CPT, Lorentz invariance and anomalous clash of symmetries

Alberto Salvio

EPF of Lausane and IFAE, Barcelona

Discrete 08, Valencia, December 2008

Based on
A. S., Phys. Rev. D 78, 085023 (2008)

Outline

Motivations and assumptions

What is the role of Lorentz (and CPT) symmetry in the anomalies?

Motivations and assumptions

What is the role of Lorentz (and CPT) symmetry in the anomalies?

Motivations:

- **Conceptual issue:** we'd like to understand if the Lorentz invariance participates in the clash of symmetries, which leads to the anomalies.
- Possible applications to the Standard Model extended to describe Lorentz violation: a framework to check and constrain Lorentz invariance **[Colladay and Kostelecky (1997, 1998); Coleman and Glashow (1998)]**

Motivations and assumptions

What is the role of Lorentz (and CPT) symmetry in the anomalies?

Motivations:

- **Conceptual issue:** we'd like to understand if the Lorentz invariance participates in the clash of symmetries, which leads to the anomalies.
- Possible applications to the Standard Model extended to describe Lorentz violation: a framework to check and constrain Lorentz invariance **[Colladay and Kostelecky (1997, 1998); Coleman and Glashow (1998)]**

Strategy and assumptions in my analysis:

(i) EFT approach **assuming locality, translational invariance and barring irrelevant operators**, but relaxing Lorentz invariance
$\Rightarrow$ Extra CPT-even and CPT-odd operators (with *tiny* coefficients): the assumptions of the CPT theorem are not satisfied.

(ii) Perturbative approach to Lorentz violation (still possible to have the fields in irreps of the Lorentz group)

(iii) Only **perturbative nongravitational anomalies and one family of fermions** are considered

The fermion sector

ψ_j: *one family* of fermions in a *general* representation of an *internal* symmetry gauge group

$$\delta\psi_j = i\Omega_{jk}\psi_k \equiv i\Omega^b \left(t^b_{Ljk}P_L + t^b_{Rjk}P_R\right)\psi_k, \quad P_{L(R)} \equiv (1 \pm \gamma_5)/2$$

To study anomalies couple ψ to (not necessarily dynamical) A^b_μ

- $\delta A^b_\mu = f^{cdb}\Omega^d A^c_\mu + \partial_\mu \Omega^b$
- $\partial_\mu\psi \to D_\mu\psi \equiv \left[\partial_\mu - iA^b_\mu\left(t^b_L P_L + t^b_R P_R\right)\right]$

Anomalies occur only in the presence of chiral reps. $\rightarrow t^b_L \neq t^b_R$ for some b

The fermion sector

ψ_j: *one family* of fermions in a *general* representation of an *internal* symmetry gauge group

$$\delta\psi_j = i\Omega_{jk}\psi_k \equiv i\Omega^b \left(t^b_{Ljk}P_L + t^b_{Rjk}P_R\right)\psi_k, \qquad P_{L(R)} \equiv (1 \pm \gamma_5)/2$$

To study anomalies couple ψ to (not necessarily dynamical) A^b_μ

- $\delta A^b_\mu = f^{cdb}\Omega^d A^c_\mu + \partial_\mu \Omega^b$
- $\partial_\mu\psi \to D_\mu\psi \equiv \left[\partial_\mu - iA^b_\mu\left(t^b_L P_L + t^b_R P_R\right)\right]$

Anomalies occur only in the presence of chiral reps. $\to t^b_L \neq t^b_R$ for some b

Fermion action [Colladay and Kostelecky (1998)]

$$S = \int d^4x \left(i\,\overline{\psi}\Gamma^\mu D_\mu\psi - \overline{\psi}M\psi\right)$$

$$\Gamma^\mu \equiv c^\mu{}_\nu\gamma^\nu + d^\mu{}_\nu\gamma_5\gamma^\nu + e^\mu + i\,f^\mu\gamma_5 + \frac{1}{2}g^{\mu\nu\rho}\sigma_{\nu\rho}$$

$$M \equiv m + \frac{1}{2}H^{\mu\nu}\sigma_{\mu\nu} + a_\mu\gamma^\mu + b_\mu\gamma_5\gamma^\mu$$

The presence of chiral reps implies $\Gamma^\mu \equiv c^\mu{}_\nu\gamma^\nu + d^\mu{}_\nu\gamma_5\gamma^\nu$.

Physical constraints

Experimental limits (coming from analysis involving e.g. baryons, electrons, photons, mesons, etc...)

$$\Rightarrow \left|\delta\Gamma^{\mu}_{\alpha\beta}\right| << 1, \quad |\delta M_{\alpha\beta}| << m,$$

where

$$\delta\Gamma^{\mu} \equiv \Gamma^{\mu} - \gamma^{\mu}, \quad \delta M \equiv M - m$$

(in a frame in which the earth is not relativistic)

For tables containing constraints coming from experimental and theoretical considerations see e.g. **[Kostelecký and Russell, 2008].**

$\rightarrow$ *validity of the perturbation theory w.r.t. the Lorentz violating parameters*

Consistency: the model should be considered as **an EFT valid at low enough energies** because we have instabilities and/or lack of microcausality at high energies (or for large boosts).
[Kostelecky and Lehnert, 2000]

Functional formalism

$$e^{iW[A]} \equiv \int \delta\psi\delta\overline{\psi} e^{iS[A]}, \quad \text{with} \;\; e^{iW[0]} = 1$$

In the absence of anomalies $\delta W[A] = 0 + M$*-terms*

$\Longleftrightarrow$ Ward identities for

$$\langle J^{\mu_1}_{b_1}(x_1)...J^{\mu_n}_{b_n}(x_n)\rangle = \int \delta\psi\delta\overline{\psi}\, e^{i\,S[A=0]} J^{\mu_1}_{b_1}(x_1)...J^{\mu_n}_{b_n}(x_n)$$

Here we *change the definition of currents* $(\gamma^\mu \to \Gamma^\mu)$*:*

$$J^\mu_b \equiv \overline{\psi}\,\Gamma^\mu T_b \psi, \qquad T_b \equiv t^b_L P_L + t^b_R P_R.$$

In the Lorentz invariant case $\delta W[A]_{anom}$ can be expressed as follows and cannot be canceled by a local counterterm.

$$\delta W[A]_{anom} \propto \mathrm{Tr} \int d^4x\, \epsilon^{\mu\nu\lambda\rho}\Omega_L\, \partial_\mu \left(2A^L_\nu \partial_\lambda A^L_\rho - iA^L_\nu A^L_\lambda A^L_\rho\right) - (L \to R),$$

where $\quad \Omega_{L(R)} \equiv \Omega^b t^b_{L(R)},\; A^{L(R)}_\mu \equiv A^b_\mu t^b_{L(R)}$

Functional formalism

$$e^{iW[A]} \equiv \int \delta\psi\delta\overline{\psi} e^{iS[A]}, \quad \text{with} \;\; e^{iW[0]} = 1$$

In the absence of anomalies $\delta W[A] = 0 + M\text{-terms}$
$\Longleftrightarrow$ Ward identities for

$$\langle J^{\mu_1}_{b_1}(x_1)...J^{\mu_n}_{b_n}(x_n)\rangle = \int \delta\psi\delta\overline{\psi}\, e^{i\,S[A=0]} J^{\mu_1}_{b_1}(x_1)...J^{\mu_n}_{b_n}(x_n)$$

Here we *change the definition of currents* $(\gamma^\mu \to \Gamma^\mu)$:

$$J^\mu_b \equiv \overline{\psi}\,\Gamma^\mu T_b \psi, \qquad T_b \equiv t^b_L P_L + t^b_R P_R.$$

In the Lorentz invariant case $\delta W[A]_{anom}$ can be expressed as follows and cannot be canceled by a local counterterm.

$$\delta W[A]_{anom} \propto \text{Tr} \int d^4x\, \epsilon^{\mu\nu\lambda\rho}\Omega_L\, \partial_\mu \left(2A^L_\nu \partial_\lambda A^L_\rho - iA^L_\nu A^L_\lambda A^L_\rho\right) - (L \to R),$$

where $\quad \Omega_{L(R)} \equiv \Omega^b t^b_{L(R)},\; A^{L(R)}_\mu \equiv A^b_\mu t^b_{L(R)}$

What happens if we allow for Lorentz violating terms?
We have more freedom to write $\delta W[A]_{anom}$, but the usual anomaly cancellation conditions remain necessary to have anomaly freedom.

A semi-classical argument (I)

Consider the case

$$m + H^{\mu\nu}\sigma_{\mu\nu}/2 = 0, \quad (\text{“massless case”})$$

The “massive case” will be considered later.

By applying the Clifford algebra for γ^μ

$$\begin{aligned}\mathcal{L} &= i\overline{\psi}\Gamma^\mu D_\mu\psi - \overline{\psi}M\psi \\ &= i\overline{\psi_L}L^\mu{}_\nu\gamma^\nu\left[D_\mu + iL^{(-1)\rho}{}_\mu\left(a_\rho - b_\rho\right)\right]\psi_L \\ &\quad + i\overline{\psi_R}R^\mu{}_\nu\gamma^\nu\left[D_\mu + iR^{(-1)\rho}{}_\mu\left(a_\rho + b_\rho\right)\right]\psi_R,\end{aligned}$$

where $L^\mu{}_\nu \equiv c^\mu{}_\nu - d^\mu{}_\nu$ and $R^\mu{}_\nu \equiv c^\mu{}_\nu + d^\mu{}_\nu$

A semi-classical argument (I)

Consider the case

$$m + H^{\mu\nu}\sigma_{\mu\nu}/2 = 0, \quad (\text{“massless case”})$$

The “massive case” will be considered later.

By applying the Clifford algebra for γ^μ

$$\begin{aligned}\mathcal{L} &= i\overline{\psi}\Gamma^\mu D_\mu\psi - \overline{\psi}M\psi \\ &= i\overline{\psi_L}L^\mu{}_\nu\gamma^\nu\left[D_\mu + iL^{(-1)\rho}{}_\mu(a_\rho - b_\rho)\right]\psi_L \\ &\quad + i\overline{\psi_R}R^\mu{}_\nu\gamma^\nu\left[D_\mu + iR^{(-1)\rho}{}_\mu(a_\rho + b_\rho)\right]\psi_R,\end{aligned}$$

where $L^\mu{}_\nu \equiv c^\mu{}_\nu - d^\mu{}_\nu$ and $R^\mu{}_\nu \equiv c^\mu{}_\nu + d^\mu{}_\nu$

So the differences in the classical action are

1. two independent non singular linear and homogeneous transformations of γ^μ (*coordinate transformations*)

A semi-classical argument (I)

Consider the case

$$m + H^{\mu\nu}\sigma_{\mu\nu}/2 = 0, \quad (\text{“massless case”})$$

The “massive case” will be considered later.

By applying the Clifford algebra for γ^μ

$$\begin{aligned}\mathcal{L} &= i\overline{\psi}\Gamma^\mu D_\mu\psi - \overline{\psi}M\psi \\ &= i\overline{\psi_L}L^\mu{}_\nu\gamma^\nu\left[D_\mu + iL^{(-1)\rho}{}_\mu\left(a_\rho - b_\rho\right)\right]\psi_L \\ &\quad + i\overline{\psi_R}R^\mu{}_\nu\gamma^\nu\left[D_\mu + iR^{(-1)\rho}{}_\mu\left(a_\rho + b_\rho\right)\right]\psi_R,\end{aligned}$$

where $L^\mu{}_\nu \equiv c^\mu{}_\nu - d^\mu{}_\nu$ and $R^\mu{}_\nu \equiv c^\mu{}_\nu + d^\mu{}_\nu$

So the differences in the classical action are

1. two independent non singular linear and homogeneous transformations of γ^μ (*coordinate transformations*)
2. two independent constant shifts of the gauge fields (due to the CPT violating terms in the action)

A semi-classical argument (II)

In the Lorentz invariant case we know that we may write

$$\delta W[A]^{(2)}_{anom} \propto \mathrm{Tr} \int d^4x \, \epsilon^{\mu\nu\lambda\rho} \Omega_L \, \partial_\mu A^L_\nu \partial_\lambda A^L_\rho - (L \to R)$$

This contains a L-part and a R-part, **separately** invariant under

1. general coordinate transformations
2. constant shifts of the gauge fields

A semi-classical argument (II)

In the Lorentz invariant case we know that we may write

$$\delta W[A]^{(2)}_{anom} \propto \mathrm{Tr} \int d^4x \, \epsilon^{\mu\nu\lambda\rho} \Omega_L \, \partial_\mu A^L_\nu \partial_\lambda A^L_\rho - (L \to R)$$

This contains a L-part and a R-part, **separately** invariant under

1. general coordinate transformations
2. constant shifts of the gauge fields

Remembering

$$\begin{aligned} \mathcal{L} \quad = \quad & i\overline{\psi_L} L^\mu{}_\nu \gamma^\nu \left[D_\mu + i L^{(-1)\rho}{}_\mu \, (a_\rho - b_\rho) \right] \psi_L \\ & + i\overline{\psi_R} R^\mu{}_\nu \gamma^\nu \left[D_\mu + i R^{(-1)\rho}{}_\mu \, (a_\rho + b_\rho) \right] \psi_R , \end{aligned}$$

this *argues* that there is a regularization in which $\delta W[A]^{(2)}_{anom}$ is equal to the Lorentz invariant case.

[A generalization of the argument provided by Coleman and Glashow, 1998]

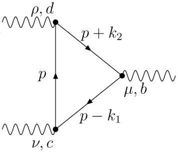
For a proof, which fully takes into account quantum mechanics and general (not necessarily Lorentz invariant) counterterms, see **[A. S., Phys. Rev. D 78, 085023 (2008)]**.

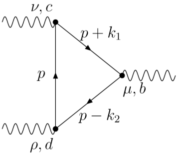
The massive case

$M_1 \equiv m + H^{\mu\nu}\sigma_{\mu\nu}/2 \neq 0$, but $c^\mu{}_\nu = \delta^\mu_\nu$ and $d^\mu{}_\nu = 0$ *for simplicity*

Consider the VVA graphs

$\langle j_b^\mu(x) j_c^\nu(y) j_{5d}^\rho(z)\rangle$, represented by





$j_b^\mu \equiv \overline{\psi}\gamma^\mu t^b \psi, \quad j_{b5}^\mu \equiv \overline{\psi}\gamma^\mu\gamma_5 t_5^b \psi$

Set $t^b \equiv (t_L^b + t_R^b)/2 = 1$ and $t_5^b \equiv (t_L^b - t_R^b)/2 = 1$

(the other cases can be treated in similar ways)

$$\frac{\partial}{\partial x^\mu}\langle j^\mu(x) j^\nu(y) j_5^\rho(z)\rangle = -\int \frac{d^4k_1}{(2\pi)^4}\frac{d^4k_2}{(2\pi)^4}\frac{d^4p}{(2\pi)^4}\, e^{i[(k_1+k_2)x - k_1 y - k_2 z]}$$

$$\left[h^{\nu\rho}(p+q_1) - h^{\nu\rho}(p) + \tilde{h}^{\nu\rho}(p+q_2) - \tilde{h}^{\nu\rho}(p)\right]$$

where $q_1 \equiv 2\alpha - k_1$ and $q_2 \equiv -2\alpha - k_2$ and

$$h^{\nu\rho}(p) \equiv \mathrm{Tr}\left(\gamma^\rho\gamma_5 \frac{1}{\not p - \not\alpha - M_1}\gamma^\nu \frac{1}{\not p + \not k_1 - \not\alpha - M_1}\right)$$

$$\tilde{h}^{\nu\rho}(p) \equiv \mathrm{Tr}\left(\gamma^\rho\gamma_5 \frac{1}{\not p + \not k_2 + \not\alpha - M_1}\gamma^\nu \frac{1}{\not p + \not\alpha - M_1}\right)$$

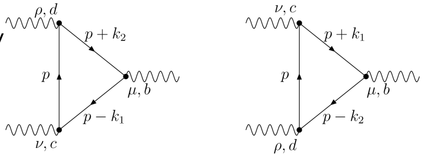
The massive case

$M_1 \equiv m + H^{\mu\nu}\sigma_{\mu\nu}/2 \neq 0$, but $c^\mu{}_\nu = \delta^\mu_\nu$ and $d^\mu{}_\nu = 0$ *for simplicity*

Consider the VVA graphs

$\langle j^\mu_b(x) j^\nu_c(y) j^\rho_{5d}(z)\rangle$, represented by

$$j^\mu_b \equiv \overline{\psi}\gamma^\mu t^b \psi, \quad j^\mu_{b5} \equiv \overline{\psi}\gamma^\mu \gamma_5 t^b_5 \psi$$



Set $t^b \equiv (t^b_L + t^b_R)/2 = 1$ and $t^b_5 \equiv (t^b_L - t^b_R)/2 = 1$
(the other cases can be treated in similar ways)

$$h_1^{\nu\rho}(p) \equiv \mathrm{Tr}\left(\gamma^\rho\gamma_5 \frac{1}{\not p - \not q - m}\delta M_1 \frac{1}{\not p - \not q - m}\gamma^\nu \frac{1}{\not p + \not k_1 - \not q - m}\right.$$
$$\left. + \gamma^\rho\gamma_5 \frac{1}{\not p - \not q - m}\gamma^\nu \frac{1}{\not p + \not k_1 - \not q - m}\delta M_1 \frac{1}{\not p + \not k_1 - \not q - m}\right),$$

where $\delta M_1 \equiv H^{\mu\nu}\sigma_{\mu\nu}/2$

The (dangerous) $m = 0$ term vanishes because δM_1 contains an **even** number of Dirac matrices. $\rightarrow$ The δM_1 independence arises for the same reason why the usual m-independence does.

Final remarks

Conclusions

- We have discussed the role of Lorentz invariance in the anomalies: *the usual anomaly cancellation conditions remain necessary for anomaly freedom even though* $\delta W[A]_{anom}$ *may assume a more general form.*

- Our approach has been intrinsically perturbative w.r.t the Lorentz violating parameters.

- We have provided an intuitive semi-classical argument and shown an explicit computation in the massive case.

Outlook

- Generalization to arbitrary mixings between families

- Analysis of anomalies associated with non internal symmetries