Experimental Tests of Vacuum Energy

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The Evolution of vacuum energy

- The cosmological constant is very small today

$$\Lambda \sim (10^{-3}\ \mathrm{eV})^4$$

- Expectation is that microscopic origin of cc is vacuum energy of quantum field theory

- Why is it so small vs. $(TeV)^4, M_{Pl}^4$

- If it is so small why is it not zero?

- Is it always very small (ie. is there an adjustment mechanism)?

The Evolution of vacuum energy

- If CC result of microphysics, in traditional picture cc should undergo a series of jumps at every phase transition

- Expectation $\Delta\Lambda_i \propto T_{c,i}^4$

- Want CC to NOT dominate AFTER phase transition (otherwise Universe accelerates too early)

- CC AFTER PT should be of order of T_c of NEXT phase transition

- eg. before EWPT $\Lambda \sim M_W^4$

The Evolution of vacuum energy

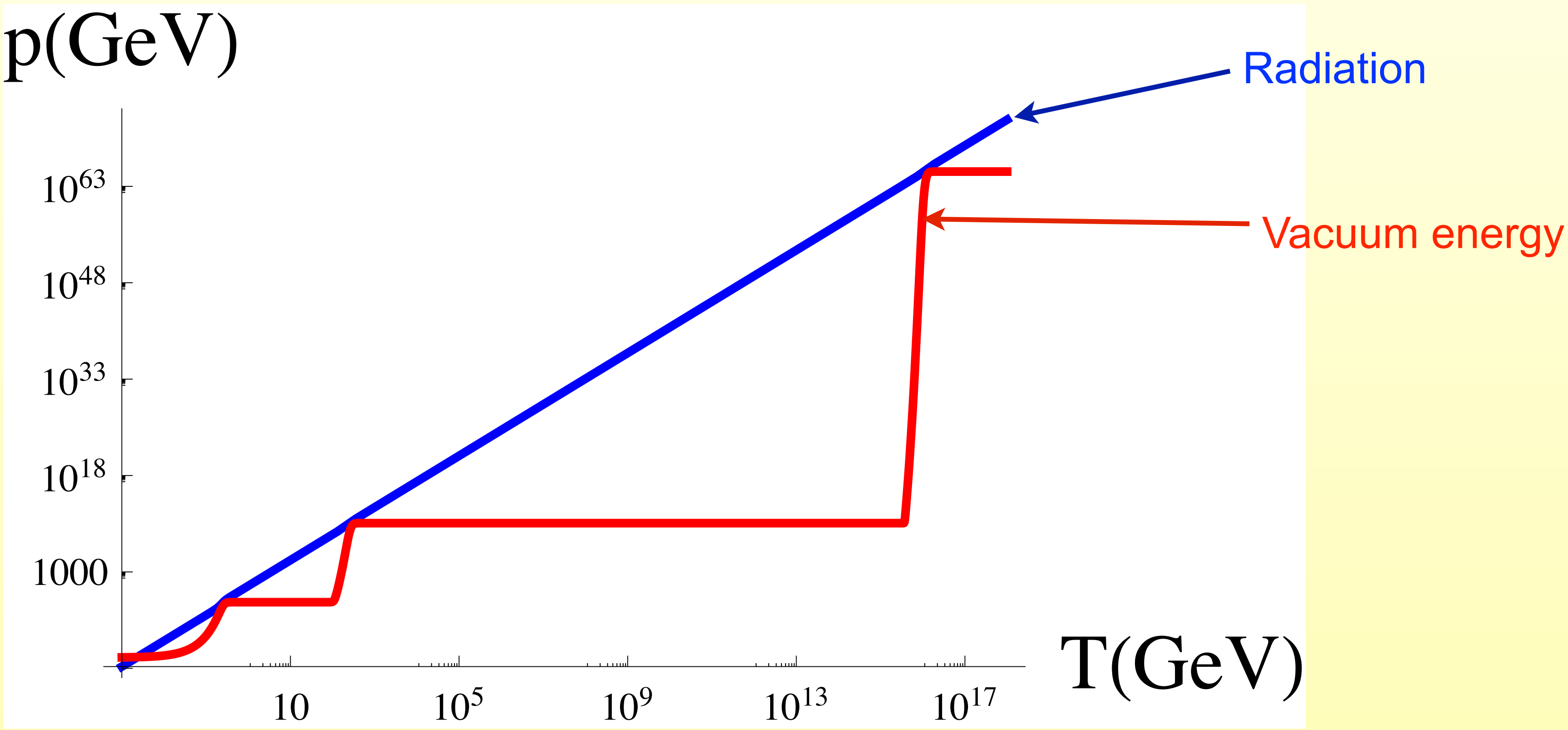
- $\Delta\Lambda \sim M_W^4$ so tuning $\Lambda + \Delta\Lambda \sim \mathcal{O}(\Lambda_{QCD}^4)$

•At one phase transition Universe already ``knows" where the next phase transition will be

•At least QCD, EW PT, potentially also SUSY and/or GUT phase transition (if SUSY changes GUT expectations)

•In previous history Λ was much larger than now, but never dominated previously!

A simple sketch of the evolution of Λ



The Evolution of vacuum energy

- Λ goes through steps during phase transitions

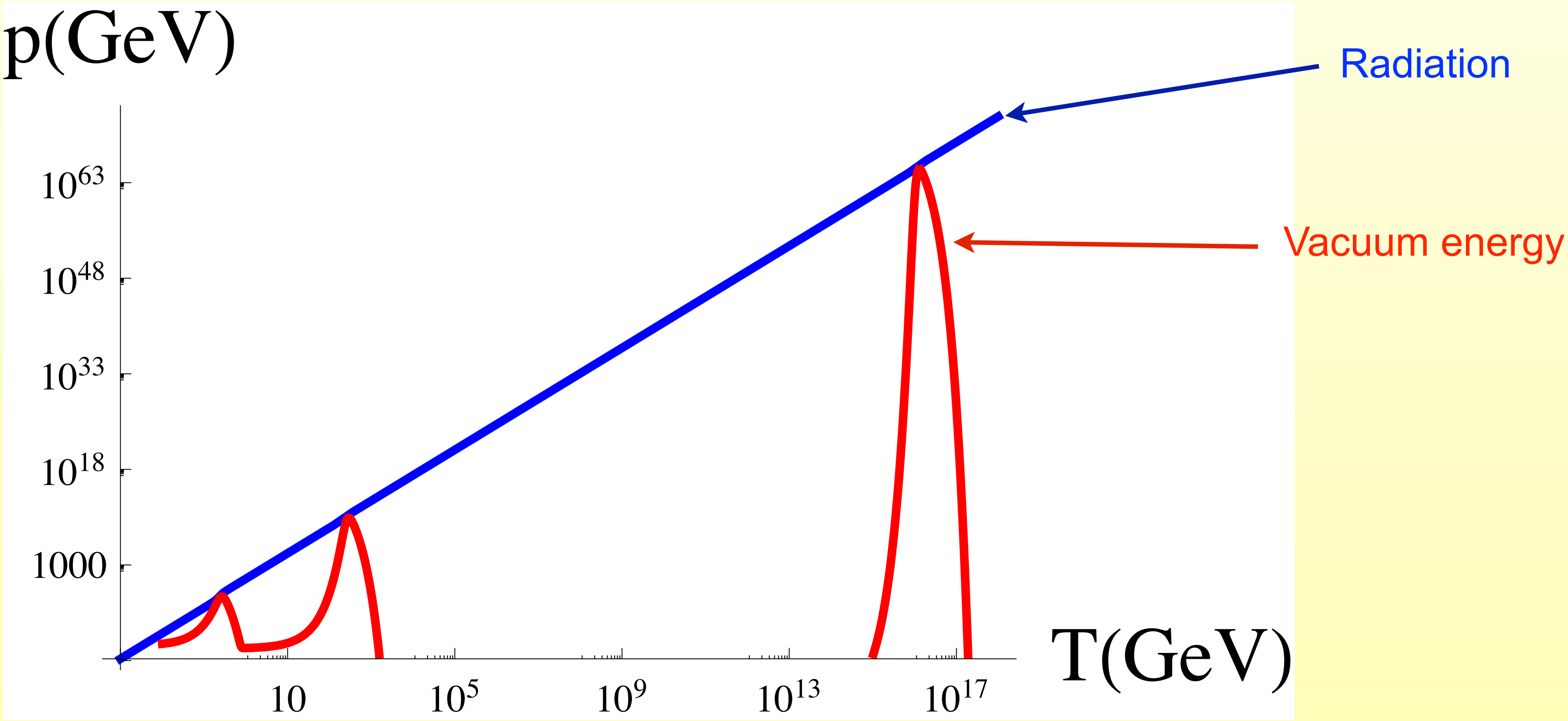
- Whenever Λ would start to dominate a new phase transition happens

- Λ is always subleading even though it was much bigger than it currently is - challenging to find experimental tests of this picture

- Size of step of order $(T_c^{(i)})^4$

- Amount of tuning given by $(T_c^{(i+1)})^4$

Alternative evolution of Λ: with adjustment



Alternative evolution of Λ: with adjustment

- Λ is always small except around PT's
- When PT starts Λ starts growing
- Adjustment mechanism kicks in and drives Λ small again
- Will have its own timescale Δt_{adj}
- Heights will depend on details of adjustment, PT

Steps or adjustment?

- Important goal: to determine experimentally which of these pictures is right one

- If steps: lends more credence to anthropic arguments

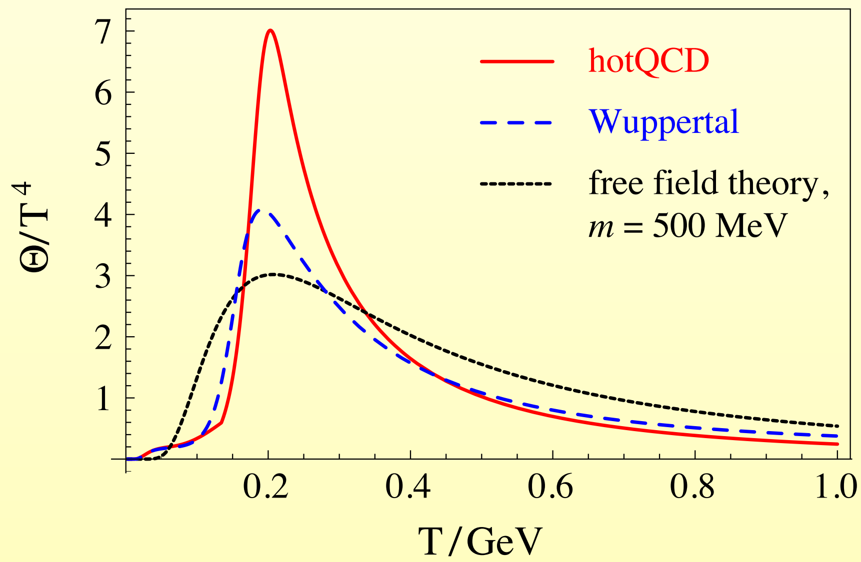
- If adjustment need to find mechanism

- Difficulty: Λ always sub-dominant

- Last of these transitions occurred at Λ_{QCD} : Above CMB, BBN, etc. Not much precision results from that period

Difficulty of finding effects

Example: QCD PT from lattice



(From Caldwell & Gubser 2013)

Deviation from radiation domination only during short period during PT...

Steps or adjustment?

- Further complication: neither EW nor QCD PT first order (at least in SM with 125 GeV Higgs) - no gravitational waves produced from bubble collisions...

- NEED: System where vacuum energy $\mathcal{O}(1)$ fraction of total energy

↓

Neutron star

Epochs where vacuum energy is comparable to radiation

↓

Cosmic phase transitions & effects on primordial gravitational waves

Goal

- Establish experimentally that vacuum energy of microscopic physics is actually what show up in Einstein eq - or there is an adjustment mechanism

- Only care about PT's that actually change VEVs of fields

- For example recombinations at z~ 1100 is a PT where $e+p \rightarrow H$, with binding energy 13.6 eV

- Decrease of energy density of matter, but not a change in vacuum energy - this energy density gets diluted with expansion, while ve does not

1. Neutron stars for testing vacuum energy

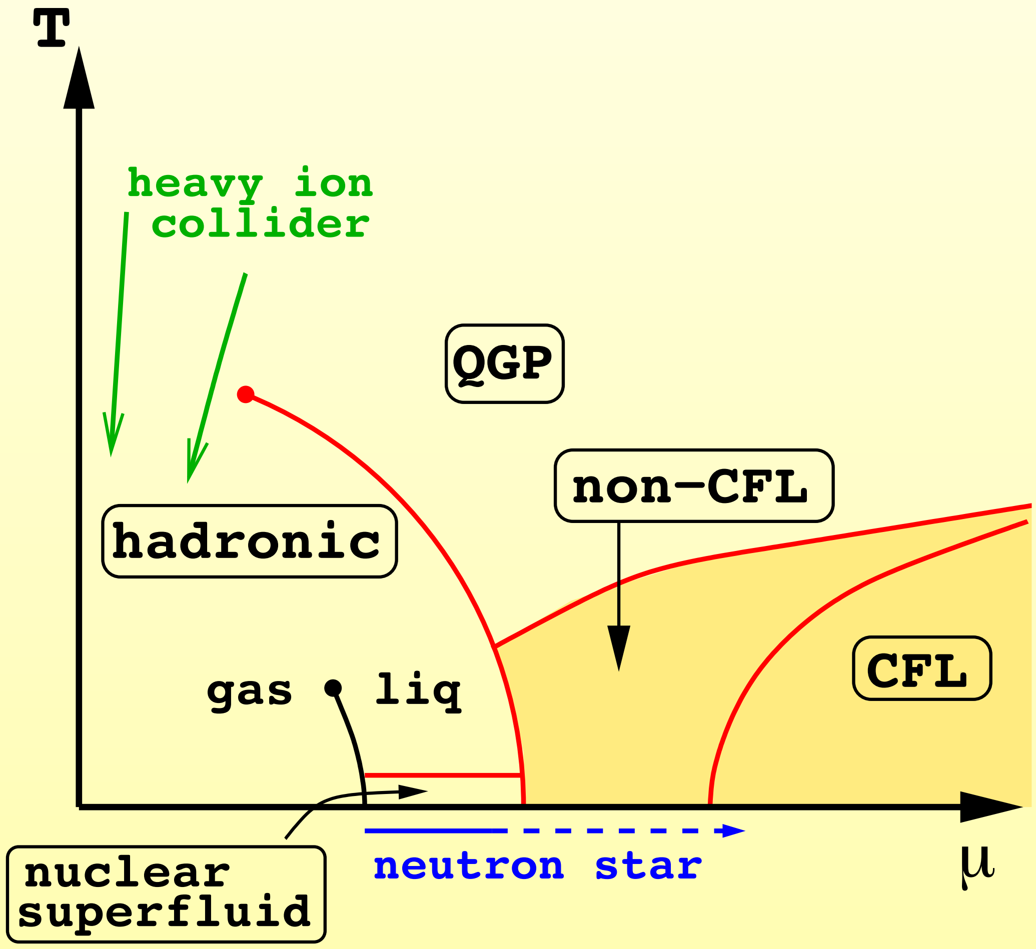
- Need a system which is in different phase of matter

- QCD at large densities probably has those phases: at low T but large chemical potential CFL phase, and non-CFL phase, both with VEVs different from QCD condensates

- Core of neutron star may have this unconventional QCD phase

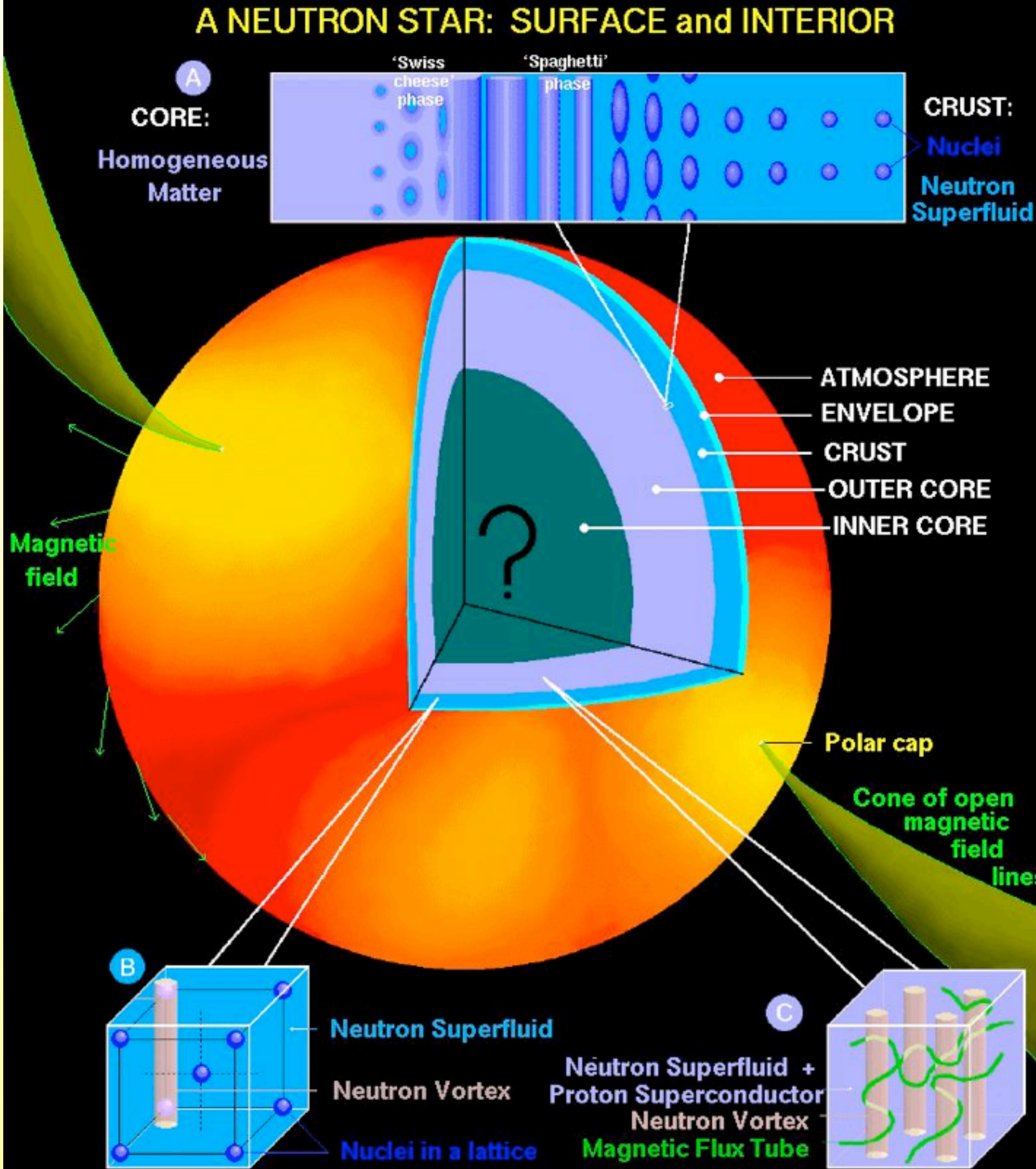
- If adjustment mechanism at play, expect to cancel effect of additional cc in the core. Will modify the structure and M(R) relation of ns's

The phases of QCD



From Alford, Schmitt, Rajagopal, Schaefer 2008

Neutron Stars



From Coleman Miller

Toy model for neutron stars

- Will just consider two phases, inner and outer core

- Neglect crust, envelope, athmosphere...

- Take simple polytropic EOS's for inner and outer cores

- Match them up at critical pressure for phase transition

- Add vacuum energy in inner core (and compare to case w/o vacuum energy)

Toy model for neutron stars

- At zero temperature, gravitational pressure balanced by pressure of fluid. Metric:

$$ds^2 = e^{\nu(r)}dt^2 - (1 - 2GM(r)/r)^{-1}\,dr^2 - r^2 d\Omega^2$$

- Einstein eq's (aka Tolman-Oppenheimer-Volkoff eq):

$$\begin{aligned} M'(r) &= 4\pi r^2 \rho(r)\,, \\ p'(r) &= -\frac{p(r)+\rho(r)}{r^2\,(1-2GM(r)/r)}\left[GM(r)+4\pi r^3 p(r)\right]\,, \\ \nu'(r) &= -\frac{2p'(r)}{p(r)+\rho(r)}\,, \end{aligned}$$

Toy model for neutron stars

- Radius determined by position of vanishing pressure p(R)=0

- Assume phase transition happens at p_{crit}

- Two different EOS's

$$p = p_{(-)}(\rho)\,, \qquad \rho = \rho_{(-)}\,, \qquad p \geq p_{cr}\,, \qquad r \leq r_{cr}$$
$$p = p_{(+)}(\rho)\,, \qquad \rho = \rho_{(+)}\,, \qquad p < p_{cr}\,, \qquad r \geq r_{cr}\,.$$

- Junction condition: $\nu'(r), M(r)$ continuous, thus $p(r)$ also cont.

Toy model for neutron stars

- For inner core use polytropic with cc:

$$p_{(-)}(\rho) = p_f(\rho) - \Lambda = K_-\rho_f^{\gamma_-} - \Lambda$$
$$\rho_{(-)} = \rho_f + \Lambda$$

- For outer core just polytropic

$$p_{(+)}(\rho) = p_f(\rho) = K_+\rho_f^{\gamma_+}$$
$$\rho_{(+)} = \rho_f \, .$$

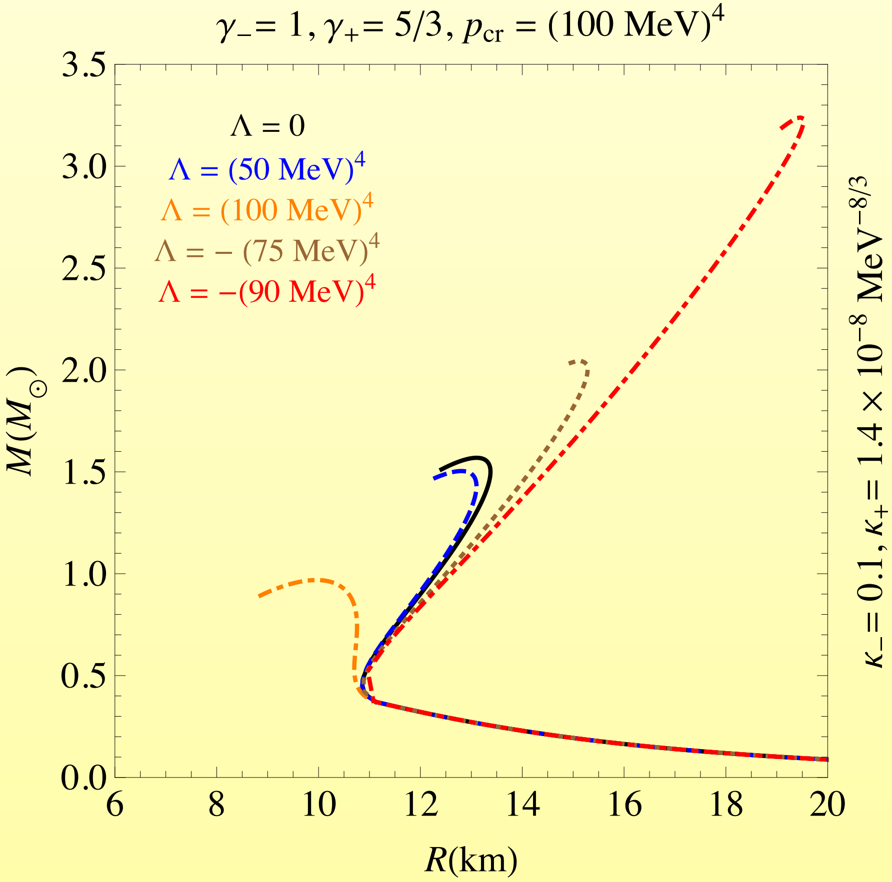
- The value $\gamma_+ = 5/3$ reproduces the small pressure limit of a Fermi fluid

- The cc can not be too large negative: $\Lambda > -p_{cr}$
Otherwise partial pressure of QCD fluid negative

Toy model for neutron stars

- Likely also a thermodynamic upper bound to satisfy $dG = 0$ for Gibbs free energy in equilibrium between phases. Will limit upper value of Λ to few $\cdot 100\ \mathrm{MeV}$

- Checked nicely reproduce the characteristic M(R) curves for neutron stars



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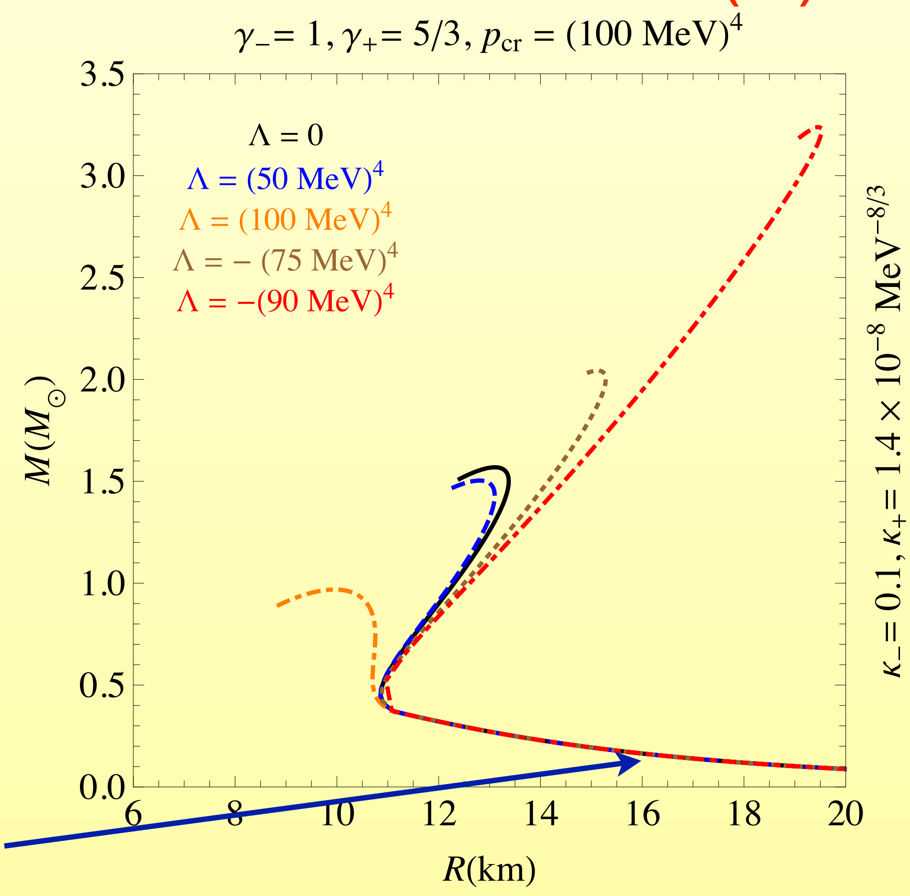
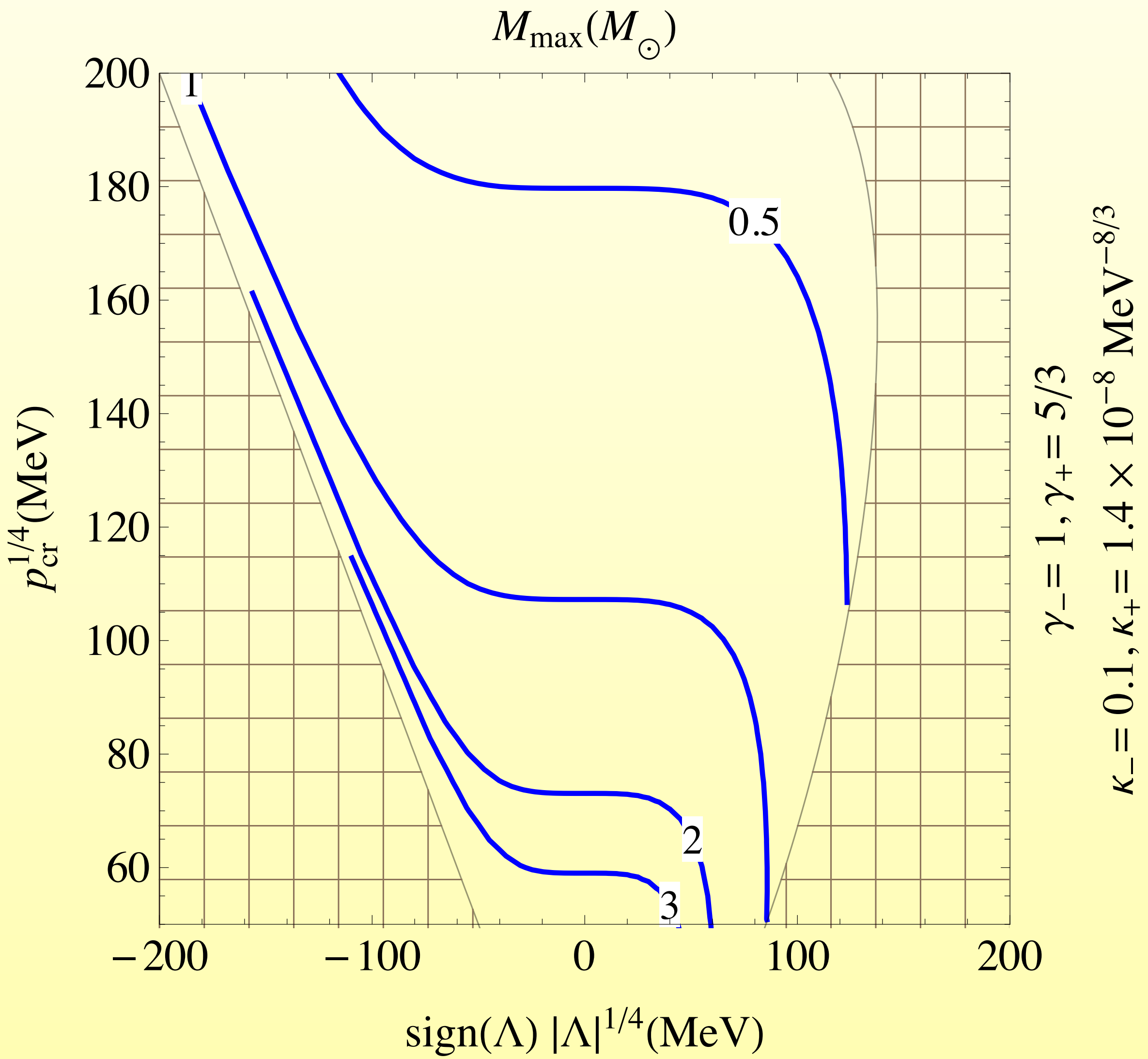


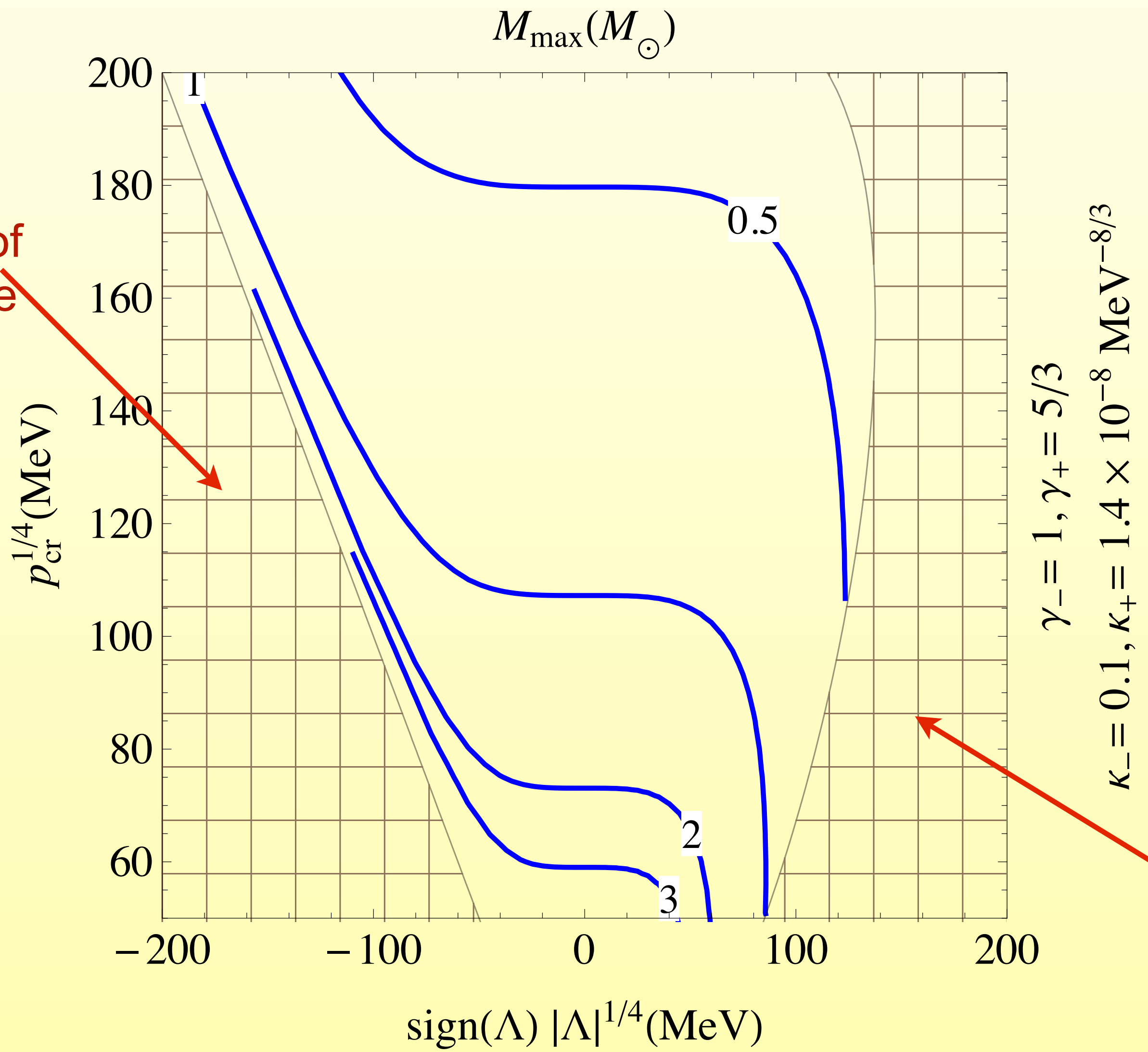
Sensitivities of NS's to vacuum energy



Maximum mass varying Λ and p_{cr}

Sensitivities of NS's to vacuum energy

Instability of fluid at core



Instability of neutron star

Maximum mass varying Λ and p_{cr}

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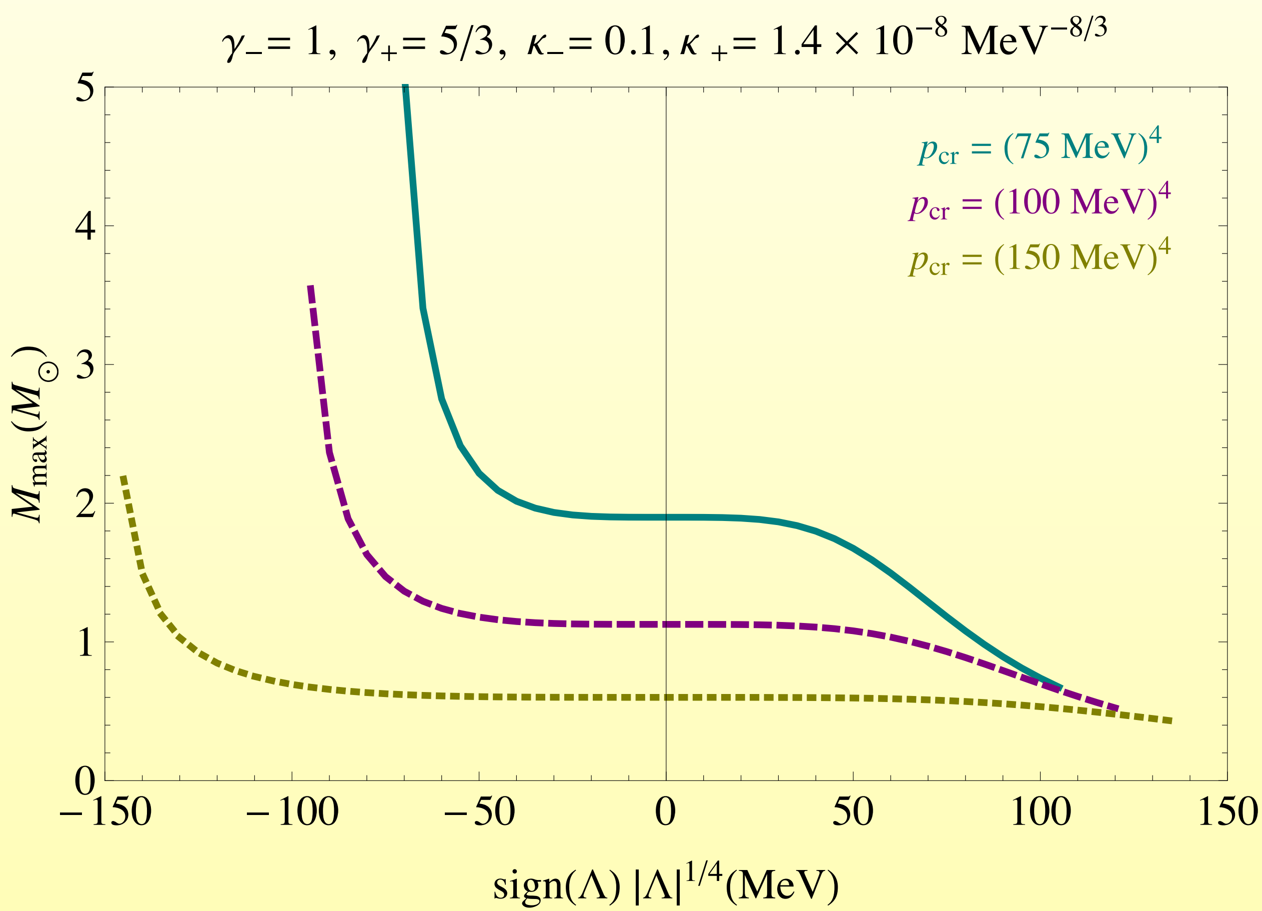


Effect on maximal mass by changing Λ for fixed p_{cr}

Sensitivities of NS's to vacuum energy

- Check effect of changing Λ on M(R) curve

- Depending on parameters maximal mass can change significantly

- But depends very strongly on equations of state parameters, critical pressure...

- Status: maximal mass appears to be bigger than $2M_\odot$

- For now radius measurements difficult, only few known from X-ray measurements.

- Promising: GW from inspiralling ns binaries - should imprint M(R), EOS on chirp...

2. Effect of PT's on Primordial GW's

- Can we possibly say something about the actual vacuum energy of the Universe?
- Need to look for periods around phase transitions
- That is only time when vacuum energy might be sizable
- Especially QCD PT might be interesting
- Case study: look at effect of PT's on primordial gravitational waves, assuming no GW's produced during PT itself

Propagation of primordial gw's

- Tensor perturbations h_{ij} transverse traceless

$h^i_i = 0, \text{ and } \partial_k h^k_i = 0$

- Perturbation of metric in expanding Universe

$$ds^2 = a(\tau)^2 \left(d\tau^2 - (\delta_{ij} + h_{ij})dx^i dx^j\right)$$

- Usually conformal time τ is used $a(\tau)d\tau = dt$

where expansion equation

$$a' = a\dot{a} = a^2 H \,, \qquad \frac{a''}{a} = a^2\left(\frac{\ddot{a}}{a} + \frac{\dot{a}^2}{a^2}\right) = \frac{4\pi G}{3} a^2 T^\mu_\mu$$

Propagation of primordial gw's

- Einstein equation: $h''_{ij} + 2\mathcal{H}h'_{ij} - \nabla^2 h_{ij} = 0$
- Expand in modes: $h_{ij} = \sum_{\sigma=+,-} \int \frac{d^3k}{(2\pi)^3} \epsilon_{ij}^{(\sigma)} h_k^{(\sigma)}(\tau) e^{ikx}$
- Rescaled modes: $\chi_k \equiv a h_k$
- Satisfy very simple equation:

$$\chi''_k + (k^2 - \frac{a''}{a})\chi_k = \chi''_k + \left[k^2 - \frac{4\pi G}{3} a^2 T^\mu_\mu\right]\chi_k = 0$$

- Exciting: equation depends on trace of EM tensor!
- Might think (we did for a while) that VE will have big effect before PT - NOT true

Propagation of primordial gw's

$$\chi_k'' + (k^2 - \frac{a''}{a})\chi_k = \chi_k'' + \left[k^2 - \frac{4\pi G}{3} a^2 T^\mu_\mu\right]\chi_k = 0$$

- Interpretation: if $k^2 > \frac{a''}{a}$ just free plane wave for χ
- But actual mode is χ/a getting damped by 1/a
- Interpretation: if $k^2 < \frac{a''}{a}$ then equation $\frac{\chi''}{\chi} = \frac{a''}{a}$

has solution $\chi \propto a$ and actual mode χ/a is frozen

- If mode outside horizon it is frozen. Once it enters horizon it is damped by 1/a

Propagation of primordial gw's

- What sets the horizon?
- Naively: $$\frac{a''}{a} = \frac{4\pi G}{3} a^2 T^\mu_\mu$$
- This horizon is larger than Hubble horizon - suggests can not have any physical effect
- Indeed when entering this ``naive horizon" velocity of solution still very large - will keep expanding until reaches actual Hubble horizon
- Real condition: rate of entering actual horizon

Energy density in GW's

- The physical quantity:

$$\rho_h(\tau) = \frac{1}{16\pi G a^2(\tau)} \int \frac{d^3k}{(2\pi)^3} |h'_{\sigma,k}|^2$$

- The power spectrum:

$$\Delta_h^2 = \frac{4k^3}{2\pi^2}|h_k|^2 \,, \quad |h_k|^2 = |h_{\sigma,k}|^2 \,.$$

- Transfer function $\mathcal{T}$: $\quad h_k(\tau) \equiv h_k^P \mathcal{T}(\tau, k)$

- h_k^P is the primordial amplitude, usually assumed to have constant power

$$(\Delta_h^P)^2 = \frac{4k^3}{2\pi^2}|h_k^P|^2 \simeq \frac{2}{\pi^2}\frac{H_\star^2}{M_P^2}$$

Energy density in GW's

•The energy density can then be written in terms of the transfer function

$$\rho_h(\tau) = \frac{1}{32\pi G a^2(\tau)} \int d\ln k (\Delta_h^P)^2 \mathcal{T}'^2(\tau,k)$$

•The most commonly used quantity: energy density per log scale normalized to critical density

$$\Omega_h(\tau,k) \equiv \frac{\tilde{\rho}_h(\tau,k)}{\rho_c(\tau)} \;, \qquad \tilde{\rho}_h(\tau,k) = \frac{d\rho_h(\tau,k)}{d\ln k}$$

•Most useful expression:

$$\Omega_h(\tau,k) = \frac{(\Delta_h^P)^2}{12} \frac{1}{H^2(\tau)} \frac{1}{a^2(\tau)} \mathcal{T}'^2(\tau,k)$$

Energy density in GW's

- Assuming mode deep inside horizon:

$$\mathcal{T}'^2(\tau,k) \simeq k^2\,\mathcal{T}^2(\tau,k)$$

- Given our previous discussion, after inflation modes start out outside the horizon and are frozen

- Mode enters at $\tau = \tau_{hc}$ after which energy density gets diluted as radiation

$$\mathcal{T}^2(\tau < \tau_{hc}, k) \simeq \frac{a^2(\tau_{hc})}{a^2(\tau)}$$

- Approximate expression:

$$\Omega_h(\tau,k) \simeq \frac{(\Delta_h^P)^2}{12}\,\frac{k^2}{H^2(\tau)}\,\frac{a^2(\tau_{hc})}{a^4(\tau)}$$

Modes entering during RD

- This is the most relevant case for studying PT's, both QCD and EW happen in that epoch

- Condition for entering: $(aH)^{-2}(\tau_{hc}) \simeq 1/k^2$

- During RD $H^2 \propto 1/a^4$

- Thus $k^2 a^2(\tau_{hc}) \propto const.$

- Spectrum for modes entering during RD constant!

Effect of Phase transition

- Depart from pure RD during PT

- Traditional description: changing number of rel. degrees of freedom in equilibrium

$$g_{\star,a} \equiv g_\star(\tau > \tau_t) \neq g_\star(\tau < \tau_t) \equiv g_{\star,b}$$

- Assuming PT is second order adiabatic (entropy conserved):

$$S = \frac{\rho + p}{T} a^3 = const.$$

- For radiation

$$\rho + p \propto g_* T^4$$

Effect of Phase transition

- Expansion rate: $$a \propto T^{-1} g_*^{-1/3}$$
- Hubble: $$H^2 \propto \rho \propto \frac{1}{a^4} g_*^{-1/3}$$
- Energy density:

$$\Omega_h \propto k^2 a^2(\tau_{hc}) \propto a^4(\tau_{hc}) H_{hc}^2 \propto g_*^{-1/3}$$

- Depends only on # of DOF's
- Expect to see a step in GW density

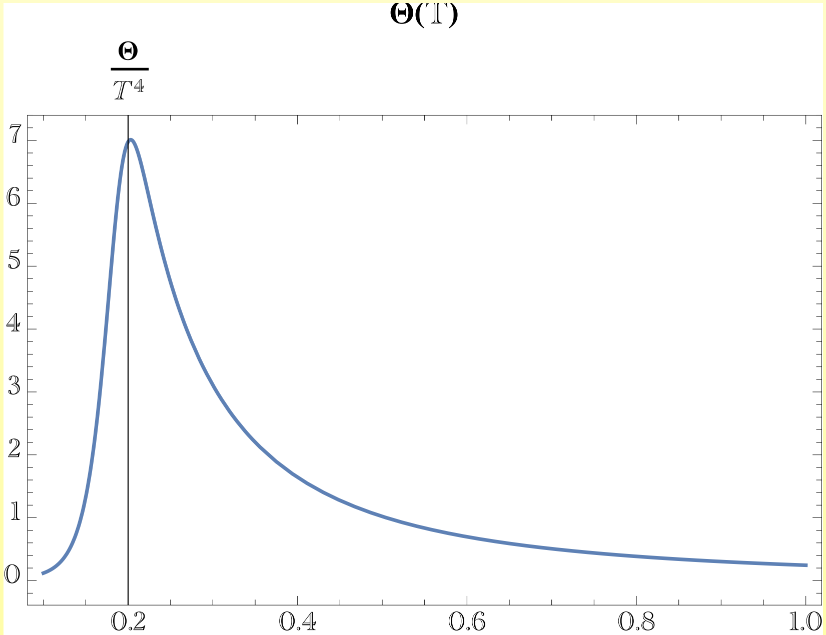
QCD Phase transition

- Numerical evaluation

- Lattice simulations:

$$\Theta = \mathrm{Tr}\, T = T^4 \left(1 - \frac{1}{(1 + e^{(T-c_1)/c_2})^2}\right) \left(\frac{d_2}{T^2} + \frac{d_4}{T^4}\right)$$

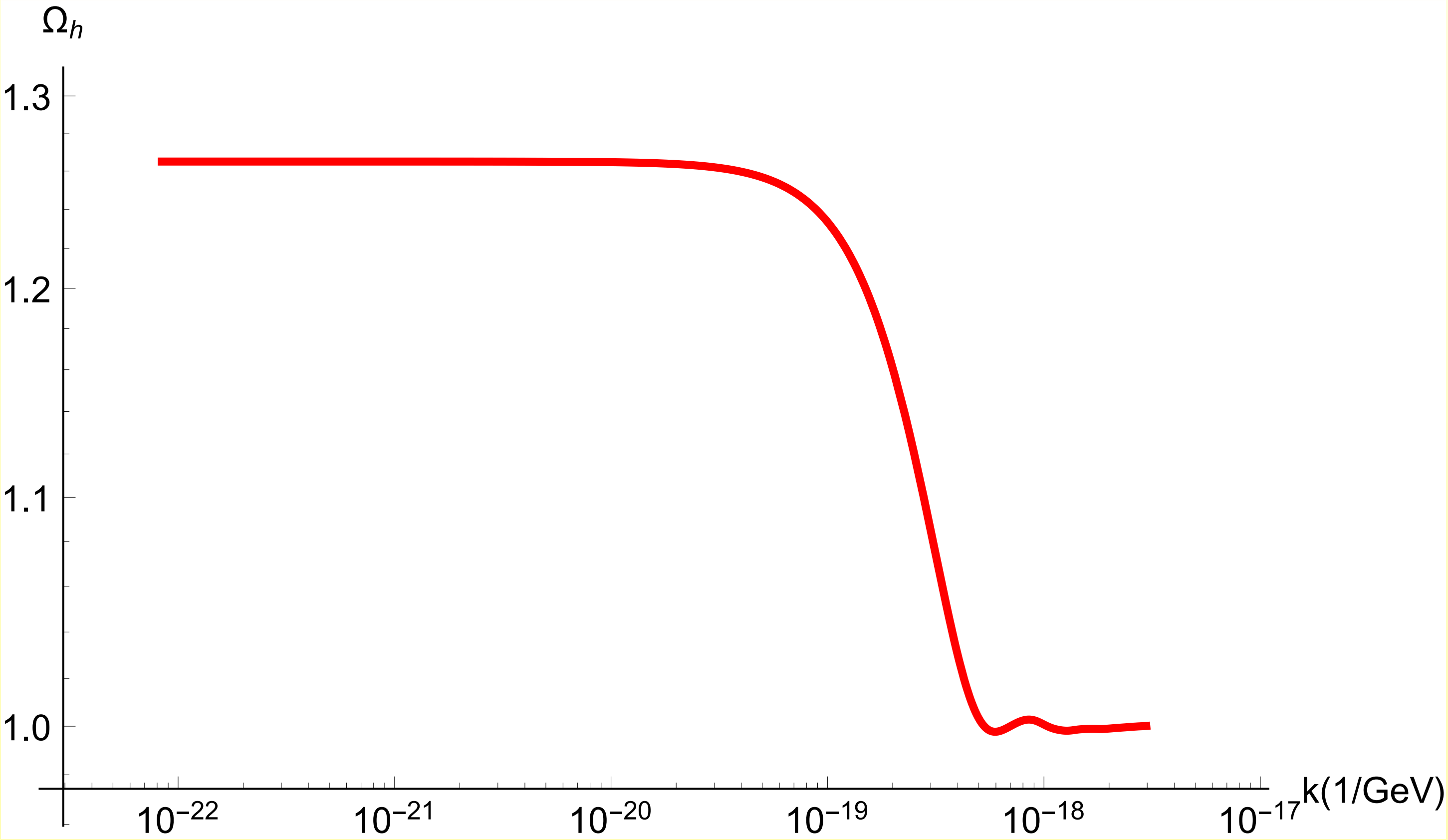
- d_4 is vacuum energy that is changing from $\mathcal{O}(\Lambda^4_{QCD})$ to almost zero



- Valid between 100 MeV and 1 GeV

QCD Phase transition

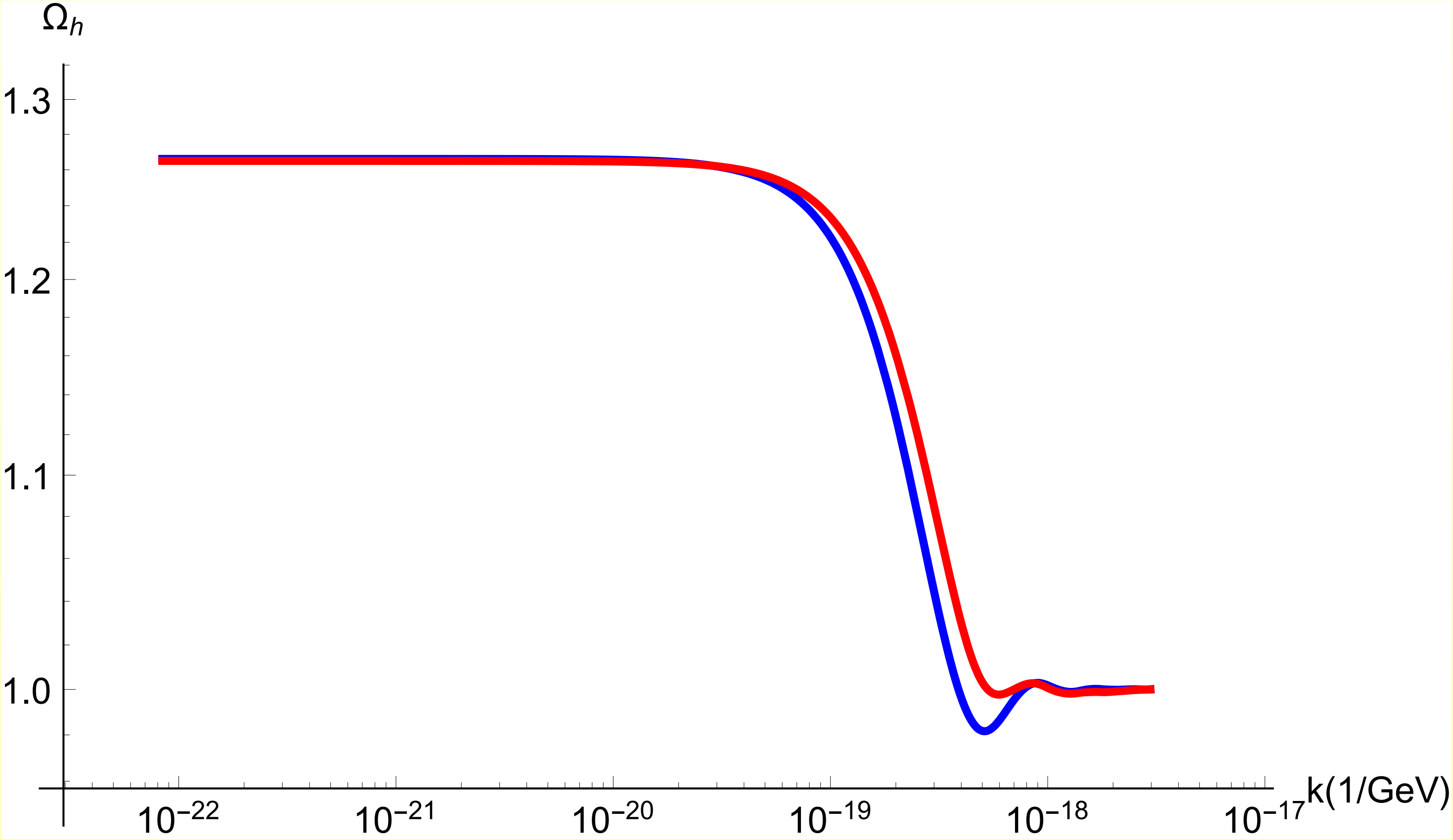
•A typical result:



•Size of step given by change in DOF 60→20 under QCD, HUGE step

QCD Phase transition

- If all VE (ie. change EOS for Θ)



- Almost no difference. Effect of VE (vs. changing DOF’s) not measurable in QCD PT

Condition for strong effect of VE

- Simplified discussion: assume VE jumps at some time τ_t

- Assume (for now) horizon monotonic (no mode re-enters)

- Expansion $$a^2H^2 \propto \rho_\Lambda a^2 + \bar{\rho}_R a^{-2}$$

$$a^2 \propto \frac{2\bar{\rho}_R}{(aH)^2}\left(1+\sqrt{1-4\frac{\rho_\Lambda}{\bar{\rho}_R}\left(\frac{\bar{\rho}_R^2}{(aH)^4}\right)}\right)^{-1}$$

- Modes entering much before & after transition unaffected

$$\Omega_h(\tau > \tau_t, k \gg k_t) \propto g_{\star,b}^{-1/3}$$

$$\Omega_h(\tau > \tau_t, k \ll k_t) \propto g_{\star,a}^{-1/3}$$

Condition for strong effect of VE

- However modes entering around transition $k_t = (aH)(\tau_t)$

- Enhancement of modes

$$\Omega_h(\tau > \tau_t, k \gtrsim k_t) \propto g_{\star,b}^{-1/3}\left[1 + \xi\left(\frac{k_t}{k}\right)^4\right]$$

$$\Omega_h(\tau > \tau_t, k_t' \lesssim k \lesssim k_t) \propto g_{\star,b}^{-1/3}\frac{k^2}{k_t^2}(1+\xi)$$

- Magnitude of peak set by ξ the maximal ratio of VE to radiation

$$\rho_{\Lambda,b}/\xi = \bar{\rho}_{R,b}/a^4$$

- However if step much larger than peak, peak will be washed out, case for QCD - VE never becomes comparable to radiation & large change in DOF's

Condition for strong effect of VE

- Even for other (hypothetical) PT's form strongly constrained by basic conditions if adiabatic

- Positive entropy $\rho + p \geq 0 \rightarrow \frac{dp}{dT} \geq 0$

- Positive energy densities $\rho > 0$

- No contracting solutions $\rho_\Lambda \geq 0$

- Apply first condition:

$$\frac{\Delta p_R/\Delta T}{\Delta p_\Lambda/\Delta T} = -\frac{1}{3}(T\frac{\Delta g}{\Delta T} + 4g)\frac{\Delta T}{T}\frac{1}{g\Delta V/\rho_R} > 1$$

Condition for strong effect of VE

- Two options:

- $$\frac{\Delta g}{\Delta T} T \gg 4g \;\Rightarrow\; \rho_\Lambda/\rho_R < \frac{g_i - g_f}{3 g_i}$$ no peak at all

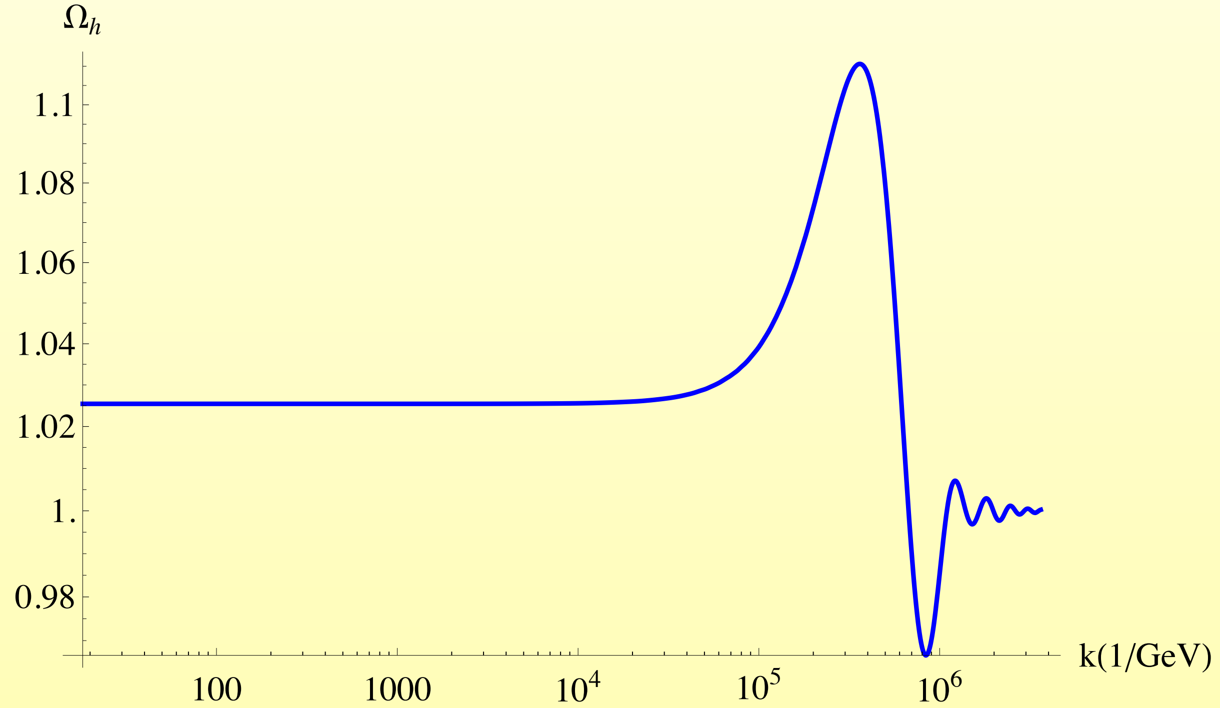
- $$\frac{\Delta g}{\Delta T} T \ll 4g \Rightarrow \rho_\Lambda/\rho_R < \frac{4}{3}\frac{\Delta T}{T} \;\gg \frac{1}{3}\frac{\Delta g}{g} \simeq (\frac{g_i}{g_f})^{1/3}$$ possibly limited peak

- Best example we found so far for a peak due to VE:

- Hypothetical PQ PT with few scalars, one sets VEV other sets quartic (and hence critical Temp).

A peak in the GW spectrum

•Best example with peak so far:



•PQ-like PT at 10^{11} GeV, DOF 119→108, Λ as large as possible >1/3 of radiation

Effect of adjustment mechanism

- Depends on the time scale for the adjustment

- If very quick - might just set VE to zero always. In this case hard to make any distinction in QCD & EW

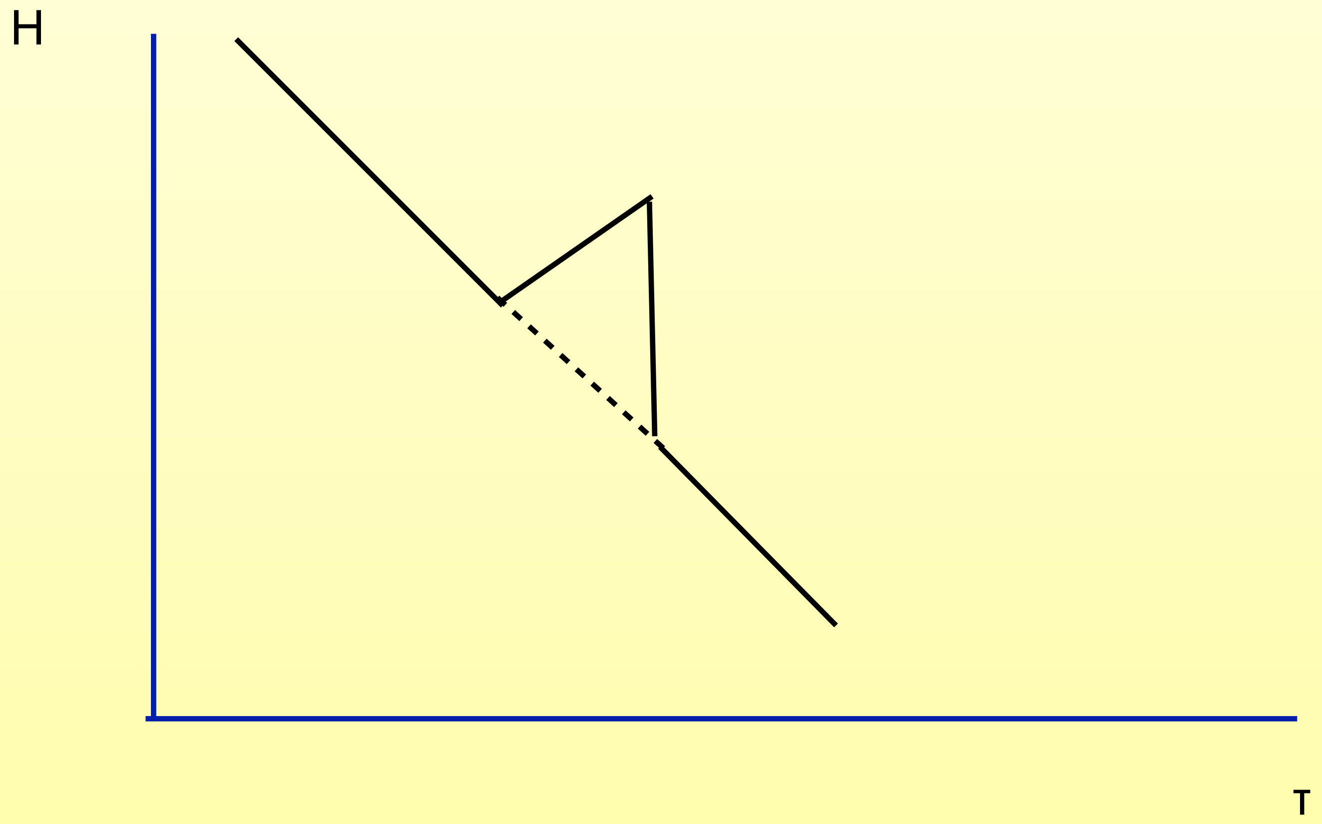
- Other possibility: adjustment time scale somewhat larger than that of PT

- In this case expect a period where VE dominates after the PT

- Could have a short inflation-like period after PT

Effect of short inflation

- Some of the modes that entered will leave again
- Some modes will only enter later



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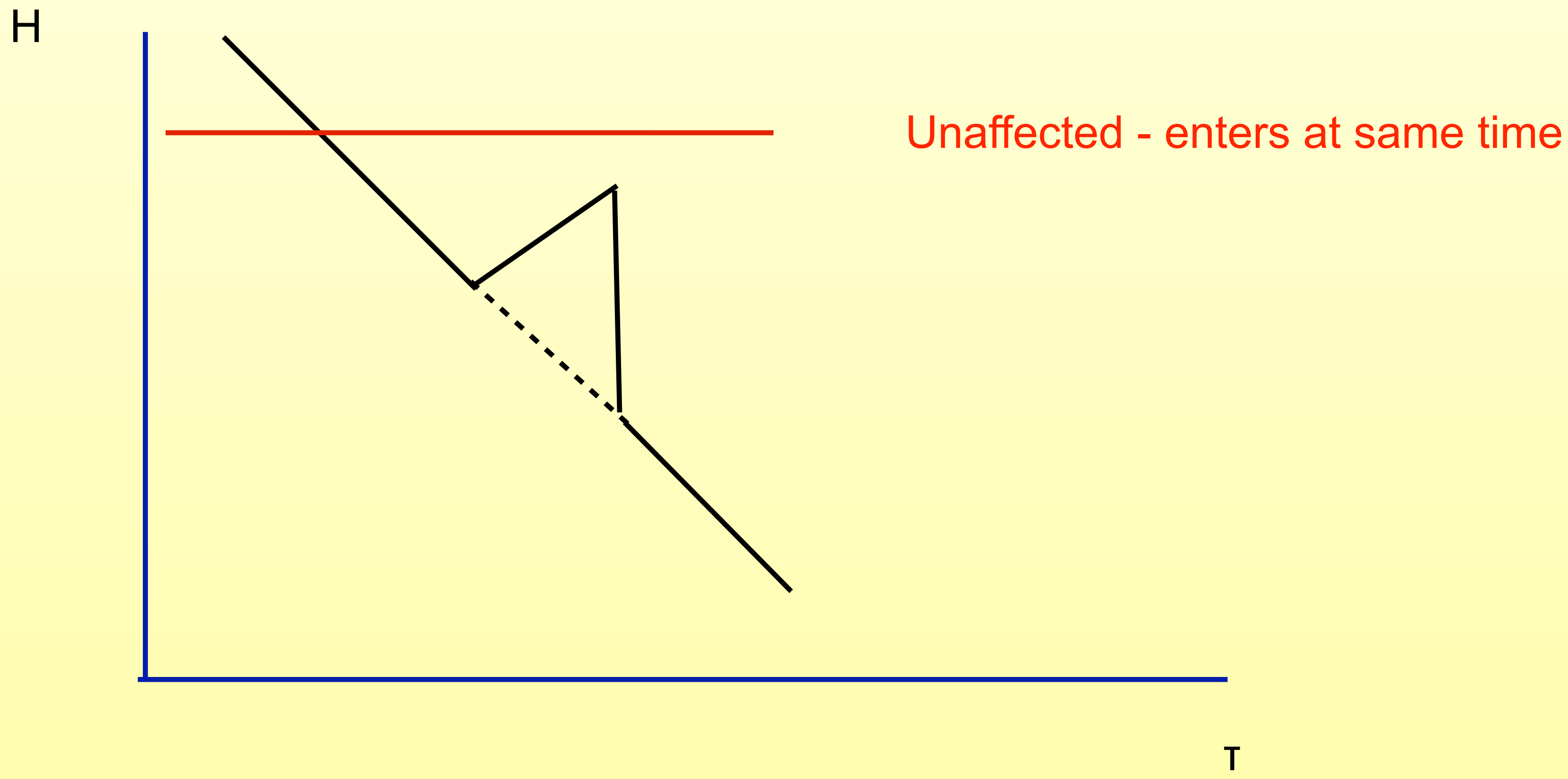

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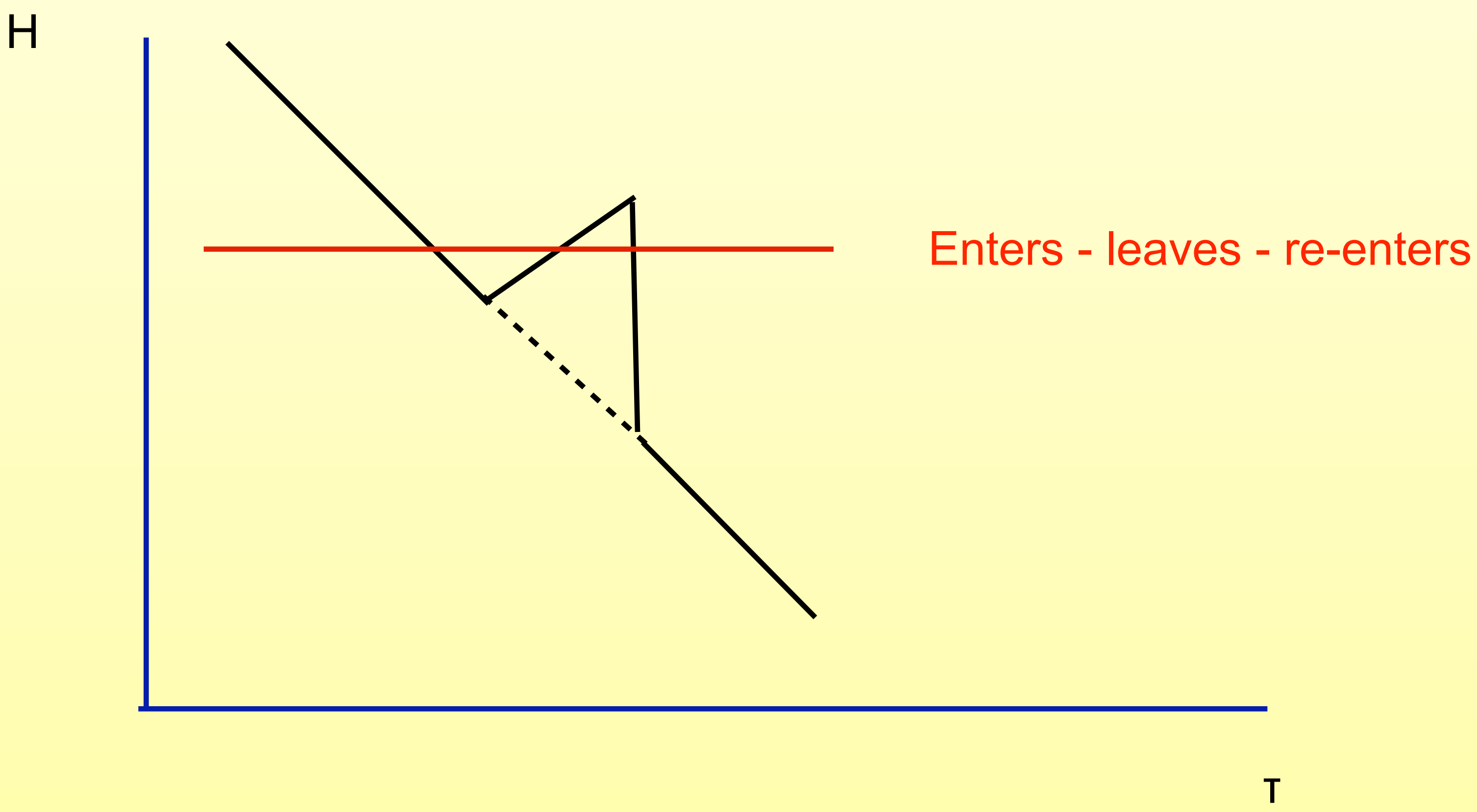

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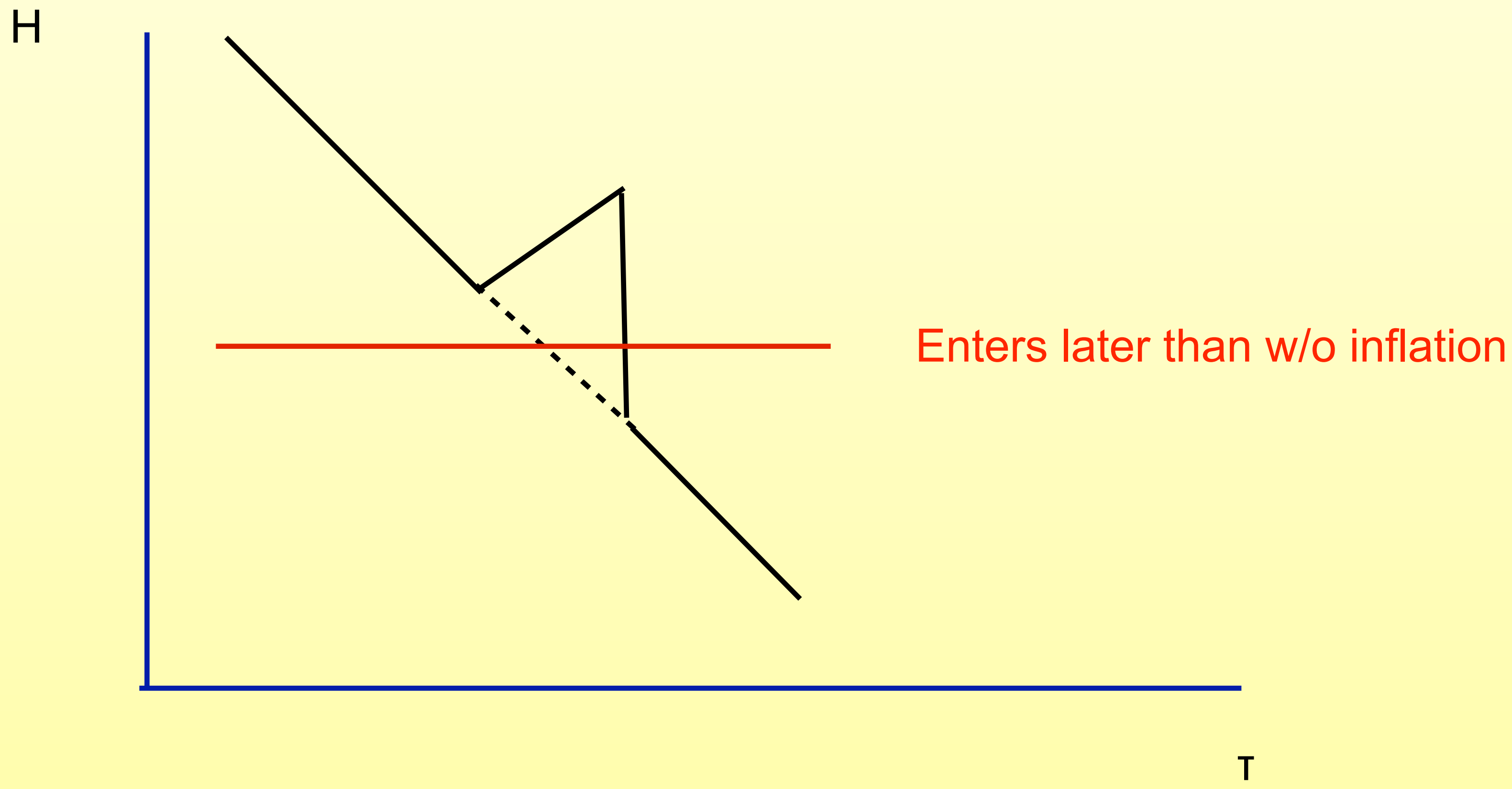

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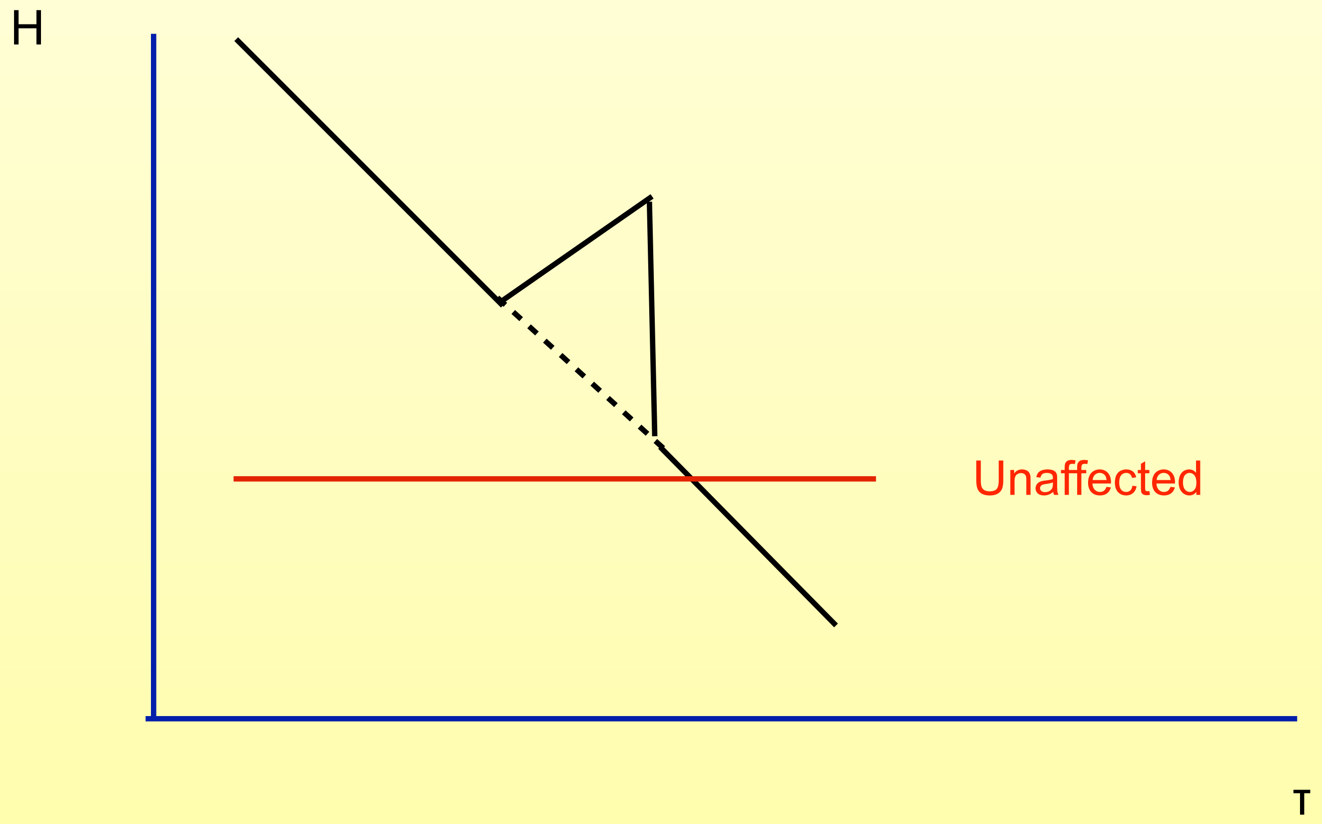
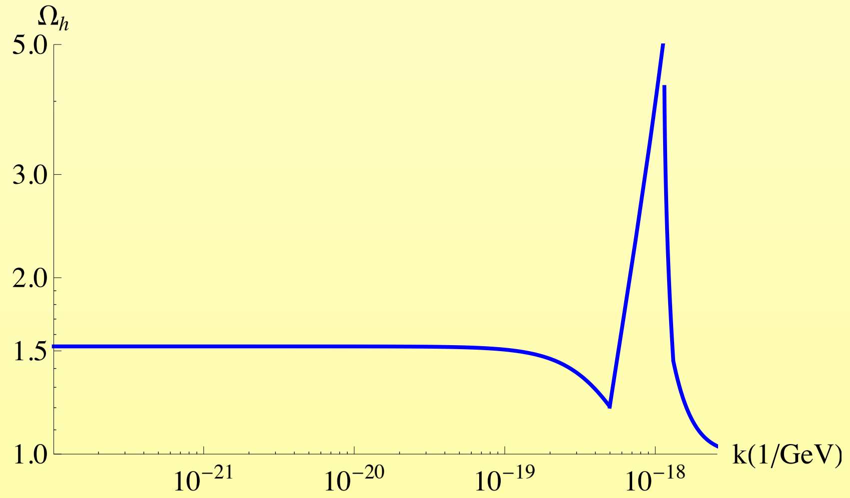


Effect of short inflation

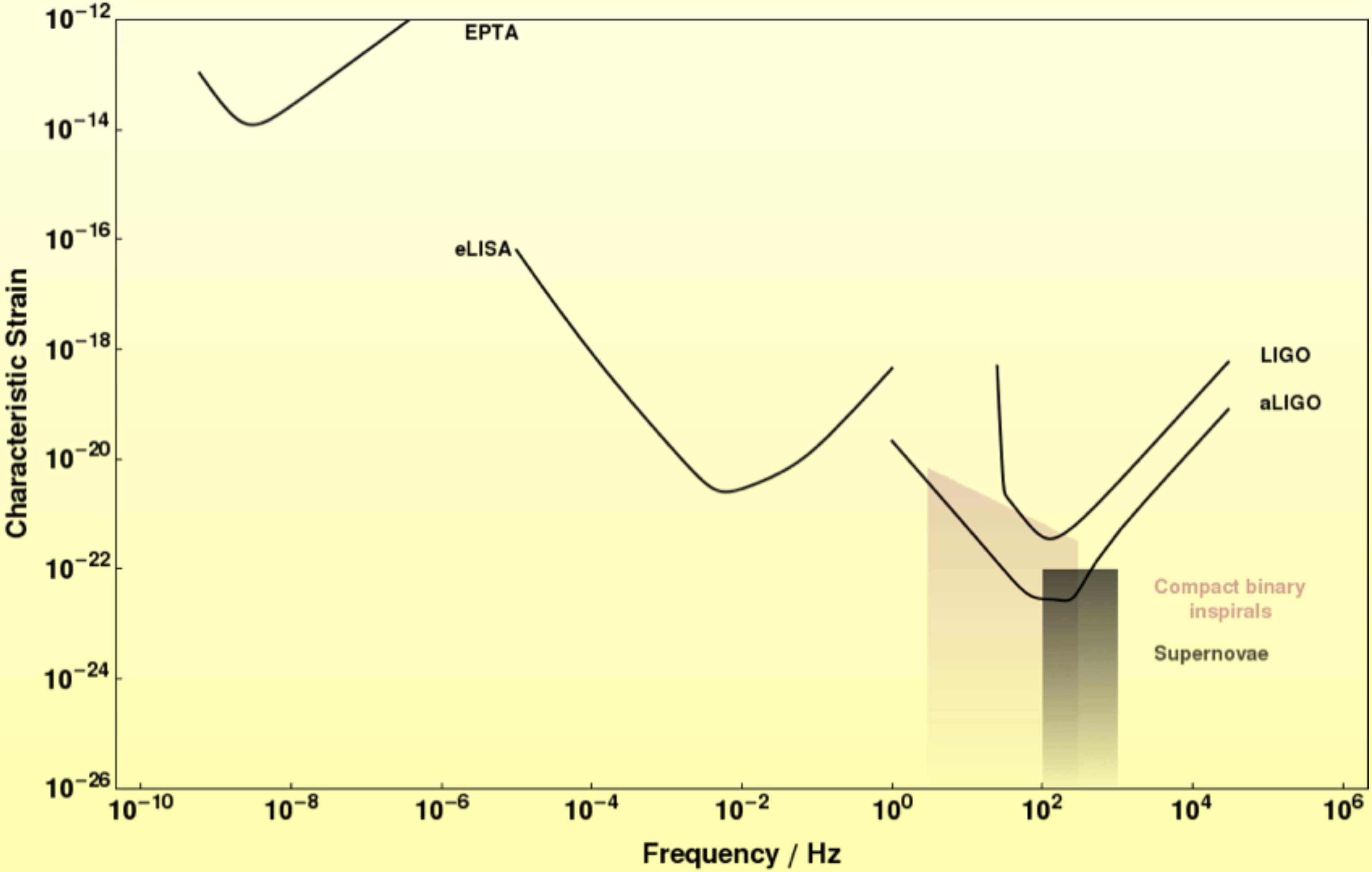
- Affected modes will be enhanced

- Expect to see a peak which can be large depending on the duration of the VE domination

- Very crude sketch for QCD with adjustment (preliminary!)



Sensitivity of future experiments



Summary

- An important part of our standard picture of cosmology & particle physics: VE should change during PT's

- Never dominates - how could we check experimentally?

- Look for systems where vacuum energy is sizeable fraction

Neutron stars - should cause measurable deviation in maximal mass of NS's

- Look for effect during PT where VE sizable:

Primordial gravitational waves - hard to see VE in standard scenario, possible peaks with adjustment?