Modelling tri-bimaximal mixing

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The Flavor Problem:

- -why fermion mass hierarchies?
- -why flavor eigenstates are mixed?
- -why quarks and leptons mixing are so different?

⇒ Is there some symmetry G_f Between families?

Neutrino physics is one of the most important source of new physics beyond the standard model

Here we try to give some hints on G_f by means of the peculiar neutrino mixing: Tri-Bimaximal Mixing

Tri-Bimaximal lepton Mixing (TBM)

Harrison,Perkins,Scott Phys.Lett.B530(02)

$$\sin^2\theta_{12} = 0.32 - (25\%\ @3\sigma)$$

$$\sin^2\theta_{23} = 0.50 - (32\%\ @3\sigma)$$

$$\sin^2\theta_{12} \leq 0.05\ (@3\sigma)$$

Maltoni et al.,0405172v6

$$\begin{array}{cccccc} & & \nu_1 & \nu_2 & \nu_3 & \\ U_{\rm HPS} = & \Bigg(& \begin{matrix} \sqrt{2/3} \\ -1/\sqrt{6} \\ -1/\sqrt{6} \end{matrix} & \begin{matrix} 1/\sqrt{3} \\ 1/\sqrt{3} \\ 1/\sqrt{3} \end{matrix} & \begin{matrix} 0 \\ -1/\sqrt{2} \\ 1/\sqrt{2} \end{matrix} & \Bigg) \begin{matrix} e \\ \mu \\ \tau \end{matrix} \end{array}$$

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\end{array}
$$

Tri-maximal $\nu_2 = \nu_e + \nu_\mu + \nu_\tau$

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Bi-maximal

$$\nu_3 = \nu_\mu - \nu_\tau$$

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$$\begin{array}{cccc} & \nu_1 & \nu_2 & \nu_3 \end{array}$$

$$U_{\mathrm{HPS}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -1/\sqrt{6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -1/\sqrt{6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix} \begin{matrix} e \\ \mu \\ \tau \end{matrix}$$

Dirac phase δ unphysical

Two Majorana phases undeterminated

Tribimaximal and neutrino mass matrix

Let us assume charged lepton diagonal and:

$$M_\nu = \begin{pmatrix} x(1+\alpha) & y(1+\alpha) & y(1+\alpha) \\ y(1+\alpha) & x+\alpha y & y+\alpha x \\ y(1+\alpha) & y+\alpha x & x+\alpha y \end{pmatrix}$$

$\nu_\mu \leftrightarrow \nu_\tau$ Z_2

$$V_\nu = \begin{pmatrix} -c & s & 0 \\ s/\sqrt{2} & c/\sqrt{2} & 1/\sqrt{2} \\ s/\sqrt{2} & c/\sqrt{2} & -1/\sqrt{2} \end{pmatrix} \Rightarrow$$

$\theta_{13} = 0$

θ_{atm} **maximal**

Tribimaximal and neutrino mass matrix

charged lepton assumed diagonal

$$M_\nu = \begin{pmatrix} x(1+\alpha) & y(1+\alpha) & y(1+\alpha) \\ y(1+\alpha) & x+\alpha y & y+\alpha x \\ y(1+\alpha) & y+\alpha x & x+\alpha y \end{pmatrix}$$

$$\sin^2 2\theta_{12} = \frac{8 M^{\nu 2}_{13}}{(M^\nu_{11} - M^\nu_{22} - M^\nu_{23})^2 + 8 M^{\nu 2}_{13}}$$

Tribimaximal and neutrino mass matrix

charged lepton assumed diagonal

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$$M_{11} = M_{22} + M_{23} - M_{13}$$

We have a further relation

$$\sin^2 2\theta_{12} = \frac{8 M_{13}^{\nu 2}}{(M_{11}^\nu - M_{22}^\nu - M_{23}^\nu)^2 + 8 M_{13}^{\nu 2}}$$

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$$\sin^2 2\theta_{12} = \frac{8 M_{13}^{\nu 2}}{(M_{11}^\nu - M_{22}^\nu - M_{23}^\nu)^2 + 8 M_{13}^{\nu 2}}$$ **=8/9** ⇨ $\sin^2\theta_{12} = 1/3$

Mass spectrum

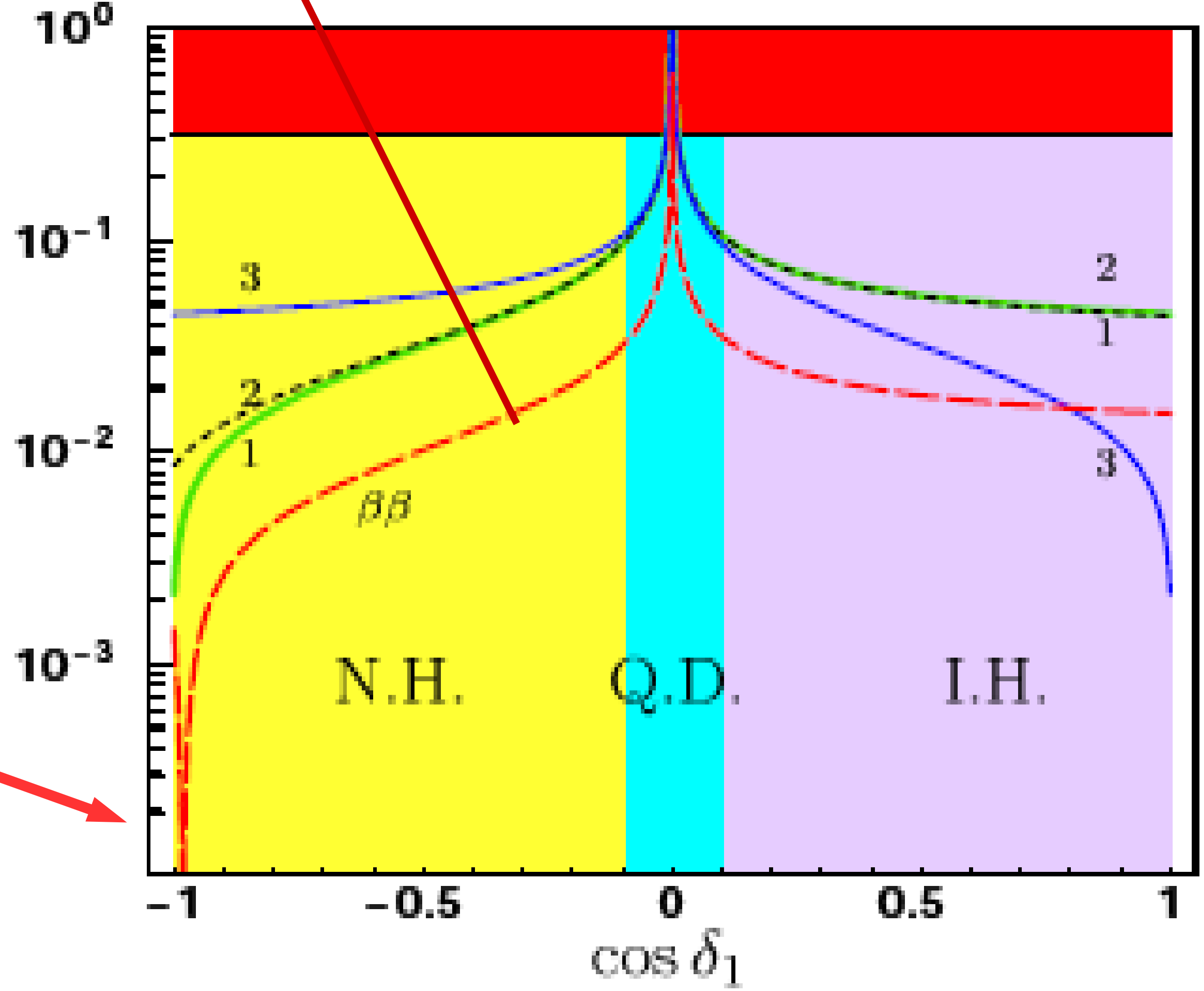
3 real par

&

2 Major.phases

$$|m_{ee}|^2 = |x|^2(1 + 2|\alpha|\cos\delta_1 + |\alpha|^2)$$

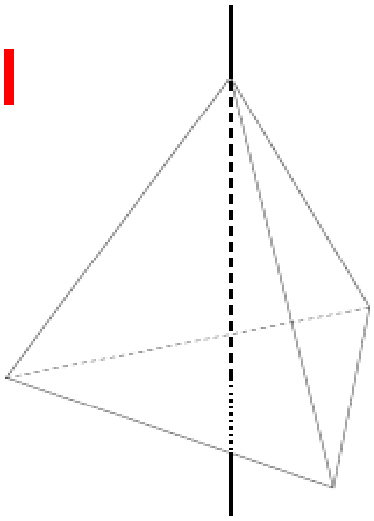
$$\cos\delta_1 = -\frac{3(1+|\alpha|^2)}{2|\alpha|(3+2r)}$$



$$|m_3|^2 - |m_1|^2 = \Delta m^2_{atm} =$$
$$= -4|\alpha|\cos\delta_1(|x|^2 + |y|^2 - 2|x||y|\cos\delta_2)$$

An A4 realization of Tri-Bimaximal

A4 group ⟺ Isomorphic to group of thetraedron rotations



It contains 1, 1', 1'' and 3 irrep → **Smallest non-Abelian**

3 x 3 = 1+1'+1''+3+3

$$\mathcal{L} = y_{1_{ij}}(L_i l_j^c)h_1 + y_{2_{ij}}(L_i l_j^c)' h_2 + y_{3_{ij}}(L_i l_j^c)'' h_3 + \\ + y_{ijk}^{\Delta_s}(L_i L_j)\Delta_{s_k}$$

	L_i	l_i^c	$h_{1,2,3}$	Δ_s
$SU(2)$	2	1	2	3
A_4	3	3	$1, 1'', 1'$	1

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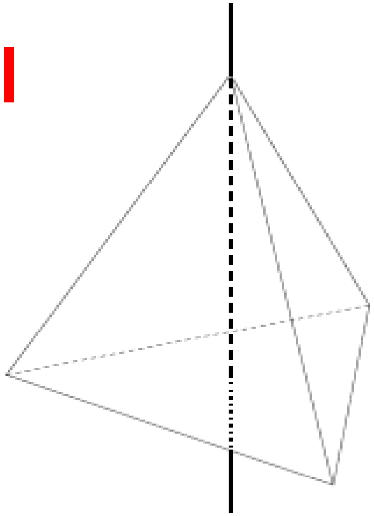

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$$M_{lep}$$

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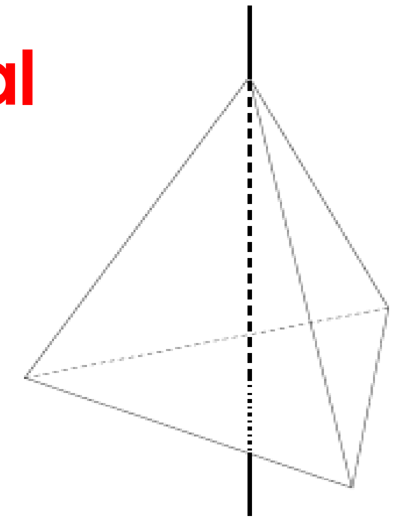

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$$+ y_{ijk}^{\Delta_s}(L_i L_j)\Delta_{s_k}$$

Neutrino

	L_i	l_i^c	$h_{1,2,3}$	Δ_s
$SU(2)$	2	1	2	3
A_4	3	3	$1, 1'', 1'$	1

Charged lepton sector

$$U_\omega^+ M_l U_\omega^+ \sim \begin{pmatrix} a+b+c & 0 & 0 \\ 0 & a+\omega b+\omega^2 c & 0 \\ 0 & 0 & a+\omega^2 b+\omega c \end{pmatrix}$$

$a \sim y_1 \langle h_1 \rangle$

$b \sim y_2 \langle h_2 \rangle$

$c \sim y_3 \langle h_3 \rangle$

3 different Higgs

$$U_\omega = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 & 1 & 1 \\ 1 & \omega & \omega^2 \\ 1 & \omega^2 & \omega \end{pmatrix}$$

$\omega^3 = 1$

"magic" matrix

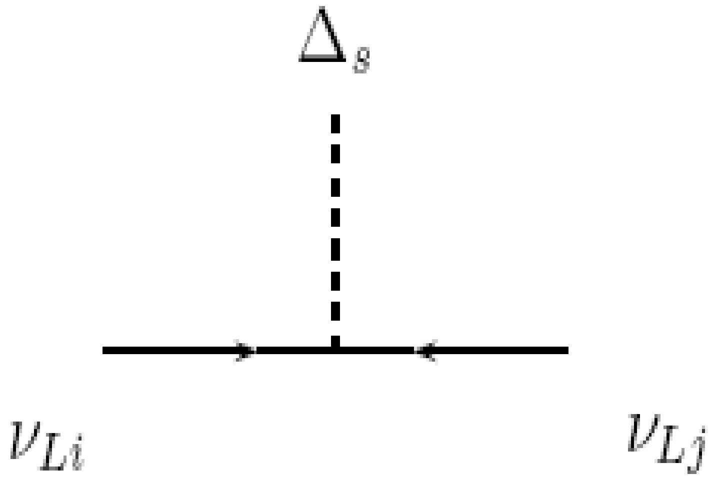
$\Rightarrow$ $m_e \quad m_\mu \quad m_\tau$ **Free parameter as in SM**

$m_e, m_\mu \ll m_\tau$ **IF** $b \sim \omega a, c \sim \omega^2 a$

Neutrino sector

At tree level we have only TYPE-II seesaw contribution

$\Delta_s \sim 1$ **Higgs triplet**



$$M_\nu^0 = \begin{pmatrix} z & 0 & 0 \\ 0 & 0 & z \\ 0 & z & 0 \end{pmatrix}$$

Two degenerate eigenstates

Diagonalizing matrix not fixed

We introduce

3 fermion-isotriplet $\Sigma_{\overline{k}} = \begin{pmatrix} \Sigma^+ \\ \Sigma^0 \\ \Sigma^- \end{pmatrix}$ Higgs doub. $\eta = \begin{pmatrix} \eta^+ \\ \eta^0 \end{pmatrix}$

Both caring an extra Z_2

$$y_{\Sigma_{ik}} (L_i \Sigma_k)\eta + \mu_{kl} \Sigma_k \Sigma_l$$

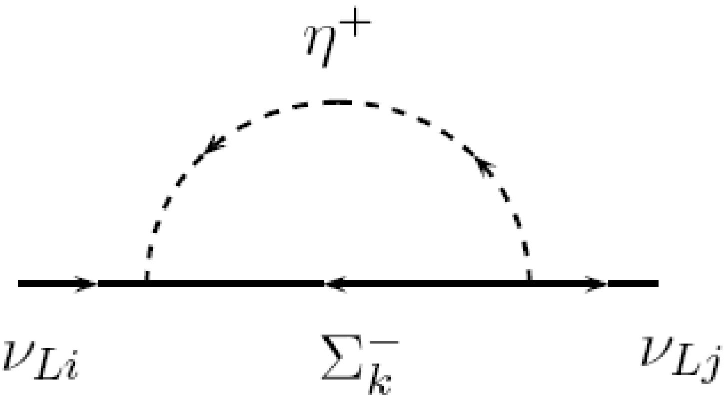
Majorana mass

We assume Σ mass term to break softly A4

$$\mu = m_\Sigma \begin{pmatrix} 1 & 0 & 0 \\ 0 & \xi & 1 \\ 0 & 1 & \xi \end{pmatrix}$$

Wave function renormalization

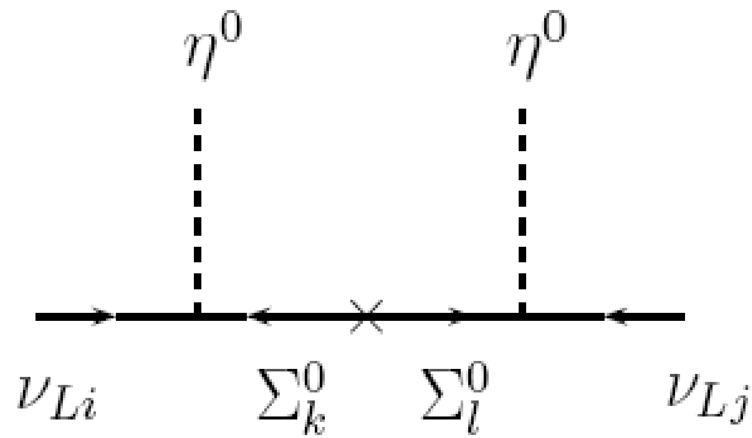
$R=$



$$M_\nu = M_\nu^0 + R M_\nu^0 + M_\nu^0 R^T$$

Type-III seesaw

Foot et al,ZPC44



$$-y_\Sigma \cdot \mu^{-1} \cdot y_\Sigma^T$$

If Z_2 **is exact** $\langle \eta^0 \rangle = 0$ **Only wave func renor**

$\xi \neq 0$ ➡ $m_{\Sigma 1} \neq m_{\Sigma 2} \neq m_{\Sigma 3}$

$$M_\nu \sim \begin{pmatrix} A & 0 & 0 \\ 0 & B & C \\ 0 & C & B \end{pmatrix}$$ with $$U_\omega^+ M_l \ldots = \textbf{Diag}$$

$$\Rightarrow \quad U_\omega^T M_\nu U_\omega$$ diagonalized by TBM

For example

$$M_\nu = M_\nu^0 + RM_\nu^0 + M_\nu^0 R^T$$

$$M_\nu^0 = \begin{pmatrix} z & 0 & 0 \\ 0 & 0 & z \\ 0 & z & 0 \end{pmatrix} \qquad R = \begin{pmatrix} \delta_1 & 0 & 0 \\ 0 & \delta_2 & \delta_3 \\ 0 & \delta_3 & \delta_2 \end{pmatrix}$$

LHC TEST

Higgs triplet

$$\Delta^{++} \rightarrow l^+ l^+$$

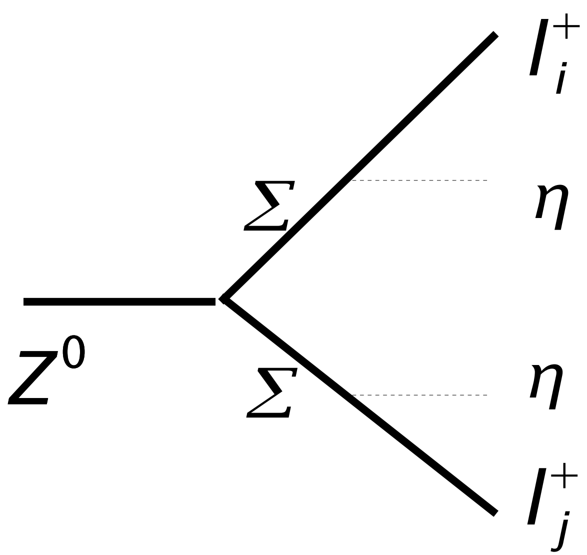
$$Br(\Delta^{++} \rightarrow e^+ e^+) \simeq Br(\Delta^{++} \rightarrow \mu^+ \mu^+) \simeq Br(\Delta^{++} \rightarrow \tau^+ \tau^+)$$

with

$$Br(\Delta^{++} \rightarrow l^+_i l^+_j) = 0$$

From kinetic term

$$\bar{\Sigma} D \Sigma$$



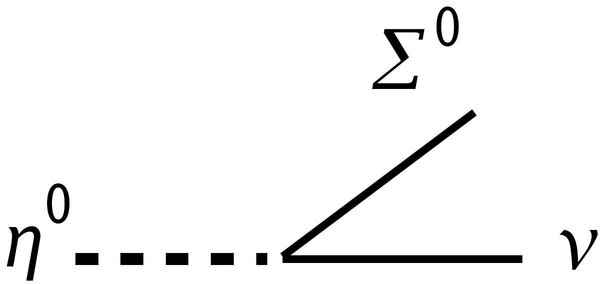
IF $m_\Sigma < 1\, TeV$ → $\Sigma^+ \Sigma^-$ produced

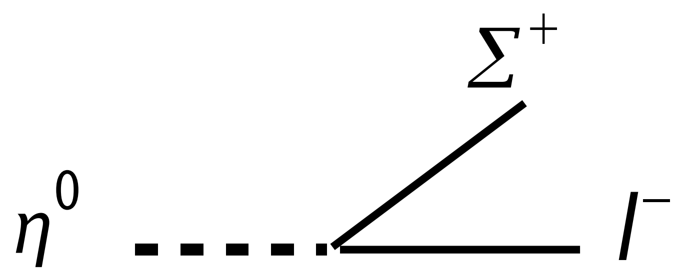
Bajc et al,PRD76, Franceschini et al, PRD78

$$Br(e\mu) \simeq Br(e\tau), Br(\mu\mu) \simeq Br(\tau\tau)$$

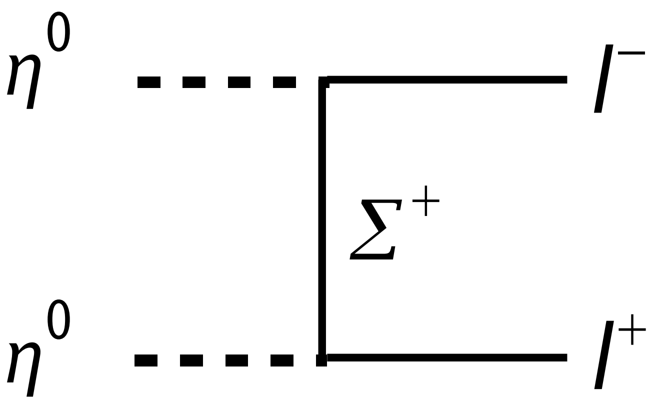
$$Br(ee) \simeq Br(\mu\mu) + Br(\mu\tau) - Br(e\tau)$$

Cold Dark Matter Candidate (work in progress)





- Σ^0 **Stable if** $m_\Sigma < m_{\eta^0}$ **and could be DM**
- η^0 **Stable if** $m_{\eta^0} < m_\Sigma$ **and could be DM**



Possible explanation of positron/electron PAMELA exes

Conclusions

- We have studied a model for Tri-Bimaximal
- Few scalar Higgs doublets and one triplet
- Softly broken A4 by triplet-fermion mass term
- Testable at LHC if $m_{\Sigma} < 1\,TeV$
- Scalar doublet as Dark Matter candidate...

We assume Σ-Mass term $\mu = m_\Sigma \begin{pmatrix} 1 & 0 & 0 \\ 0 & \xi & 1 \\ 0 & 1 & \xi \end{pmatrix}$

To break softly A4

Diagonalized by

$$R^\mu = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1/\sqrt{2} & 1/\sqrt{2} \\ 0 & -1/\sqrt{2} & 1/\sqrt{2} \end{pmatrix}$$

$$R_{ij} = \frac{{y_\Sigma}^2}{16\pi^2} \sum_k R^\mu_{ik} R^{\mu *}_{jk} B_1(m^2_{\Sigma_k}, m^2_\eta)$$

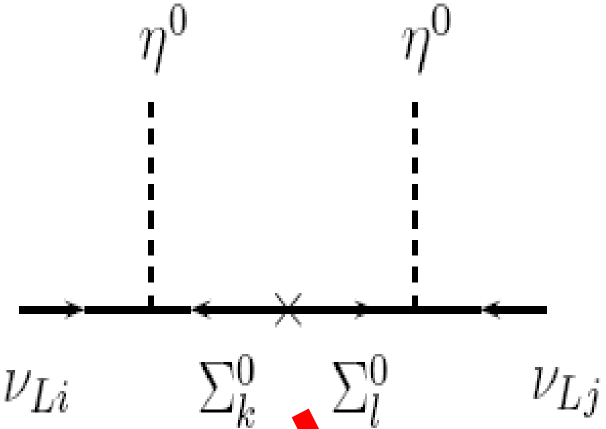
Wave function corr

If $\xi \neq 0$ → $m_{\Sigma 1}$ $m_{\Sigma 2}$ $m_{\Sigma 3}$

Not degenerate

$$\Rightarrow R = \begin{pmatrix} \delta_1 & 0 & 0 \\ 0 & \delta_2 & \delta_3 \\ 0 & \delta_3 & \delta_2 \end{pmatrix}$$

With arbitrary parameters



Σ , η Carry extra Z_2

Z_2 Broken ➡ $\langle \eta \rangle \neq 0$

$$M_\nu = \begin{pmatrix} z+\beta & 0 & 0 \\ 0 & \frac{\beta\xi}{\xi^2-1} & z-\frac{\beta}{\xi^2-1} \\ 0 & z-\frac{\beta}{\xi^2-1} & \frac{\beta\xi}{\xi^2-1} \end{pmatrix}$$

In the basis where charged leptons are diagonal

TBM

$$\begin{cases} m_1 = \frac{\beta+z(1+\xi)}{1+\xi} \\ m_2 = \beta+z \\ m_3 = \frac{\beta+z(1-\xi)}{1-\xi} \end{cases}$$

➡

$$4r = |1-\xi|^2 \quad \textbf{N.H.}$$

$$r = 1-\frac{4}{|1-\xi|^2} \quad \textbf{I.H}$$