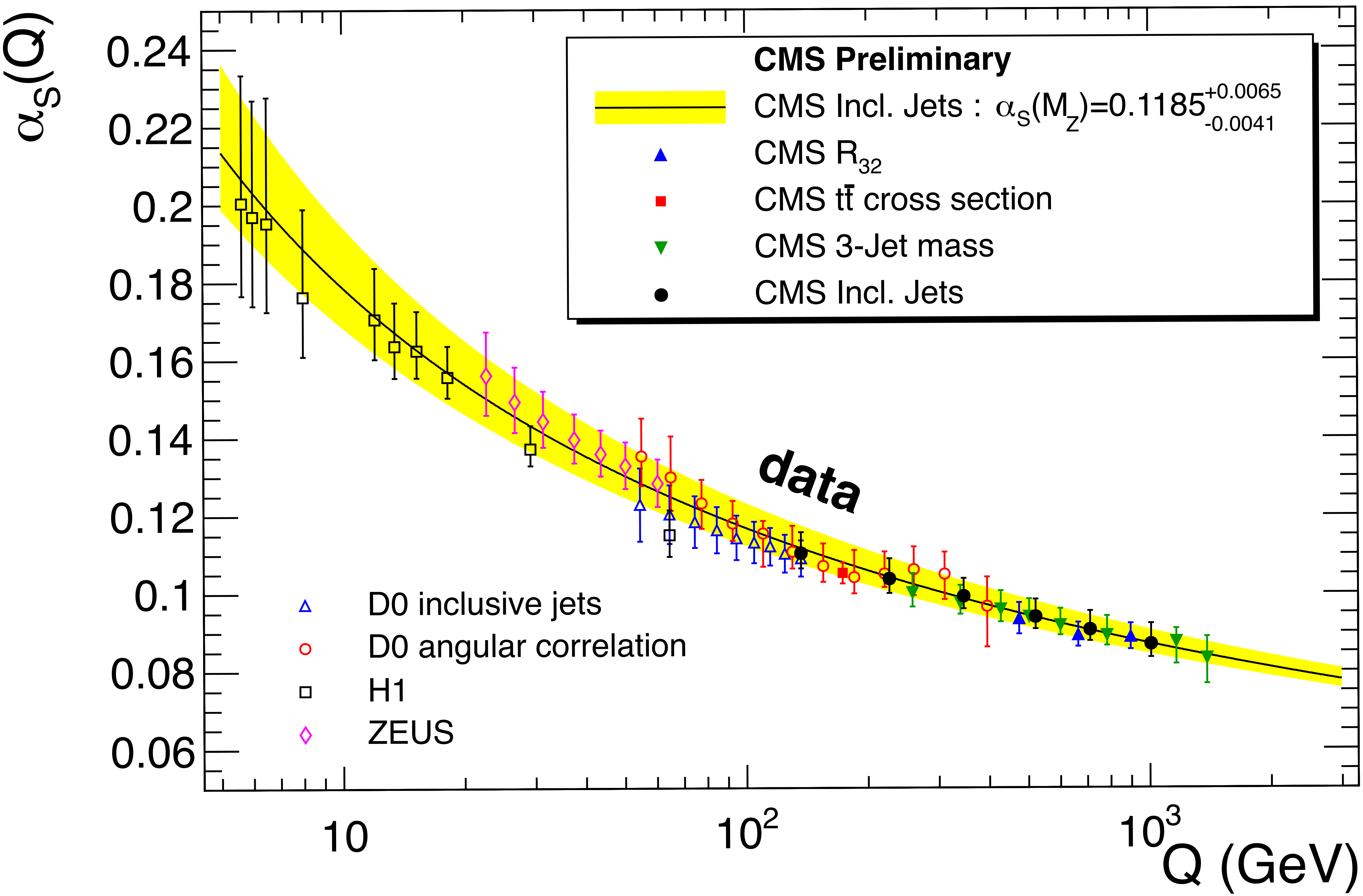
from asymptotic freedom to asymptotic safety

with F Sannino, 1406.2337

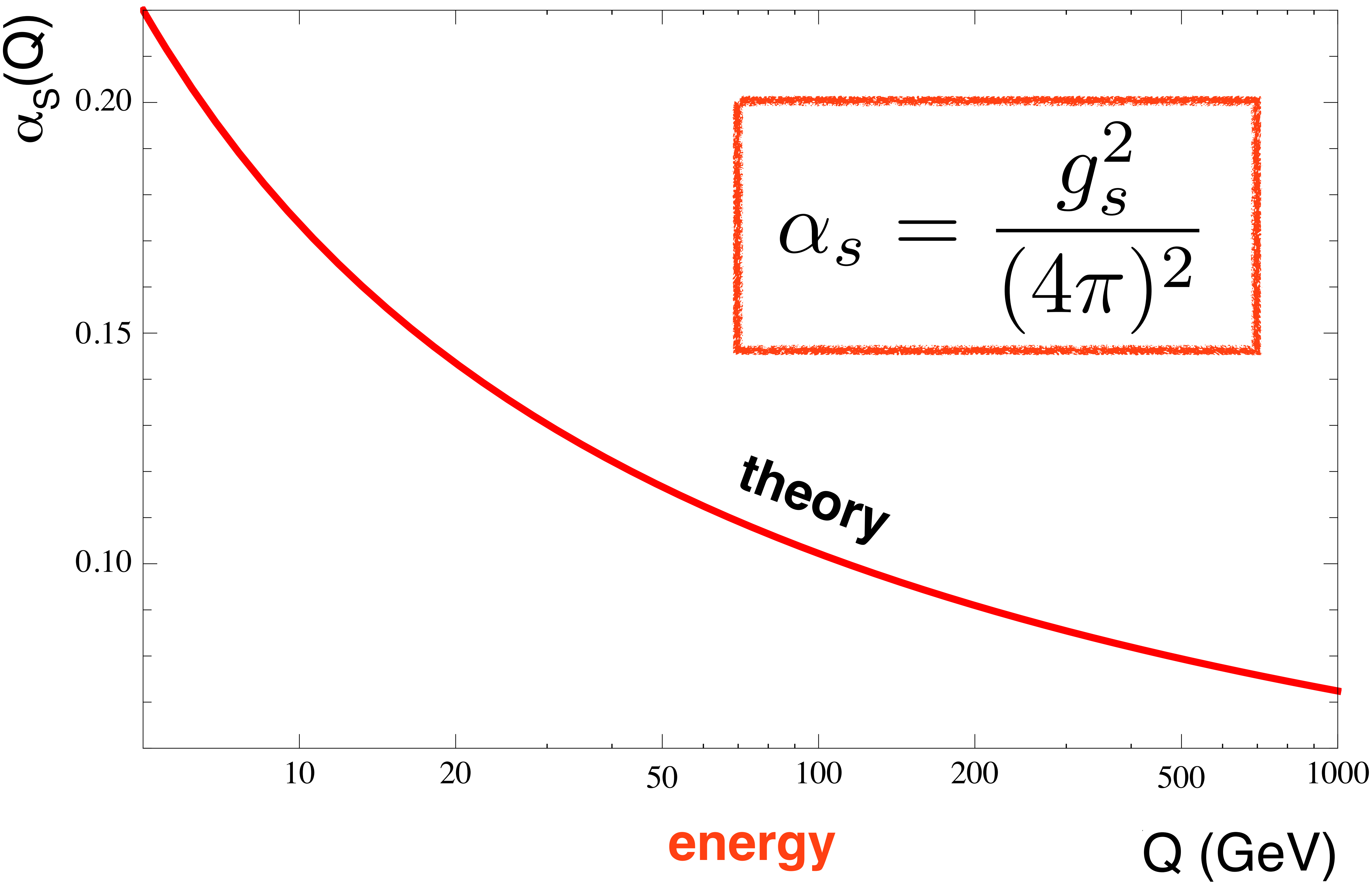
Daniel F Litim

University of Sussex

asymptotic freedom



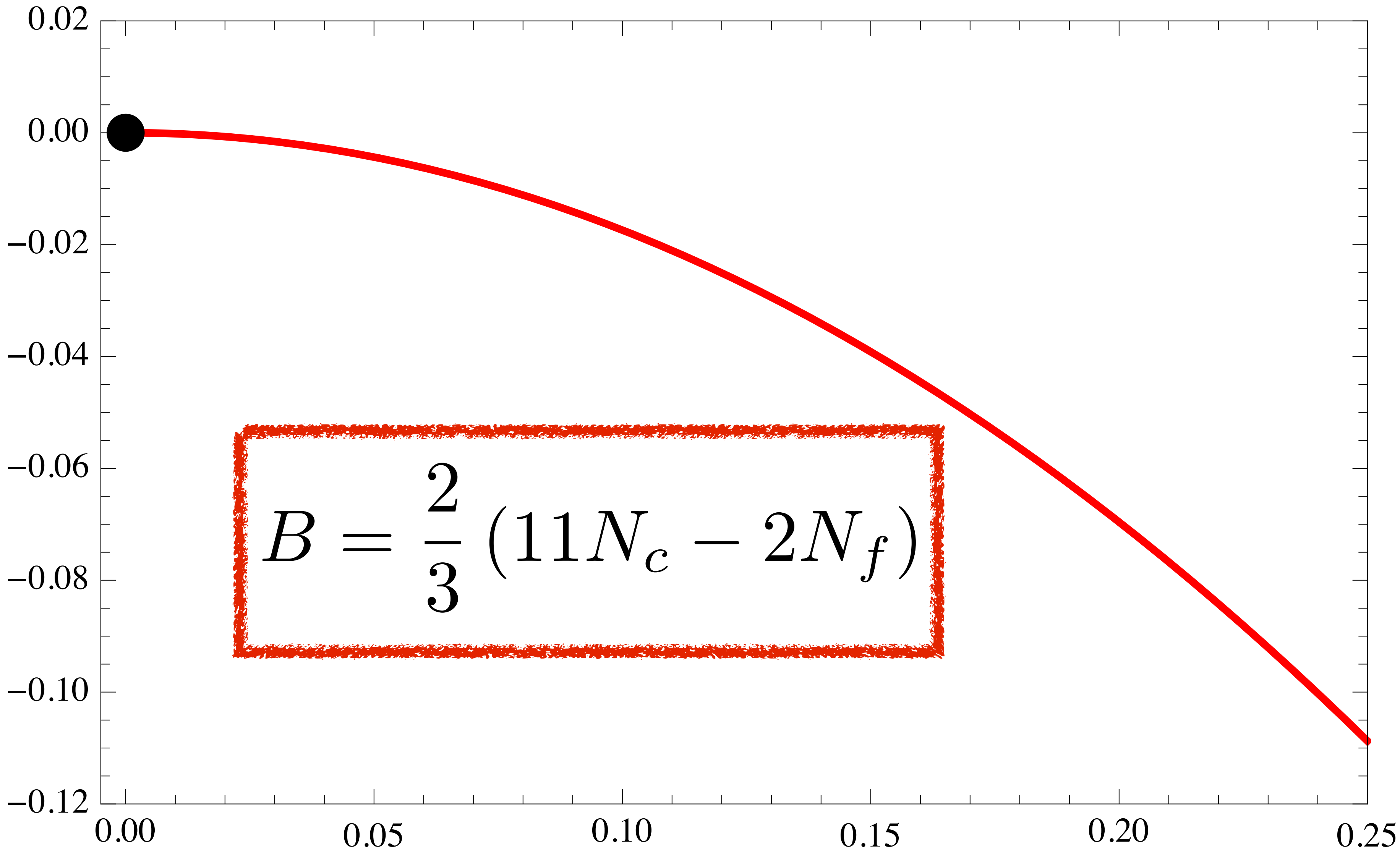
asymptotic freedom



asymptotic freedom

$$\mu \frac{d\alpha_s}{d\mu} = -B\alpha_s^2 < 0$$

QCD beta function



$$B = \frac{2}{3}(11N_c - 2N_f)$$

gauge α_s coupling

asymptotic freedom

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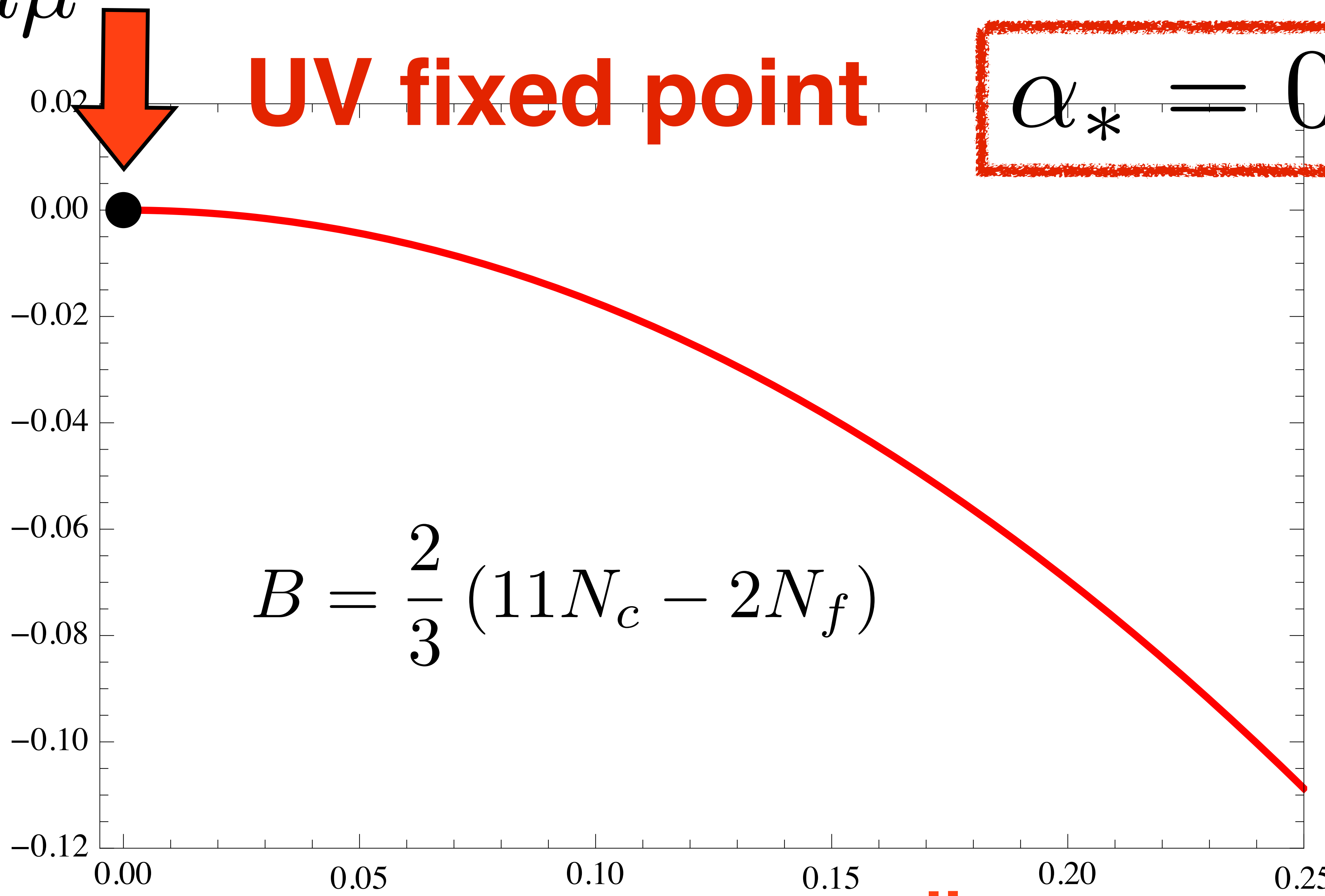


UV fixed point

$$\alpha_* = 0$$

$$B = \frac{2}{3}\left(11N_c - 2N_f\right)$$

gauge α_s coupling

asymptotic freedom

$$\mu \frac{d\alpha_s}{d\mu} = -B\alpha_s^2 < 0$$

QCD **beta function**

UV fixed point $\alpha_* = 0$

fundamental definition of QFT ⟺ **UV fixed point**

Wilson ’71

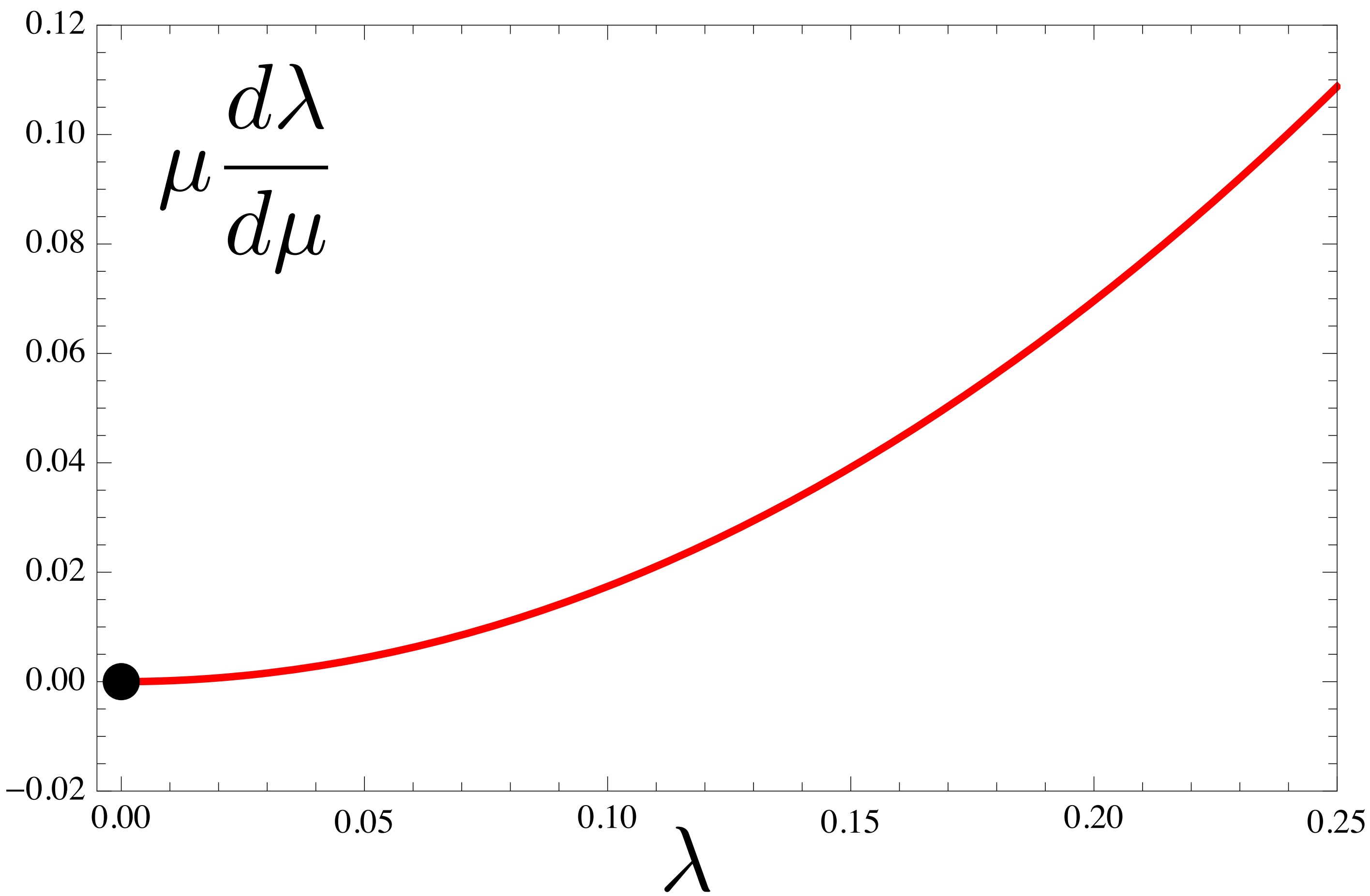
asymptotic freedom

$$\mu \frac{d\alpha_{e.m.}}{d\mu} \propto \alpha_{e.m.}^2 > 0$$

QED beta function

$$\mu \frac{d\lambda}{d\mu} \propto \lambda^2 > 0$$

Higgs self-coupling
Yukawa couplings



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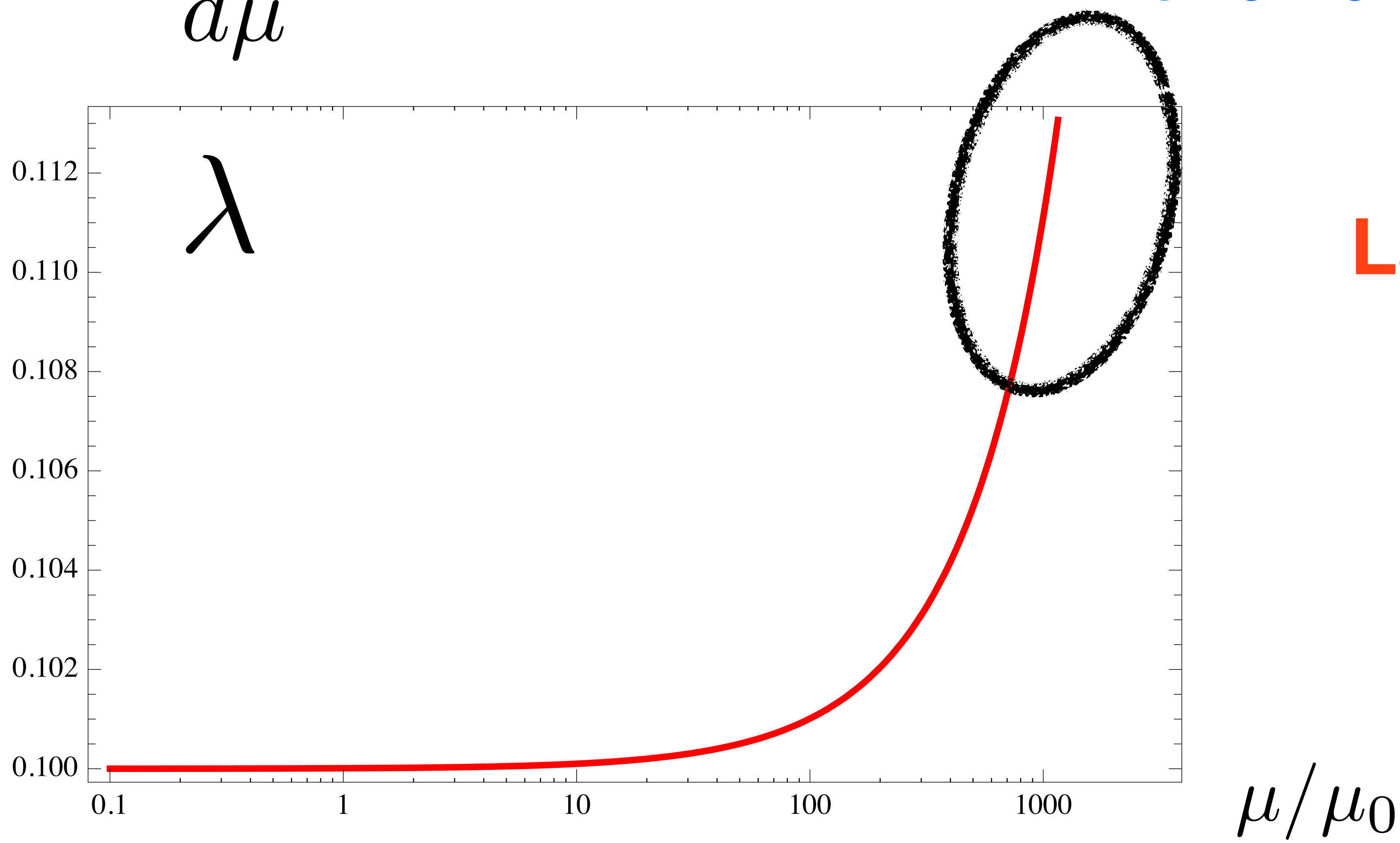

Landau pole

asymptotic freedom

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QED **beta function**

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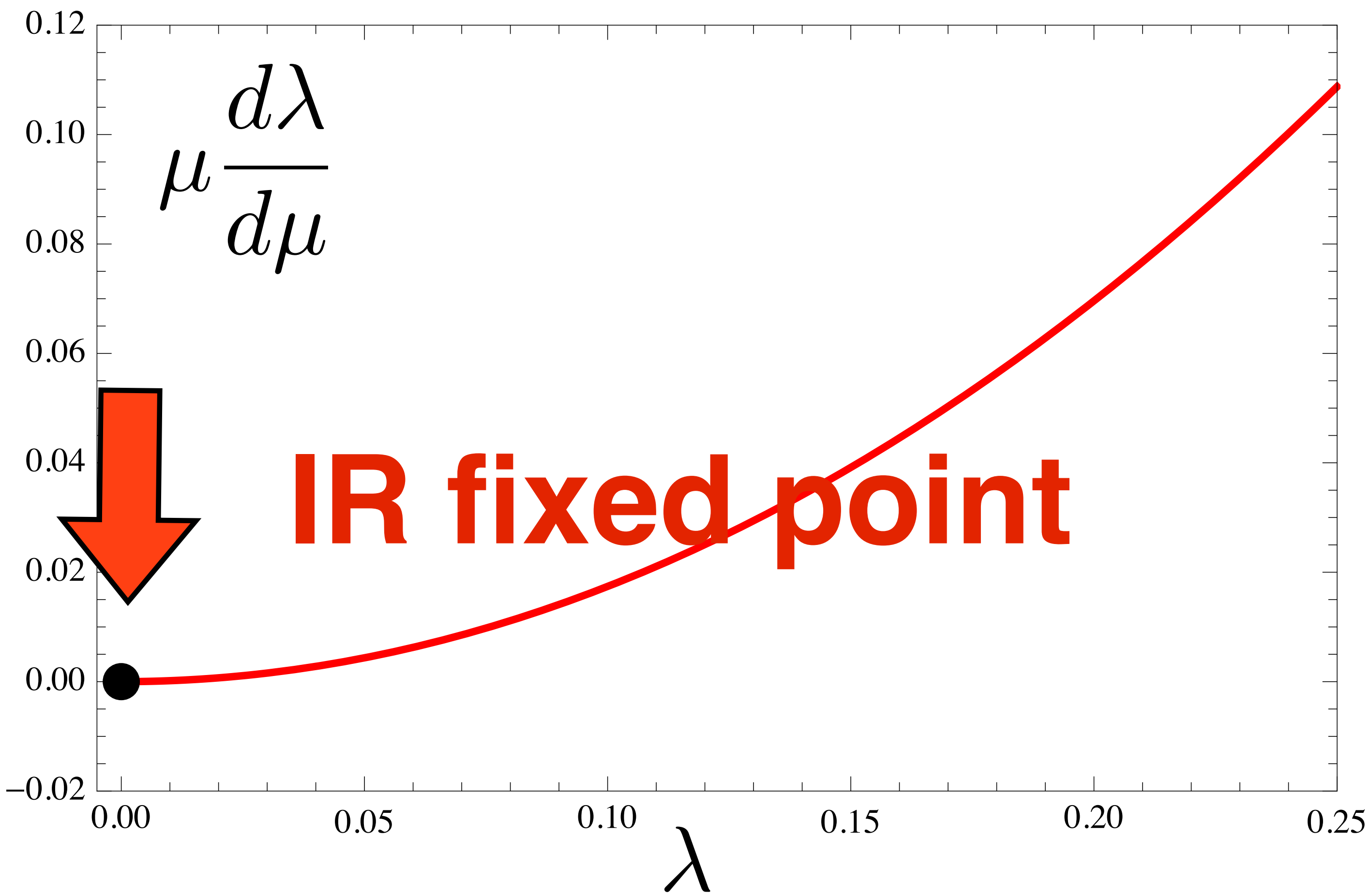
Higgs **self-coupling**
Yukawa **couplings**



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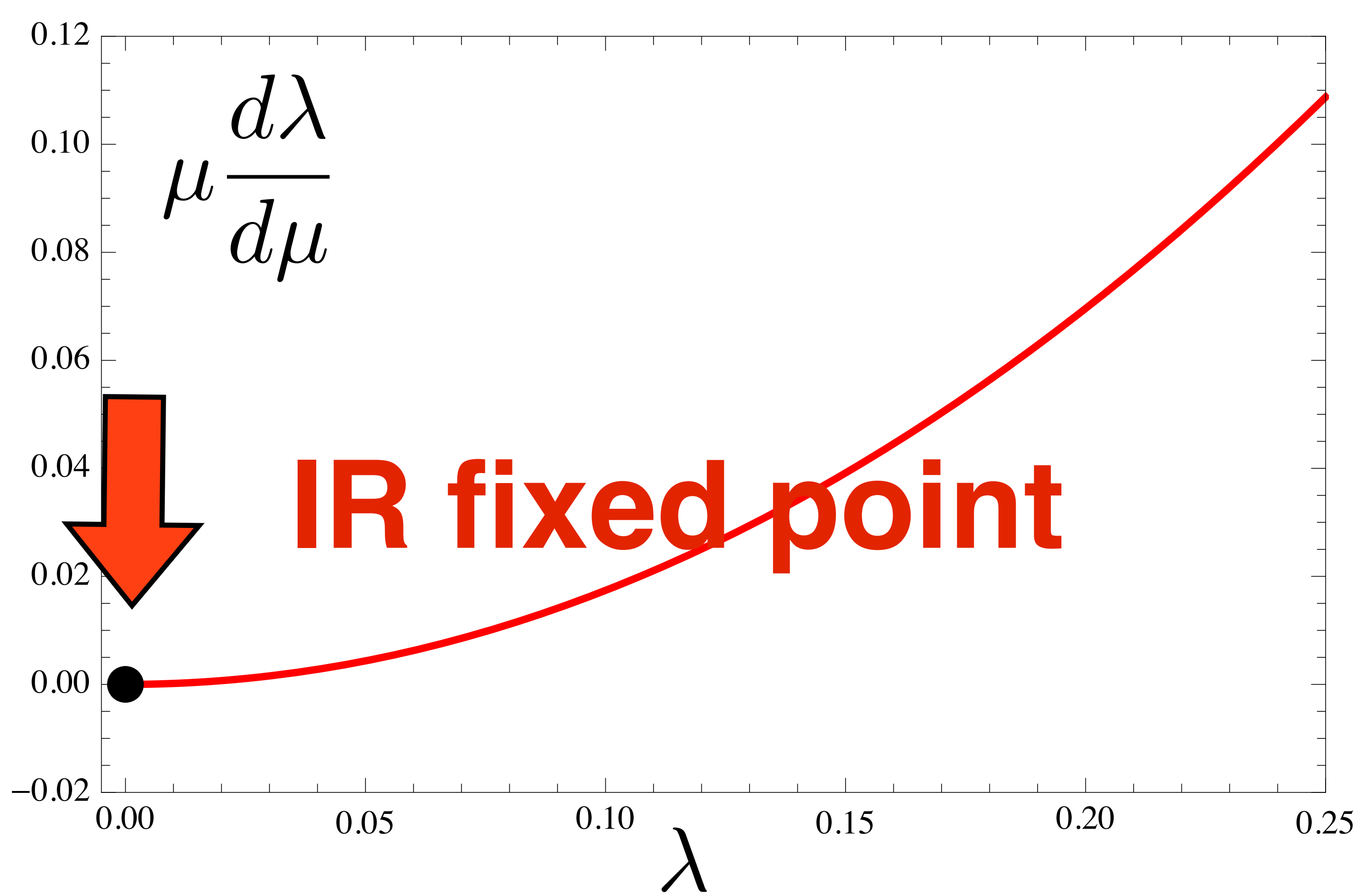

... but no UV fixed point

asymptotic freedom

$$\mu \frac{d\alpha_{e.m.}}{d\mu} \propto \alpha_{e.m.}^2 > 0$$

QED beta function

$$\mu \frac{d\lambda}{d\mu} \propto \lambda^2 > 0$$

Higgs self-coupling
Yukawa couplings

complete asymptotic freedom:

fully coupled (gauge-Yukawa-scalar) theory
all couplings achieve non-interacting UV fixed point

asymptotic safety

idea:

some or all couplings achieve interacting UV fixed point

Wilson '71
Weinberg '79

models for scenarios of BSM physics

asymptotic safety

idea:

some or all couplings achieve
interacting UV fixed point

Wilson ’71
Weinberg ’79

models for scenarios of BSM physics

today:

fully coupled gauge-Yukawa-scalar theory

Litim, Sannino ’14

gauge-Yukawa theory

Litim, Sannino '14

Lagrangean

$$L_{\rm YM} = -\frac{1}{2}{\rm Tr}\, F^{\mu\nu}F_{\mu\nu}$$

$$L_F = {\rm Tr}\left(\overline{Q}\, i\not{D}\, Q\right)$$

$$L_Y = y\,{\rm Tr}\left(\overline{Q}\, H\, Q\right)$$

$$L_H = {\rm Tr}\,(\partial_\mu H^\dagger\, \partial^\mu H)$$

$$L_U = -u\,{\rm Tr}\,(H^\dagger H)^2$$

$$L_V = -v\,({\rm Tr}\, H^\dagger H)^2\,.$$

gauge **Nc colours**

Yukawa **Nf flavours**

Higgs

gauge-Yukawa theory

Litim, Sannino '14

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couplings

$$\alpha_g = \frac{g^2\, N_C}{(4\pi)^2}\,, \quad \alpha_y = \frac{y^2\, N_C}{(4\pi)^2}$$

$$\alpha_h = \frac{u\, N_F}{(4\pi)^2}\,, \quad \alpha_v = \frac{v\, N_F^2}{(4\pi)^2}\,.$$

gauge-Yukawa theory

Litim, Sannino '14

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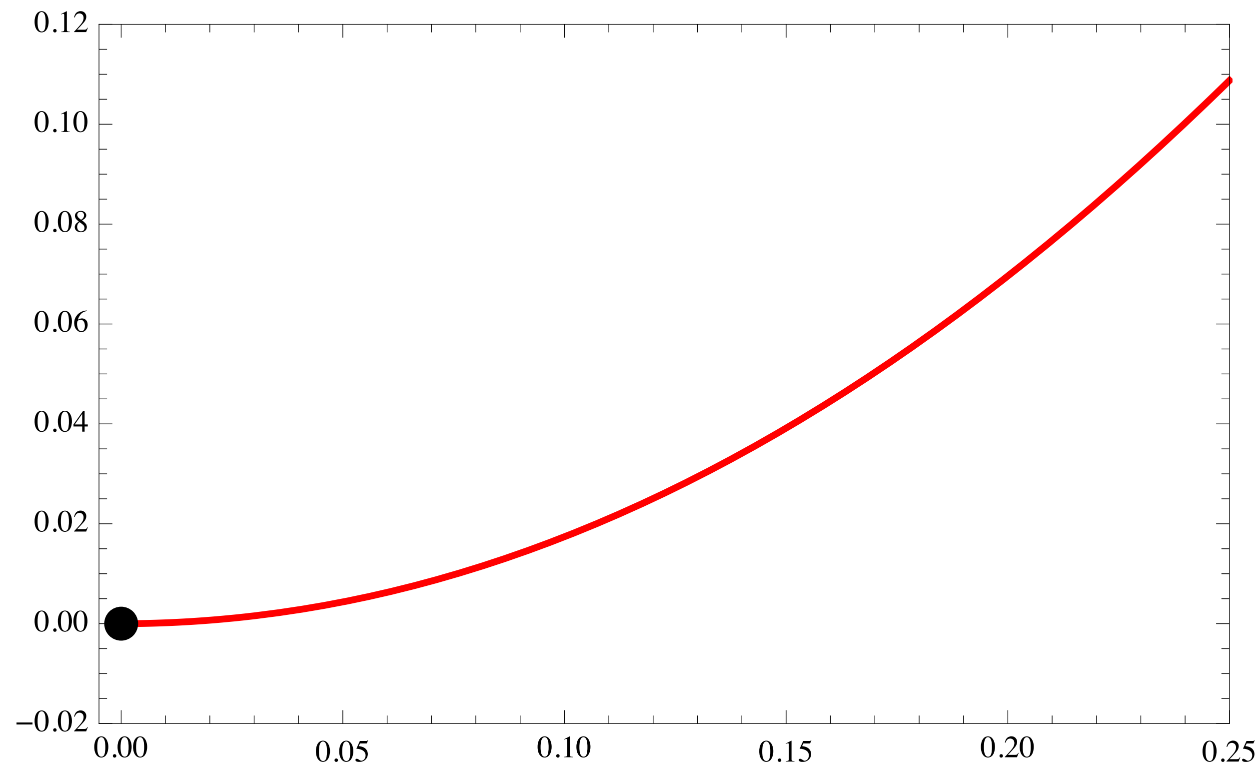
no asymptotic freedom

gauge beta function



gauge-Yukawa theory

Litim, Sannino '14

Lagrangean

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no asymptotic freedom

small parameter

$$0 < \epsilon \ll 1 \qquad \epsilon = \frac{N_f}{N_c} - \frac{11}{2}$$

technical trick: large-Nf &Nc (Veneziano) limit

gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$

gauge

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right\} .$$

Yukawa

$$\beta_h = -(11 + 2\epsilon)\,\alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y)\ .$$

Higgs

gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon \right) \alpha_g - 2 \left(\frac{11}{2} + \epsilon \right)^2 \alpha_y \right\}$$

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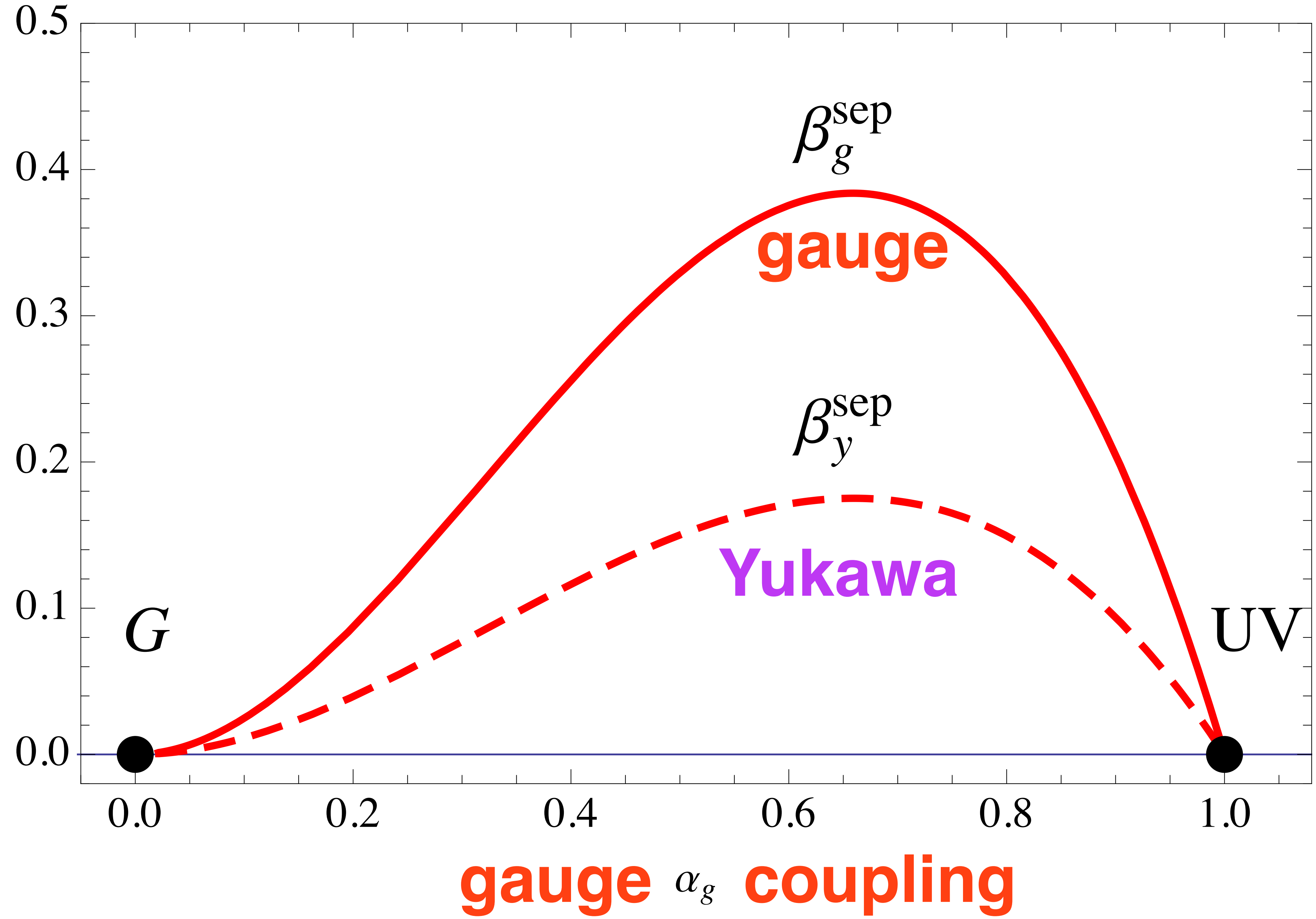
results



UV fixed point
entirely due to `fluctuations'

gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$

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$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y) .$$

exact
UV fixed point

$$\begin{aligned} \alpha_g^* &= 0.4561\,\epsilon + 0.7808\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_y^* &= 0.2105\,\epsilon + 0.5082\,\epsilon^2 + \mathcal{O}(\epsilon^3) \\ \alpha_h^* &= 0.1998\,\epsilon + 0.5042\,\epsilon^2 + \mathcal{O}(\epsilon^3) . \end{aligned}$$

vacuum stability

$$\alpha_h^* + \alpha_{v2}^* < 0 < \alpha_h^* + \alpha_{v1}^*$$

gauge-Yukawa theory

$$\beta_g = \alpha_g^2 \left\{ \frac{4}{3}\epsilon + \left(25 + \frac{26}{3}\epsilon\right)\alpha_g - 2\left(\frac{11}{2} + \epsilon\right)^2 \alpha_y \right\}$$

$$\beta_y = \alpha_y \left\{ (13 + 2\epsilon)\,\alpha_y - 6\,\alpha_g \right\} .$$

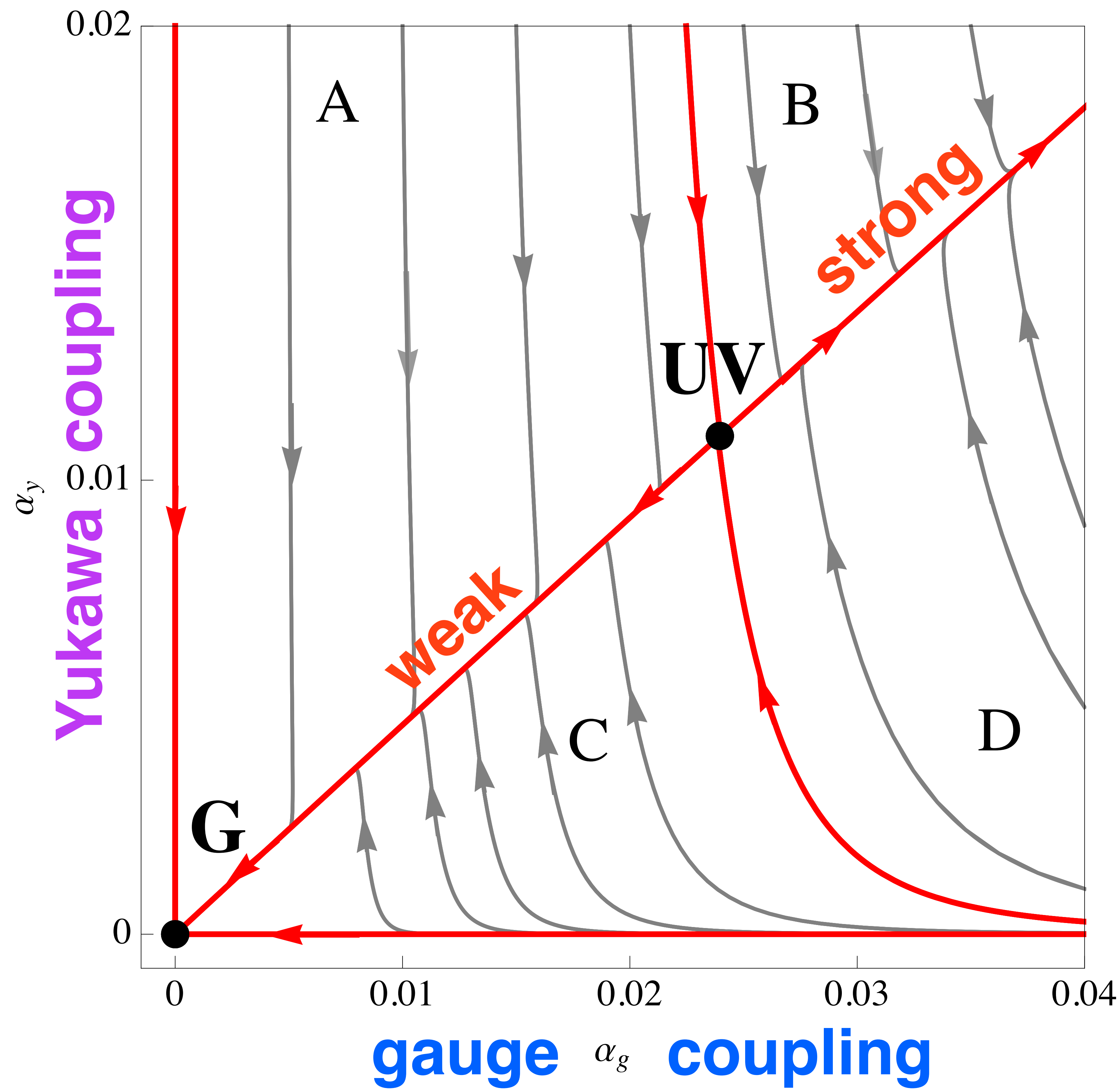
$$\beta_h = -(11 + 2\epsilon)\,\alpha_y^2 + 4\alpha_h(\alpha_y + 2\alpha_h)$$

$$\beta_v = 12\alpha_h^2 + 4\alpha_v\,(\alpha_v + 4\alpha_h + \alpha_y) \ .$$

→ **sensible interacting UV fixed point**

results

phase diagram



exact UV FP

strict perturbative control

what can we do with it?

interacting UV fixed points in 4D

exact proof of existence
exact quantum scale invariance
generic

what can we do with it?

interacting UV fixed points in 4D

exact proof of existence
exact quantum scale invariance
generic

UV-complete extensions of the Standard Model?

new handle on
hierarchy problem
feature: scalars are elementary

what can we do with it?

interacting UV fixed points in 4D

exact **proof of existence**
exact quantum **scale invariance**
generic

UV-complete extensions of the Standard Model

new handle on
hierarchy problem
feature: scalars are **elementary**

quantum gravity and the Standard Model

UV fixed point in 4D gravity
feature: gravitons are **elementary**
incorporate gravity into SM

overviews:
DL **0810.3675** and **1102.4624**
recent results:
Falls, DL, Nikolakopoulos, Rahmede
1410.4815