Standard(-like) Model from an SO(12) Grand Unified Theory in six-dimensions with S^2 extra space

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Outline

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1. Introduction

Higgs sector of the Standard Model(SM) plays important roles

- Leading Spontaneous Symmetry Breaking of $SU(2)_L \times U(1)_Y$
- Leading particle masses

However….

SM itself does not give a prediction about Higgs sector

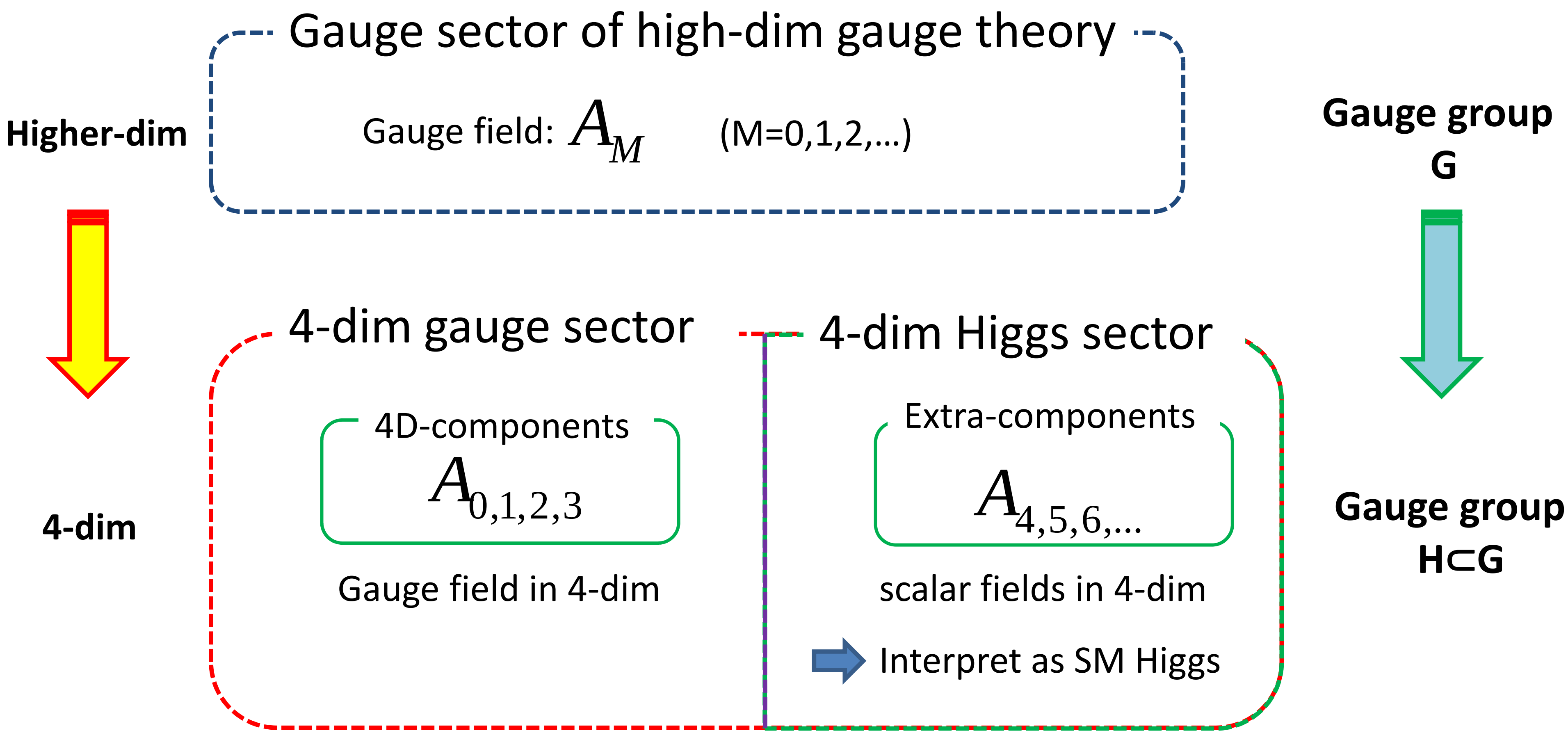
We can not predict the value of Higgs mass or Higgs self-coupling constant

Physics beyond the SM would provide the prediction ?

Gauge Higgs Unification scenario is the attractive candidate

1. Introduction

Gauge Higgs Unification(GHU)



- Gauge group G should be $G \supset G_{SM}$ to provide SM Higgs doublet
- Model with Simple group G is especially interesting = GUT-like GHU

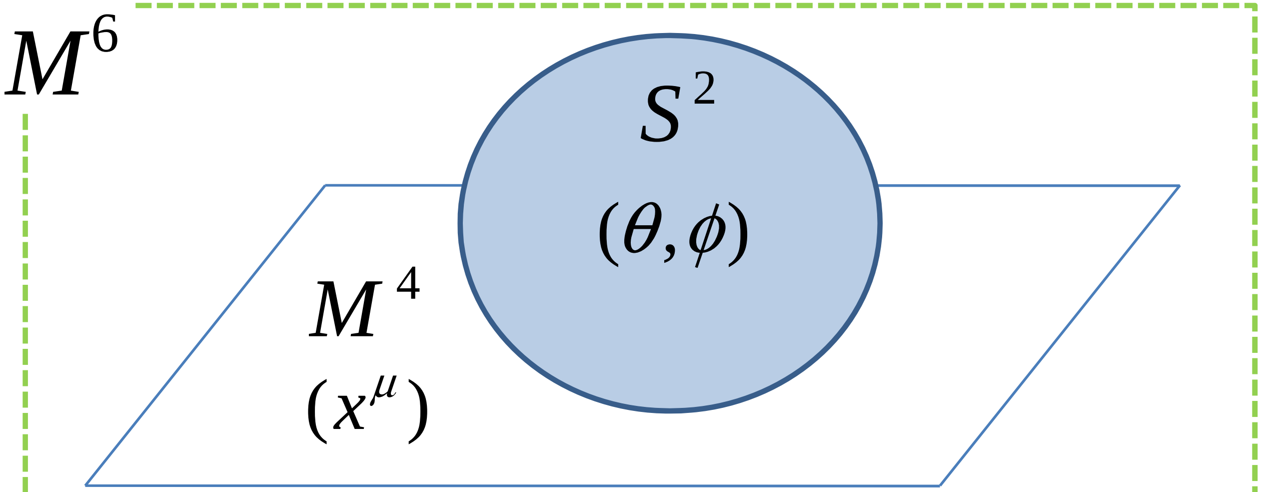
2.Our approach to GHU

Basic theory

Gauge theory on 6-dim space-time $M^6 = M^4 \otimes S^2$

M^4 : 4-dim Minkowski space-time

S^2 : 2-sphere extra space



M^6 Coordinates: $X^M = (x^\mu, \theta, \phi)$

★ S^2 has coset space structure $SU(2)_I / U(1)_I \quad (SU(2)_I \supset U(1)_I)$

➡ S^2 has isometry group $SU(2)_I$

Fields in the theory

● Left handed Weyl fermion of SO(1,5)

$$\Psi(X) = \begin{pmatrix} \psi_L(X) \\ \psi_R(X) \end{pmatrix}$$

ψ_L :Left handed Weyl fermion of SO(1,3)

ψ_R :Right handed Weyl fermion of SO(1,3)

● Gauge field

$$A_M(X) = (A_\mu(X), A_\theta(X), A_\phi(X))$$

☆ We introduce a background gauge field A_ϕ^B

⇨ It is necessary to obtain massless chiral fermion

Action of the theory

$$S = \int dx^4 \sin\theta d\theta d\phi (\Psi i \Gamma^M D_M \Psi - \frac{1}{4g^2} g^{MN} g^{KL} Tr[F_{MK} F_{NL}])$$

$$F^{MN}(X) = \partial_M A_N(X) - \partial_N A_M(X) - [A_M(X), A_N(X)]$$

$$g_{MN} = diag(1,-1,-1,-1,-R^{-2},-R^{-2}\sin^{-2}\theta)$$: M^6 metric

(R:radius)

$$\Gamma^M : \begin{cases} \Gamma^\mu = \gamma^\mu \otimes I_2 \\ \Gamma^4 = \gamma^5 \otimes \sigma_1 \\ \Gamma^5 = \gamma^5 \otimes \sigma_2 \end{cases}$$

:6-dim gamma matrix

$$D^M : \begin{cases} D_\mu = \partial_\mu - A_\mu \\ D_\theta = \partial_\theta - A_\theta \\ D_\phi = \partial_\phi - i\frac{\Sigma_3}{2}\cos\theta - A_\phi \end{cases} \quad (\Sigma_3 = I_4 \otimes \sigma_3)$$

:covariant derivative

Spin connection term (for fermion)

Reduction of the theory to 4-dim effective theory

In reducing high-dim theory, we impose these conditions

- **Symmetry condition of gauge field** (Manton & Forgacs (1980))

  SU(2) isometry transformation of S^2 is compensated
  by Gauge transformation

  ⇒ Coset space dimensional reduction of gauge sector
  (D. Kapetanakis and G. Zoupanos, Phys. Rept. 219(1992) 1.)

- **The condition to obtain massless fermions**

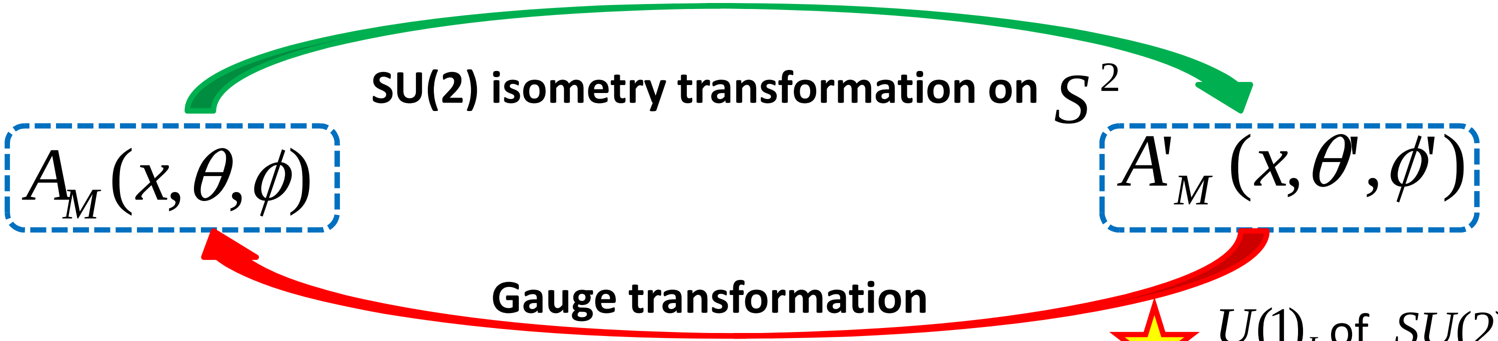
  ⇒ due to existence of background gauge field

- **The non-trivial boundary condition of S^2**

  ⇒ Leading reduction of gauge symmetry

2. Our approach to GHU

Symmetry condition of the gauge field



★ $U(1)_I$ of $SU(2)_I/U(1)_I$ should be embedded in G

Infinitesimal form

$$\xi_i^\beta \partial_\beta A_\mu = \partial_\mu W_i + [W_i, A_\mu]$$

$$\xi_i^\beta \partial_\beta A_\alpha + \partial_\alpha \xi_i^\beta A_\beta = \partial_\alpha W_i + [W_i, A_\alpha]$$

ξ_i^β :Killing vector of S^2

W_i :fields generating gauge transformation

2. Our approach to GHU

Symmetry condition of the gauge field

SU(2) isometry transformation on S^2

$A_M(x,\theta,\phi)$ → $A'_M(x,\theta',\phi')$

Gauge transformation

★ $U(1)_I$ of $SU(2)_I/U(1)_I$ should be embedded in G

Infinitesimal form

$$\xi_i^\beta \partial_\beta A_\mu = \partial_\mu W_i + [W_i, A_\mu]$$

$$\xi_i^\beta \partial_\beta A_\alpha + \partial_\alpha \xi_i^\beta A_\beta = \partial_\alpha W_i + [W_i, A_\alpha]$$

Isometry transformation on S^2 | Gauge transformation

ξ_i^β :Killing vector of S^2

W_i :fields generating gauge transformation

Symmetry condition of the gauge field

SU(2) isometry transformation on S^2

$A_M(x,\theta,\phi)$ → $A'_M(x,\theta',\phi')$

Gauge transformation

$A'_M(x,\theta',\phi')$ → $A_M(x,\theta,\phi)$

★ $U(1)_I$ of $SU(2)_I/U(1)_I$ should be embedded in G

Infinitesimal form

$$\xi_i^\beta \partial_\beta A_\mu = \partial_\mu W_i + [W_i, A_\mu]$$

$$\xi_i^\beta \partial_\beta A_\alpha + \partial_\alpha \xi_i^\beta A_\beta = \partial_\alpha W_i + [W_i, A_\alpha]$$

Isometry transformation on S^2 Gauge transformation

ξ_i^β :Killing vector of S^2

W_i :fields generating gauge transformation

By the condition

$$L_{6D}^{(gauge)}(x,\theta,\phi) \longrightarrow L_{6D}^{(gauge)}(x)$$

Independent of S^2 coordinate

⇒ Dimensional reduction can be carried out

2. Our approach to GHU

Manton (1979)

Solutions of the condition

$$A_\mu = A_\mu(x) \qquad W_1 = -\Phi_3 \cos\phi / \sin\theta$$

$$A_\theta = -\Phi_1(x) \qquad W_2 = -\Phi_3 \sin\phi / \sin\theta$$

background

$$A_\phi = \Phi_2(x)\sin\theta - \Phi_3 \cos\theta \qquad W_3 = 0$$

$\Phi_{1,2}$:scalar field

$-i\Phi_3 : U(1)_I$ generator embedded in G

Constraints for the solutions

$[\Phi_3, A_\mu(x)] = 0$ **4-dim gauge group has to commute with $U(1)_I$ in G**

$[-i\Phi_3, \Phi_i] = i\varepsilon_{3ij}\Phi_j$ $U(1)_I$ **charges of the 4-dim scalars are restricted**

$-i\Phi_3 : U(1)_I$ generator embedded in G

Dimensional reduction of the gauge sector

Applying solution of the condition,

➡ Extra-space can be integrated and we obtain…

4-dim gauge-Higgs sector

$$S_{4D}^{(gauge-Higgs)} = \int dx^4 \Big(-\frac{1}{4g^2} Tr[F^{\mu\nu} F_{\mu\nu}] - \frac{1}{2g^2} Tr[D_\mu \Phi_1(x) D^\mu \Phi_1(x) + D_\mu \Phi_2(x) D^\mu \Phi_2(x)] - \frac{1}{2g^2} Tr[(\Phi_3 + [\Phi_1(x), \Phi_2(x)])^2]$$

4-dim Higgs sector with potential term

★ **No massive KK mode**

2. Our approach to GHU

The condition to obtain massless fermions

Fermions on $M^6 = M^4 \otimes S^2$ have no massless mode without background gauge field

➡ due to the existence of positive cavature of S^2

In Lagrangian, it is expressed as existence of spin connection term

Spin connection term should be canceled by background gauge field

⬇

$$-i\Phi_3 \Psi(X) = \frac{\Sigma_3}{2}\Psi(X)$$

➡ $U(1)_I$ **charges of the 4-dim massless fermion contents are restricted**

4-dim fermion sector is obtained by expanding to normal modes

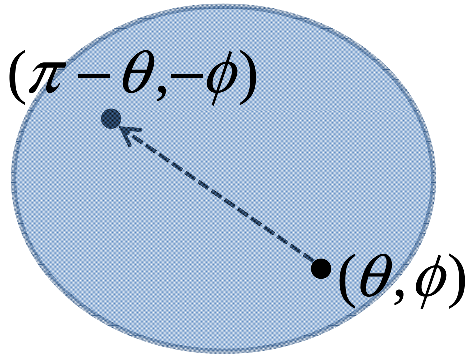
➡ Massive KK modes appear

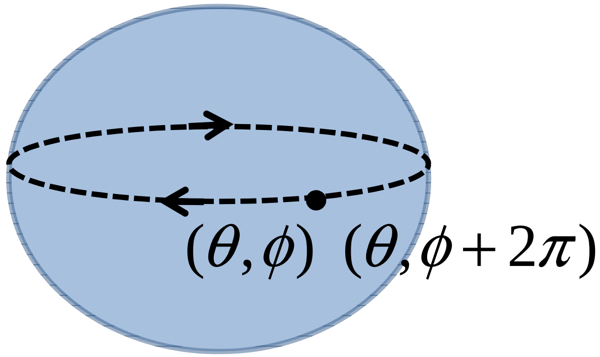
2. Our approach to GHU

The non-trivial boundary condition of S^2

We impose non-trivial boundary condition for

$$(\theta,\phi) \to (\pi-\theta,-\phi) \qquad\qquad (\theta,\phi) \to (\theta,\phi+2\pi)$$





$(\theta,\phi) \to (\pi-\theta,-\phi)$

$$\Psi(x,\pi-\theta,-\phi)=\gamma_5 P\Psi(x,\theta,\phi)$$
$$A_\mu(x,\pi-\theta,-\phi)=PA_\mu(x,\theta,\phi)P$$
$$A_\theta(x,\pi-\theta,-\phi)=-PA_\theta(x,\theta,\phi)P$$
$$A_\phi(x,\pi-\theta,-\phi)=-PA_\phi(x,\theta,\phi)P$$

$(\theta,\phi) \to (\pi,\phi+2\pi)$

$$\Psi(x,\theta,\phi+2\pi)=\gamma_5 \bar{P}\Psi(x,\theta,\phi)$$
$$A_\mu(x,\theta,\phi+2\pi)=\bar{P}A_\mu(x,\theta,\phi)\bar{P}$$
$$A_\theta(x,\theta,\phi+2\pi)=-\bar{P}A_\theta(x,\theta,\phi)\bar{P}$$
$$A_\phi(x,\theta,\phi+2\pi)=-\bar{P}A_\phi(x,\theta,\phi)\bar{P}$$

- ★ P, $\bar{\text{P}}$: matrices acting on the representation space of gauge group
- ★ Components of P, $\bar{\text{P}}$ is +1 or -1 and $P^2=1(\bar{P}^2=1)$

2. Our approach to GHU

Boundary condition for the solution of the symmetry condition

$$A_\mu(x) = P A_\mu(x) P$$
$$A_\mu(x) = \bar{P} A_\mu(x) \bar{P}$$

Only even components under the parity remain in 4-dim

$$\Phi_{1,2}(x) = -P \Phi_{1,2}(x,\theta,\phi) P$$
$$\Phi_{1,2}(x) = \bar{P} \Phi_{1,2}(x,\theta,\phi) \bar{P}$$

Boundary condition for the massless fermion modes

$$\Psi(x) = \gamma_5 P \Psi(x)$$
$$\Psi(x) = \bar{P} \Psi(x)$$

Only even components under the parity remain massless

Massless particle contents in 4D are restricted by the conditions

3.The SO(12) model

Set up

- For gauge group
  - Gauge group in 6-dim G=SO(12)
  - $U(1)_I$ of $SU(2)_I/U(1)_I$ is embedded into SO(12) as

    $SO(12) \supset SO(10) \otimes U(1)_I$
- For fileds
  - Two types of left-handed weyl fermion in 32 reps of SO(12)

    $\Psi^{(\bar{P})}(X) : \Psi^{(\bar{P})}(x,\theta,\phi+2\pi) = \bar{P}\Psi^{(\bar{P})}(x,\theta,\phi)$

    $\Psi^{(-\bar{P})}(X) : \Psi^{(-\bar{P})}(x,\theta,\phi+2\pi) = -\bar{P}\Psi^{(-\bar{P})}(x,\theta,\phi)$
  - The parity assignment for P and $\bar{P}$ in SO(12) spinor basis

$$
\begin{aligned}
32 = & (3,2)^{(+,-)}(1,-1,1) + (\bar{3},2)^{(+,-)}(1,-1,1) + (3,1)^{(-,-)}(4,1,-1) + (\bar{3},1)^{(-,-)}(-4,-1,1) \\
& + (\bar{3},1)^{(-,+)}(2,3,1) + (3,1)^{(-,+)}(-2,-3,-1) + (1,2)^{(+,+)}(3,-3,-1) + (1,2)^{(+,+)}(-3,3,1) \\
& + (1,1)^{(-,-)}(6,-1,1) + (1,1)^{(-,-)}(-6,1,-1) + (1,1)^{(-,+)}(0,-5,1) + (1,1)^{(-,+)}(0,5,-1)
\end{aligned}
$$

(P assignment, $\bar{P}$ assignment)

$(SU(3), SU(2))(U(1)_Y, U(1)_X, U(1)_I)$

Constraints from the symmetry condition

● For gauge field and gauge group

Constraints: $[\Phi_3, A_\mu(x)] = 0$ ($-i\Phi_3 : U(1)_I$ generator)

Gauge group should commute with $U(1)_I$

➡ $SO(12) \supset SO(10) \otimes U(1)_I$

● For scalar field contents

Constraints: $[-i\Phi_3, \Phi_i] = i\varepsilon_{3ij}\Phi_j$

adj rep of $SU(2)_I$

$$adj SU(2)_I = 3 \to 1(0) + 1(2) + 1(-2) \quad (SU(2)_I \supset U(1)_I)$$

$$adj SO(12) = 66 \to 45(0) + 1(0) + 10(2) + 10(-2) \quad (SO(12) \supset SO(10) \otimes U(1)_I)$$

4-dim scalar contents should be contained in $10(2) + 10(-2)$

● For massless modes of fermion

Constraints: $-i\Phi_3 \Psi(X) = \frac{\Sigma_3}{2} \Psi(X)$

$$\left(\begin{array}{l} \Sigma_3 = I_4 \otimes \sigma_3 \\ -i\Phi_3 : U(1)_I \text{ charge} \end{array} \right)$$

$U(1)_I$ charge of the massless fermion contents should be…

- +1 for left handed fermion
- - 1 for right handed fermion

$$\left(\Psi(X) = \begin{pmatrix} \psi_L(X) \\ \psi_R(X) \end{pmatrix} \right)$$

For 32 rep of SO(12)

$$32 \to 16(1) + \overline{16}(-1) \quad (SO(12) \supset SO(10) \otimes U(1)_I)$$

⇒ Fermion contents in 4-dim should be contained in

$$16(1)_L + \overline{16}(-1)_R$$

Particle contents in 4-dim

Particle contents in four-dimensions are determined by

The constraints from symmetry condition

+

The parity assignment for non-trivial boundary condition

The particle contents have to..

- satisfy the constraints
- be even under the parity assignment

3. The SO(12) model

●Particle contents of gauge field

Parity assignment for adj SO(12)=66 under

$$A_\mu(x) \to P A_\mu(x) P, A_\mu(x) \to \bar{P} A_\mu(x) \bar{P}$$

$$\begin{aligned}66 = & (8,1)^{(+,+)}(0,0,0) + (1,3)^{(+,+)}(0,0,0) + (1,1)^{(+,+)}(0,0,0) + (1,1)^{(+,+)}(0,0,0) + (1,1)^{(+,+)}(0,0,0) \\ & + (3,2)^{(-,+)}(-5,0,0) + (\bar{3},2)^{(-,+)}(5,0,0) + (3,2)^{(-,-)}(1,4,0) + (\bar{3},2)^{(-,-)}(-1,-4,0) \\ & + (3,1)^{(+,-)}(4,-4,0) + (\bar{3},1)^{(+,-)}(-4,4,0) + \underline{(3,1)^{(+,-)}(-2,2,2) + (\bar{3},1)^{(+,-)}(2,-2,-2)} \\ & \underline{+ (3,1)^{(+,+)}(-2,2,-2) + (\bar{3},1)^{(+,+)}(2,-2,2) + (1,2)^{(-,-)}(3,2,2) + (1,2)^{(-,-)}(-3,-2,-2)} \\ & \underline{+ (1,2)^{(-,+)}(3,2,-2) + (1,2)^{(-,+)}(-3,-2,2)} + (1,1)^{(+,-)}(6,4,0) + (1,1)^{(+,-)}(-6,-4,0)\end{aligned}$$

10(2)+10(-2) (66=45(0)+1(0)+10(2)+10(-2)) $(SU(3), SU(2))(U(1)_Y, U(1)_X, U(1)_I)$

Do not satisfy constraints: $[\Phi_3, A_\mu(x)] = 0$

3. The SO(12) model

●Particle contents of gauge field

Parity assignment for adj SO(12)=66 under

$$A_\mu(x) \to P A_\mu(x) P, A_\mu(x) \to \overline{P} A_\mu(x) \overline{P}$$

$$66 = (8,1)^{(+,+)}(0,0,0) + (1,3)^{(+,+)}(0,0,0) + (1,1)^{(+,+)}(0,0,0) + (1,1)^{(+,+)}(0,0,0) + (1,1)^{(+,+)}(0,0,0)$$
$$+ (3,2)^{(-,+)}(-5,0,0) + (\overline{3},2)^{(-,+)}(5,0,0) + (3,2)^{(-,-)}(1,4,0) + (\overline{3},2)^{(-,-)}(-1,-4,0)$$
$$+ (3,1)^{(+,-)}(4,-4,0) + (\overline{3},1)^{(+,-)}(-4,4,0) + (3,1)^{(+,-)}(-2,2,2) + (\overline{3},1)^{(+,-)}(2,-2,-2)$$
$$+ (3,1)^{(+,+)}(-2,2,-2) + (\overline{3},1)^{(+,+)}(2,-2,2) + (1,2)^{(-,-)}(3,2,2) + (1,2)^{(-,-)}(-3,-2,-2)$$
$$+ (1,2)^{(-,+)}(3,2,-2) + (1,2)^{(-,+)}(-3,-2,2) + (1,1)^{(+,-)}(6,4,0) + (1,1)^{(+,-)}(-6,-4,0)$$

$$(SU(3), SU(2))(U(1)_Y, U(1)_X, U(1)_I)$$

10(2)+10(-2)

Do not satisfy constraints: $[\Phi_3, A_\mu(x)] = 0$

●Particle contents of gauge field

Parity assignment for adj SO(12)=66 under

$$A_\mu(x) \to P A_\mu(x) P,\ A_\mu(x) \to \bar{P} A_\mu(x) \bar{P}$$

$$\begin{aligned}
66 = \ & (8,1)^{(+,+)}(0,0,0) + (1,3)^{(+,+)}(0,0,0) + (1,1)^{(+,+)}(0,0,0) + (1,1)^{(+,+)}(0,0,0) + (1,1)^{(+,+)}(0,0,0) \\
& + (3,2)^{(-,+)}(-5,0,0) + (\bar{3},2)^{(-,+)}(5,0,0) + (3,2)^{(-,-)}(1,4,0) + (\bar{3},2)^{(-,-)}(-1,-4,0) \\
& + (3,1)^{(-,-)}(4,-4,0) + (\bar{3},1)^{(-,-)}(-4,4,0) + (3,1)^{(-,-)}(-2,2,2) + (\bar{3},1)^{(-,-)}(2,-2,-2) \\
& + (3,1)^{(-,+)}(-2,2,-2) + (\bar{3},1)^{(-,+)}(2,-2,2) + (1,2)^{(+,-)}(3,2,2) + (1,2)^{(+,-)}(-3,-2,-2) \\
& + (1,2)^{(+,+)}(3,2,-2) + (1,2)^{(+,+)}(-3,-2,2) + (1,1)^{(-,-)}(6,4,0) + (1,1)^{(-,-)}(-6,-4,0)
\end{aligned}$$

$$(SU(3), SU(2))(U(1)_Y, U(1)_X, U(1)_I)$$

Symmetry breaking

$$SO(12) \supset SO(10) \otimes U(1)_I \supset SU(3) \otimes SU(2) \otimes U(1)_Y \otimes U(1)_X \otimes U(1)_I$$

3. The SO(12) model

● Particle contents of scalar field

66 ⇨ **{(1,2)(3,2,-2), (1,2)(-3,-2,2)}**

Only SM Higgs doublet remain!

● Particle contents of massless fermion

$$32^{(\bar{P})} \Rightarrow \{\underset{l_L}{(1,2)(-3,3,1)_L}, \underset{d_R}{(3,1)(-2,-3,-1)_L}, \underset{\nu_R}{(1,1)(0,-5,1)_R}\}$$

$$32^{(-\bar{P})} \Rightarrow \{\underset{u_R}{(3,1)(4,1,-1)_R}, \underset{e_R}{(1,1)(-6,1,-1)_R}, \underset{q_L}{(3,2)(1,-1,1)_L}\}$$

One generation of massless fermion modes!

Analysis of the Higgs potential

The Higgs sector in terms of $\Phi_i(x)$

$$L_{Higgs} = -\frac{1}{2g^2} Tr[D_\mu \Phi_1(x) D^\mu \Phi_1(x) + D_\mu \Phi_2(x) D^\mu \Phi_2(x)]$$ **KE term**

$$-\frac{1}{2g^2} Tr[(\Phi_3 + [\Phi_1(x), \Phi_2(x)])^2]$$ **potential term**

Rewrite in terms of Higgs doublet ϕ

$$V = -\frac{2}{R^2} |\phi|^2 + \frac{3g^2}{2} |\phi|^4$$

(R: radius of the two-sphere)

This potential leads electroweak symmetry breaking!

Prediction

Vacuum expectation value of Higgs field are obtained

in terms of
- The radius of the two-spere
- Gauge coupling constant

$$<\phi>=\frac{1}{\sqrt{2}}\begin{pmatrix}0\\v\end{pmatrix} \qquad v=\sqrt{\frac{4}{3}}\frac{1}{gR}$$

W boson mass and Higgs mass are also given as

$$m_W=\sqrt{\frac{2}{3}}\frac{1}{R}$$

$$m_H=\sqrt{4}\frac{1}{R}$$

$$\Rightarrow \quad \frac{m_H}{m_W}=\sqrt{6}$$

Summary

●We analyzed Gauge Higgs Unification for gauge theory on $M^6 = M^4 \otimes S^2$ with symmetry condition and non-trivial boundary condition

●We construct SO(12) model and obtained

◆One SM generation of massless fermion mode (with RH neutrino)

$$\{\underset{l_L}{(1,2)(-3,3,1)_L}, \underset{d_R}{(3,1)(-2,-3,-1)_L}, \underset{\nu_R}{(1,1)(0,-5,1)_R}\}$$

$$\{\underset{u_R}{(3,1)(4,1,-1)_R}, \underset{e_R}{(1,1)(-6,1,-1)_R}, \underset{q_L}{(3,2)(1,-1,1)_L}\}$$

◆SM Higgs field and prediction for Higgs mass

$$m_H = \sqrt{6} m_W$$

Future work

- Extra U(1) symmetry
  - → Break by some mechanism
- Realistic Yukawa coupling
  - → Realistic mass hierarchy
  - → Realistic fermion mixing
- Fermion mass spectrum
  - → Search for a dark matter candidate

Appendix

- Vielbein

$$e_\theta^1 = 1, \qquad e_\phi^2 = \sin\theta$$

$$e_\theta^2 = e_\phi^1 = 0$$

- Killing vectors

$$\xi_1^\theta = \sin\phi, \qquad \xi_1^\phi = \cot\theta\cos\phi$$

$$\xi_2^\theta = -\cos\phi, \qquad \xi_2^\phi = \cot\theta\sin\phi$$

$$\xi_3^\theta = 0, \qquad \xi_3^\phi = 0$$