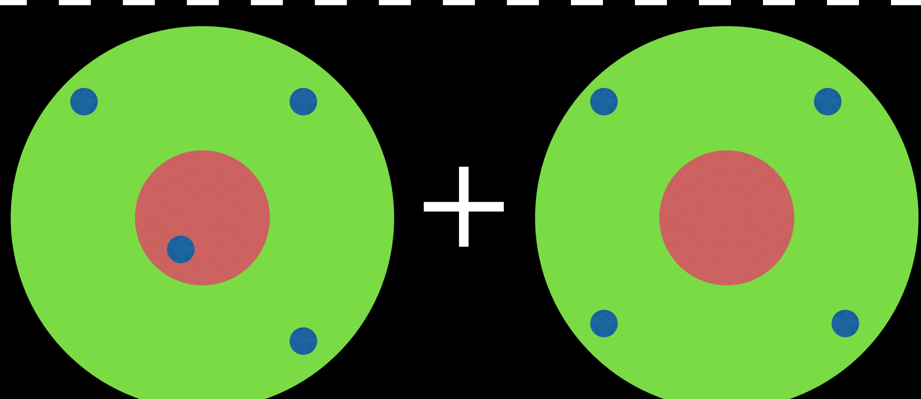
All 4 originating from the core (short-lived emitters)



3 originating from the core



2 originating from the core



1 or 0 originating from the core.
<u>No observable</u> Bose-Einstein correlations

Multi-pion Distributions

4-pion Distributions

$N_4(p_1, p_2, p_3, p_4)$ — 4 pions from same event

$N_3(p_1, p_2, p_3)\ N_1(p_4)$ — 3 pions from same event

$N_2(p_1, p_2)\ N_1(p_3)\ N_1(p_4)$ — 2 pions from same event

$N_2(p_1, p_2)\ N_2(p_3, p_4)$ — 2 pairs from same event

$N_1(p_1)\ N_1(p_2)\ N_1(p_3)\ N_1(p_4)$ — All from different events

$K_4 = K_2^{12}\ K_2^{13}\ K_2^{14}\ K_2^{23}\ K_2^{24}\ K_2^{34}$ — 4-body Final-State-Interaction

Isolation of 4-pion QS

Quantity of Interest

$$\begin{aligned}
N_4(p_1,p_2,p_3,p_4) &= f_{41}N_1(p_1)N_1(p_2)N_1(p_3)N_1(p_4) \\
&+ f_{42}N_2(p_1,p_2)N_1(p_3)N_1(p_4) \\
&+ f_{43}N_3(p_1,p_2,p_3)N_1(p_4) \\
&+ f_{44}K_4(q_{12},q_{13},q_{14},q_{23},q_{24},q_{34})N_4^{QS}(p_1,p_2,p_3,p_4)
\end{aligned}$$

$$\begin{aligned}
f_{41}^{Core/Halo} &= -3(1-f_c)^4 - 8f_c(1-f_c)^3 + 6(1-f_c^2)(1-f_c)^2 \\
f_{42}^{Core/Halo} &= -6(1-f_c)^2 \\
f_{43}^{Core/Halo} &= 4(1-f_c) \\
f_{44}^{Core/Halo} &= f_c^4.
\end{aligned}$$

f_c^2 = “lambda” = 0.7 +- 0.05 (fraction of correlated pairs)

Data and Track Selection

Collision types

pp $\sqrt{s}$ = 7 TeV

p-Pb $\sqrt{s_{NN}}$ = 5.02 TeV

Pb-Pb $\sqrt{s_{NN}}$ = 2.76 TeV

Track Selection

• **Pions** selected based on their specific energy loss in the Time Projection Chamber. Time of Flight also used for p > 0.6 GeV/c.

• p_T > 0.16 GeV/c

• p < 1.0 GeV/c

• $|\eta|$ < 0.8

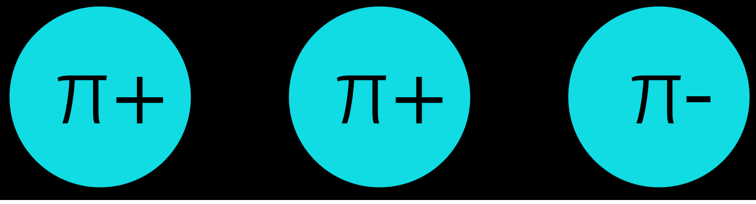
Pair Cuts

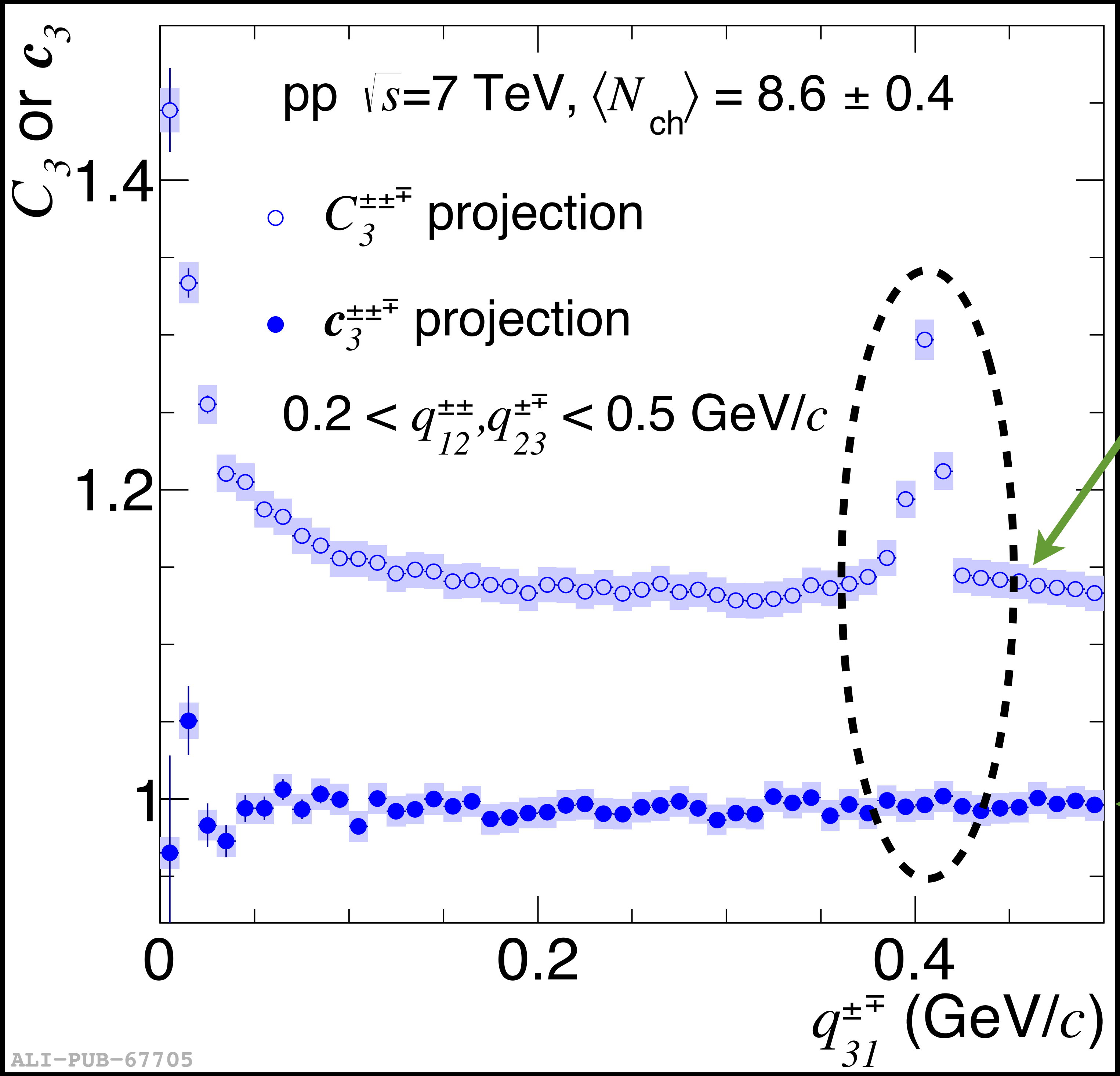
• Track merging and splitting: pair angular separation

• For 3 (4) pions, pair cuts applied to all 3 (6) pairs in the triplet (quadruplet).

Freeze-out Radii Extracted from 3-pion Bose-Einstein Cumulants

3-pion cumulants remove 2-pion correlations





Hollow points:
K^0_s peak visible.

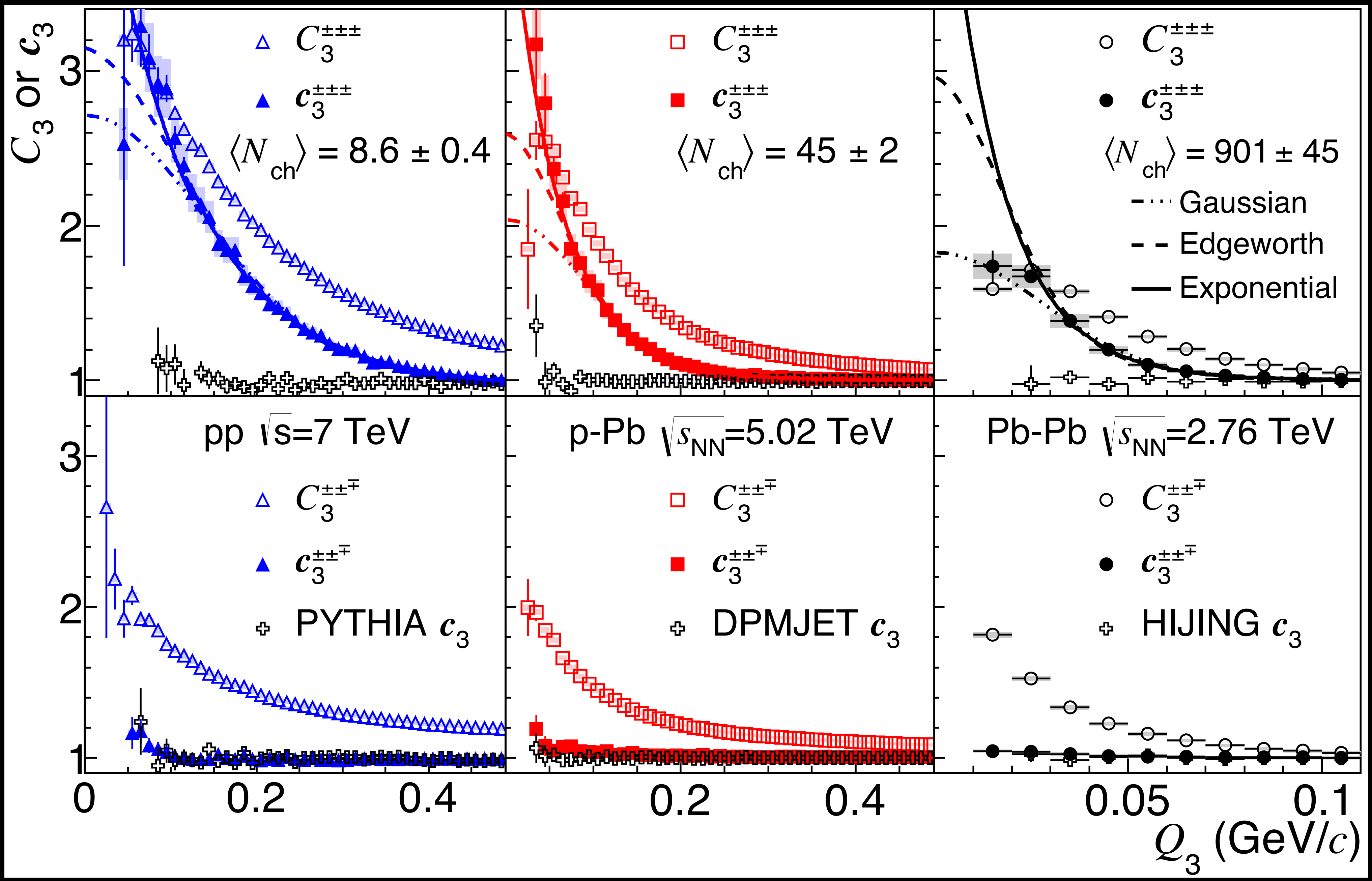
Solid points:
Cumulant.
K^0_s peak removed

ALICE
Phys. Lett. B accepted
arXiv:1404.1194 (2014)

3-pion Correlation Functions

Same Charge

Mixed Charge



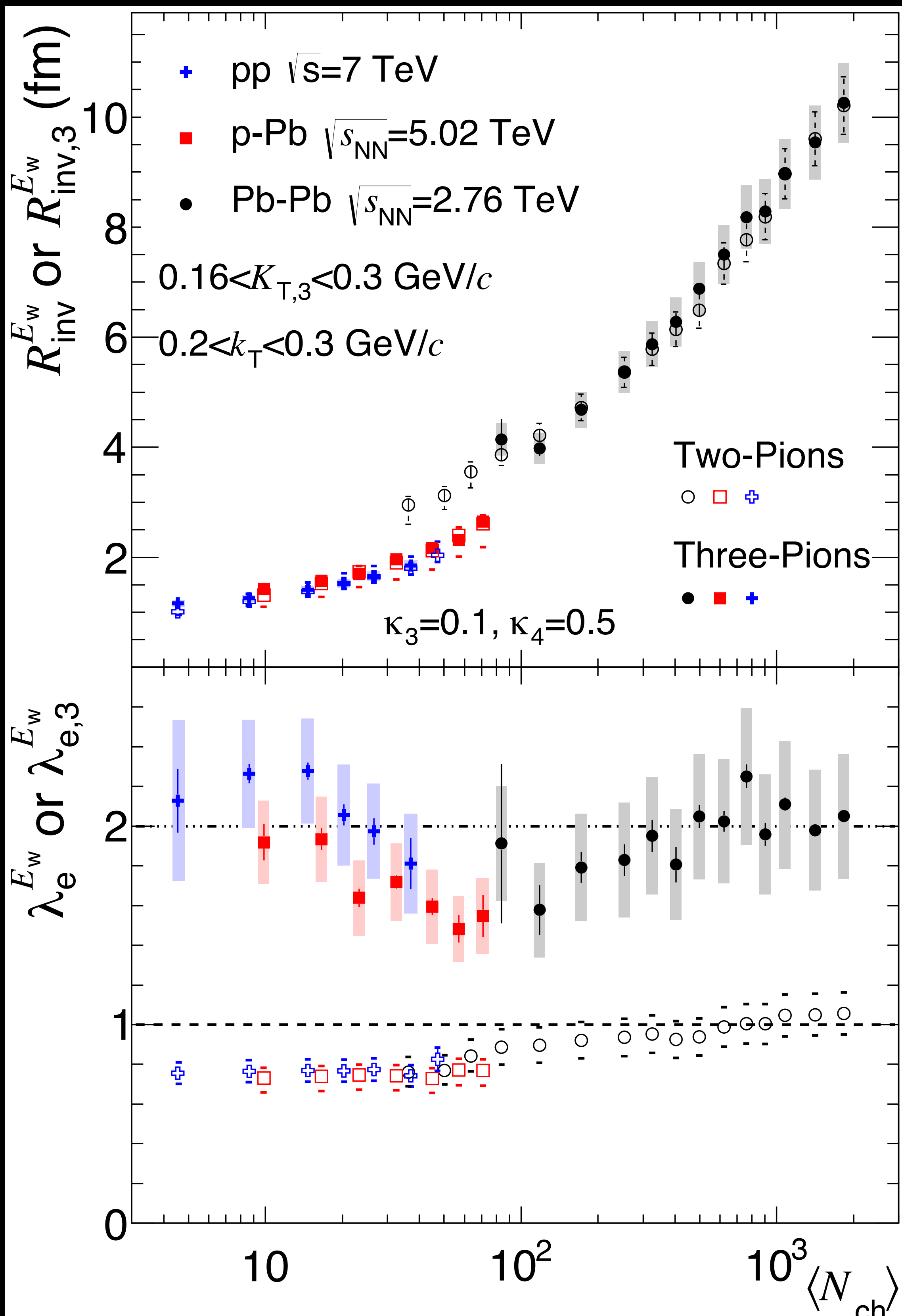
pp
low multiplicity

p-Pb
mid multiplicity

Pb-Pb
high multiplicity

ALICE
Phys. Lett. B accepted
arXiv:1404.1194 (2014)

Edgeworth Radii and Intercepts



- Non-Gaussian fits (Edgeworth or Exponential) provide a better fit of the correlation function.

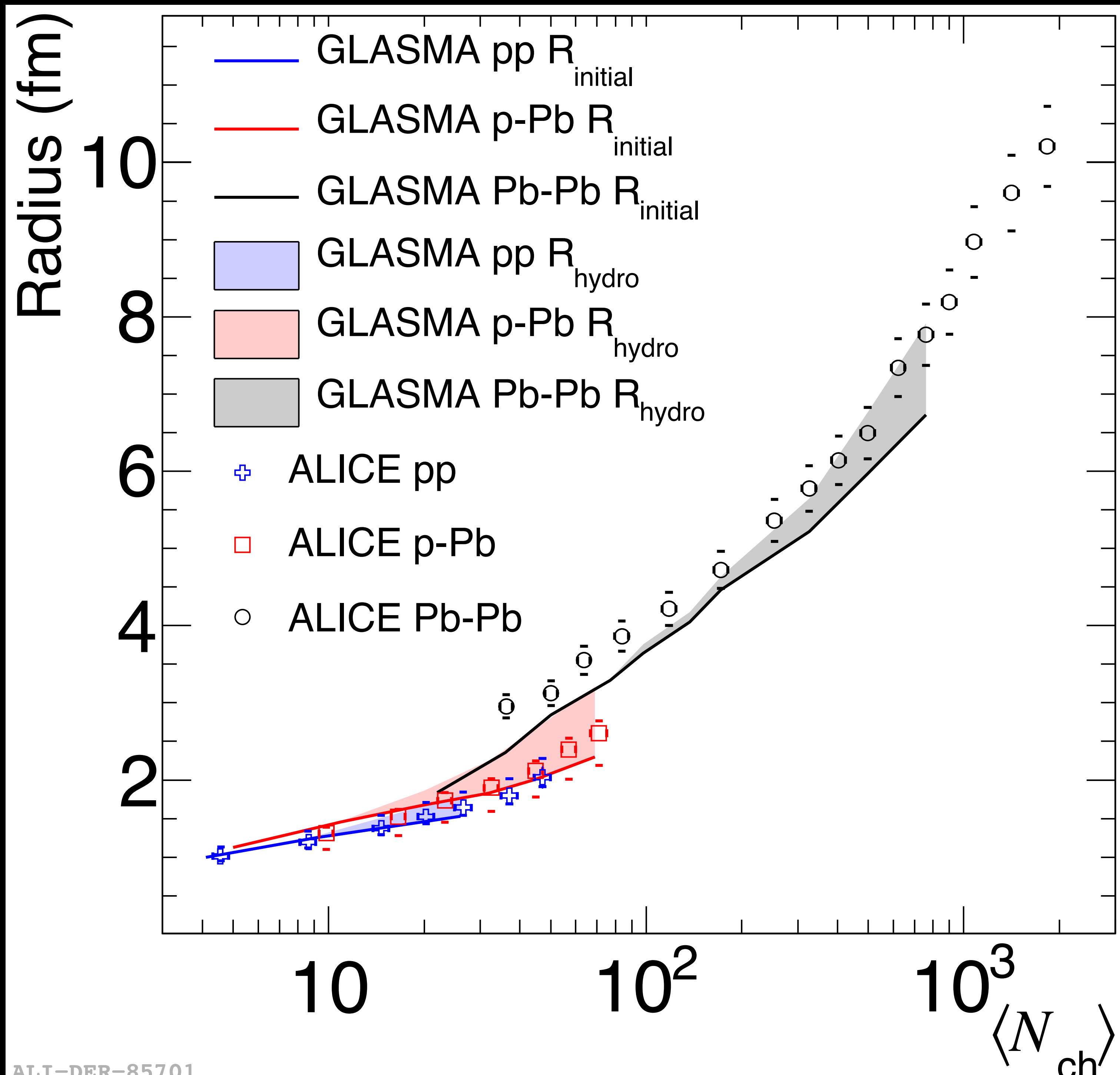
- Radii report the 2nd cumulant of the Edgeworth correlation function.

- p-Pb similar to pp.
- Pb-Pb not similar to pp/p-Pb.

- Intercept parameters much closer to their chaotic limits.

ALICE
Phys. Lett. B accepted
arXiv:1404.1194 (2014)

Radii Comparison with IP-GLASMA



<u>Message:</u>

Similarity of ALICE radii in p-Pb and pp can be reproduced with GLASMA initial conditions alone.

They can also be reproduced with a hydrodynamic phase in p-Pb.

Schenke & Venugopalan
arXiv:1405.3605

- GLASMA points are first scaled such that the calculations in pp match the ALICE pp data. Scale = 1.15. GLASMA calculations have uncertainty due to infrared cutoff (m=0.1 GeV).

Coherence Measurements from 3-pion Cumulants

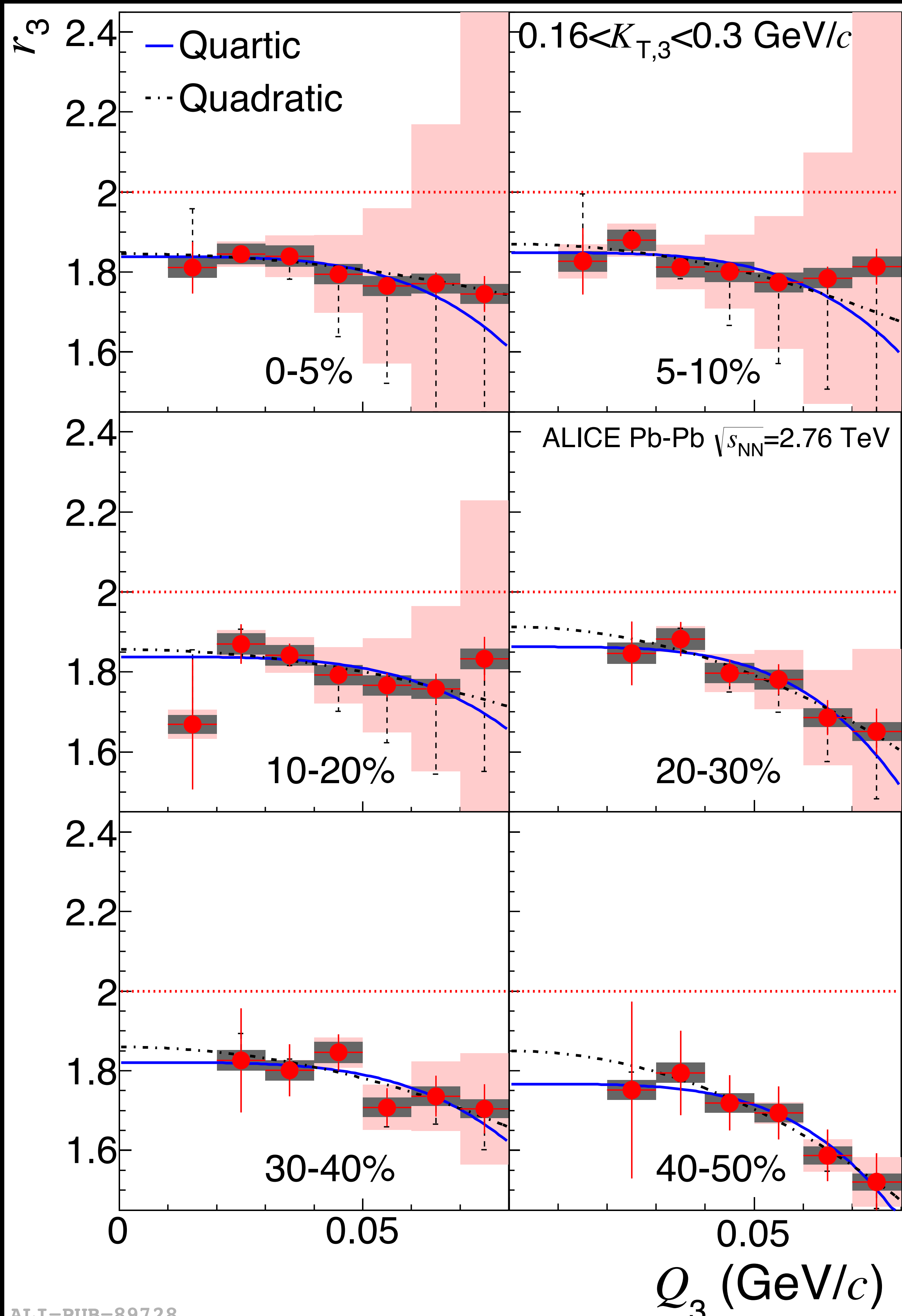
r_3

A comparison of 3-pion to 2-pion
Bose-Einstein correlation strengths

$$r_3(Q_3) = \frac{c_3(q_{12}, q_{23}, q_{31}) - 1}{\sqrt{(C_2(q_{12}) - 1)(C_2(q_{13}) - 1)(C_2(q_{23}) - 1)}}$$

$r_3(0) = 2.0$ for no coherence
$r_3(Q_3) = 2.0$ additionally for no 3-pion phase

r_3 for 6 centrality bins in Pb-Pb



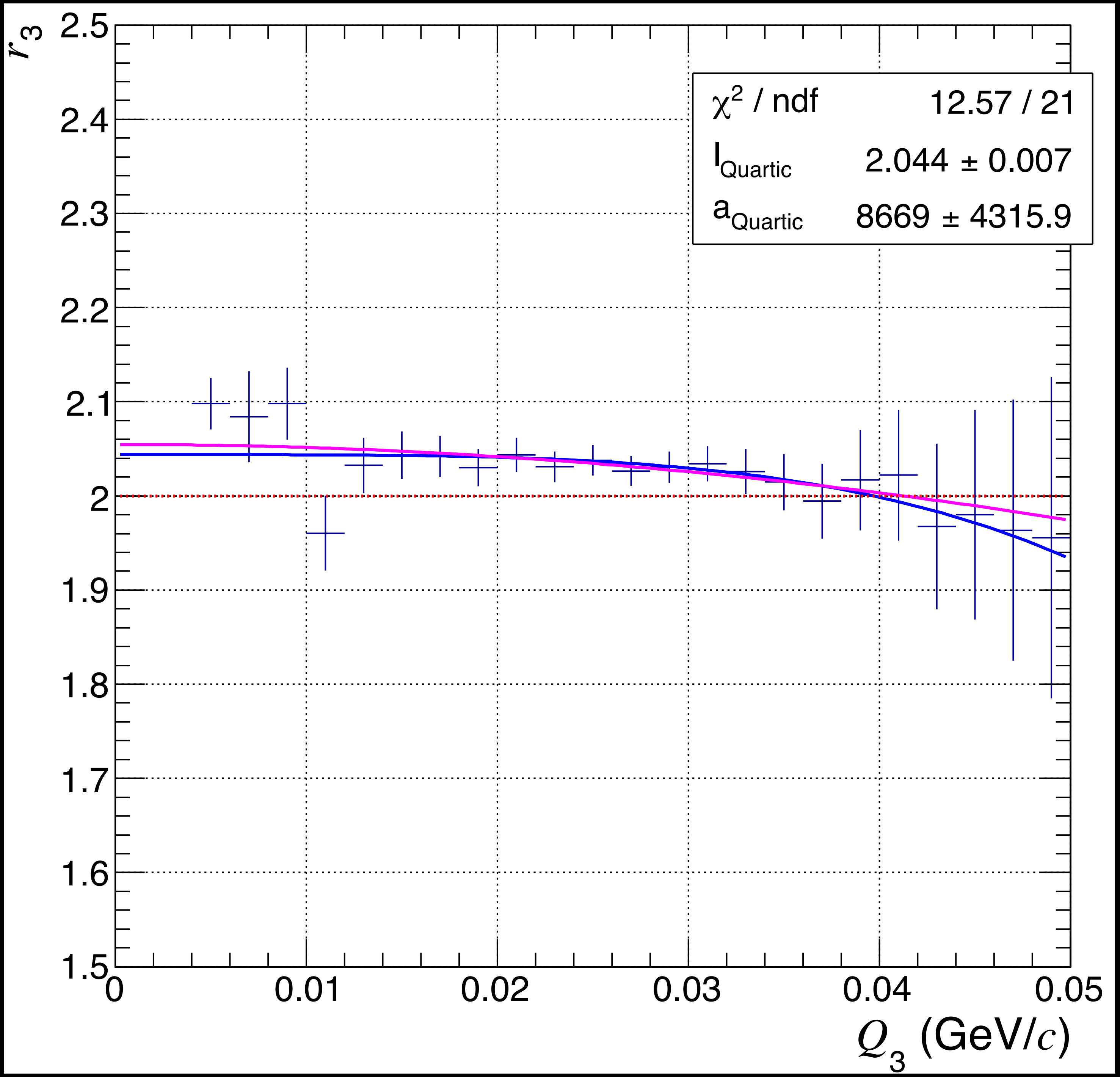
$$r_3(Q_3) = \frac{c_3(q_{12}, q_{23}, q_{31}) - 1}{\sqrt{(C_2(q_{12}) - 1)(C_2(q_{13}) - 1)(C_2(q_{23}) - 1)}}$$

All correlations are first Coulomb corrected.

r_3 is suppressed below 2.0. Intercept corresponds to **23% ± 8% coherence at low p_T.**

ALICE PRC 89 024911 (2014)

r_3 Calculation in Therminator



Therminator model calculation without coherence.

No Q_3 dependence in this model.
= No effect of the 3-pion phase.

Therminator 2 model:
Kisiel et al.,
Comput. Phys. Commun. 174, 669 (2006)

Coherence Measurements from 4-pion Correlations

Equations to Build QS correlations with coherence

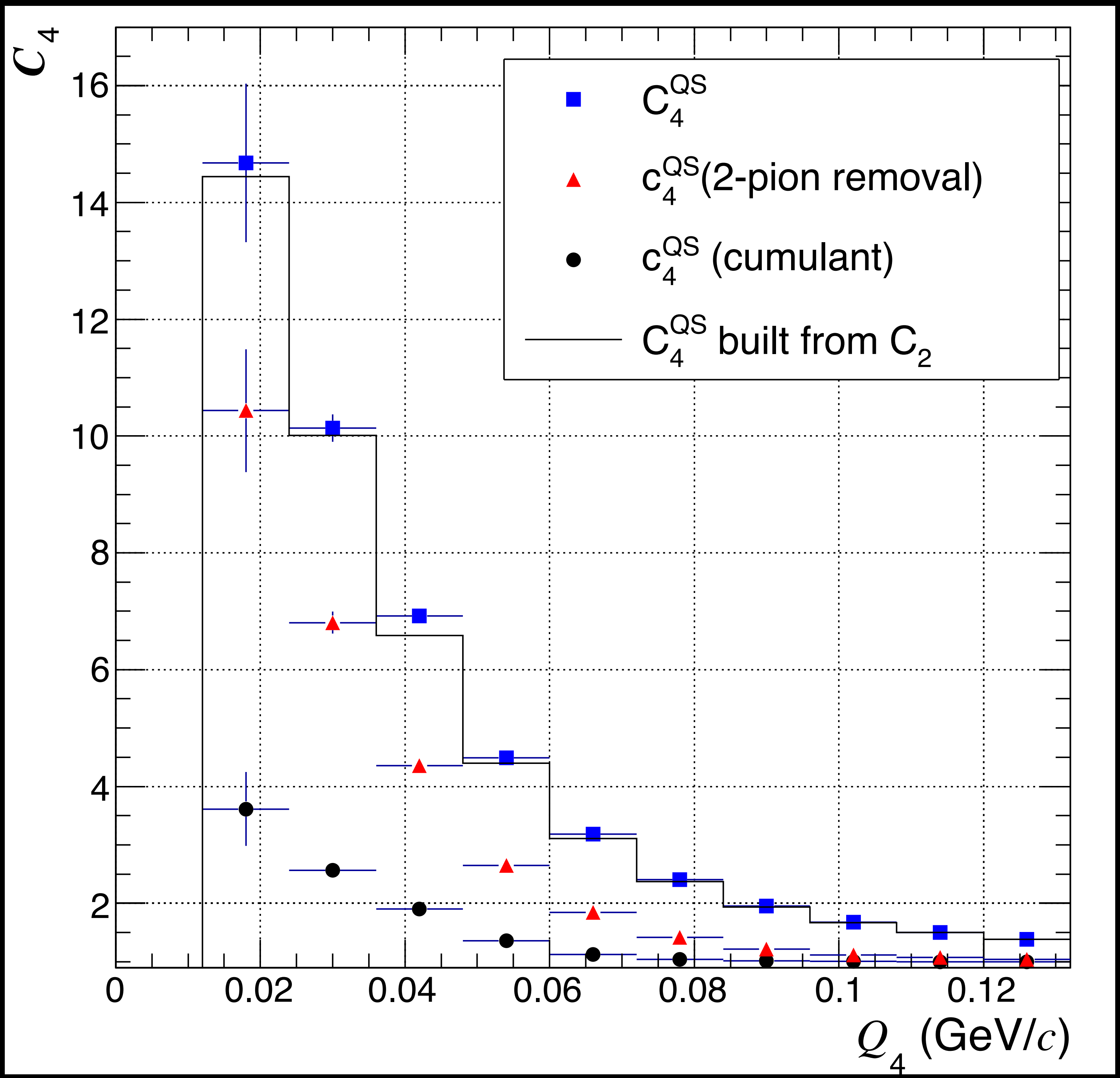
G = coherent fraction of pions

$$\boxed{C_2^{QS} - 1 = (1-G^2)T_{12}^2} \quad \text{Extract building block, } T_{ij}\text{, from here} \tag{52}$$

$$\begin{aligned}
C_3^{QS} - 1 &= (1-G)^2(T_{12}^2 + T_{13}^2 + T_{23}^2) && (53)\\
&+ (6G(1-G)^2 + 2(1-G)^3)T_{12}T_{13}T_{23} && (54)\\
C_4^{QS} - 1 &= (1-G^2)(T_{12}^2 + T_{13}^2 + T_{14}^2 + T_{23}^2 + T_{24}^2 + T_{34}^2) && (55)\\
&+ (4G(1-G)^3 + (1-G)^4(T_{12}^2T_{34}^2 + T_{13}^2T_{24}^2 + T_{14}^2T_{23}^2) && (56)\\
&+ (6G(1-G)^2 + 2(1-G)^3)(T_{12}T_{13}T_{23} + T_{12}T_{14}T_{24} + T_{13}T_{14}T_{34} + T_{23}T_{24}T_{34}) && (57)\\
&+ (8G(1-G)^3 + 2(1-G)^4)(T_{12}T_{13}T_{24}T_{34} + T_{12}T_{14}T_{23}T_{34} + T_{13}T_{14}T_{23}T_{24}) && (58)
\end{aligned}$$

These equations valid for R_{coh}=R_{ch} (coherent Radius = chaotic Radius).
We will also consider R_{coh}=0 (point source).

Proof of Principle



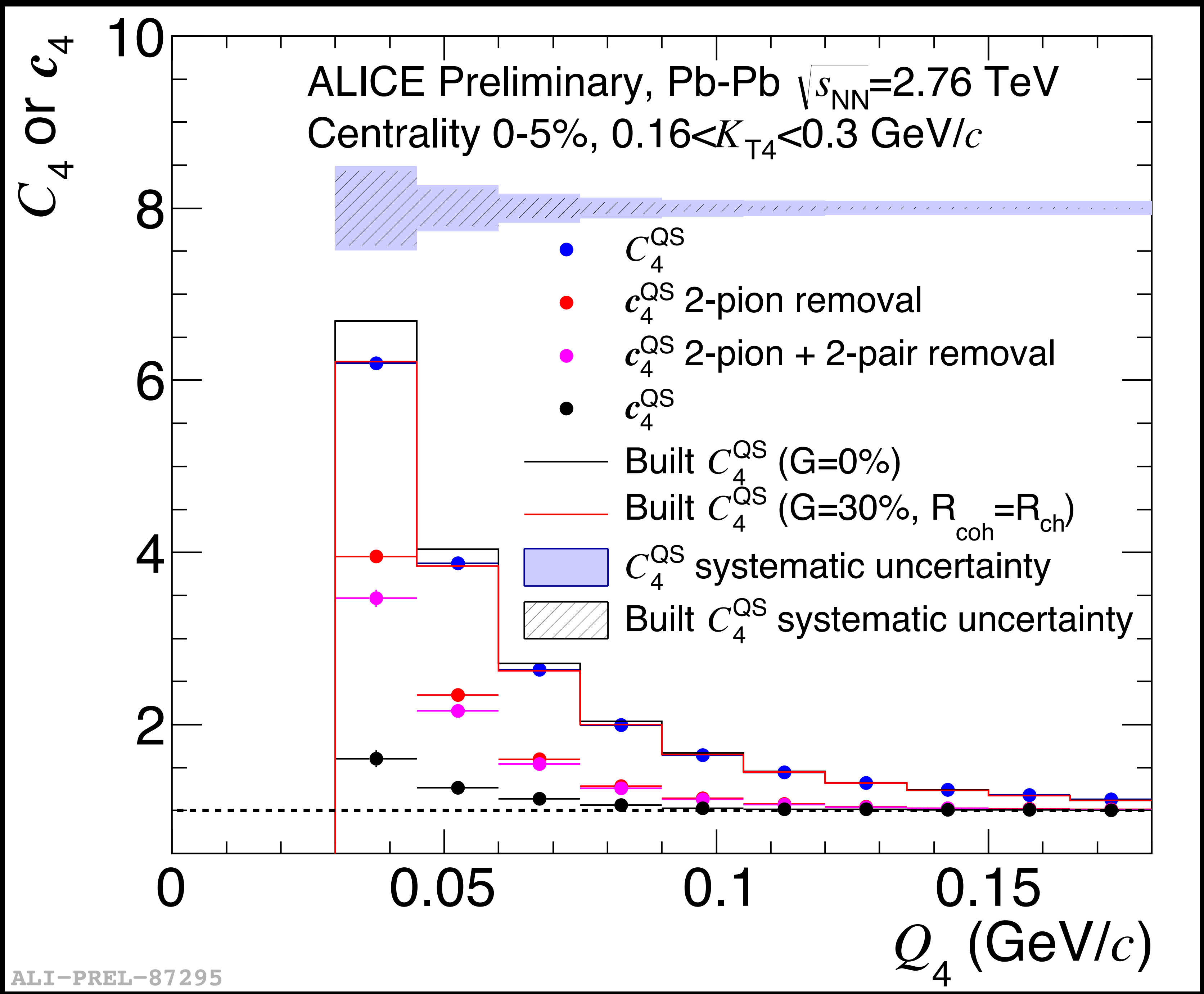
Therminator model calculation without coherence.

Measured 4-pion correlation is close to the “built” correlation.

Therminator 2 model:
Kisiel et al.,
Comput. Phys. Commun. 174, 669 (2006)

New: 4-pion Bose-Einstein

π+ π+ π+ π+



Extracted coherent fractions are again non-zero:
~30% for R_{coh} = R_{ch}
~15% for R_{coh} = 0

The Goal with Coherence Studies

We need a consistent picture from the comparison of all available orders of Bose-Einstein correlations:
3-to-2 (done)
4-to-2 (done)
4-to-3 (ongoing)

Consistent coherent fractions from each type makes a convincing case!

Work is ongoing to extract coherent fractions in pp and p-Pb.

Summary

Use 1

Freeze-out Radii:

- We have extracted freeze-out radii from 3-pion Bose-Einstein cumulants in pp, p-Pb, and Pb-Pb collisions.
- Radii in pp and p-Pb are quite similar, at similar multiplicity.
- Radii in Pb-Pb are quite different from pp and p-Pb, at similar multiplicity.
- Radii are consistent with initial conditions alone without a hydrodynamic phase. However, they do not rule out hydrodynamic expansion in all 3 systems.

Summary

Use 2

<u>Quantum Coherence at Freeze-out:</u>

- Our results indicate that 15-30% of charged pions may be coherent at freeze-out.
- First seen with the 3-to-2 comparison (r_3).
- Confirmed with the 4-to-2 comparison.
- Ongoing work to check 4-to-3 comparison.
- Ongoing work to extract coherent fractions in pp and p-Pb.

Survival of partial coherence would imply:
→2 disjunct particle-emitting sources!
→Local thermal equilibrium at most.
→Hydrodynamics not applicable to entire system of low p_T pions?

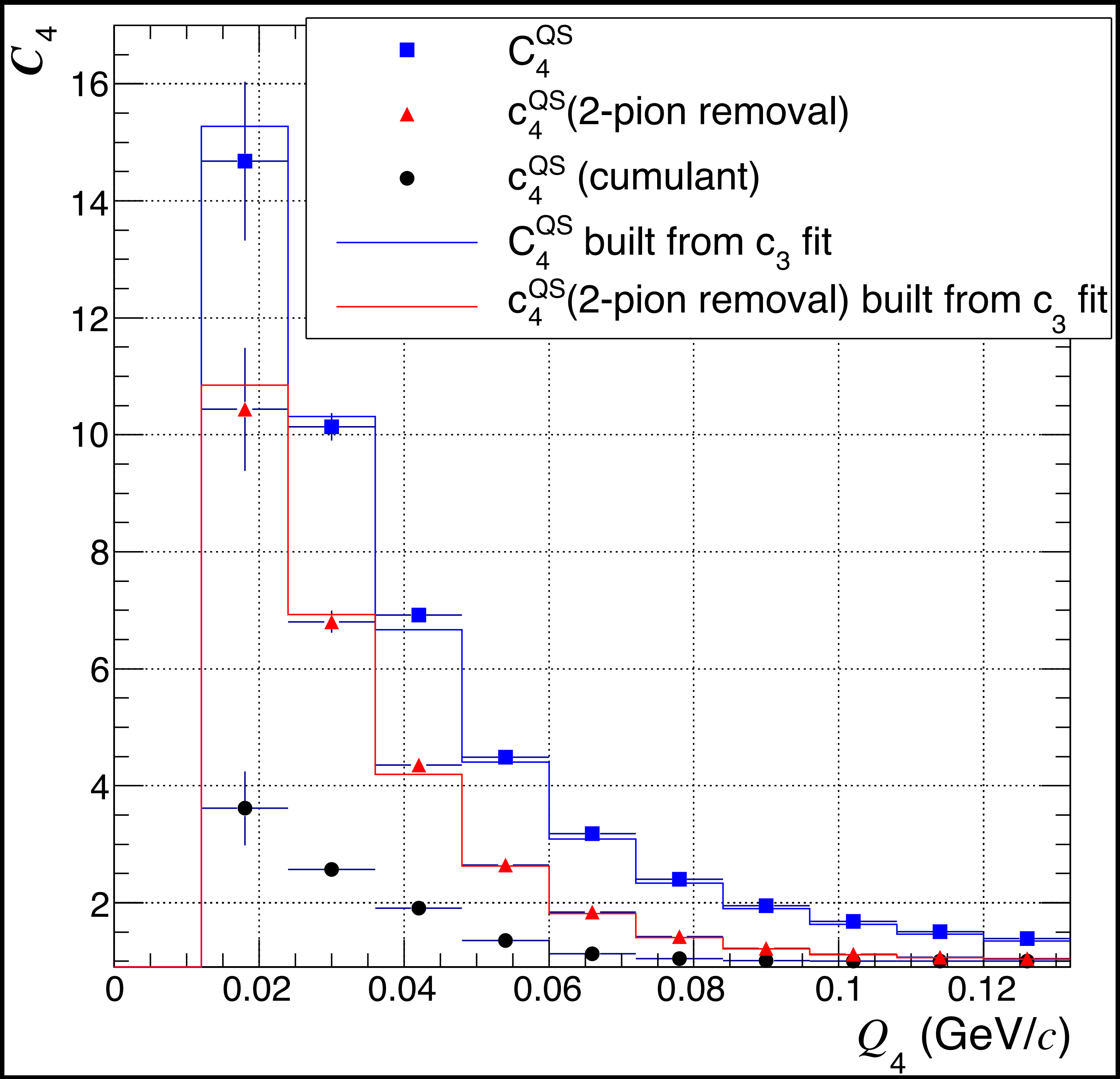
Supporting ALICE Publications

"Two- and three-pion quantum statistics correlations in Pb-Pb collisions at $\sqrt{s_{NN}}$ = 2.76 TeV at the CERN Large Hadron Collider"
Phys. Rev. C **89** 024911 (2014)

"Freeze-out radii extracted from three-pion cumulants in pp, p-Pb and Pb-Pb collisions at the LHC"
Accepted by Phys. Lett. B. arXiv: 1404.1194 (2014)

Supporting Slides

Proof of Principle for building C_4 from c_3 fits

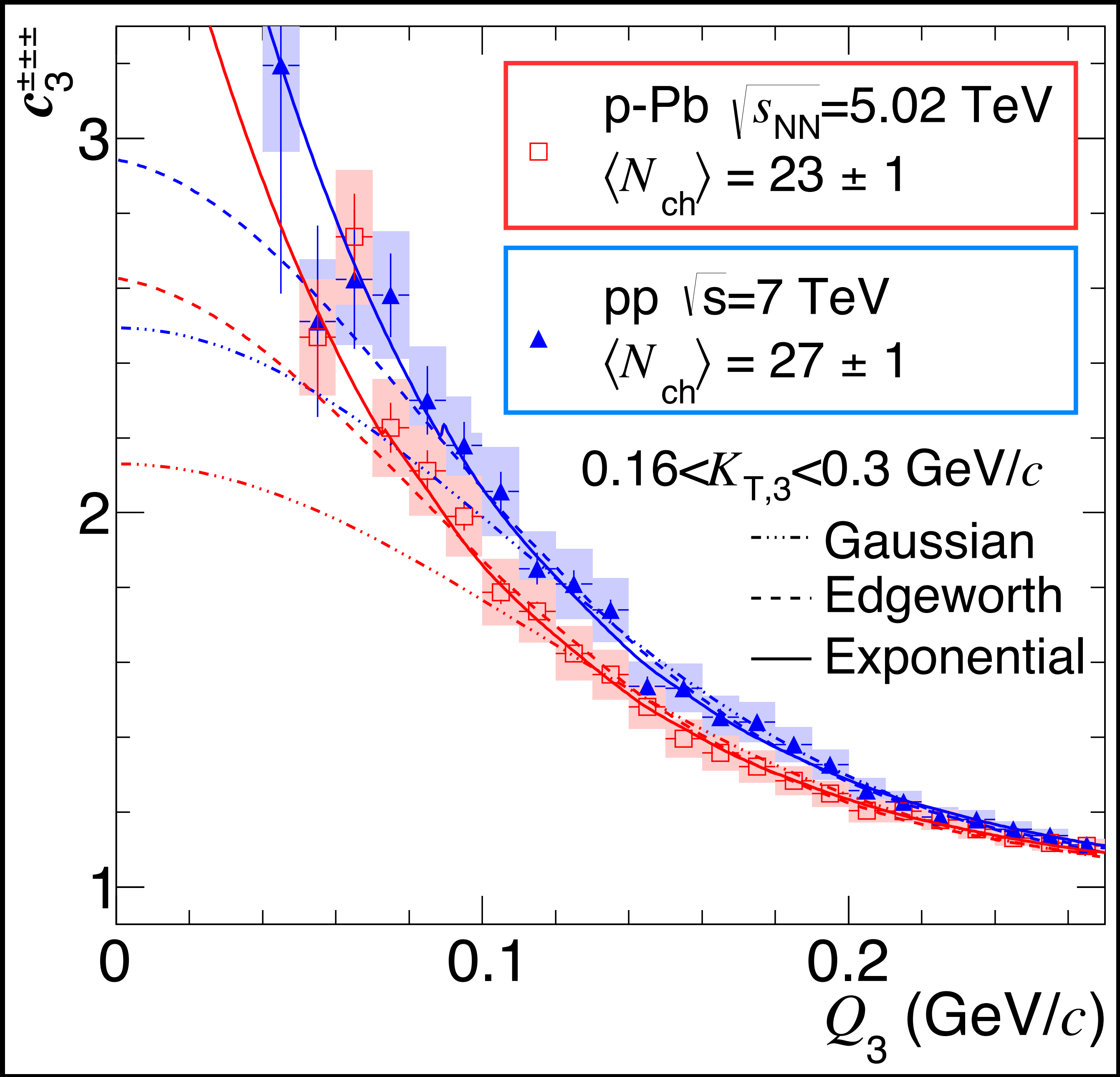


Thermintor model calculation without coherence.

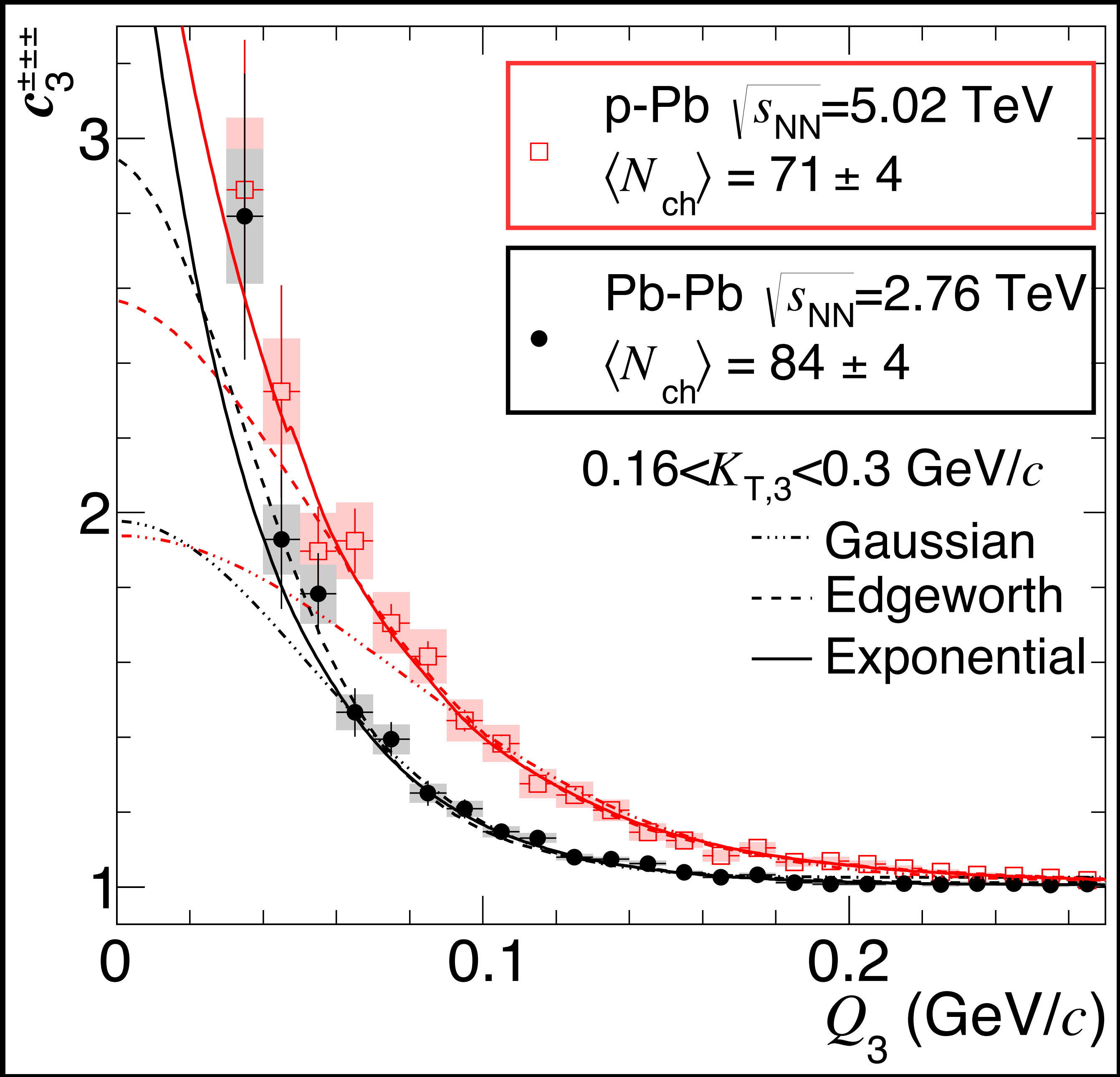
Measured C_4 and the partial cumulant c_4 (2-pion removal) are close to their "built" versions.

Therminator 2 model:
Kisiel et al.,
Comput. Phys. Commun. 174, 669 (2006)

Comparison of c_3 at similar N_{ch}



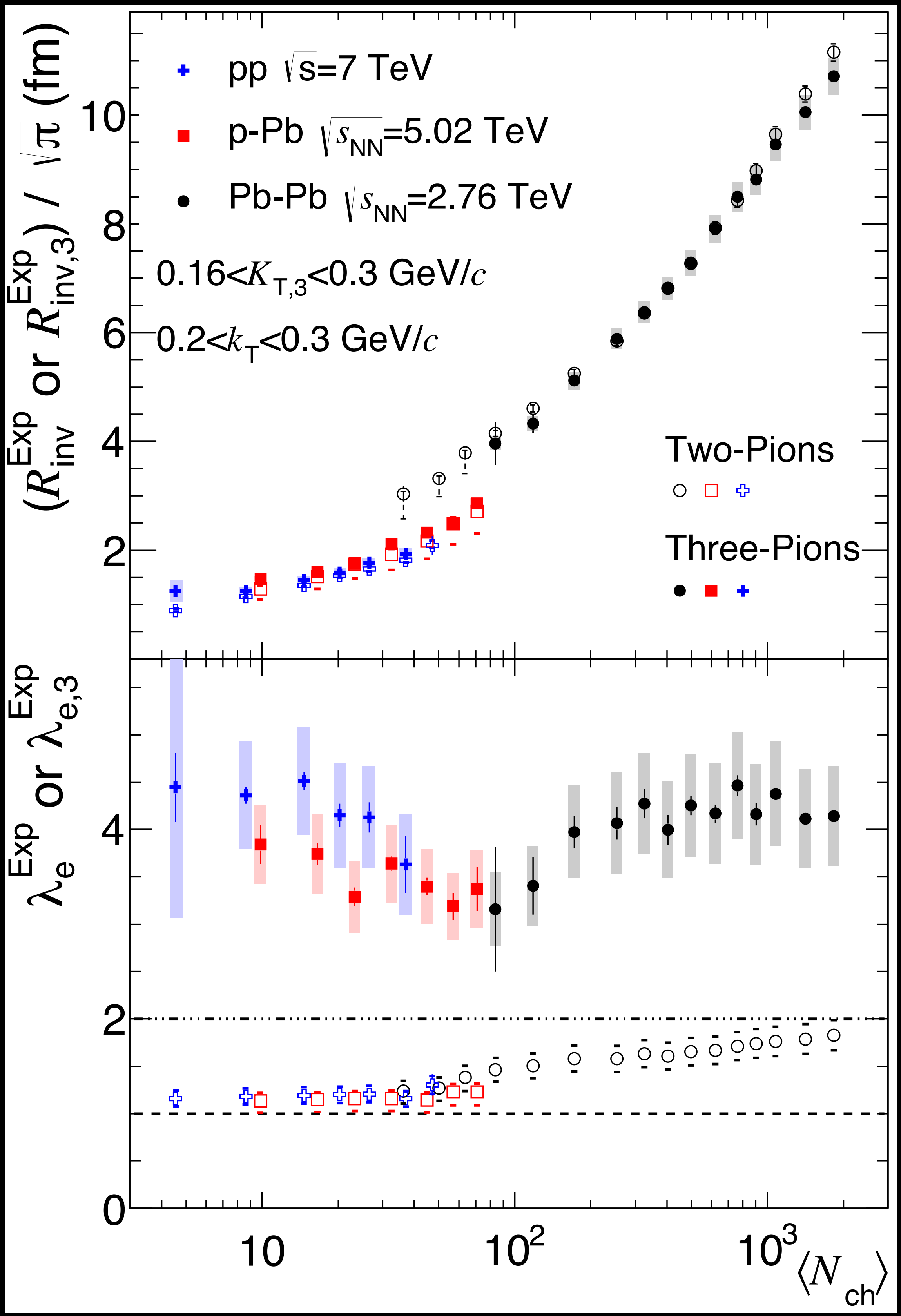
Correlation functions <u>similar</u>



Correlation functions <u>different</u>

ALICE
Phys. Lett. B accepted
arXiv:1404.1194 (2014)

Exponential Radii and Intercepts



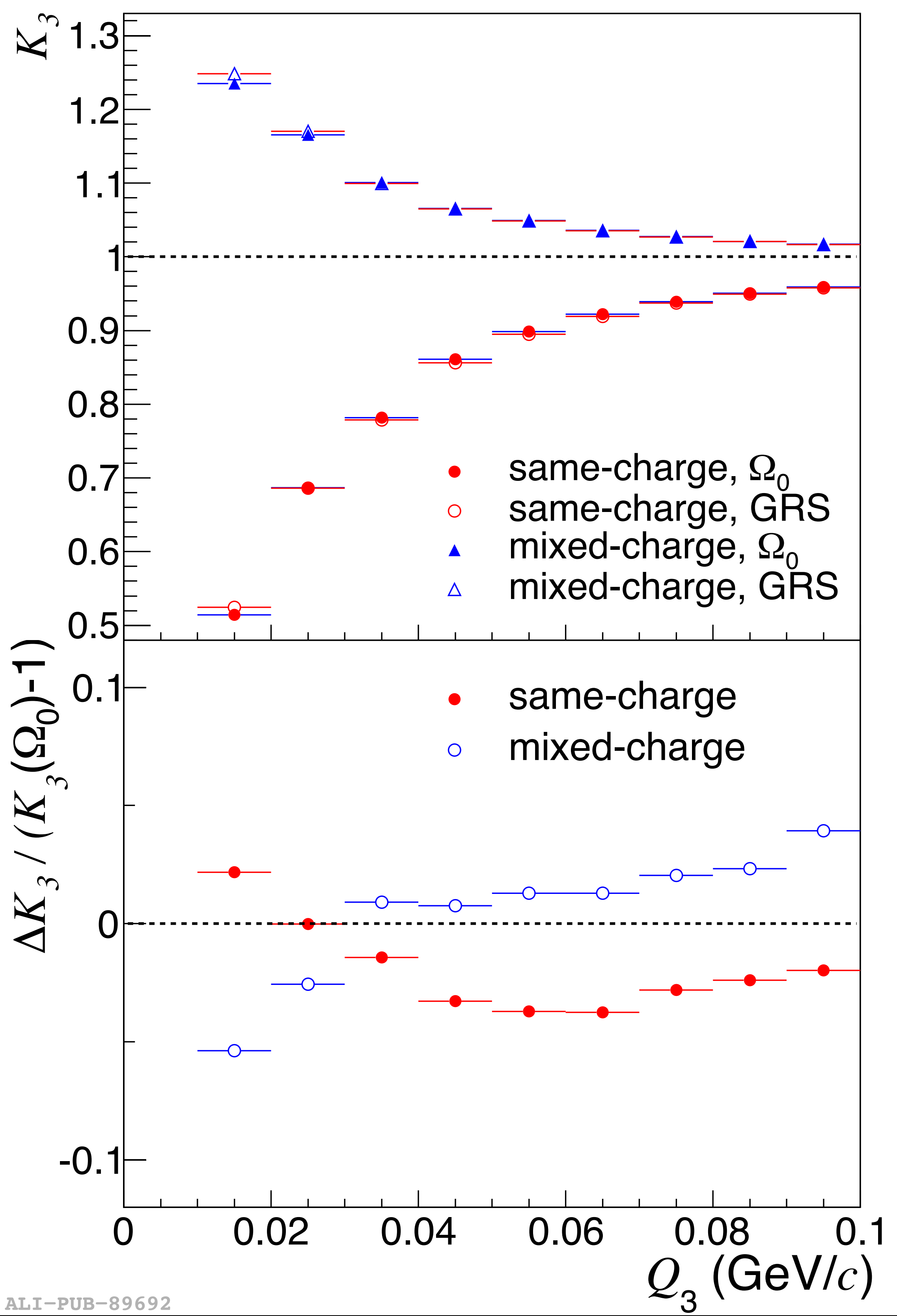
- Exponential fits generally better fit low q part of the correlation function.

- Radii report FWHM of a Cauchy (Lorentzian) source profile.
- p-Pb similar to pp.

- Intercept parameters exceed the chaotic limits. Source profile cannot be entirely Cauchy.

ALICE
Phys. Lett. B accepted
arXiv:1404.1194 (2014)

Near Equivalence between 2 types of Coulomb Calculations

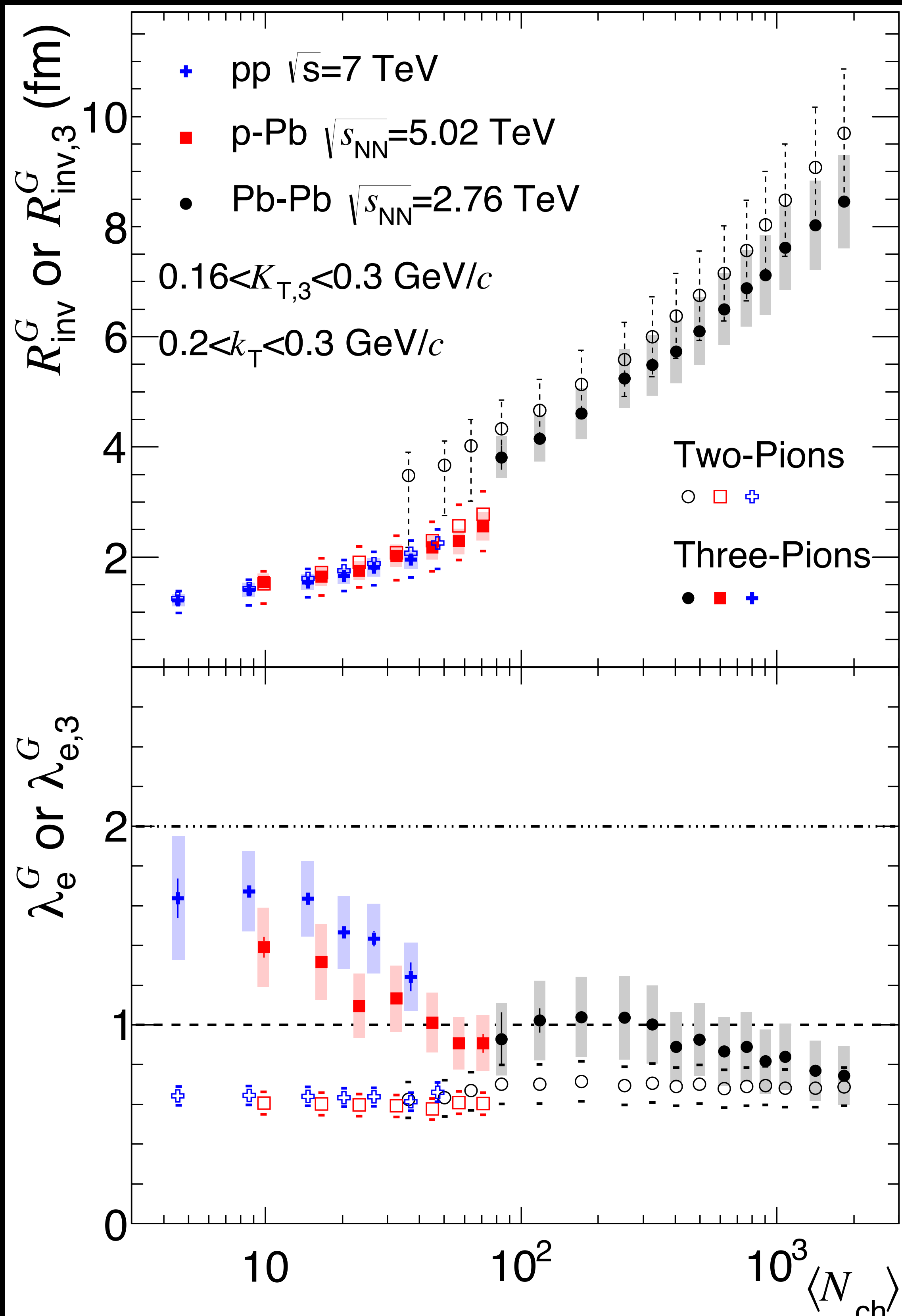


Ω_0 = Full Asymptotic wave-function calculation.

GRS = Generalized Riverside $= K_{12}K_{13}K_{23}$

ALICE
PRC 89 024911 (2014)

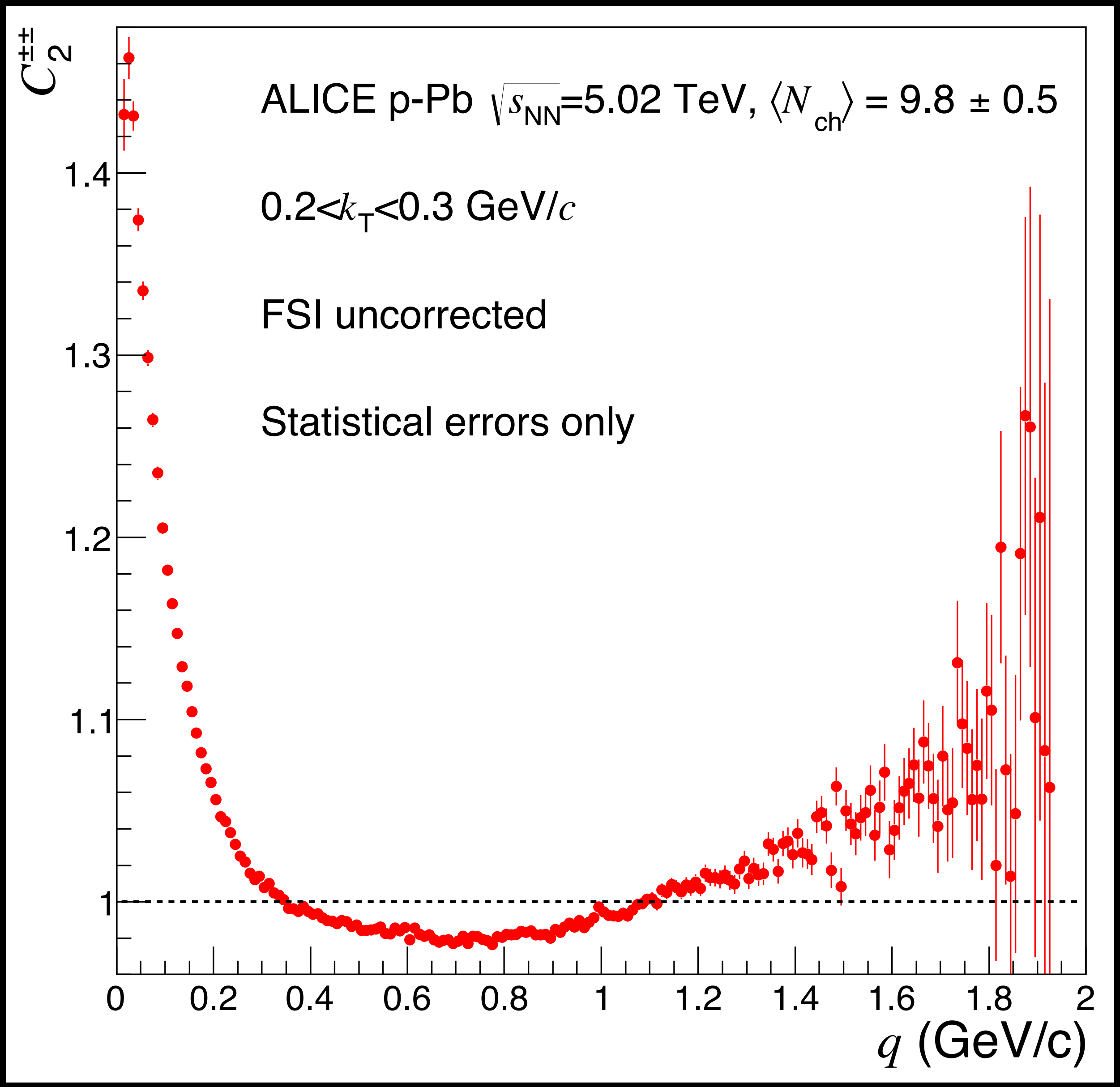
Gaussian Radii and Intercepts

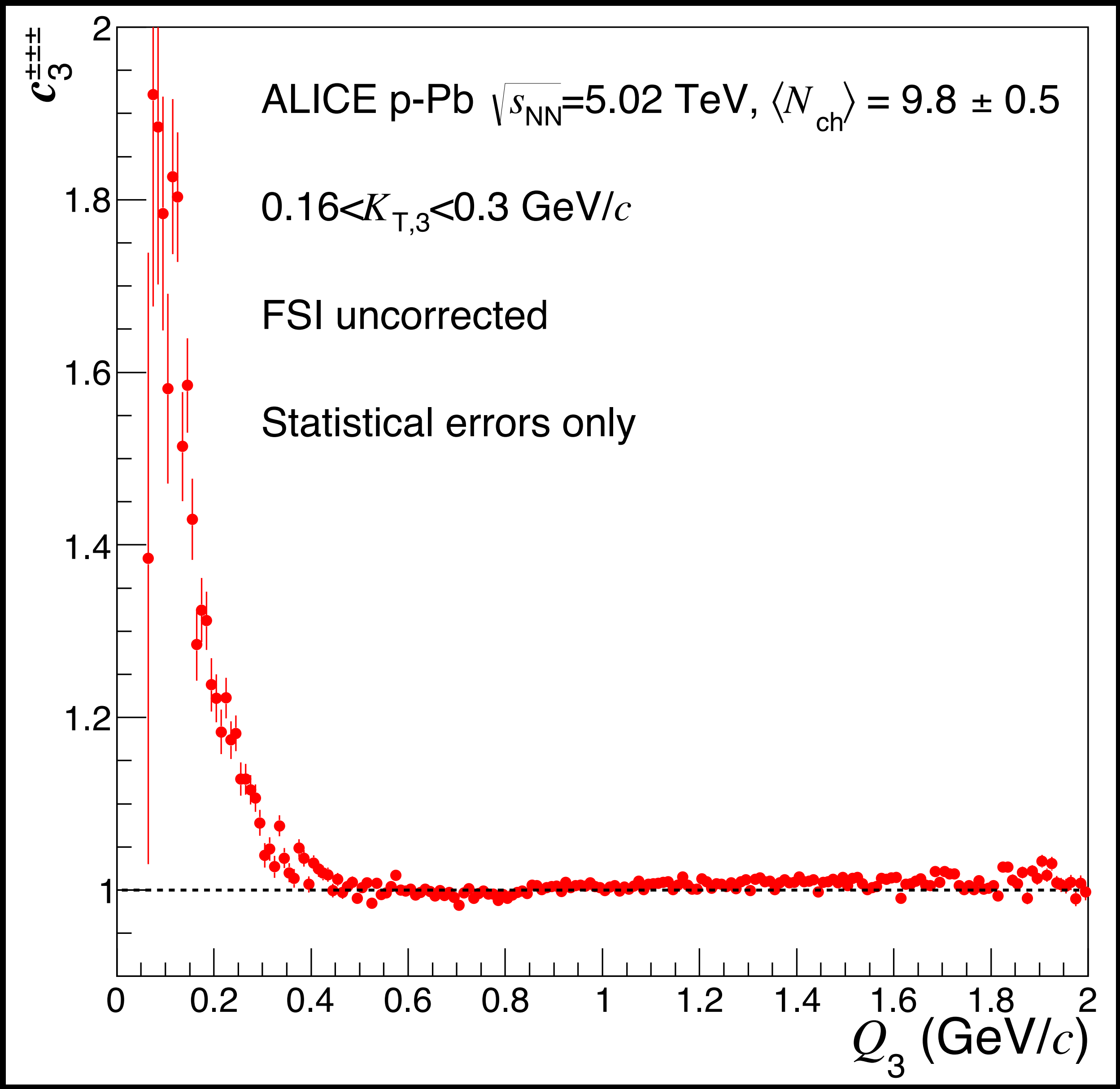


- Gaussian fits were the worst at describing the correlation function.
- p-Pb similar to pp.
- Intercept parameters far below the chaotic limits. Source profile cannot be entirely Gaussian.

ALICE
Phys. Lett. B accepted
arXiv:1404.1194 (2014)

C_2 & c_3 in an extended range





The baseline for 3-pion cumulants is more flat
than for 2-pion correlations.

2-boson Symmetrization

We consider 2 extreme cases for the size of the coherent source radius

Point source

$R_{coh} = 0$

$$C_2(1,2) = 1 + (1-G)^2 (T_{12}^{ch})^2 + 2G(1-G)T_{12}^{ch}$$

Full size source

$R_{coh} = R_{ch}$

$$C_2(1,2) = 1 + (1-G)^2 (T_{12}^{ch})^2 + 2G(1-G)(T_{12}^{ch})^2$$

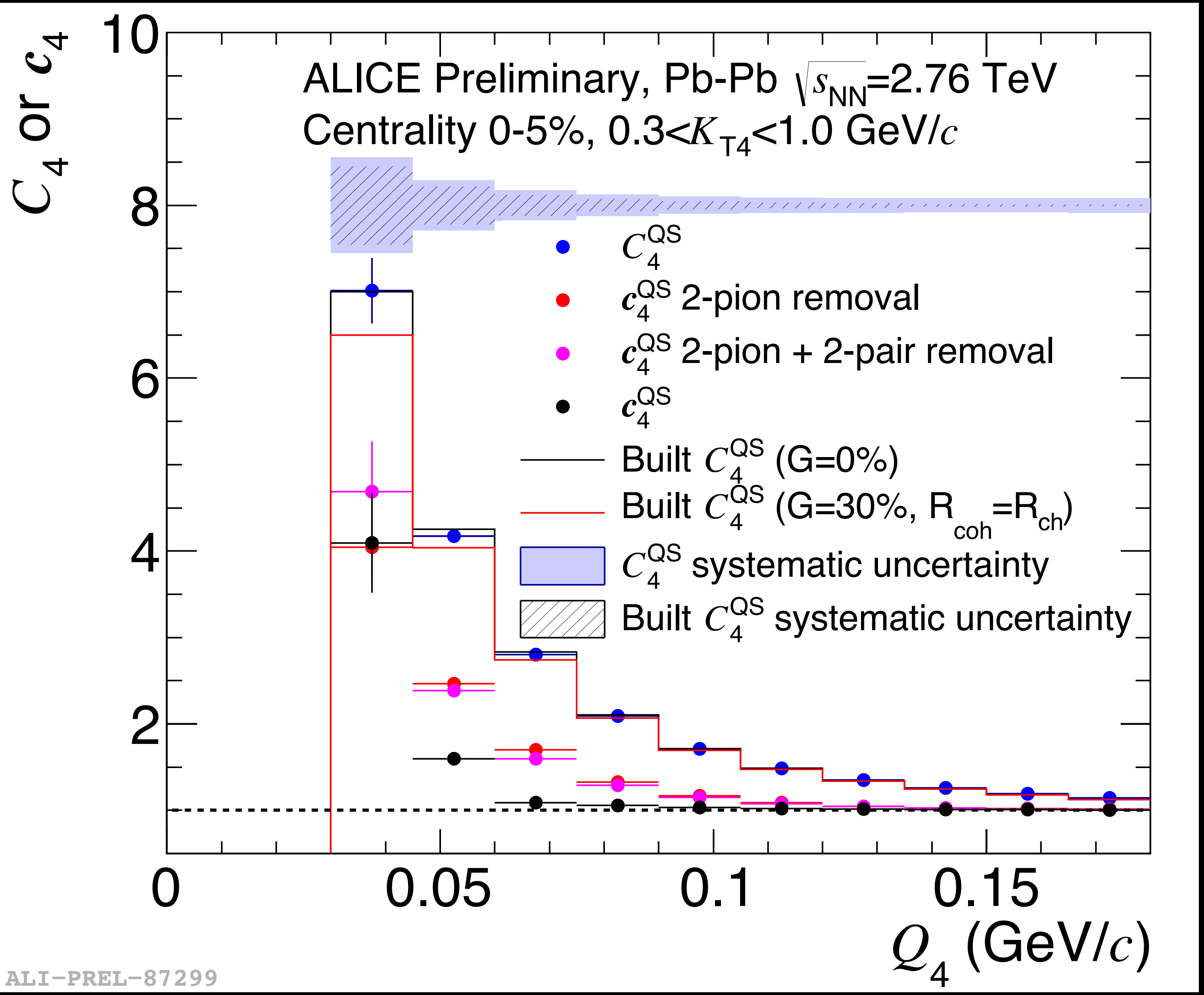
Measured

Assumed value

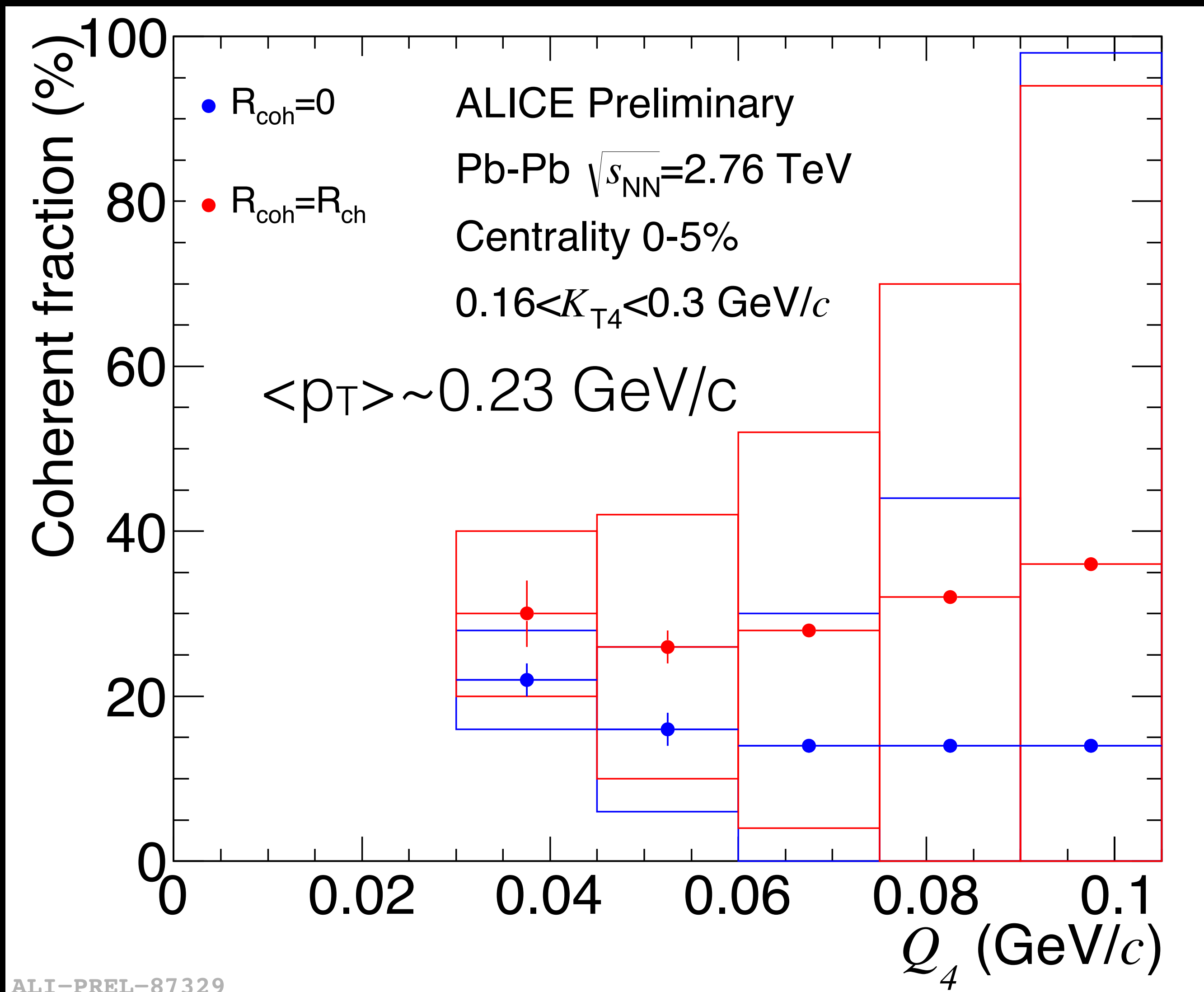
Extracted

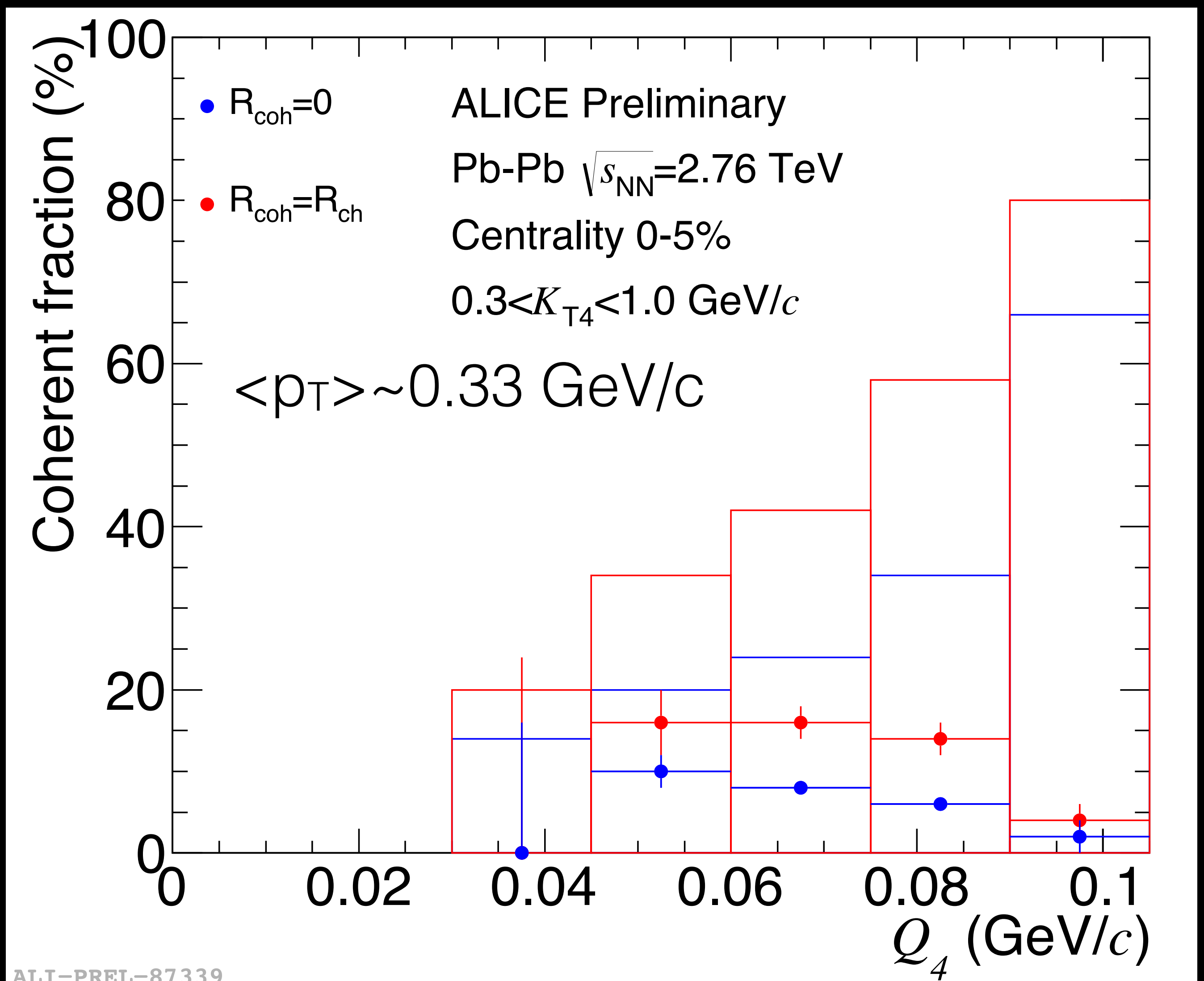
*Equations derived from I. Andreev et al. Int. J. Mod. Phys. A **8** 4577*

4-pion Bose-Einstein at High K_{T4}



Coherent Fractions vs. Q_4

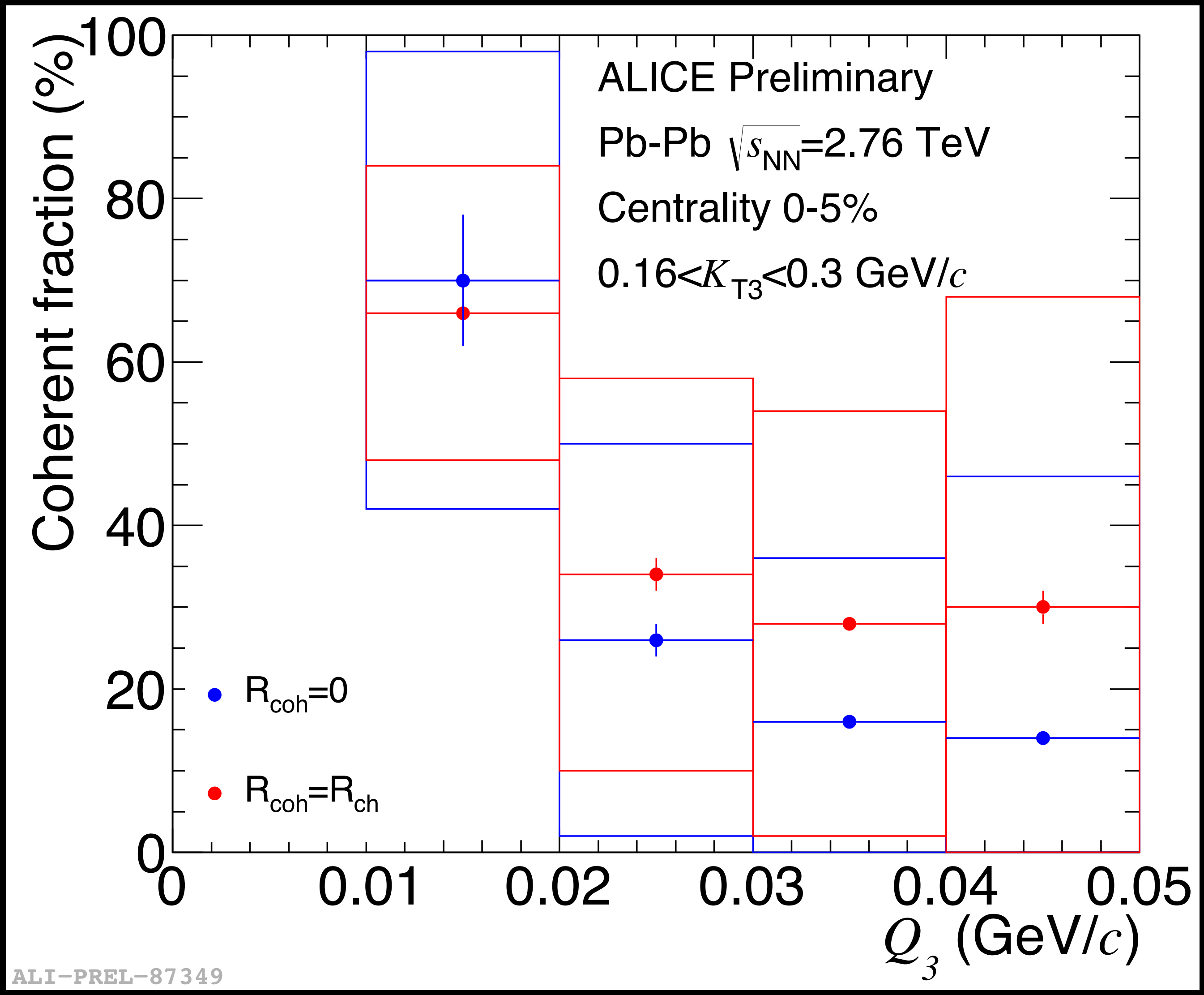


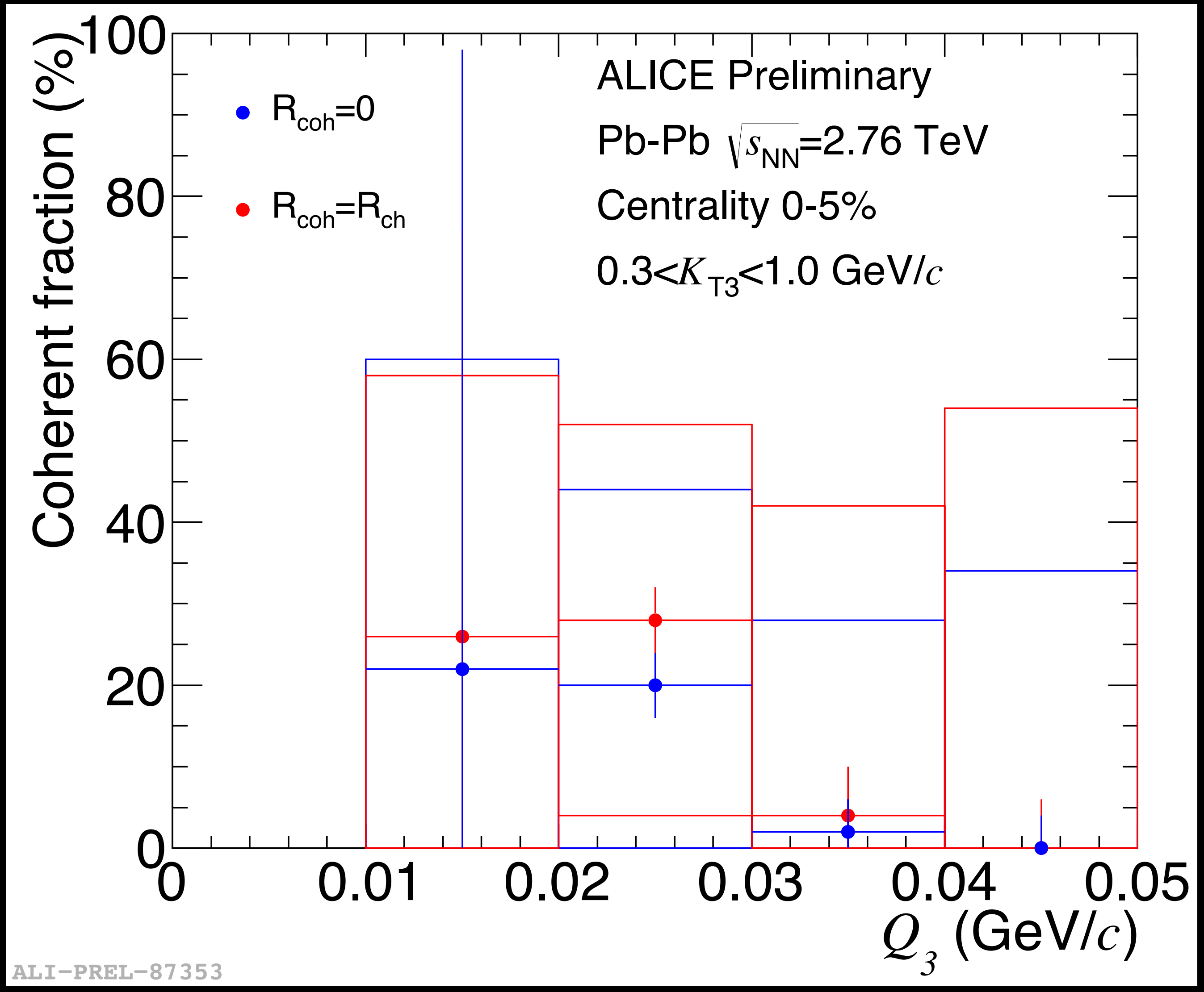


Low K_{T4} **High K_{T4}**

- Coherent fraction is fairly stable with Q_4.
- Systematics dominated by - - - + residual correlation.

3-pions (C_3^{QS}) : Minima vs. Q_3





<p_T> ~0.23 GeV/c

Low K_{T3}

<p_T> ~0.33 GeV/c

High K_{T3}

- Coherent fraction is fairly stable with Q_4.
- Systematics dominated by - - - + residual correlation.

Isolation of 2-pion Quantum Statistics (QS)

Quantity of Interest

$$N_2(p_1,p_2) = f_{21} N_1(p_1) N_1(p_2) + f_{22} K_2(q_{12}) N_2^{QS}(p_1,p_2)$$

Sinyukov et al.,
Phys. Lett. B 432, 249 (1998)

f_{22} estimated to be 0.7 +- 0.05 for this analysis

$f_{21} = 1 - f_{22}$

f_{22} previously estimated in
ALICE 2014
PRC 89 024911 (2014)

Isolation of 3-pion QS

ALICE 2014
PRC 89 024911 (2014)

Quantity of Interest

$$
\begin{aligned}
N_3(p_1,p_2,p_3) &= f_{31}N_1(p_1)N_1(p_2)N_1(p_3)\\
&+ f_{32}N_2(p_1,p_2)N_1(p_3)\\
&+ f_{33}K_3(q_{12},q_{13},q_{23})N_3^{QS}(p_1,p_2,p_3)
\end{aligned}
$$

f coefficients derived in the core/halo picture as:

$$
\begin{aligned}
f_{31} &= (1-f_c)^3+3f_c(1-f_c)^2-3(1-f_c)(1-f_c^2)\\
f_{32} &= 3(1-f_c)\\
f_{33} &= f_c^3
\end{aligned}
$$

f_c^2 = "lambda" = 0.7 +- 0.05 (fraction of correlated pairs)

Systematics Checked

Those which pertain to both measured and built C_4^{QS}

Systematics are Q_4 dependent

- - vs. + pions — 0.1%.
- TPC B field orientation — negligible.
- Tracking efficiency — 0.4% at low Q_4.
- variation of f_c^2 (pair dilution). Default = 0.7, tried 0.65 and 0.75 — 6% at low Q_4
- Momentum resolution corrections — 1% at low Q_4
- Muon correction uncertainties — 2% at low Q_4.

High degree of correlation between measured and built C_4^{QS} for each of these variations.

Systematics Checked

Measured C_4^{QS} only

Systematics are Q_4 dependent

- variation of f_{41}, f_{42}, f_{43}, f_{44} from Therminator as compared to Core/Halo prescription — 0.4% at high Q_4
- Residue of mixed-charge (- - - +) cumulant — 5%
- K_4 FSI factor — 1% uncertainty at low Q_4.
  test of factorization: $K_4 = K_2^{12} K_2^{13} K_2^{14} K_2^{23} K_2^{24} K_2^{34}$

These systematics are the least understood sources of uncertainties. Future studies may reveal smaller values.

Systematics Checked

<u>Built</u> C_4^{QS} only

<u>Systematics are Q_4 dependent</u>

- Interpolator of 2-particle weights (C_2-1 = T_{ij}) — 0.7% at low Q_4.

Cubic interpolation used in between bins of q_{out}, q_{side}, q_{long} by default. Linear interpolation used as a variation.

- 2-particle weight problem at high q_{inv}

Statistical fluctuations at high q_{inv} can give a negative T_{ij} which is not allowed in theory (Bose-Einstein correlations are positive). In these cases Tij is set to zero.

— 0.3% at high Q_4, Low K_{T4}

— 4% at high Q_4, High K_{T4}

Equations to Build QS correlations with coherence, $R_{coh}=0$

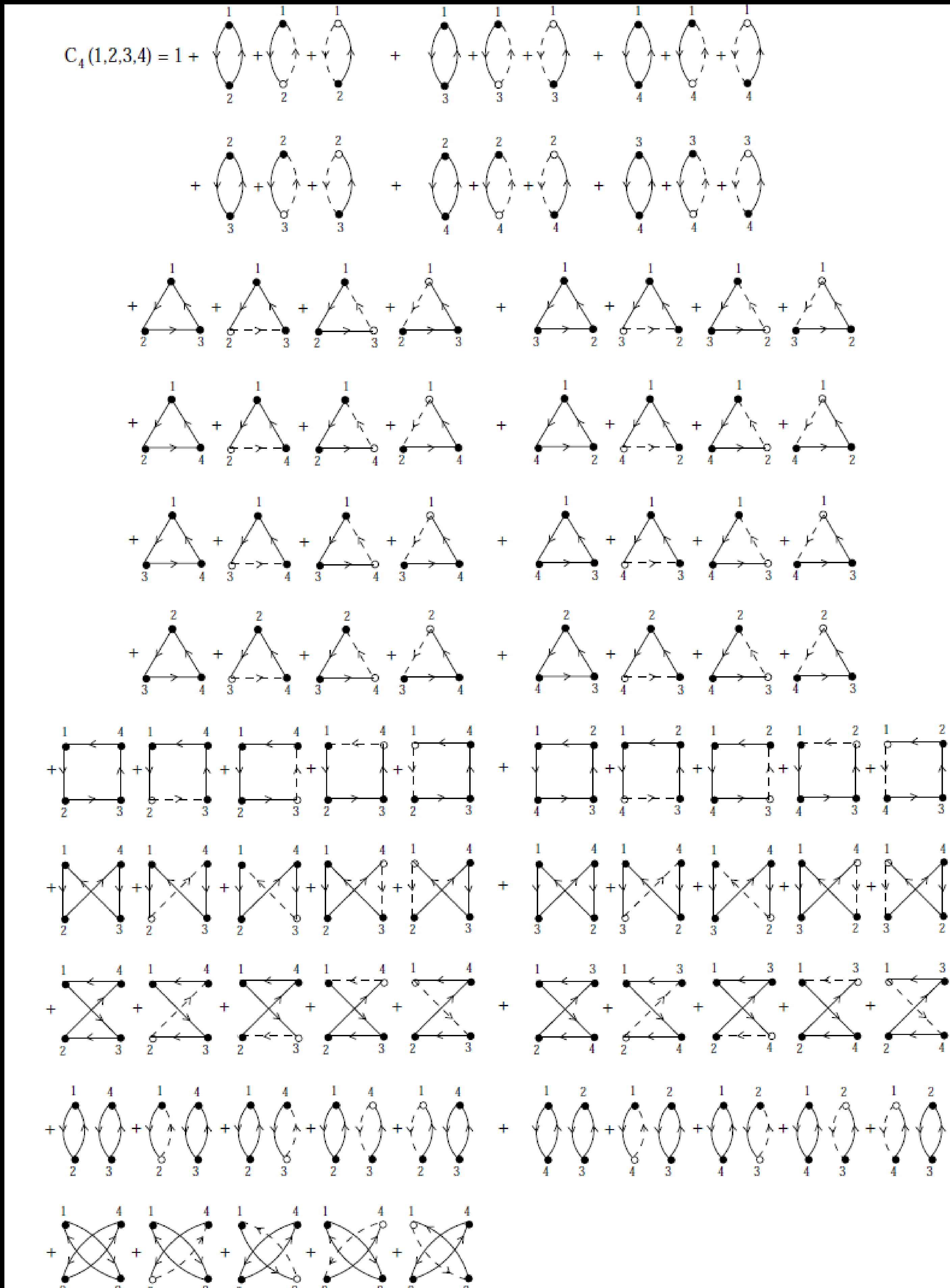
$$C_2^{QS}-1 = 2G(1-G)T_{12}+(1-G)^2T_{12}^2$$

$$\begin{aligned}
C_3^{QS}-1 &= 2G(1-G)(T_{12}+T_{13}+T_{23})+(1-G)^2(T_{12}^2+T_{13}^2+T_{23}^2)\\
&+ 2G(1-G)^2(T_{12}T_{13}+T_{12}T_{23}+T_{13}T_{23})+2(1-G)^3(T_{12}T_{13}T_{23})\\
C_4^{QS}-1 &= 2G(1-G)(T_{12}+T_{13}+T_{14}+T_{23}+T_{24}+T_{34})\\
&+ (1-G)^2(T_{12}^2+T_{13}^2+T_{14}^2+T_{23}^2+T_{24}^2+T_{34}^2)\\
&+ 2G(1-G)^3(T_{12}T_{34}^2+T_{12}^2T_{34}+T_{13}T_{24}^2+T_{13}^2T_{24}+T_{14}T_{23}^2+T_{14}^2T_{23})\\
&+ (1-G)^4(T_{12}^2T_{34}^2+T_{13}^2T_{24}^2+T_{14}^2T_{23}^2)\\
&+ 2G(1-G)^2(T_{12}T_{13}+T_{12}T_{23}+T_{13}T_{23}+T_{12}T_{14}+T_{12}T_{24}+T_{14}T_{24})\\
&+ 2G(1-G)^2(T_{13}T_{14}+T_{13}T_{34}+T_{14}T_{34}+T_{23}T_{24}+T_{23}T_{34}+T_{24}T_{34})\\
&+ 2(1-G)^3(T_{12}T_{13}T_{23}+T_{12}T_{14}T_{24}+T_{13}T_{14}T_{34}+T_{23}T_{24}T_{34})\\
&+ 2G(1-G)^3(T_{12}T_{14}T_{34}+T_{12}T_{14}T_{23}+T_{12}T_{23}T_{34}+T_{14}T_{23}T_{34})\\
&+ 2G(1-G)^3(T_{12}T_{13}T_{34}+T_{12}T_{34}T_{24}+T_{12}T_{24}T_{13}+T_{13}T_{24}T_{34})\\
&+ 2G(1-G)^3(T_{14}T_{13}T_{23}+T_{14}T_{13}T_{24}+T_{13}T_{23}T_{24}+T_{14}T_{24}T_{23})\\
&+ 2(1-G)^4(T_{12}T_{13}T_{24}T_{34}+T_{12}T_{14}T_{23}T_{34}+T_{13}T_{14}T_{23}T_{24})
\end{aligned}$$

G = coherent fraction of pions

Weiner et al.
Int.J.Mod.Phys.A.
26 4577 (1993)

T. Csorgo.
Heavy Ion Phys.
15 1 (2002)

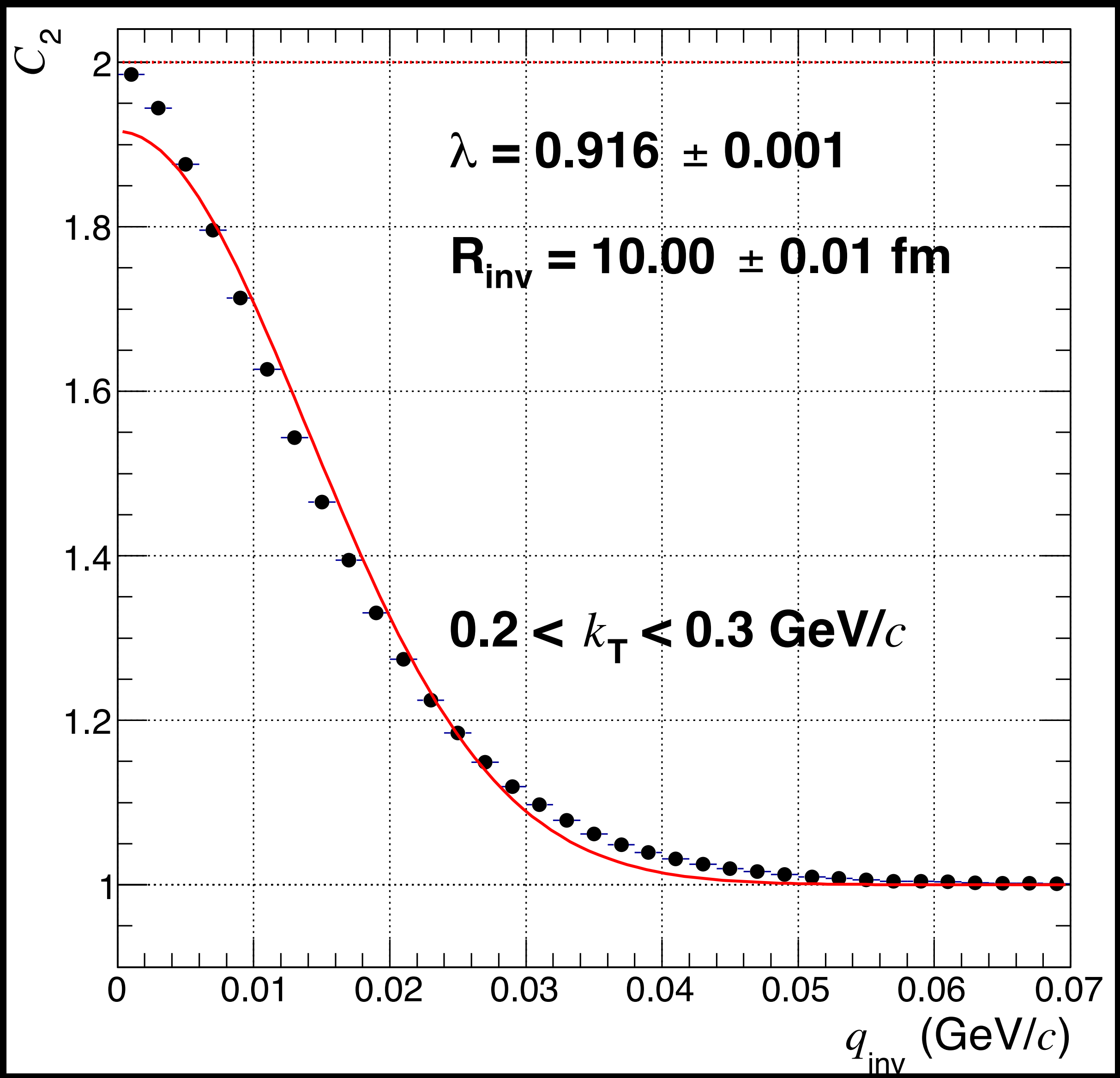


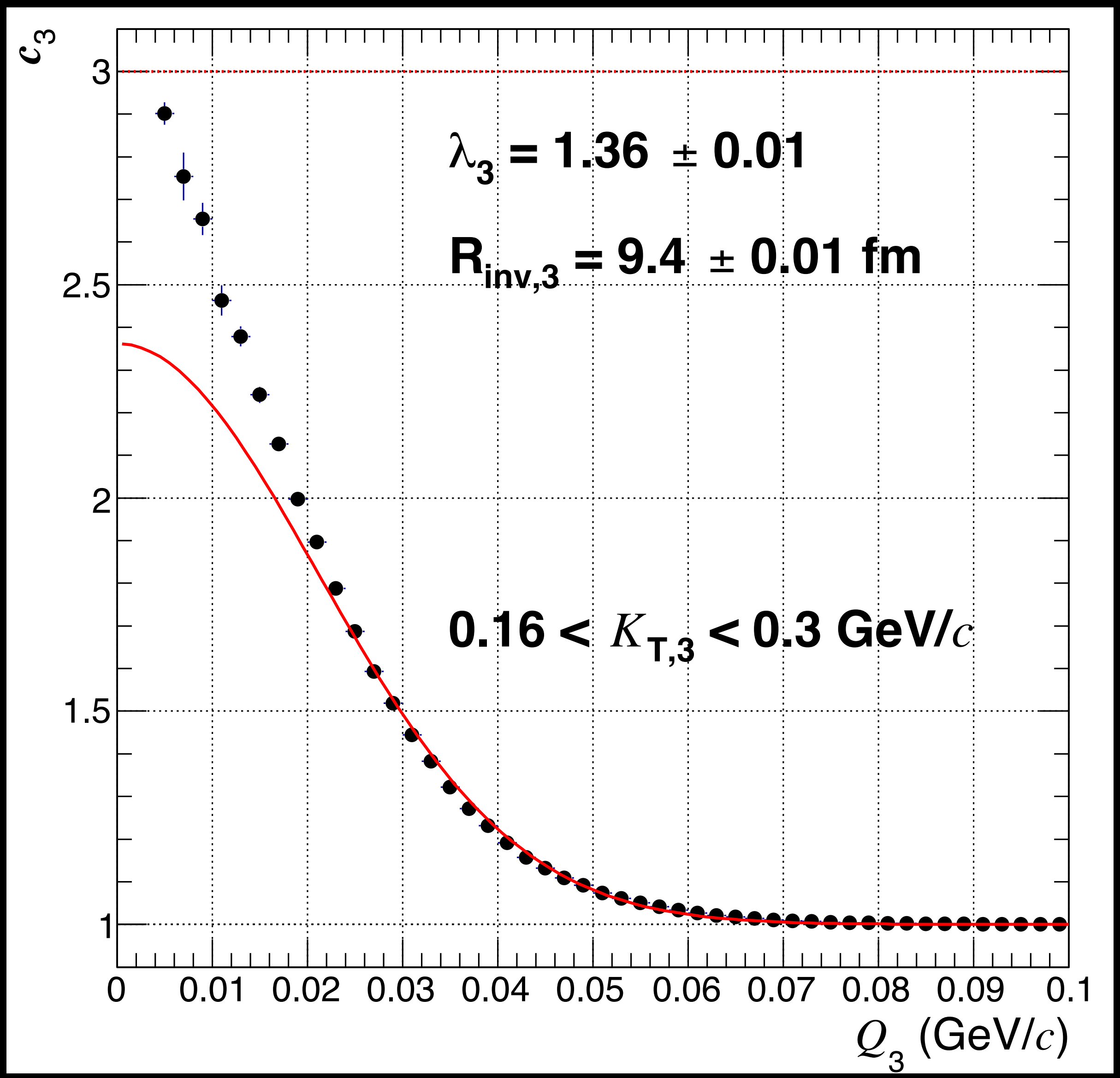
Full 4-pion Quantum Interference Diagrams

T. Csorgo
Heavy Ion Physics ***15*** *1-80*

Therminator2 calculations of 2- and 3-pion Bose-Einstein correlations

Therminator2
0-5% Pb-Pb



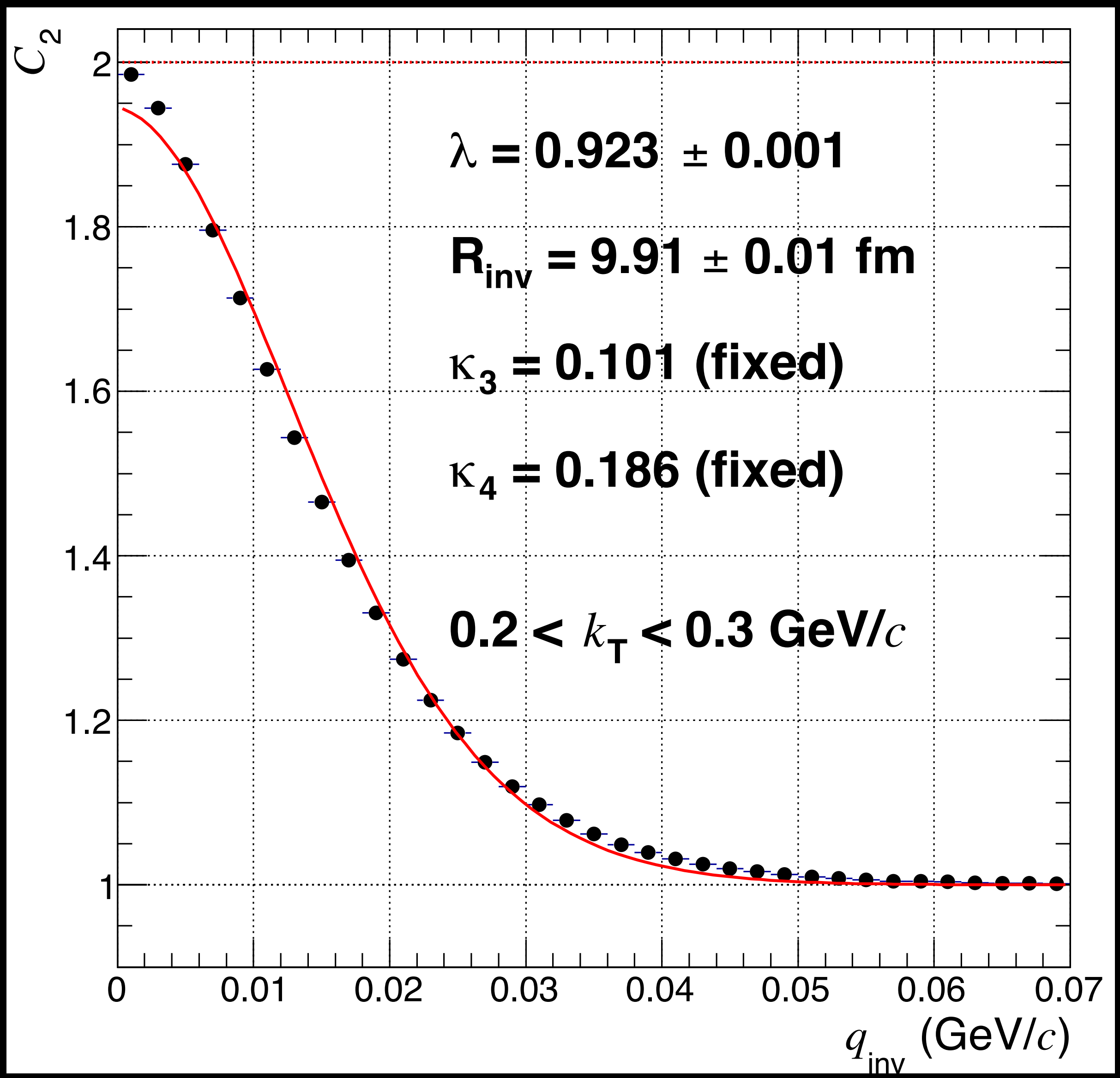


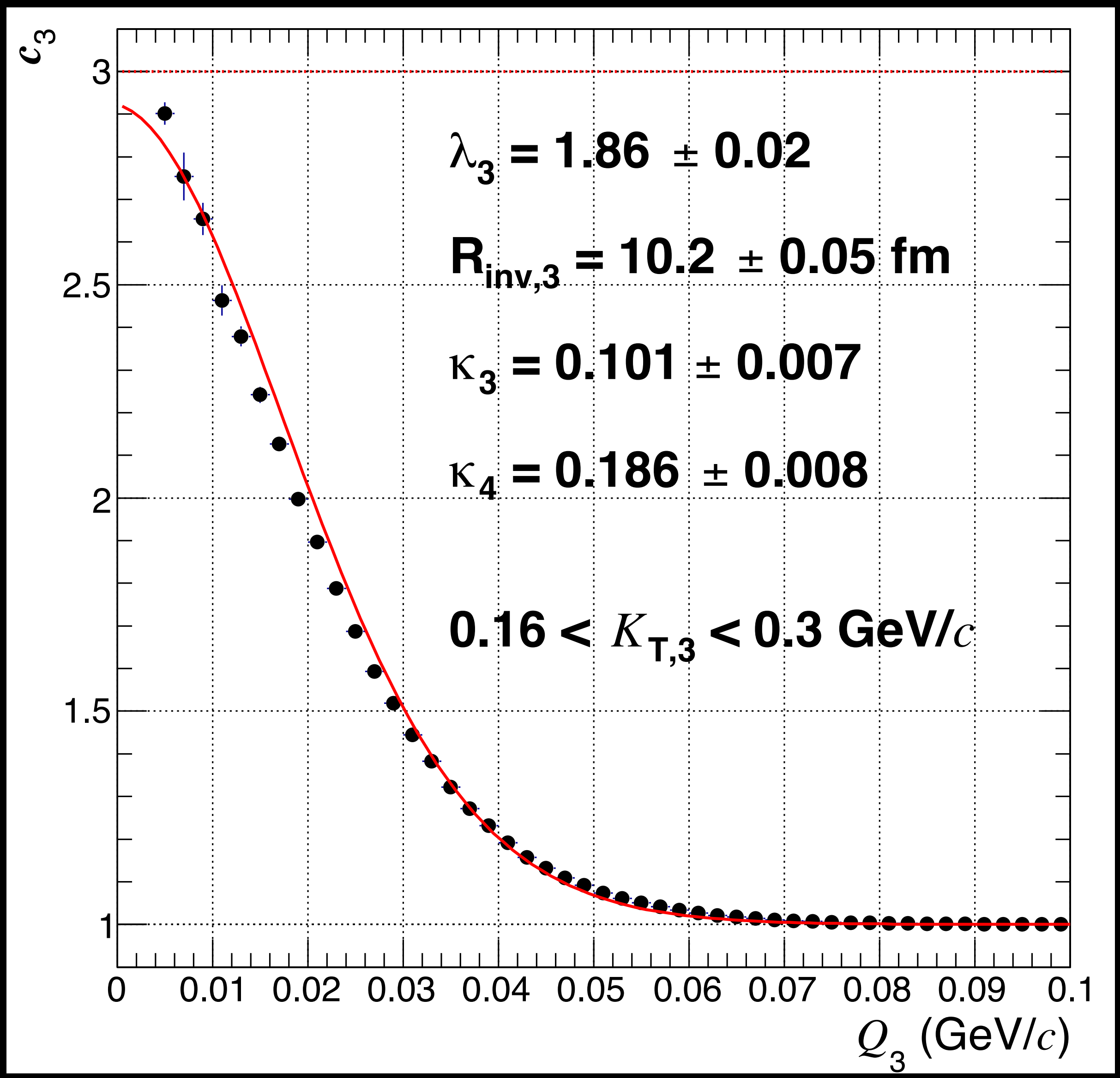
Gaussian fits in red.
$R_{inv,3}$ smaller than R_{inv} by ~0.6 fm (6%).

Therminator 2 model:
Kisiel et al.,
Comput. Phys. Commun. 174, 669
(2006)

Therminator2
0-5% Pb-Pb

Therminator2 calculations of 2- and 3-pion Bose-Einstein correlations

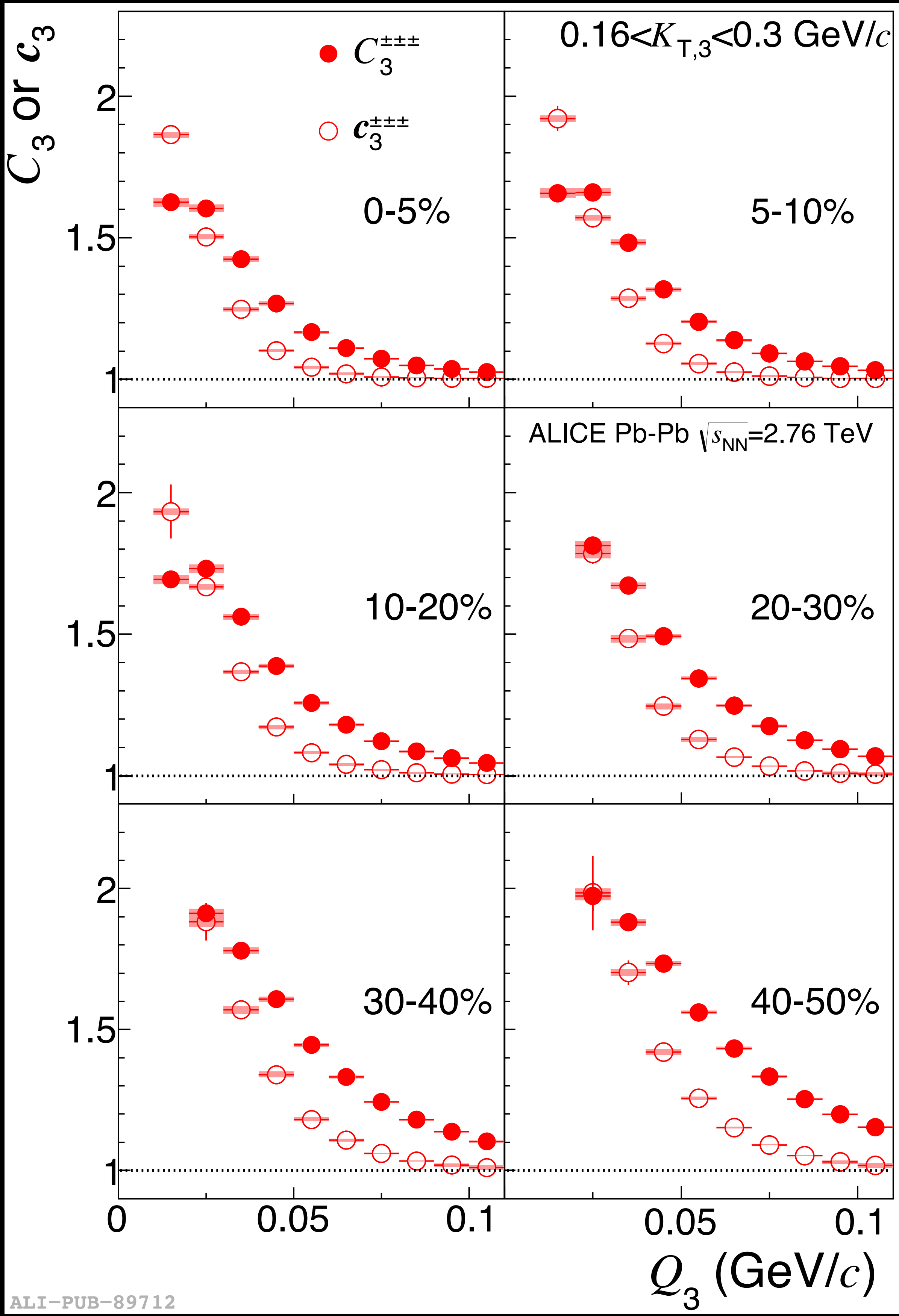




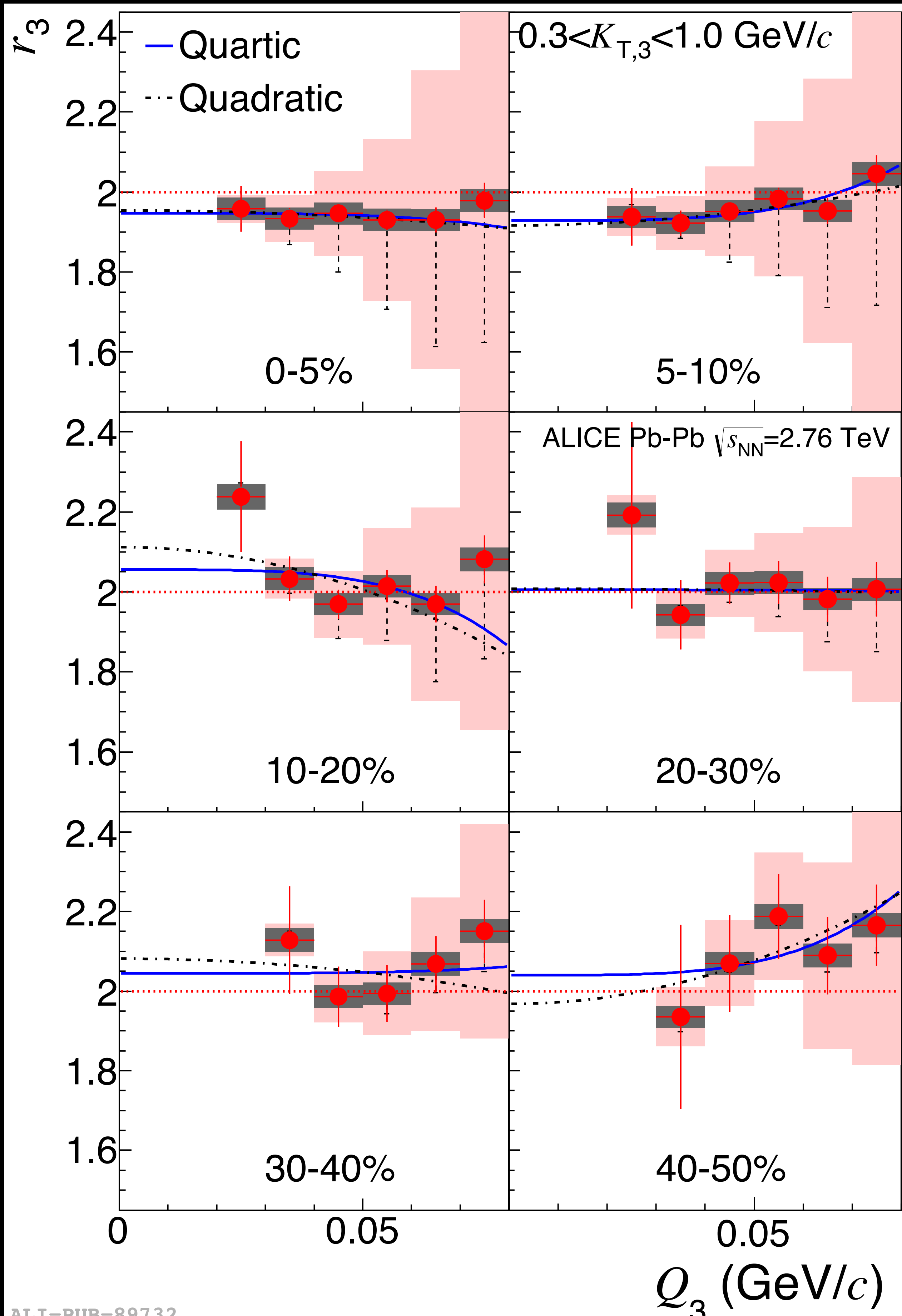
Edgeworth fits in red.
$R_{inv,3}$ similar to R_{inv} within ~0.3 fm (3%).

Therminator 2 model:
Kisiel et al.,
Comput. Phys. Commun. 174, 669 (2006)

3-pion Correlation Functions in Pb-Pb



ALICE
PRC 89 024911 (2014)



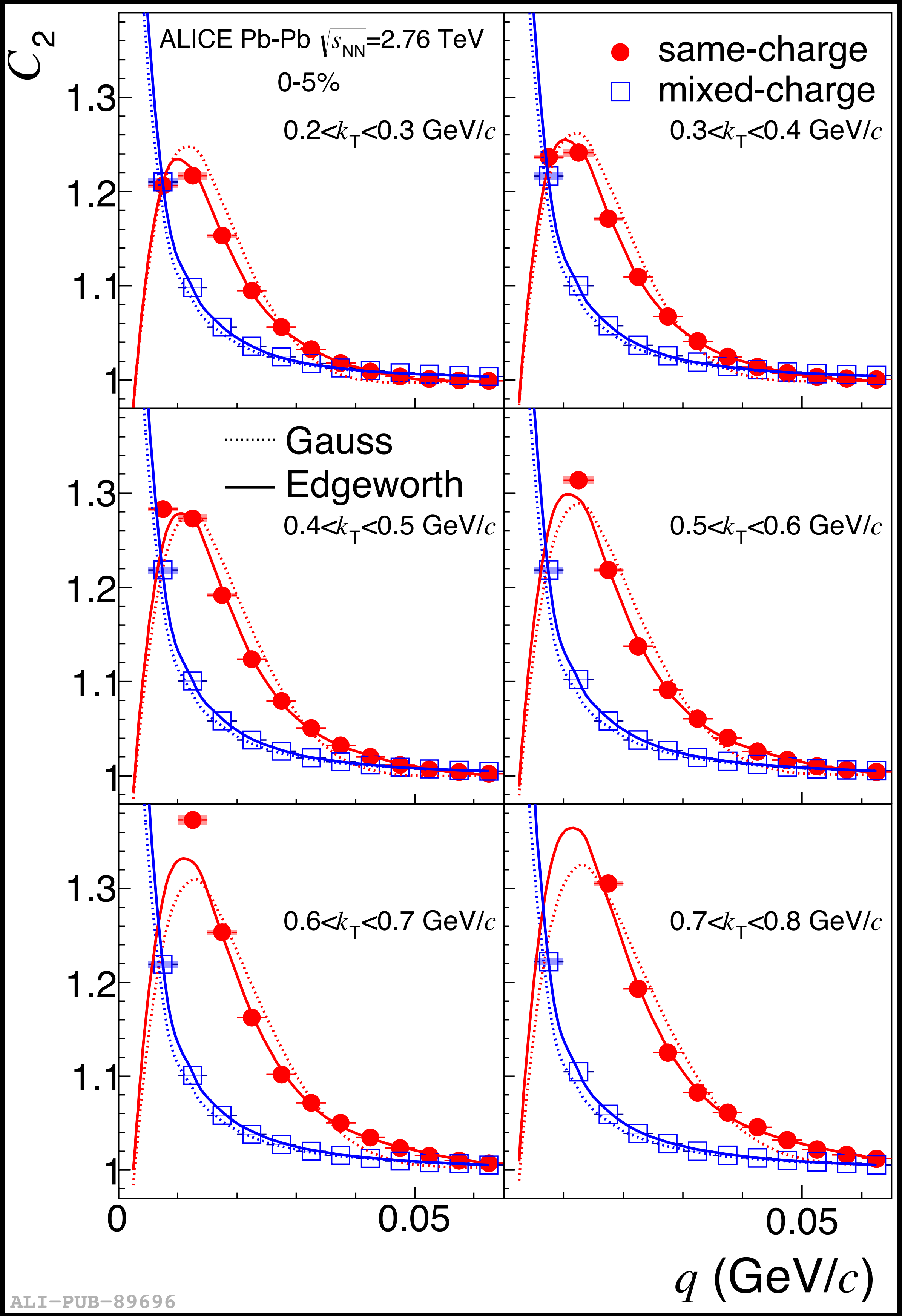
Measure of coherent fraction by comparing 3-pion to 2-pion correlation strength

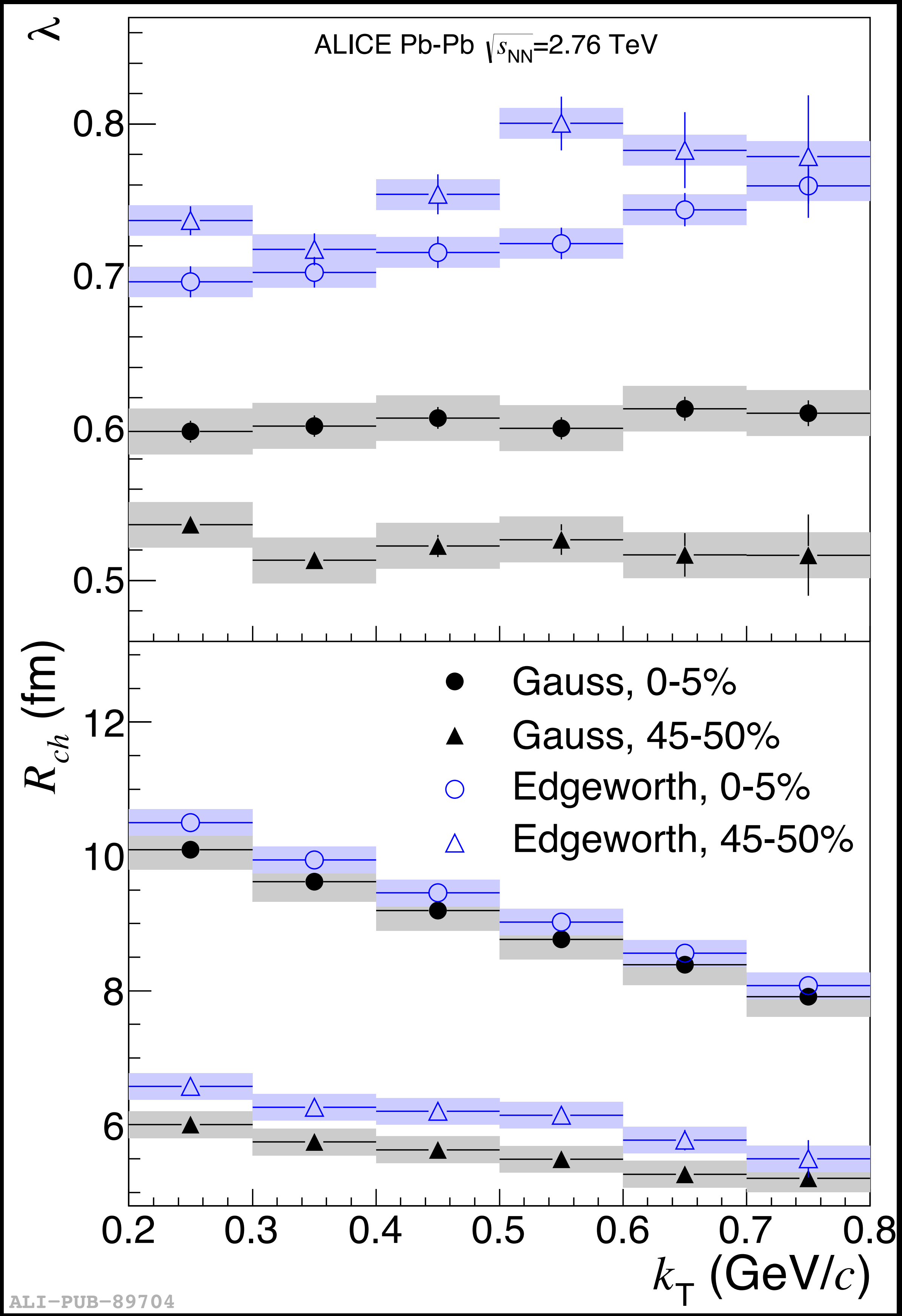
$$r_3 = \frac{c_3(q_{12}, q_{23}, q_{31}) - 1}{\sqrt{(C_2(q_{12}) - 1)(C_2(q_{13}) - 1)(C_2(q_{23}) - 1)}}$$

All Correlations are first Coulomb corrected.

r_3 is consistent with 2.0.
Intercept is consistent with
0% coherence at high p_T

ALICE
PRC 89 024911 (2014)





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PRC 89 024911 (2014)

Quadruplet Fractions in Therminator