POWHEG MiNLO

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Plan of the talk

- Basics of MiNLO
- Accuracy for inclusive observables
- MiNLO': merging with no merging
- NNLO+PS

MiNLO

- Started from a study seeking an optimal scale choice for NLO calculations in processes with associated jets (Hamilton,Zanderighi,P.N. 2012)
- Turned out to be particularly suited for NLO+PS applications. Implemented as automatic feature in the POWHEG-BOX (HNZ 2012).
- An avenue towards merging without merging scales, fully demonstrated for processes of production of a massive colourless system in association with one jet (Hamilton,Zanderighi,P.N. 2013). Implemented in the POWHEG BOX for Higgs and Z/W (HNZ 2013), $H+Z/W$ (Luisoni,Oleari,Tramontano,P.N. 2013).
- Used to build NNLO+PS generators (Hamilton,Re,Zanderighi,P.N. 2013; Karlberg,Re,Zanderighi 2014)

Basics

Extension to NLO of CKKW (Catani,Krauss,Kuhn,Webber,2001):
CKKW basics:

- Use LO matrix elements with up to a given number of associated partons
- Consider only configurations with the smallest relative transverse momentum $> Q_0$.
- Reconstruct a branching history from the kinematic of the event, using a clustering algorithm (for example, by recursively merging the pair of partons with smallest relative transverse momentum).
- Assign running couplings and Sudakov form factors as in the Parton Shower approximation to include LL virtual corrections.
- Feed the kinematics of the event to a parton shower to generate splittings with transverse momenta below Q_0.

The CKKW procedure is improved to deal with NLO accuracy in `MiNLO`.
The most important items are:

- The scale Q_0 is set to the last clustering scale for Born and Virtual contributions, and to the second last clustering scale for the Real (i.e. is not an externally imposed scale).

- The renormalization scale appearing explicitly in the virtual corrections is set to the geometric average of the nodal scales.
  The factorization scale is set to Q_0.

- NLO contributions introduced by the CKKW Sudakov form factors are subtracted from the virtual corrections:

$$B \Rightarrow B\left(1 - \sum_{ij}\left[\Delta^{(1)}_{f_{ij}}(Q_0, q_i) - \Delta^{(1)}_{f_{ij}}(Q_0, q_j)\right] - \sum_{l}\Delta^{(1)}_{f_l}(Q_0, q_i)\right)$$

Main features of the MiNLO improved generators:

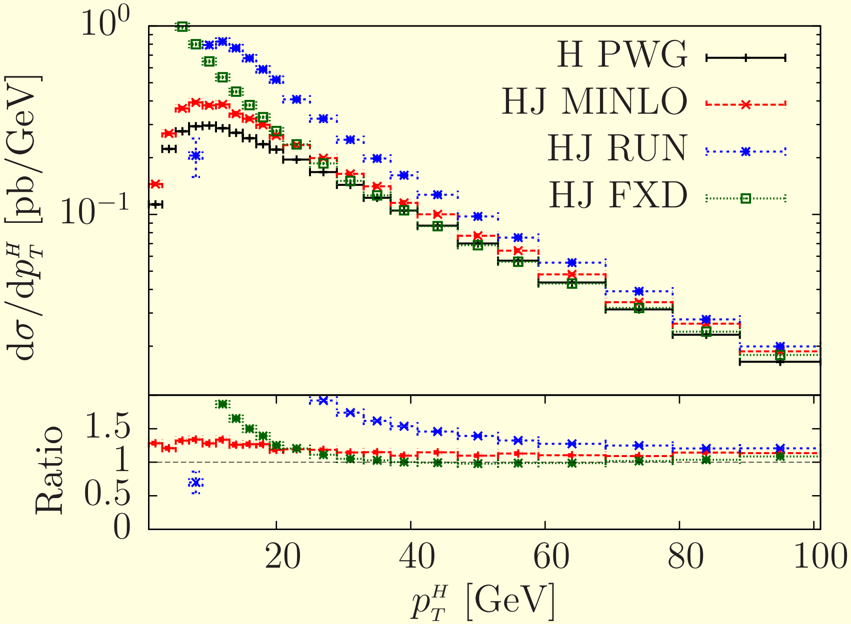
Thanks to the above subtraction and to the Sudakov form factors (that togeter amount to the exponentiation of large logs appearing in the inclusive cross section), the MiNLO improved generators are well behaved also for inclusive quantities.
(In what follows by "inclusive quantities" we mean in the "QCD theorists" sense: cross sections where radiation is integrated out, with full real-virtual cancellation of divergences)

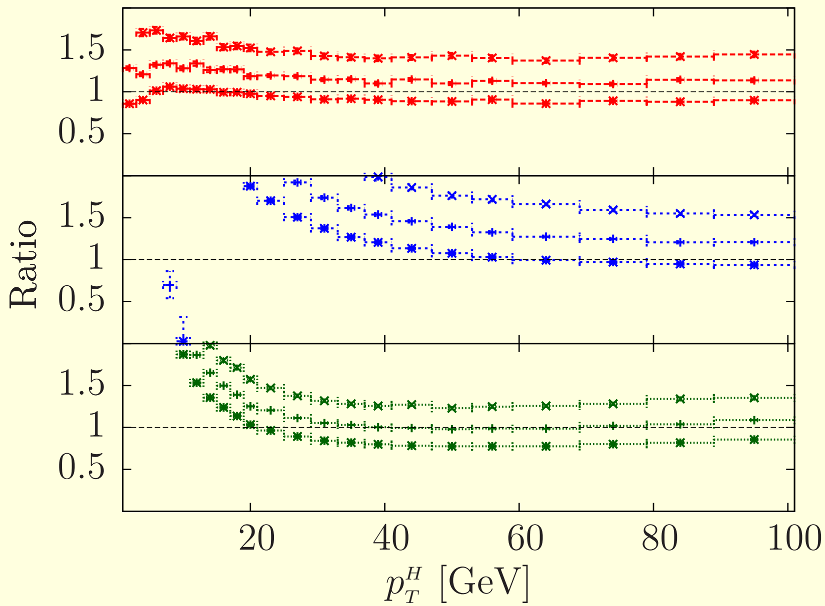
Example: H, HJ and HJJ MiNLO generators:

	H	HJ	HJJ
Total Higgs σ_{H} at 8 TeV, pb	13.2	16.2	17.8

Without MiNLO the HJ and HJJ results are undefined: the HJ result divergent already at the Born level, and the HJJ result divergent already if we require one jet, even worse if we don't require jets.

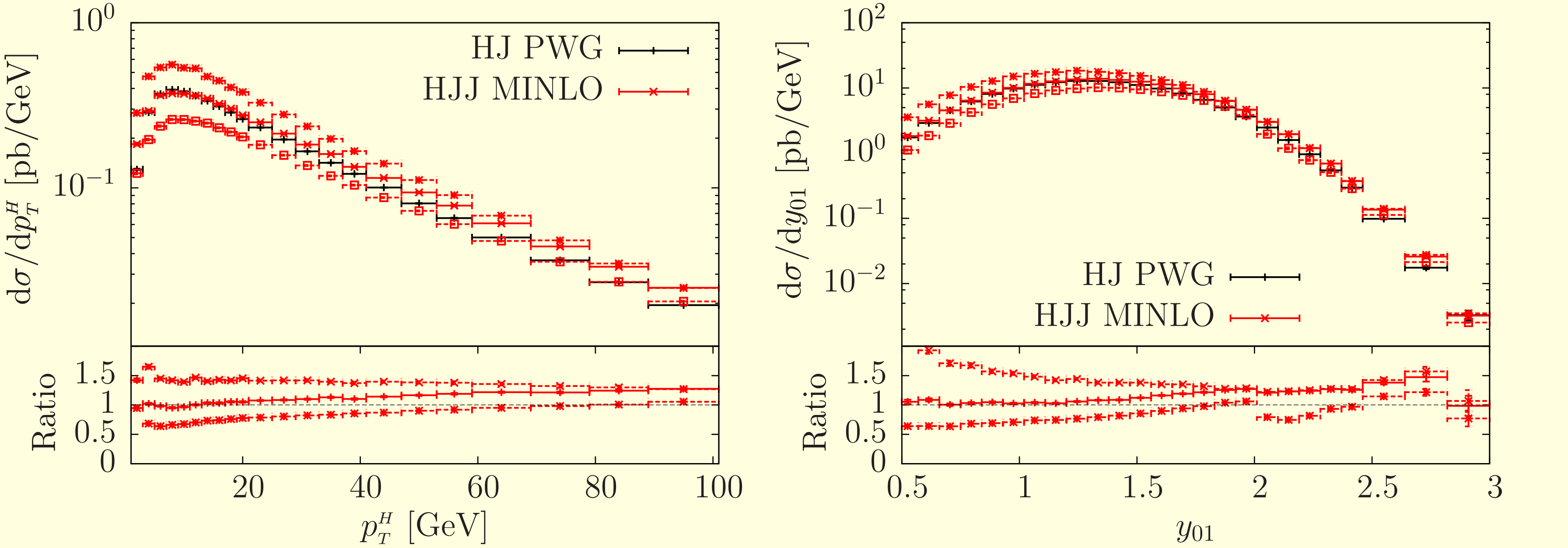
HJ results





- H PWG: the (showered) gg_H POWHEG BOX result.
- FXD and RUN (NLO with fixed and running scales) need a generation cut (or Born suppression) at small p_T.
  The MiNLO result is instead FINITE (up to a cut-off $\approx\Lambda$).

HJJ results



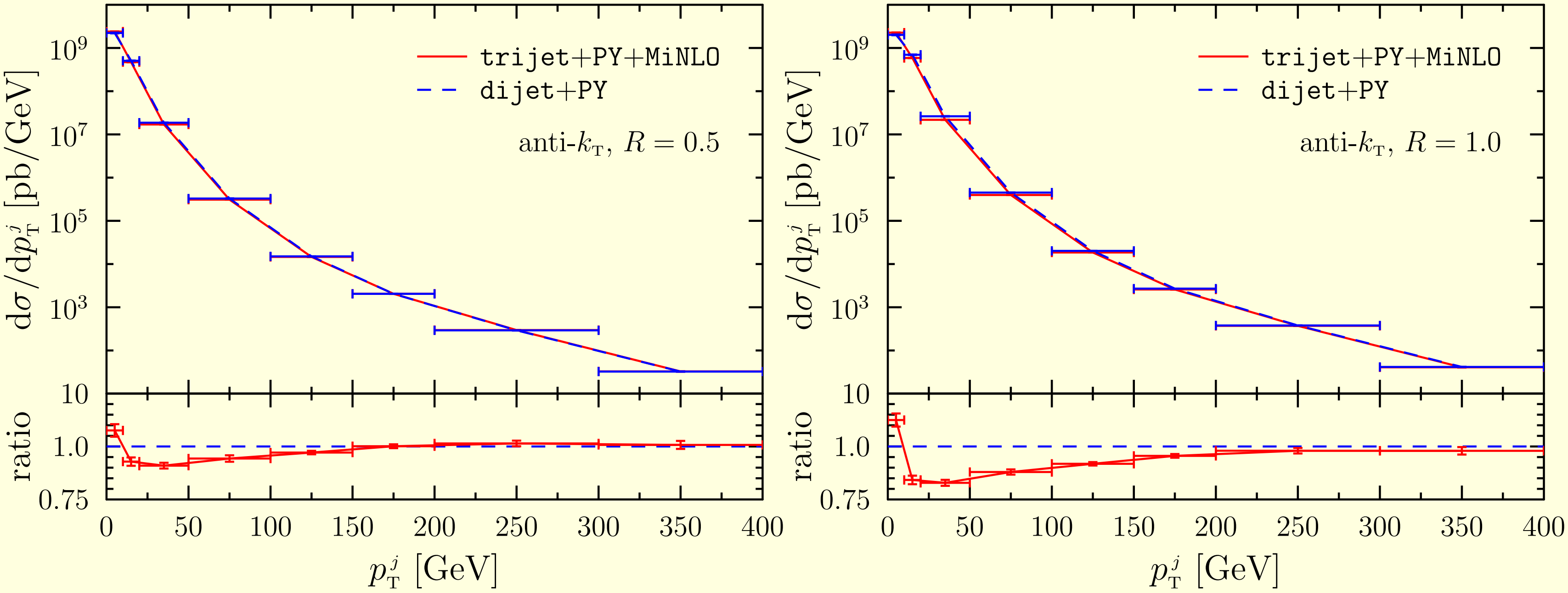
Notice: Fixed order results not available for these distributions.The Born term alone is divergent, since it is integrated in the second jet with no restriction.

However: the `MiNLO` result is still sensible, and is sensible also at very low $p_{\rm T}$

(although with larger scale uncertainty: no longer NLO!)

The HJ PWG result here refers to the fully showered `HJ MiNLO POWHEG` result.

An interesting example: `dijets` vs. `thrijet` with `MiNLO`:



`dijet`: Alioli,Hamilton,Oleari,Re,P.N. 2011;
threejet: Kardos, Oleari, P.N. 2014;

How accurate is the inclusive cross section in MiNLO?

Preliminary formula:

$$\int_{\Lambda}^{M_{\rm H}} \frac{{\rm d}p_T}{p_T} L^m \alpha_s^n \, \Delta(L) \approx \int_{\Lambda}^{M_{\rm H}} {\rm d}L \, L^m \alpha_s^n \, \Delta(L) \approx \left[\alpha_s(M_{\rm H}^2)\right]^{n-\frac{m+1}{2}}$$

where $L = \log M_{\rm H}/p_T$.

So: each power of L counts as $\alpha^{-1/2}$.
Can be easily proved rigorously.

Also obvious intuitively: $\Delta(L) \approx \exp\left[-\alpha L^2\right]$, so the dominant contribution to the integral is when $L \approx \alpha^{-1/2}$

HJ: how accurate are inclusive cross sections?

The MiNLO improved cross section, differential in the Higgs p_T (and in other inclusive variables, like the Higgs rapidity), has the expression

$$\frac{d\sigma}{dp_T} = \Delta(p_T)\,\sigma_0\left[\alpha\frac{L}{p_T} + \alpha\frac{1}{p_T} + \alpha + \alpha^2\frac{L^3}{p_T} + \alpha^2\frac{L^2}{p_T} + \alpha^2\frac{L}{p_T} + \alpha^2\frac{1}{p_T} + \alpha^2\right],$$

The grey terms are those that are subtracted away from the expansion of Δ:

$$\Delta(p_T) \approx 1 + \alpha(L^2 + L) + \mathcal{O}(\alpha^2),$$

or in other words, the grey terms are exponentiated in Δ. An inaccuracy of NLL order in Δ leads to an incomplete cancellation of the $\alpha^2 L^2/p_T$ term, that upon integration counts as $\alpha^2\, L^3 \approx \alpha^2\alpha^{-1.5} \approx \alpha^{0.5}$ times the Born term σ_0. So, the integrated cross section is not even LO accurate in the usual sense, since we have leftover of order $\alpha^{0.5}$ rather than α.

What do we need to reach NLO accuracy?

We don't include terms of order α^3. These can yield terms like

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = \Delta(p_T)\,\sigma_0\left[\alpha\frac{L}{p_T} + \ldots + \alpha^3\frac{L^5}{p_T} + \alpha^3\frac{L^4}{p_T} + \alpha^3\frac{L^3}{p_T} + \alpha^3\frac{L^2}{p_T} + \alpha^3\frac{L}{p_T}\ldots.\right],$$

All grey terms must already be included in Δ. In fact, $\alpha^3 L/p_T$ is the first term that upon integration yields a contribution like $L^2\alpha^3 \approx \alpha^2$, beyond NLO accuracy, and the Sudakov form factor must be capable to give the correct $\alpha^3 L^2/p_T$ term.

$$\Delta(p_T) = -\int_{p_T^2}^{M_\mathrm{H}^2} \frac{\mathrm{d}q^2}{q^2}\left[A(\alpha_s(q^2))\log\frac{M_\mathrm{H}^2}{q^2} + B(\alpha_s(q^2))\right],$$

where $A(\alpha_s) = A_1\alpha_s + A_2\alpha_s^2 + \ldots$ and $B(\alpha_s) = B_1\alpha_s + B_2\alpha_s^2 + \ldots$
Must include up to the B_2 term to achieve NLO accuracy! In fact:

$$\exp\left[-\int_{p_T^2}^{M_H^2} \frac{\mathrm{d}q^2}{q^2}\,B_2\,\alpha_s^2(q^2)\right] \approx 1 + \alpha_s^2 L$$

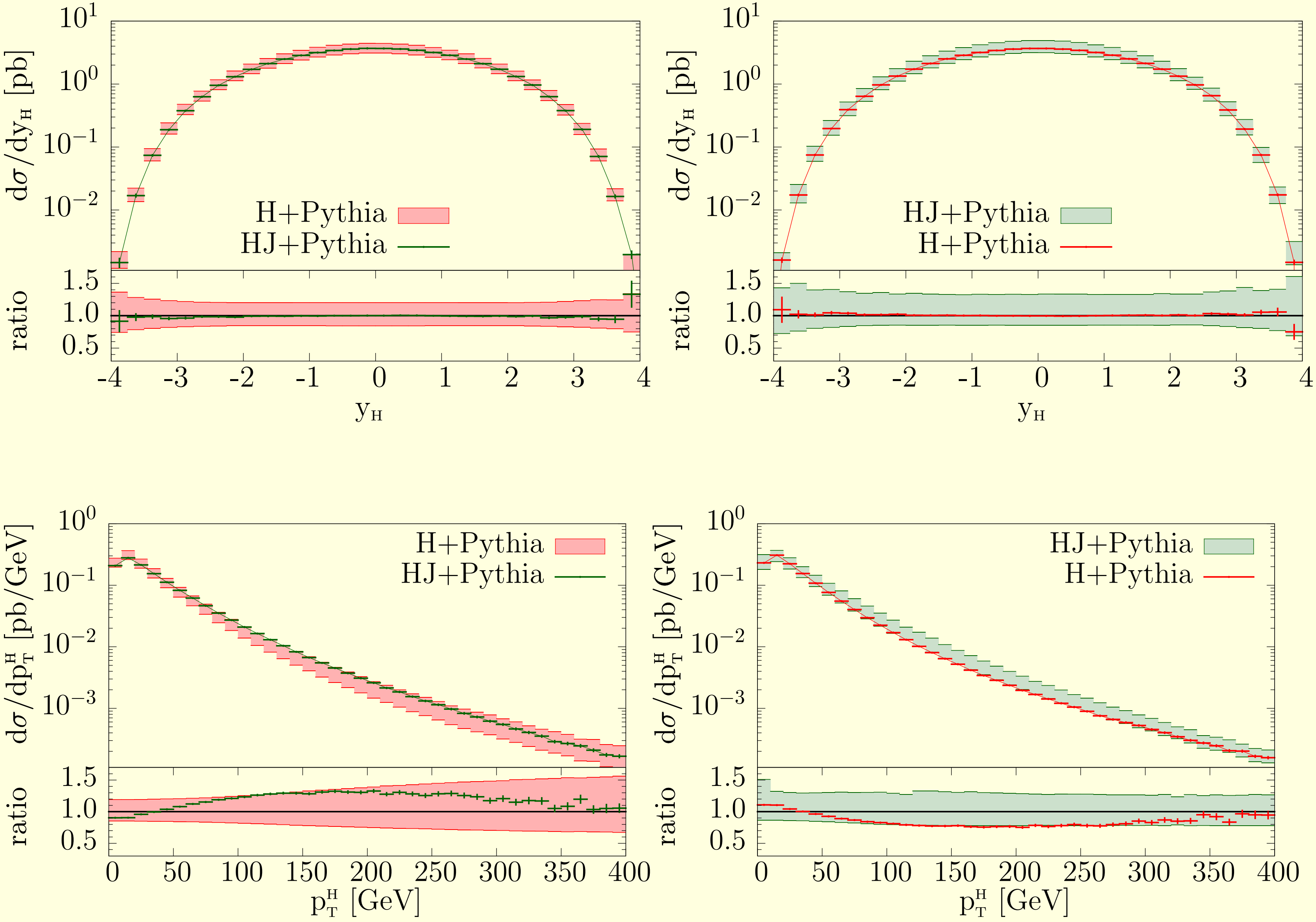
Combined with $\alpha L/p_T$ yields $\alpha^3 L^2/p_T \to \alpha^3 L^3 \approx \alpha^{1.5}$ (relative $\alpha^{0.5}$ w.r.t. NLO).

Summarizing:

- If Sudakov's are not NLL accurate, inclusive cross section are LO accurate up to corrections of relative order $\alpha_s^{0.5}$ (not quite LO).
- If Sudakov are NLL accurate (and include A_2), inclusive cross sections are NLO up to corrections of relative order $\alpha_s^{0.5}$ (not quite NLO).
- If Sudakov are NLL accurate and include A_2 and B_2, inclusive cross sections are NLO up to corrections of relative order α_s (i.e. TRUE NLO): `MiNLO`-prime, or `MiNLO`' in this talk.

In case of $H/W/Z + 1\,\mathrm{jet}$, it is in fact possible to modify the MiNLO Sudakov form factor by carefully including the B_2 term, so that integrating over the radiated jet we achieve NLO accuracy for inclusive $H/W/Z$ distributions. (Hamilton,Oleari,Zanderighi,P.N. 2012)

Also applied to HV production (Luisoni,Oleari,Tramontano,P.N. 2013)


H+Pythia
HJ+Pythia
dσ/dy_H [pb]
ratio
y_H
HJ+Pythia
H+Pythia
dσ/dp_T^H [pb/GeV]
p_T^H [GeV]

Merging without merging

Normally we think of merging as taking

- `H` (NLO accurate for inclusive observables) for $p_T^{\mathrm{H}} < Q_0$;
- `HJ` (NLO accurate for $H + j$ observables) for $p_T^{\mathrm{H}} > Q_0$;
- Merge the two samples

in order to achive NLO accuracy both for inclusive and one jet observables.

The `MiNLO`' generators achieves this accuracy without doing any merging.

Arguments similar to those that have lead to `MiNLO`' have convinced us that standard merging using a separation scale cannot really achieve NLO accuracy for both inclusive and 1-jet inclusive observables.

Difficulties in establishing rigorous merging procedures

If we introduce a matching cut Q_0 below which we use the `H` generator, and above which we use the `HJ` one, we have for the `H` generator:

$$\int^{Q_0} \mathrm{d}p_T \frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = \mathrm{NLO} \times (1 + \mathcal{O}(\alpha_s)) - \int_{Q_0} \mathrm{d}p_T \frac{\mathrm{d}\sigma}{\mathrm{d}p_T}$$

If Q_0 is large ($\sim M_{\mathrm{H}}$), the subtraction has $\mathrm{NLO} \times (1 + \mathcal{O}(\alpha_s))$ accuracy. But if $Q_0 \ll M_{\mathrm{H}}$ (isn't this what we want?) the accuracy is always less than that. We have

$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T} = \Delta(L)\, \sigma_0 \left(\underbrace{\alpha \frac{L}{p_T} + \alpha \frac{1}{p_T} + \alpha}_{\text{present}} + \underbrace{\alpha^2 \frac{L^3}{p_T} + \alpha^2 \frac{L^2}{p_T}}_{\text{exponentiated in } \Delta} + \underbrace{\alpha^2 \frac{L}{p_T}}_{\text{missing}} \right)$$

Missing term: $\alpha^2 \log^2 M_{\mathrm{H}}/Q_0$; not of order α_s^2 unless $Q_0 \approx M_{\mathrm{H}}$!!!

Notice that the unrestricted integral of the H transverse momentum distribution is forced to be NLO accurate by unitarity constraints in the NLO+PS matching.

MiNLO and NNLO+PS

At the present stage, MiNLO for processes with one associated jet production of colour neutral systems (Higgs, W/Z, HW, etc), allows for full NLO accuracy for inclusive quantities involving and not involving the extra jet. It is possible (and not difficult) to extend the present framework in such a way that a true NNLO+PS generator is built, without the use of any matching scales.

In the example of Higgs production: the MiNLO HJ generator can be promoted to a NNLO+PS generator by simply reweighting the events with the factor

$$\frac{\mathrm{d}\sigma^{(\mathrm{NNLO})}}{\mathrm{d}y_{\mathrm{H}}} \Big/ \frac{\mathrm{d}\sigma^{(\mathtt{HJ})}}{\mathrm{d}y_{\mathrm{H}}} = 1 + \mathcal{O}(\alpha_S^2)$$

or with alternative procedures where the reweighting takes place mostly for events with $p_T^{(\mathrm{H})} \lesssim M_{\mathrm{H}}$.

NNLO+PS generator with MiNLO

Given an NLO+PS Higgs merged generator, accurate at order α_s^3 for fully inclusive quantities, and at order α_s^4 for Higgs plus one jet observables, it is easy to prove that NNLO accuracy can be achieved as follows:

- Generate events with the NLO+PS merged generator
- Reweight the event cross section with the factor

$$\frac{\frac{\mathrm{d}\sigma^{\mathrm{NNLLO}}}{\mathrm{d}\,y_{\mathrm{H}}}}{\frac{\mathrm{d}\sigma^{\mathrm{NLO+PS}}}{\mathrm{d}\,y_{\mathrm{H}}}}$$

where y_{H} is the Higgs rapidity in the generated event.

In order for the proof to work, it is essential that

$$\frac{\frac{d\sigma^{\mathrm{NNLLO}}}{d\,y_{\mathrm{H}}}}{\frac{d\sigma^{\mathrm{NLO+PS}}}{d\,y_{\mathrm{H}}}} = \frac{\sigma^{(0)}_{\mathrm{NNLO}} + \alpha_s \sigma^{(1)}_{\mathrm{NNLO}} + \alpha_s^2 \sigma^{(2)}_{\mathrm{NNLO}}}{\sigma^{(0)}_{\mathrm{NLO+PS}} + \alpha_s \sigma^{(1)}_{\mathrm{NLO+PS}}} = 1 + \mathcal{O}(\alpha_s^2)$$

i.e. that $d\sigma_{\mathrm{NLO+PS}}/dy_{\mathrm{H}}$ is NLO accurate.

If the ratio was $1 + \mathcal{O}(\alpha_s)$, distributions like the Higgs transverse momentum, that have the expansion:

$$\frac{d\sigma}{dp_t} = \alpha_s^3 \frac{d\sigma^{(3)}}{dp_t} + \alpha_s^4 \frac{d\sigma^{(3)}}{dp_t}$$

so that by reweighting:

$$(1 + \mathcal{O}(\alpha_s)) \times \frac{d\sigma}{dp_t} = \alpha_s^3 \frac{d\sigma^{(3)}}{dp_t} + \alpha_s^4 \frac{d\sigma^{(3)}}{dp_t} + \underbrace{\alpha_s^3 \frac{d\sigma^{(3)}}{dp_t} \times \mathcal{O}(\alpha_s)}_{\mathcal{O}(\alpha_s^4)}$$

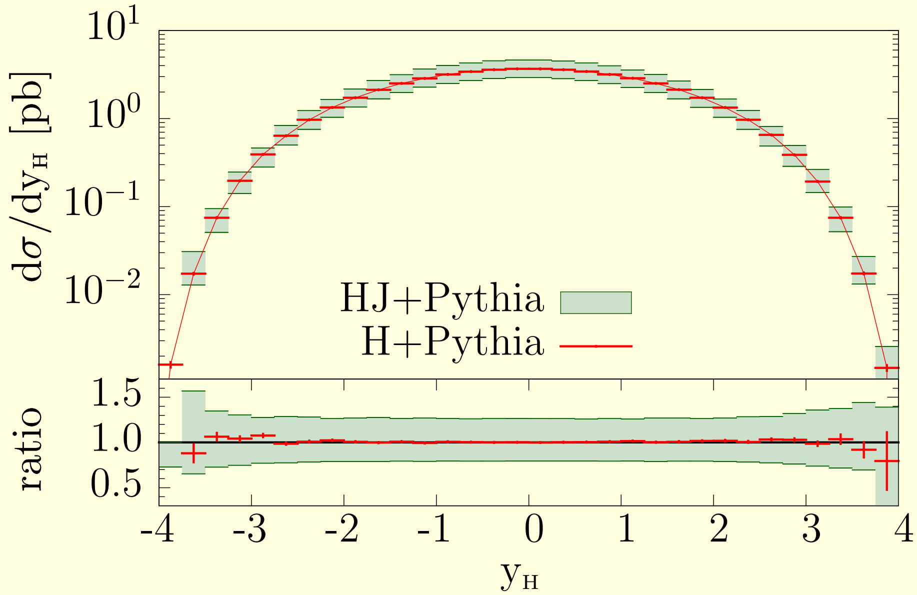
we get spurious terms of order α_s^4, spoiling its α_s^4 accuracy.

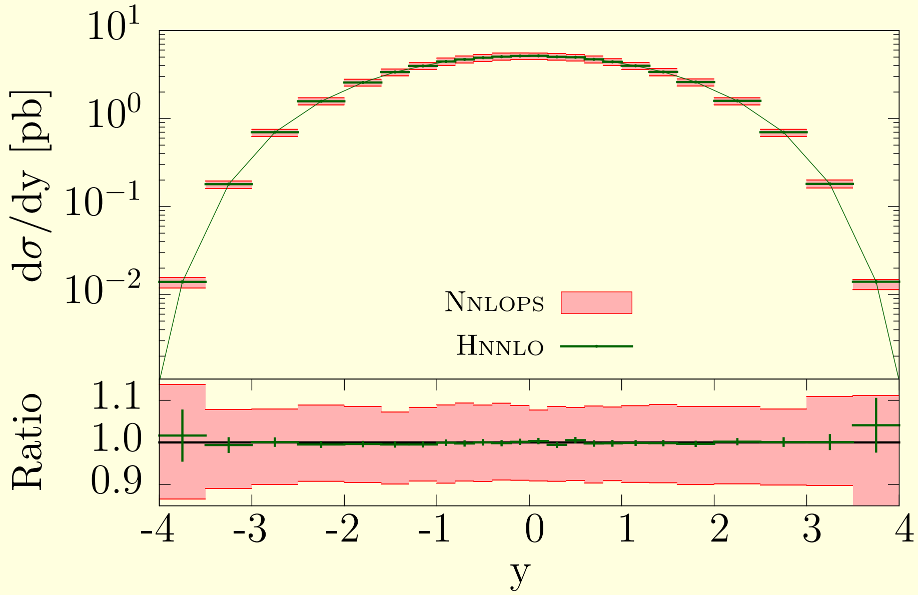
Variants

At fixed order, NNLO corrections (α_s^4) enter with Born level kinematics.
Away from Born level kinematics we just Higgs plus 1 parton at NLO (again of order α_s^4).
In NNLO+PS, the Born kinematic region is depressed by Sudakov form factor.
NNLO correction canno sit there; they must be spread over (at least) the Sudakov region. Can be spread uniformly over the whole phase space without violating α_s^4 accuracy. Or can be spread over a transverse momentum region of size of the order of the Higgs mass. This is a true uncertainty that must be explored.
Rather than reweighting the full HJ generator, one can reweight a fraction of its events of order $h^2/(p_T^2 + h^2)$, where p_T is the Higgs transverse momentum, or the transverse momentum of the hardest jet, using the NNLO cross section reweighted in the same way. The parameter h (slightly reminiscent of the h parameter in POWHEG) can be taken from somewthing of the order of the Higgs mass, up to infinity.

Applied to Higgs production at NNLO+PS:

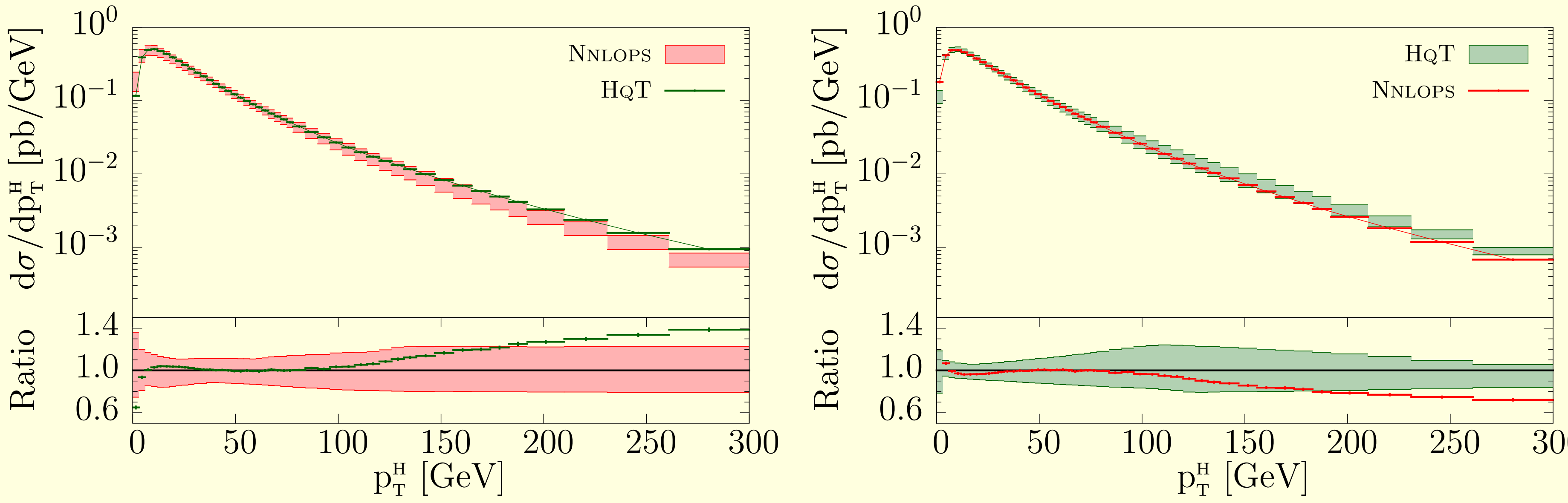
Zanderighi,Hamilton,Re,P.N. Aug. 2013, reweighting MiNLO generator from Zanderighi,Hamilton,Oleari,P.N. 2012; hnnlo (Grazzini)





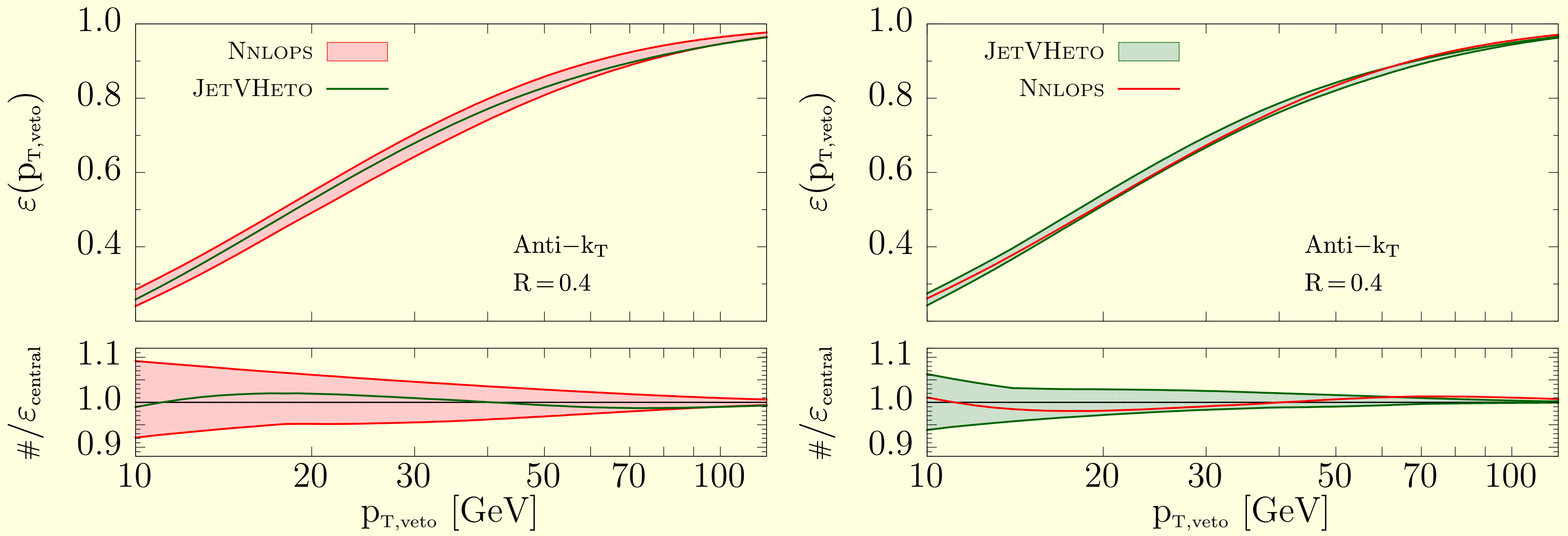
Accuracy: (left) NLO+PS: ~30%, (right) NNLO+PS:~10%

Higgs transverse momentum comparison to HqT



- HqT: dedicated program for NNLO+NNLL calculation of $\mathrm{d}\sigma^{\mathrm{H}}/\mathrm{d}p_{\mathrm{T}}$, Bozzi,Catani,De Florian,Ferrera,Grazzini,Tommasini
- Good agreement at small/moderate p_{T}
- Large p_{T}: it will be interesting to compare to $H+1j$ NNLO calculation by Boughezal,Caola,Melnikov,Petriello,Schulze Feb. 2013

Jet-veto efficiency: comparison to JetVHeto



- JetVHeto: dedicate NNLO+NNLL, Banfi,Salam,Monni,Zanderighi 2012
- good agreement everywhere

Conclusions

- `MiNLO` method: improve multi-jet generators near phase space edges
- `MiNLO`': merging without merging; for now only for colourless systems plus one jet
- `MiNLO`': NNLO+PS possible, with no matching/separation scales. Applied to `HJ`, Drell-Yan (Karlberg,Re,Zanderighi,2014)
- Can `MiNLO`' be extended to all processes? (I believe so)