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MSSM and 2HDM tools

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Particles, Strings, and the Early Universe
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Outline

Higgs sector of the SM: One $SU(2)$ doublet $\Phi = (\phi^+, \phi^0)^T$ with $Y = \frac{1}{2}$
4 degrees of freedom:

- 3 Goldstone bosons 'eaten up' by the long. components of $W^\pm, Z$.
- 1 scalar particle (SM Higgs), free parameter: Higgs mass m_H

It couples to the heavy gauge bosons and via Yukawa couplings to the fermions:

$$\mathcal{L}_H \supset -\frac{m_f}{v} g_f^H f\bar{f}H - \frac{2m_V^2}{v} g_V^H V^\mu V_\mu H$$

For the SM Higgs we have: $g_f^H = g_V^H = 1$

With $\cos\theta_W = \sqrt{g^2 + g'^2}/g^2$ we define

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \qquad \text{(at tree-level).}$$

How can the Higgs sector in a local $SU(2) \times U(1)$ theory be extended?

Measurement of $\rho \sim 1.0004$ [PDG]

Introduce n scalar multiplets ϕ_i with:

- weak Isospin T_i
- weak hypercharge Y_i
- VEVs v_i of neutral comp.

$$\rho = \frac{\sum_{i=1}^{n} [T_i(T_i+1) - Y_i^2] v_i}{\sum_{i=1}^{n} 2Y_i^2 v_i}$$

$\rightarrow$ Possible SM extensions: Add multiplets with $T(T+1) = 3Y^2$
e.g. $SU(2)$ singlets with $Y = 0$, $SU(2)$ doublets with $Y = \pm\frac{1}{2}$, ...

All are consistent with electroweak precision data and can host a SM Higgs:

- Standard Model: One $SU(2)$ doublet
- Singlet extended Standard Model: One $SU(2)$ doublet + $SU(2)$ singlet
- 2HDM: Two $SU(2)$ doublets
- MSSM: Two $SU(2)$ doublets
- NMSSM: Two $SU(2)$ doublets + One $SU(2)$ singlet

...

Higgs sector of the 2 Higgs Doublet Model:
Two $SU(2)$ Higgs doublets $\Phi_i = (\phi_i^+, \phi_i^0)$ with potential (no FCNCs!): [0902.0851]

$$\begin{aligned}\mathcal{V}_{\text{gen}} =& m_{11}^2\Phi_1^\dagger\Phi_1 + m_{22}^2\Phi_2^\dagger\Phi_2 - \left[m_{12}^2\Phi_1^\dagger\Phi_2 + \text{h.c.}\right] + \frac{1}{2}\lambda_1\left(\Phi_1^\dagger\Phi_1\right)^2 \\ &+ \frac{1}{2}\lambda_2\left(\Phi_2^\dagger\Phi_2\right)^2 + \lambda_3\left(\Phi_1^\dagger\Phi_1\right)\left(\Phi_2^\dagger\Phi_2\right) + \lambda_4\left(\Phi_1^\dagger\Phi_2\right)\left(\Phi_2^\dagger\Phi_1\right) \\ &+ \left\{\frac{1}{2}\lambda_5\left(\Phi_1^\dagger\Phi_2\right)^2 + \left[\lambda_6\left(\Phi_1^\dagger\Phi_1\right) + \lambda_7\left(\Phi_2^\dagger\Phi_2\right)\right]\left(\Phi_1^\dagger\Phi_2\right) + \text{h.c.}\right\}\end{aligned}$$

Electroweak symmetry breaking fixes m_{11} and m_{22}.
General input basis: $\tan\beta, \lambda_1, \ldots, \lambda_7, m_{12}^2$
CP-conserving case:
Two CP-even bosons h, H, CP-odd boson A, charged bosons $H^\pm$
Higgs basis: only one vev $\tan\beta = 0 \to \Lambda_1, \ldots, \Lambda_7, m_{H^\pm}$
Physical basis: $\tan\beta, m_h, m_H, m_A, m_{H^\pm}, \sin(\beta-\alpha), m_{12}^2, \lambda_6, \lambda_7$

Coupling to gauge bosons:
$g_V^h = \sin(\beta-\alpha) \approx 1, \quad g_V^H = \cos(\beta-\alpha), \quad g_V^A = 0$

Four types of 2HDM differ by couplings to fermions:

Model	Type I	Type II	Flipped	Lepton-specific
g_u^h	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$	$\cos\alpha/\sin\beta$
g_d^h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$
g_l^h	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$	$\cos\alpha/\sin\beta$	$-\sin\alpha/\cos\beta$
g_u^H	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$	$\sin\alpha/\sin\beta$
$\ldots$				

Special case of the MSSM (Type II 2HDM):

$$\lambda_1 = \lambda_2 = \frac{g^2 + g'^2}{4}, \qquad \lambda_3 = \frac{g^2 - g'^2}{4}, \qquad \lambda_4 = -\frac{g^2}{2},$$
$$\lambda_5 = \lambda_6 = \lambda_7 = 0, \qquad m_{12}^2 = m_A^2 \cos\beta \sin\beta.$$

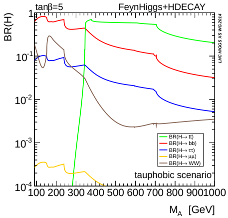
Usual input in the real CP-conserving MSSM: $m_A, \tan\beta$
In particular: $m_{h/H}$ and $m_{H^\pm}$ are predicted! $m_h \leq m_Z$ at tree-level!
Codes for Higgs mass calculation at (N)NNLO (see later!)

Outline

Branching ratios (and masses in the MSSM) for the Higgs bosons:

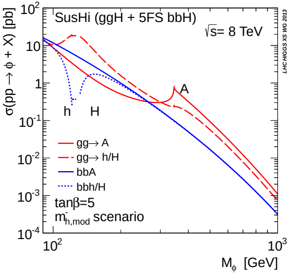
▷ Recommendation for 2HDM [1312.5571; Harlander Mühlleitner Rathsman Spira Stål]
Codes: `(HIGLU+)HDECAY` or `(SusHi+)2HDMC`
`HDECAY: tiger.web.psi.ch/hdecay/` [Djouadi Kalinowski Spira]
`2HDMC: 2hdmc.hepforge.org` [Eriksson Rathsman Stål]
Both codes can be used with the physical basis and support the four 2HDM types. They provide BRs for
$\phi_i \to q\bar{q}, gg, \gamma\gamma, Z\gamma, WW, ZZ, \phi_j\phi_k, W\phi_j, Z\phi_k$ with $\phi_i \in \{h, H, A, H^\pm\}$.
Detailed comparison taking into account different order in perturbation theory, threshold behaviour with mostly $< 1\%$ difference: [1312.5571]

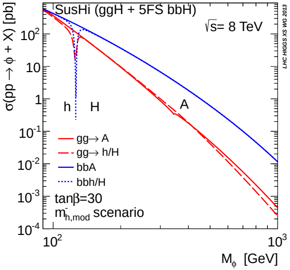
▷ MSSM: Spectrum generators like `SPheno, SOFTSUSY, Suspect, SuSeFlav,...`
Default in the LHC Higgs XS WG: `FeynHiggs` to calculate the masses and couplings, link to `HDECAY` via SLHA for branching ratios and total width.
`FeynHiggs` [Hahn Heinemeyer Hollik Rzehak Weiglein]



Outline

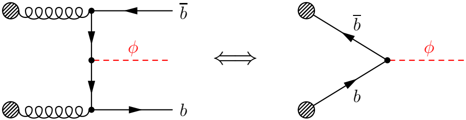
Inclusive neutral Higgs production in the real MSSM/2HDM:



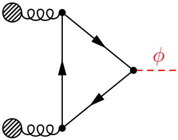


Relevant processes:

Bottom-quark annihilation:



Gluon fusion:



Gluon fusion: Calculation of 2HDM/MSSM Higgs cross sections

SM: ggh@nnlo

SM: HIGLU

$$\sigma(pp \to \phi + X) = (g_t^\phi)^2 \left(\sigma_{\text{NLO}}^{t,\text{SM}} + \Delta\sigma_{\text{NNLO}}^{t,\text{SM},0} \right) + (g_b^\phi)^2 \sigma_{\text{NLO}}^{b,\text{SM}} + (g_t^\phi)(g_b^\phi)\sigma_{\text{NLO}}^{tb,\text{SM}}$$

Couplings (including resummation in the MSSM):

$$g_t^h = \frac{\cos\alpha}{\sin\beta} \qquad g_b^h = -\frac{\sin\alpha}{\cos\beta}\frac{1}{1+\Delta_b}\left(1 - \frac{\Delta_b}{\tan\alpha\tan\beta}\right)$$

Higgs mixing angle α, $\tan\beta = v_u/v_d$, Resummation of sbottom effects in Δ_b

The above procedure misses:

- ✓ electroweak contributions by light quarks
- ✓ in the MSSM NLO third generation squark contributions (on top of Δ_b)

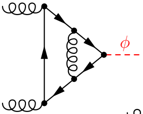
$$\sigma(pp \to \phi + X) = \sigma_{\text{NLO}}^{\text{MSSM/2HDM}}(1 + \delta_{\text{EW}}^{lq}) + (g_t^\phi)^2 \left(\Delta\sigma_{\text{NNLO}}^{t,\text{SM},0} \right)$$

$\Longrightarrow$ Included for $\phi \in \{h, H, A\}$ in SusHi (linked to FeynHiggs or 2HDMC)

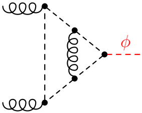
HIGLU [Spira], ggh@nnlo [Harlander Kilgore], SusHi [Harlander Liebler Mantler]

✓ **Inclusion of NLO squark contributions:**

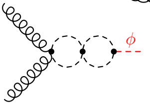
- gluon-quark: known analytically (higher orders)
  [Spira Djouadi Graudenz Zerwas ’95; Harlander Kant ’05; . . .]



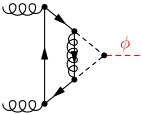
- gluon-squark: known analytically/numerically
  [Anastasiou Beerli Bucherer Daleo Kunszt ’06;
  Aglietti Bonciani Degrassi Vicini ’06; Mühlleitner Spira ’06;
  Bonciani Degrassi Vicini ’07]



- gluino-squark-quark contributions:
  semi-analytically known
  [Anastasiou Beerli Daleo ’08; Mühlleitner Spira Rzehak ’10]



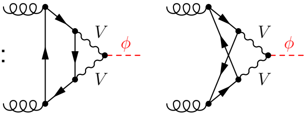
Challenge for gluino-quark-squark contributions:
Five different masses: $m_q, m_{\tilde{q}1}, m_{\tilde{q}2}, m_{\tilde{g}}, p^2 = m_\phi^2$



- Taylor expansion in small Higgs mass:
  $\longrightarrow$ top-stop-gluino contribution $m_\phi \ll m_t, m_{\tilde{t}1}, m_{\tilde{t}2}, m_{\tilde{g}}$
  [Harlander Steinhauser ’03 ’04 + Hofmann ’05; Degrassi Slavich ’08]
  (NNLO top-stop-gluino contr. [Pak Steinhauser Zerf ’10 ’12])
- Expansion in heavy SUSY masses: $m_\phi, m_q \ll m_{\tilde{q}1}, m_{\tilde{q}2}, m_{\tilde{g}}$
  $\longrightarrow$ quark-squark-gluino [Harlander Hofmann Mantler ’10; Degrassi Slavich ’10 + Di Vita ’11 ’12]

✓ **Inclusion of elw. contributions by light quarks:** [Aglietti Bonciani Degrassi Vicini '04 '10]

Relevant diagrams with $V \in \{W, Z\}$:



Definition of SUSY electroweak correction factor:

$$\delta_{\rm EW}^{lq} = \frac{\alpha_{\rm EW}}{\pi} 2\text{Re}(\mathcal{A}^\phi \mathcal{A}^{\phi,\rm EW})/|\mathcal{A}^\phi|^2$$

$$\mathcal{A}^{\phi,\rm EW} = -\frac{3}{8}\frac{x_W}{s_W^2}\left[\frac{2}{c_W^2}\left(\tfrac{5}{4} - \tfrac{7}{3}s_W^2 + \tfrac{22}{9}s_W^4\right) A[x_Z] + 4A[x_W]\right] g_V^\phi$$

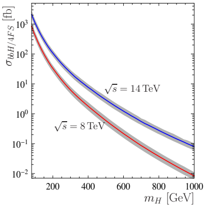
Complex mass scheme: $x_V = (m_V - i\frac{\Gamma_V}{2})^2/m_\phi^2$

Supersymmetry enters g_V^ϕ:
$g_V^h = \sin(\beta - \alpha), \quad g_V^A = 0, \quad g_V^H = \cos(\beta - \alpha)$

For moderate masses of SM-like Higgs results in similiar correction as SM electroweak correction factor [Actis Passarino Sturm Uccirati '08].

Bottom-quark annihilation in 5FS at NNLO: `bbh@nnlo` [Harlander Kilgore]
included in `SusHi`, provides 2HDM/MSSM reweighted XS through g_b^ϕ (with Δ_b)
Bottom-quark associated production in 4FS at NLO: [Dittmaier Krämer Spira]

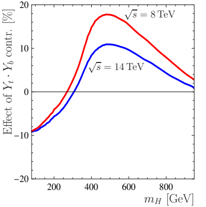
- ✓ Grids for $\sqrt{s} = 7, 8, 13$ and 14 TeV for H/A
- ✓ Grids for the $Y_b \cdot Y_t$ interference terms (interferences of $bb\phi$ in 4FS and $gg\phi$ production)



$\rightarrow$ reweighted 2HDM/MSSM XS
through g_b^ϕ (with Δ_b) and g_t^ϕ

$\longrightarrow$ Santander-matching to combine 4FS and 5FS XS: [1112.3478]

$$\sigma = \frac{\sigma^{4FS} + \omega\sigma^{5FS}}{1+\omega} \quad \text{with} \quad \omega = \ln\frac{m_\phi}{m_b} - 2$$



Combined calculation ongoing:
[Papanastasiou Bonvini Tackmann]

Usage of presented setup for MSSM benchmark scenarios:

Scenario	M_{SUSY} [GeV]	X_t [GeV]	μ [GeV]	M_2 [GeV]
m_h^{max}	1000	2000	200	200
$m_h^{\mathrm{mod}+}$	1000	1500	200	200
$m_h^{\mathrm{mod}-}$	1000	-1900	200	200
light stop	500	1000	400	400
light stau	1000	1600	500	200
tau-phobic	1500	3675	2000	200
(low MH)	1500	3675	$m_A = 110$ GeV	200

[Carena Heinemeyer Stål Wagner Weiglein '13]

Root files provided on the webpage of the LHC Higgs XS WG as a function of m_A and $\tan\beta$ for $\phi \in \{h, H, A\}$:

- ✓ Higgs masses m_ϕ (h mostly compatible with SM Higgs ~ 125 GeV)
- ✓ Gluon fusion XS + Bottom-quark annihilation XS in 4FS/5FS
- ✓ Branching ratios
- ✓ Scale and PDF$+\alpha_s$ uncertainties

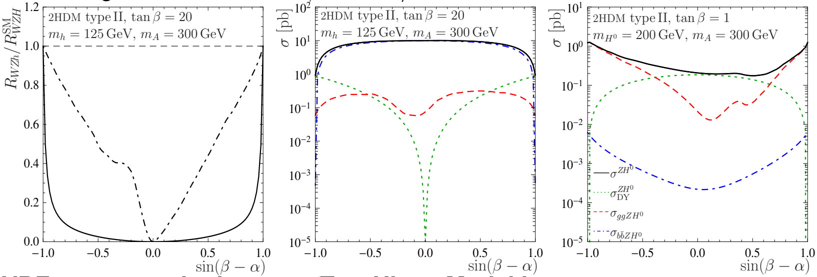
For other scenarios: Get in contact!
Future idea: Include charged Higgs XS+BRs! $\rightarrow$ Talk Maria Ubiali

Higgsstrahlung in the 2HDM in `vh@nnlo`:

`vh@nnlo` with link to `2HDMC`, available on request: [1307.8122; Harlander Liebler Zirke]

DY-initiated, gluon-initiated, bottom-quark initiated contributions

Interesting observable: $R_{WZ\phi} = \sigma^{W\phi}/\sigma^{Z\phi}$



VBF processes in the MSSM/Two Higgs Model in `VBFNLO`:

→ Talks Michael Rauch and Barbara Jäger

`gg2VV` **with additional heavy Higgs** → Talk Nikolas Kauer

Double Higgsstrahlung in the MSSM in `HPAIR`

Implementation of the 2HDM/MSSM in `MG5_aMC@NLO`:

used for a study $gg \to \phi_i\phi_j$ in 2HDM at NLO with PS [Hespel Lopez-Val Vryonidou '14]

`SusHi`-amplitudes to `MG5_aMC@NLO` [Wiesemann]

Outline

Transverse momentum distributions in gluon fusion:

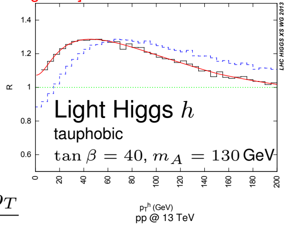
p_T resummation due to divergent logarithms $\log(p_T/m_\phi)$

$\rightarrow$ Talks Hayk Sargsyan and Emanuele Bagnaschi

Treatment of heavy quark masses has particular relevance in BSM models.
[Mantler Wiesemann '12, Grazzini Sargsyan '13, Banfi Monni Zanderighi '13]

Available codes:

▷ Implementation of gluon fusion in the 2HDM/MSSM in POWHEG−BOX:
gg_H_2HDM, gg_H_MSSM
[1111.2854; Bagnaschi Degrassi Slavich Vicini]
Status at the level of YR3

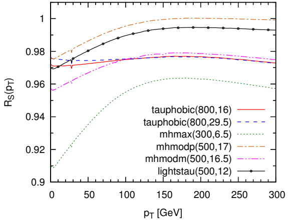


Red: SusHi FO, Black: POWHEG FO,
Blue: POWHEG method: Resummation
via Sudakov form factor and PS)

$$R = \frac{d\sigma^{\mathrm{SUSY}}/dp_T}{d\sigma^{\mathrm{SM}}/dp_T}$$

▷ Add-on to SusHi:
[1409.0531; Harlander Mantler Wiesemann]

Momentum-resummed SusHi MoRe−SusHi
with analytic p_T resummation for 2HDM/MSSM
SusHi-amplitudes to MG5_aMC@NLO (PS)
$\rightarrow$ check sushi.hepforge.org



Exclusive bottom-quark associated Higgs production:

`SusHi` allows for basic cuts with respect to $bb\phi$ in 5FS

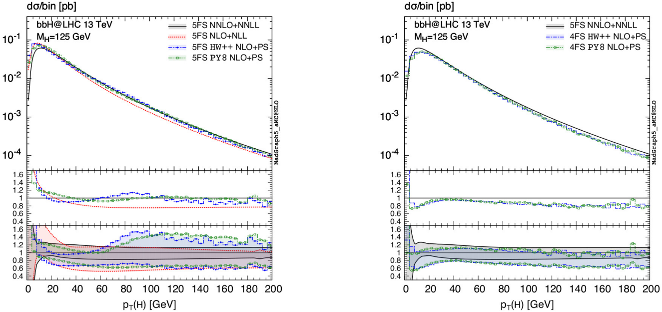
Divergence $p_T \to 0$ asks for resummation in 5FS at NNLO+NNLL (NLO+NLL [Belyaev Nadolsky Yuan '06]) [1403.7196; Harlander Tripathi Wiesemann]

Recent implementation of $bb\phi$ in 4FS/5FS in `MG5_aMC@NLO`

Discussion of exclusive observables

[1409.5301; Wiesemann Frederix Frixione Hirschi Maltoni Torrielli]

$\rightarrow$ Talk Marius Wiesemann



Outline

Detailed uncertainty estimation for gluon fusion:

[1404.0327; Bagnaschi Harlander Liebler Mantler Slavich Vicini]

- ▷ Inclusion of approx. NNLO stop contributions (implemented in SusHi).
- ▷ Uncertainties in the gluon fusion XS prediction:
  - ✓ Renormalization and factorization scale uncertainties
  - ✓ PDF+α_s uncertainties
  - ✓ Uncertainty from heavy SUSY masses expansion + of approximate NNLO stop contributions
  - ✓ Uncertainty from missing contributions to Δ_b
  - ✓ Uncertainty in renormalization of Y_b

Outlook: SusHi version for NMSSM neutral Higgs production on the way.
Singlet Higgs component couples to squarks, **not** to quarks or gauge bosons.
$\longrightarrow$ Squark contributions of relevance.
Probably: Open to any NMSSM spectrum generator through SLHA input.

Outline

Conclusions:

▷ A variety of codes for Higgs production and decay are on the market for the 2HDM and the MSSM.

▷ You can get inclusive cross sections and branching ratios almost to the level as known for the SM Higgs (misses N^{3}LO and soft-gluon resummation for gluon fusion and not as sophisticated for $+n$jets).

▷ Progress for exclusive observables: p_T resummation, $bb\phi$ production.

What I could not cover:

▷ Also interesting are off-shell Higgses in the context of extended Higgs sectors, where the light SM off-shell Higgs can interfere with heavy Higgses.

→ Talks Nikolas Kauer and Christoph Englert

▷ CP-violating 2HDM/MSSM Higgs production and decays.

→ Talk Georg Weiglein

Thank you for your attention and the invitation!
And have some relaxing days (soon)!

The neutral components of the Higgs doublets $H_u = (H_u^+, H_u^0)^T$ and $H_d = (H_d^0, H_d^-)^T$ mix as follows

CP-even Higgs CP-odd Higgs

$$\begin{pmatrix} H_u^0 \\ H_d^0 \end{pmatrix} = \begin{pmatrix} v_u \\ v_d \end{pmatrix} + \frac{1}{\sqrt{2}} R_\alpha \begin{pmatrix} h \\ H \end{pmatrix} + \frac{i}{\sqrt{2}} R_\beta \begin{pmatrix} G \\ A \end{pmatrix} \quad .$$

The mixing matrix is expressed in terms of the “Higgs mixing angle α”

$$R_\alpha = \begin{pmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{pmatrix} \quad .$$

The Higgs sector at LO is determined by fixing $\tan\beta = \frac{v_u}{v_d}$ and m_A^2:

$$m_{h,H} = \frac{1}{2}\left(m_A^2 + m_Z^2 \mp \sqrt{(m_A^2 - m_Z^2)^2 + 4m_Z^2 m_A^2 \sin^2(2\beta)}\right)$$

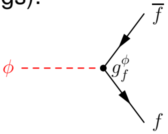
$$\tan(2\alpha) = \tan(2\beta)\frac{m_A^2 + m_Z^2}{m_A^2 - m_Z^2}$$

The lightest Higgs h mass obtains large corrections at higher orders:
`FeynHiggs` [Frank Degrassi Hahn Heinemeyer Hollik Rzehak Slavich Weiglein Williams]
3-loop [Martin '07; Kant Harlander Mihaila Steinhauser '08 '10].

In the MSSM, Higgs couplings to the b-quark can be enhanced by $\tan\beta$:
Relative strength of the Higgs boson couplings g_f^ϕ with $\phi \in \{h, H, A\}$ to the SM fermions (with respect to the SM Higgs boson couplings):

$$g_u^h = \frac{\cos\alpha}{\sin\beta} \qquad g_u^H = \frac{\sin\alpha}{\sin\beta} \qquad g_u^A = \frac{1}{\tan\beta}$$

$$g_d^h = -\frac{\sin\alpha}{\cos\beta} \qquad g_d^H = \frac{\cos\alpha}{\cos\beta} \qquad g_d^A = \tan\beta$$

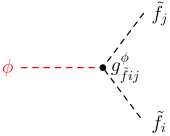


In addition, the superpartners of the quarks, the squarks, are relevant

$$\mathcal{L} \supset -(\tilde{q}_L^\dagger, \tilde{q}_R^\dagger)\mathcal{M}_{\tilde{q}}^2 \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

with the mass matrix:

$$\mathcal{M}_{\tilde{q}}^2 = \begin{pmatrix} M_{qL}^2 + m_q^2 + m_{E1}^2 & m_q(A_q - \mu\kappa) \\ m_q(A_q - \mu\kappa) & M_{qR}^2 + m_q^2 + m_{E2}^2 \end{pmatrix}$$



They form two mass eigenstates:

$$\begin{pmatrix} \tilde{q}_1 \\ \tilde{q}_2 \end{pmatrix} = \begin{pmatrix} \cos\theta_{\tilde{q}} & \sin\theta_{\tilde{q}} \\ -\sin\theta_{\tilde{q}} & \cos\theta_{\tilde{q}} \end{pmatrix} \begin{pmatrix} \tilde{q}_L \\ \tilde{q}_R \end{pmatrix}$$

$m_{E1}^2 = m_z^2 \cos(2\beta)(T_q^3 - Q_q \sin^2\theta_W)$; $m_{E2}^2 = m_Z^2 \cos(2\beta) Q_q \sin^2\theta_W$; $\kappa = \tan\beta(d), \cot\beta(u)$

Gluon fusion at LO using $\tau_\phi = m_\phi^2/s$:

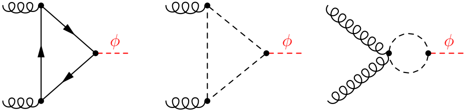
$$\sigma(pp \to \phi + X) = \sigma_0^\phi \tau_\phi \frac{d\mathcal{L}^{gg}}{d\tau_\phi}$$

Partonic → Hadronic XS:

$$\frac{d\mathcal{L}^{gg}}{d\tau} = \int_\tau^1 \frac{dx}{x} g(x) g(\tau/x)$$

LO partonic cross section (XS):

$$\sigma_0^\phi = \frac{G_F \alpha_s^2}{288\sqrt{2}\pi} |\mathcal{A}^\phi|^2$$



$$\mathcal{A}^\phi = \sum_{q \in \{t,b\}} \left(a_q^{\phi,(0)} + a_{\tilde{q}}^{\phi,(0)} \right)$$

with

$$a_q^{\phi,(0)} = g_q^\phi \frac{3\tau_q}{2} (1 + (1 - \tau_q^\phi) f(\tau_q^\phi))$$

Quark contributions

Squark contributions

$$\tilde{a}_q^{\phi,(0)} = -\frac{3\tau_q^\phi}{8} \sum_{i=1}^{2} g_{\tilde{q}ii}^\phi (1 - \tau_{\tilde{q}i}^\phi f(\tau_{\tilde{q}i}^\phi))$$

$$\tau_q^\phi = 4m_q^2/m_\phi^2, \quad \tau_{\tilde{q}i}^\phi = 4m_{\tilde{q}i}^2/m_\phi^2, \quad f(\tau) = \begin{cases} \arcsin^2 \frac{1}{\sqrt{\tau}} & \tau \geq 1 \\ -\frac{1}{4} \left(\log \frac{1+\sqrt{1-\tau}}{1-\sqrt{1-\tau}} - i\pi \right)^2 & \tau < 1 \end{cases}$$

Cancellation of logs - Bottom 2-loop contributions:

[Spira Djouadi Graudenz Zerwas '95] in the notation of [Degrassi Slavich '10]

$$G_b^{2\ell} = C_F\, G_b^{(g,C_F)} + C_A\, G_b^{(g,C_A)}$$

$$2\, m_b^2\, G_b^{(g,C_F)} = \mathcal{F}_{1/2}^{(2\ell,a)}(\tau_b) + \mathcal{F}_{1/2}^{(2\ell,b)}(\tau_b) \left(\ln \frac{m_b^2}{Q^2} - \frac{1}{3} \right)$$

$$2\, m_b^2\, G_b^{(g,C_A)} = \mathcal{G}_{1/2}^{(2\ell,C_A)}(\tau_b)$$

In the limit $\tau_b = 4m_b^2/m_\phi^2 \ll 1$ the above expressions reduce to:

$$\mathcal{F}_{1/2}^{(2\ell,a)}(\tau) = -\tau \Big[9 + \frac{9}{5}\zeta_2^2 - \zeta_3 - (1+\zeta_2+4\,\zeta_3)\ \ln(\frac{-4}{\tau}) - (1-\zeta_2)\ \ln^2(\frac{-4}{\tau}) + \frac{1}{4}\ \ln^3(\frac{-4}{\tau}) + \frac{1}{48}\ \ln^4(\frac{-4}{\tau}) \Big] + \mathcal{O}(\tau^2)$$

$$\mathcal{F}_{1/2}^{(2\ell,b)}(\tau) = 3\,\tau \left[1 + \frac{1}{2}\ \ln(\frac{-4}{\tau}) - \frac{1}{4}\ \ln^2(\frac{-4}{\tau}) \right] + \mathcal{O}(\tau^2)$$

$$\mathcal{G}_{1/2}^{(2\ell,C_A)}(\tau) = -\tau \Big[3 - \frac{8}{5}\zeta_2^2 - 3\,\zeta_3 + 3\,\zeta_3\ \ln(\frac{-4}{\tau}) - \frac{1}{4}\,(1+2\,\zeta_2)\ \ln^2(\frac{-4}{\tau}) - \frac{1}{48}\ \ln^4(\frac{-4}{\tau}) \Big] + \mathcal{O}(\tau^2)$$

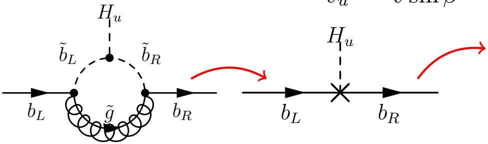
For $Q = m_b$ the various logarithms accidentally cancel in case of gluon fusion.

Resummation of large $\tan\beta$-enhanced terms in the MSSM

$$\mathcal{L} \supset -Y_t H_u Q t_R + Y_b H_d Q b_R$$

Using $\langle H_u\rangle = v_u, \langle H_d\rangle = v_d$ and $v_d^2 + v_u^2 = v^2, \tan\beta = v_u/v_d$ we define

$$Y_t = \frac{m_t}{v_u} = \frac{m_t}{v\sin\beta}, \qquad Y_b = \frac{m_b}{v_d} = \frac{m_b}{v\cos\beta}$$



$$\mathcal{L}^{\text{eff}} \supset Y_b H_d Q b_R - \tilde{Y}_b H_u^* Q b_R$$

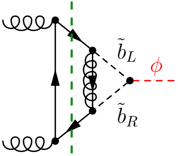
$$\Delta_b = \frac{\tilde{Y}_b v_u}{Y_b v_d} =: \epsilon \tan\beta$$

The effective Lagrangian motivates:

$$m_b = Y_b v_d + \tilde{Y}_b v_u = Y_b v_d (1 + \epsilon\tan\beta)$$

$$\Rightarrow \quad Y_b = \frac{m_b}{v_d(1+\Delta_b)}$$

This replacement implies a resummation of large $\tan\beta$-enhanced terms:

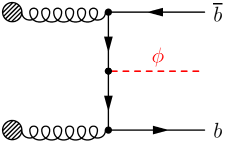


[Hall Rattazzi Sarid ’93, Hempfling ’94, Carena Garcia Nierste Wagner ’99, Guasch Häfliger Spira ’03]

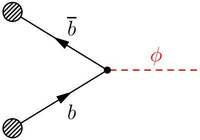
Bottom-quark annihilation:
$pp \to (b\bar{b})\phi + X$ for enhanced couplings to b-quarks relevant $\to$ MSSM!

4 flavour scheme (4FS)
Collinear logarithms $\propto \log(m_b/m_\phi)$

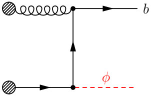
5 flavour scheme (5FS)
Resummmation of logarithms
b quarks as partons



$\Longleftrightarrow$



Calculation of inclusive cross section at NNLO in the 5FS:
`bbh@nnlo` [Harlander Kilgore '03]



Distributions for Higgs+jet(s) production in the 5FS
[Harlander Ozeren Wiesemann '10, Buehler Herzog Lazopoulos Mueller '12]