Gravity from Breaking of Local Lorentz Symmetry

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Kostelecky and Potting, GRG '05, and paper in preparation

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Outline

Masslessness from symmetry or broken symmetry ?

Gauge Symmetries

Generator of unbroken gauge symmetry $\Rightarrow$ massless vector boson

General Relativity

Diffeomorphism invariance $\Rightarrow$ massless gravitons

Spontaneously Broken Global Symmetry

Spontaneously broken global symmetry $\Rightarrow$ massless Nambu-Goldstone boson

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Linearized "Cardinal" dynamics[1]

gravitons as Nambu-Goldstone modes

$$L = \frac{1}{2} C^{\mu\nu} K_{\mu\nu\alpha\beta} C^{\alpha\beta} + V(C^{\mu\nu}, \eta_{\mu\nu})$$

$$K_{\mu\nu\alpha\beta} = -\partial^2(\eta_{\mu\alpha}\eta_{\nu\beta} - \frac{1}{2}\eta_{\mu\nu}\eta_{\alpha\beta}) + \partial_\mu \eta_{\nu\alpha}\partial_\beta + \partial_\nu \eta_{\mu\alpha}\partial_\beta$$

- $K_{\mu\nu\alpha\beta}$: ghost-free quadratic kinetic operator for spin 2
- $C^{\mu\nu}$: tensor density; $\eta_{\mu\nu}$: flat background metric
- V: scalar potential built out of the 4 independent scalars $X_1 = C^{\mu\nu}\eta_{\nu\mu}$, $X_2 = (C \cdot \eta \cdot C \cdot \eta)^\mu_\mu, \ldots$
- V acquires minimum for $C^{\mu\nu} = c^{\mu\nu} \equiv \langle C^{\mu\nu} \rangle \neq 0$: spontaneous breaking of Lorentz symmetry
- fluctuations around vev: $C^{\mu\nu} = c^{\mu\nu} + h^{\mu\nu}$

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Linearized "Cardinal" dynamics

At low energy, assume V can be approximated by sum of delta-functions that fix the 4 independent scalars: $V = \sum_{n=1}^{4} \frac{\lambda_n}{n} X_n$

linearized equations of motion:

$$K_{\mu\nu\alpha\beta} h^{\alpha\beta} - \lambda_1 \eta_{\mu\nu} - \lambda_2 (\eta c \eta)_{\mu\nu} - \lambda_3 (\eta c \eta c \eta)_{\mu\nu} - \lambda_4 (\eta c \eta c \eta c \eta)_{\mu\nu} = 0$$

constraints:

Lagrange multiplier terms force the constraints

$$h^\mu_\mu = 0 \qquad c^{\mu\nu} h_{\mu\nu} = 0 \qquad (c\eta c)^{\mu\nu} h_{\mu\nu} = 0 \qquad (c\eta c\eta c)^{\mu\nu} h_{\mu\nu} = 0$$

Low-energy dynamics of $h_{\mu\nu}$-fluctuations around v.e.v. equivalent to linearized general relativity in axial-type gauge, with possible Lorentz-violating source term from Lagrange multiplier terms

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Counting degrees of freedom

Propagating massless degrees of freedom

- Can be considered Nambu-Goldstone modes of spontanously broken Lorentz generators:

$$h_{\mu\nu} = \mathcal{E}_\mu{}^\alpha c_{\alpha\nu} + \mathcal{E}_\nu{}^\alpha c_{\mu\alpha}$$

- Equations of motion imply masslessness $\partial^2 h_{\mu\nu} = 0$ and Lorenz conditions $\partial^\mu h_{\mu\nu} = 0$
- Number of propagating massless degrees of freedom: $6 - 4 = 2$

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Linear coupling to matter

Linear coupling to matter

$$\mathcal{L} \supset h^{\mu\nu}\tau_{\mu\nu}$$

- $\tau_{\mu\nu}$: trace-inversed energy-momentum tensor
- linear coupling to EM-tensor gives rise to linearized Einstein equation

$$K_{\mu\nu\alpha\beta}h^{\alpha\beta} \equiv R^{L}_{\mu\nu} = \tau_{\mu\nu}$$

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consistent coupling

consistent coupling to total EM tensor

- require coupling to total EM tensor, including contribution of gravitational fluctuations

$$K_{\mu\nu\alpha\beta}h^{\alpha\beta} = \tau_h^{(1)}{}_{\mu\nu}$$

- $\tau_h^{(1)}{}_{\mu\nu}$ corresponds to cubic term of total lagrangian
- cubic term yields new contribution to EM-tensor $\tau_h^{(2)}{}_{\mu\nu} \Rightarrow$ quartic term in Lagrangian
- etc., etc.
- This alternative non-geometrical "bootstrap" principle to deriving GR was proposed by Kraichnan, Feynman and others in 1950's

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Bootstrap

1-step bootstrap

- bootstrap can be done in one step using procedure invested by Deser
- uses trace-reverted field: $C^{\mu\nu} \to -C^{\mu\nu} + \frac{1}{2}\eta^{\mu\nu} C^{\alpha}_{\alpha}$
- employs first-order quadratic Lagrangian

$$C^{\mu\nu}(\Gamma^{\alpha}_{\mu\nu,\alpha} - \Gamma_{\mu,\nu}) + \eta^{\mu\nu}(\Gamma^{\alpha}_{\mu\nu}\Gamma_{\alpha} - \Gamma^{\alpha}_{\beta\mu}\Gamma^{\beta}_{\alpha\nu})$$

final result for kinetic term

recursive process yields nonlinear bootstrapped kinetic action

$$S_{kin} = \int d^4x(\eta + C)^{\mu\nu} R_{\mu\nu}(\Gamma)$$

Thus $(\eta + C)^{\mu\nu}$ is naturally interpreted as curved-space metric density!

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Bootstrap of matter EM tensor and scalar potential

matter energy-momentum tensor

- Bootstrap can also be applied to flat-space matter Lagrangian
- Result: curved-space matter lagrangian, with metric density $(\eta + C)^{\mu\nu}$
- Example: $L_{EM} = -\frac{1}{4\sqrt{|\eta+C|}}(\eta + C)^{\alpha\gamma}(\eta + C)^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}$

scalar potential V

- Assume flat-space V built out of independent scalars X_1, X_2, X_3, X_4
- Bootstrap requires V satisfy integrability conditions
- Conditions satisfied only by particular solutions, e.g.:

$$1, \quad X_1, \quad X_2 - \frac{X_1^2}{2}, \quad X_3 - \frac{3X_1X_2}{4} + \frac{X_1^3}{8}, \quad \ldots$$

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- Example: $L_{EM} = -\frac{1}{4\sqrt{|\eta+C|}}(\eta + C)^{\alpha\gamma}(\eta + C)^{\beta\delta}F_{\alpha\beta}F_{\gamma\delta}$

scalar potential V

- Assume flat-space V built out of independent scalars X_1, X_2, X_3, X_4
- Bootstrap requires V satisfy integrability conditions
- Conditions satisfied only by particular solutions, e.g.:

$$1, \qquad X_1, \qquad X_2 - \frac{X_1^2}{2}, \qquad X_3 - \frac{3X_1X_2}{4} + \frac{X_1^3}{8}, \qquad \dots$$

Bootstrap of matter EM tensor and scalar potential

matter energy-momentum tensor

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Bootstrap of scalar potential

integrable scalar potentials

Particularly interesting: Scalar potentials of the form

$$V(\{X_i\}) = \frac{1}{2}\sum_{i,j} m_{ij}(X_i - x_i)(X_j - x_j) + \mathcal{O}(X_i - x_i)^3$$

with local minimum at $X_i = x_i$ $(i = 1 \ldots 4)$

- Represent possibly stable vacuum
- Integrability and stability highly nontrivial conditions (work in progress)
- Expect limit $m_{ij} \to \infty$ to correspond to bootstrap of linearized limit

$$V^L = \lambda_1 X_1 + \lambda_2(X_2 - \frac{X_1^2}{2}) + \lambda_3(X_3 - \frac{3X_1X_2}{4} + \frac{X_1^3}{8}) + \ldots$$

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Vacuum energy-momentum tensor

Bootstrapped Lagrangian

$$(\eta + C^{\mu\nu} R_{\mu\nu}(\Gamma) - \sqrt{-|\eta + C|} V(X_1, X_2, X_3, X_4) + L_{matter}(C, \eta, \phi_i, \partial_\mu \phi_i)$$

Linearized equations of motion

$$K_{\mu\nu\alpha\beta} h^{\alpha\beta} = (\eta_{\mu\nu}\partial_1 + 2\eta_{\mu\alpha} c^{\alpha\beta}\eta_{\beta\nu}\partial_2 + \ldots)V + \tau^{(m)}_{\mu\nu}(\eta, \phi_i, \partial_\mu \phi_i)$$

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"vacuum energy-momentum tensor"

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Vacuum energy-momentum tensor (cont.)

Explicit solutions

- Explicit solutions of linearized equations of motion can be obtained for $h^{\mu\nu}$ with nonzero vacuum energy-momentum tensor
- Initial/boundary values can be defined on suitable initial timelike/spacelike spacetime slices (4 independent functions)

Conservation and initial conditions

If matter EM tensor conserved independently, same is true for vacuum EM tensor

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Choosing $T_{\mu\nu}^{(vac)}$ to be zero at suitable initial timelike/spacelike spacetime slices ensures it is zero at all spacetime

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Conclusions

- Construction of alternative theory of gravity possible
- Massless gravitons can be interpreted as Nambu-Goldstone modes of spontaneously broken Lorentz symmetry
- Nonlinear lagrangian from requirement of consistent coupling to total energy-momentum tensor
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