B_s mixing phase and lepton flavor violation in supersymmetric SU(5)

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in collaboration with

Pyungwon Ko and Masahiro Yamaguchi

DISCRETE '08, IFIC, València, 15/12/2008

Based on

- JhP, M. Yamaguchi, 0809.2614, to appear in PLB
- P. Ko, JhP, M. Yamaguchi, JHEP11(2008)051

B_s mixing phase

- B_s mixing phase is theoretically clean

$$\phi_s = \arg\, \langle B_s | \mathscr{H}_{\text{eff}}^{\Delta B=2} | \overline{B_s} \rangle$$

- SM prediction

$$\phi_s^{\text{SM}} \simeq -2\eta\lambda^2 \simeq -0.04$$

- Measurements

$$\phi_s = \begin{cases} -0.57^{+0.24+0.07}_{-0.30-0.02} & \text{DØ, 0802.2255} \\ [-1.36, -0.24] \text{ or } [-2.90, -1.78] & \text{CDF, 0712.2397} \end{cases}$$

- Constrained fit by HFAG

$$\phi_s = -0.76^{+0.37}_{-0.33} \text{ or } -2.37^{+0.33}_{-0.37}$$

Consistent with SM at 2.4σ level

HFAG, 0808.1297

New sources of flavor & CP violation in MSSM

- Down-type scalar quark mass term:

$$-\mathscr{L}_{\text{soft}} \ni \widetilde{d}^*_{Ai}\,(M^2_{d,AB})_{ij}\,\widetilde{d}_{Bj}$$

$A,B = L,R$ and $i,j = 1,2,3$

- may lead to an interaction

$$\tilde{d}_{B\,j}\;\text{- - -}\times\text{- - -}\;\tilde{d}_{A\,i} \quad = -i\,(M^2_{d,AB})_{ij}$$

if M^2_d is non-diagonal in the basis where m_d is diagonal and a d–$\widetilde{d}$–$\widetilde{g}$ vertex preserves flavor

- For almost diagonal $M^2_{\tilde{d}}$, use a mass insertion parameter

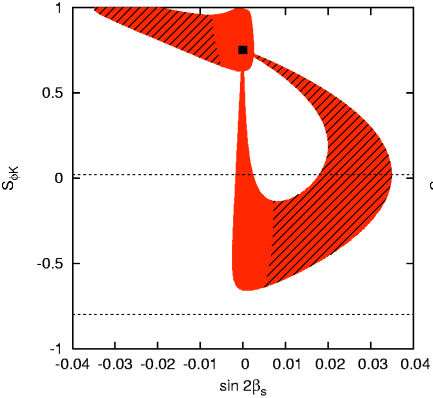
$$(\delta^d_{ij})_{AB} \equiv (\Delta^d_{ij})_{AB}/\widetilde{m}^2$$
$$(M^2_{d,AB})_{ij} = \widetilde{m}^2\,\delta_{AB}\delta_{ij} + (\Delta^d_{ij})_{AB}$$

Focus on *LL* and *RR* mixings

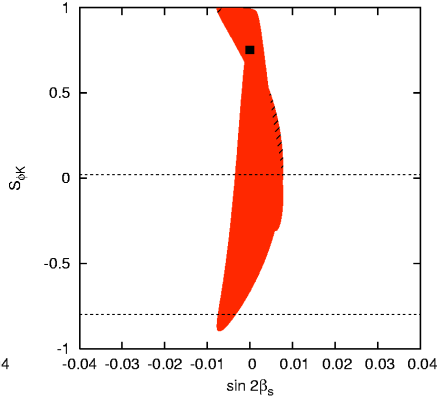
- $B \to X_s \gamma$ constraints on *LR* and *RL* insertions are too strong to make an appreciable difference in ϕ_s

With Kane, Ko, Kolda, Wang×2, PRL(2003); PRD(2004)

Ciuchini, Franco, Masiero, Silvestrini, PRD(2003)



LR



RL

Grand unification of flavor violations

Ciuchini, Masiero, Silvestrini, Vempati, Vives, PRL(2004)
Ciuchini, Masiero, Paradisi, Silvestrini, Vempati, Vives, NPB(2007)

- Superpotential of SUSY SU(5) model

$$W_{\rm GUT} = T^T \lambda_U T H + T^T \lambda_D \overline{F}\overline{H} + N^T \lambda_N \overline{F} H + N^T M_N N$$

$$T\ [\mathbf{10}] = \{Q, \overline{U}, \overline{E}\}, \quad \overline{F}\ [\overline{\mathbf{5}}] = \{\overline{D}, L\}$$

- MSSM superpotential from $W_{\rm GUT}$

$$W_{\rm SSM} = Q^T [V_Q^T \widehat{Y}_U] \overline{U} H_u + Q^T [\widehat{Y}_D] \overline{D} H_d + \overline{E}^T [\widehat{Y}_D] L H_d$$
$$+ N^T [\widehat{Y}_N V_L] L H_u + N^T M_N N$$

- Soft scalar mass terms

$$-\mathscr{L}_{\rm soft} \supset \widetilde{Q}^\dagger m^2_{\mathbf{10}} \widetilde{Q} + \widetilde{\overline{E}}^\dagger m^2_{\mathbf{10}} \widetilde{\overline{E}} + \widetilde{\overline{U}}^\dagger m^2_{\mathbf{10}} \widetilde{\overline{U}} + \widetilde{\overline{D}}^\dagger m^2_{\overline{\mathbf{5}}} \widetilde{\overline{D}} + \widetilde{L}^\dagger m^2_{\overline{\mathbf{5}}} \widetilde{L}$$

- GUT flavor relations

$$\boxed{(\delta^d_{ij})_{LL} = (\delta^l_{ij})^*_{RR}, \quad (\delta^d_{ij})_{RR} = (\delta^l_{ij})^*_{LL} \qquad \text{at } M_{\rm GUT}}$$

Caveat: $Y_D \neq Y_E^T$ at $M_{\rm GUT}$

- Fermion mass relations from $W_{\rm GUT}$,

$$m_e = m_d, \quad m_\mu = m_s, \quad m_\tau = m_b \qquad \text{at } M_{\rm GUT},$$

  are inconsistent with data
- A solution: add non-renormalizable terms
- In the basis where Y_D is diagonal,

$$Y_U = V_Q^T \widehat{Y}_U U_Q^*, \quad Y_D = \widehat{Y}_D, \quad Y_E = U_L^T \widehat{Y}_E U_R^*, \quad Y_N = U_L^T V_L^T \widehat{Y}_N$$

- GUT flavor relations

$$\delta^l_{LL} = \quad \delta^{d*}_{RR} \quad , \quad \delta^l_{RR} = \quad \delta^{d*}_{LL}$$

Transitions of L		Transitions of $\overline{D}$
$\mu \to e$	$\leftrightarrow$	$s \to d$
$\tau \to e$	$\leftrightarrow$	$b \to d$
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- GUT flavor relations are modified to

$$\delta_{LL}^l = U_L \, \delta_{RR}^{d*} \, U_L^\dagger, \quad \delta_{RR}^l = U_R \, \delta_{LL}^{d*} \, U_R^\dagger$$

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but don't give up yet!

$Y_D - Y_E^T$ from non-renormalizable operators

Ellis and Gaillard, PLB(1979)

- Non-renormalizable terms account for Yukawa difference

$$W_{\rm NR} \ni \sqrt{2}\left(h_1^{ij}\overline{H}_a \frac{\Sigma_b^a}{M_*} T_i^{bc}\overline{F}_{jc} + h_2^{ij}\overline{H}_a T_i^{ab}\frac{\Sigma_b^c}{M_*}\overline{F}_{jc}\right)$$

$$Y_D - Y_E^T = \xi h_2, \quad \xi \equiv \frac{1}{2}\frac{\langle\Sigma_1^1\rangle}{M_*} \approx \frac{M_{\rm GUT}}{M_*} \approx 10^{-2}$$

- In the basis where Y_D is diagonal,

$$\frac{Y_E}{[\widehat{Y}_E]_{33}} \sim \begin{pmatrix} \cos\beta & \cos\beta & \cos\beta \\ \cos\beta & \cos\beta & \cos\beta \\ \cos\beta & \cos\beta & 1 \end{pmatrix} \Rightarrow U_{L,R} \sim \left(\begin{array}{cc|c} \multicolumn{2}{c|}{[U_{L,R}]_{ab}} & \cos\beta \\ & & \cos\beta \\ \hline \cos\beta & \cos\beta & 1 \end{array}\right)$$

- 1–3 & 2–3 mixings suppressed by $\cos\beta$
- 1–2 mixing angles in $[U_{L,R}]_{ab}$ unlimited: may be zero, small, large

Baek, Goto, Okada, Okumura, PRD(2001)

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Exploit Planck-suppression of Yukawa difference

- GUT flavor relations broken

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- GUT flavor relations broken, **but not completely**

$$(\delta^l_{a3})_{LL} = [U_L]_{ab}\,(\delta^d_{b3})^*_{RR}\,[U_L]^*_{33} + \mathscr{O}(\cos^2\beta\;\delta^d_{RR}), \quad a,b = 1,2$$

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- 1–2 mixing and quark–lepton FCNC correlation

Transitions of $\overline{D}$		Transitions of L
$b \to d$	$\longleftrightarrow$	$\tau \to e$
$b \to s$	$\longleftrightarrow$	$\tau \to \mu$

if $[U_L]_{ab} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$

Exploit Planck-suppression of Yukawa difference

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if $[U_L]_{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

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if $\quad [U_L]_{ab} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

- Take the sum of branching fractions

$$\begin{aligned} B(\tau \to (e+\mu)\gamma) \propto{}& |(\delta^l_{13})_{LL}|^2 + |(\delta^l_{23})_{LL}|^2 \\ \approx{}& |(\delta^d_{13})_{RR}|^2 + |(\delta^d_{23})_{RR}|^2 + \mathscr{O}[\cos^2\beta\,(\delta^d_{RR})^2] \end{aligned}$$

Constraint on δ^d_{RR} roughly independent of U_L

- Suppose $U_L = U_R = \mathbf{1}$ for numerical analysis

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Constraint on δ^d_{RR} roughly independent of U_L

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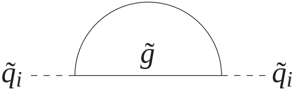
RG running of squark mass matrix below M_{GUT}

- Diagonal components

$$[m_{\tilde{q}}^2]_{ii}\,(M_{\text{SUSY}}) \approx m_0^2 + 6\,M_{1/2}^2 = (1+6x)\,m_0^2$$



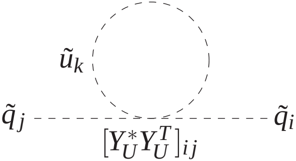
$$x \equiv M_{1/2}^2/m_0^2$$

GUT scale version of $m_{\tilde{g}}^2/m_{\tilde{q}}^2$ called x at M_Z

Gabbiani, Gabrielli, Masiero, Silvestrini, NPB(1996)

- Off-diagonal components

$$\Delta[m_Q^2]_{ij} \simeq -\frac{6}{(4\pi)^2}\,[V_Q^\dagger \widehat{Y}_U^2 V_Q]_{ij}\; m_0^2\; \ln\frac{M_{\text{GUT}}}{M_{\text{SUSY}}}$$



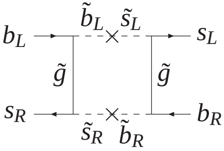
- Low & high scale mass insertions related by

$$(\delta_{ij}^d)_{LL}(M_{\text{SUSY}}) \approx \frac{(\delta_{ij}^d)_{LL}(M_{\text{GUT}}) + q_{ij}}{1+6x}, \qquad (\delta_{ij}^d)_{RR}(M_{\text{SUSY}}) \approx \frac{(\delta_{ij}^d)_{RR}(M_{\text{GUT}})}{1+6x}$$

$q_{ij} \equiv \Delta_{\text{s}}[m_Q^2]_{ij}/m_0^2$ ⟵ independent of x

x-dependence of B mixing

- B_s–$\overline{B_s}$ transition amplitude



$$\propto \left.\frac{(\delta^d_{23})_{LL}(\delta^d_{23})_{RR}}{m_S^2}\right|_{M_{\rm SUSY}}$$

$$\approx \frac{[(\delta^d_{23})_{LL}(M_{\rm GUT})+q_{ij}]\cdot(\delta^d_{23})_{RR}(M_{\rm GUT})}{(1+6x)^2 m_S^2}$$

$$\propto \frac{x}{(1+6x)^3} \quad \text{for fixed } M_{1/2} \,\&\, (\delta^d_{23})_{AB}(M_{\rm GUT})$$

$\Longrightarrow$ Maximized at $x \approx 1/12$

- $(\delta^d_{23})_{LL}(\delta^d_{23})_{LL}/m_S^2$ and $(\delta^d_{23})_{RR}(\delta^d_{23})_{RR}/m_S^2$ behave in the same way

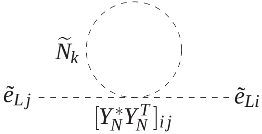
RG running of slepton mass matrix below $M_{\rm GUT}$

- Diagonal components

$$[m_{\tilde{l}}^2]_{ii}(M_{\rm SUSY}) \approx m_0^2$$

- Off-diagonal components

$$\Delta[m_{\tilde{l}}^2]_{ij} \simeq -\frac{6}{(4\pi)^2}[V_L^\dagger \widehat{Y}_N^2 V_L]_{ij}\, m_0^2\, \ln\frac{M_{\rm GUT}}{M_R}$$



- Low & high scale mass insertions related by

$$(\delta_{ij}^l)_{LL}(M_{\rm SUSY}) \approx (\delta_{ij}^l)_{LL}(M_{\rm GUT}) + l_{ij}, \qquad (\delta_{ij}^l)_{RR}(M_{\rm SUSY}) \approx (\delta_{ij}^l)_{RR}(M_{\rm GUT})$$

$$l_{ij} \equiv \Delta[m_{\tilde{l}}^2]_{ij}/m_0^2$$

- We ignore Y_N in numerical analysis, but there are cases with large Y_N to which our result can be applied

Parameter dependence of LFV

- LFV decay amplitudes

$$A(\tau \to \mu\gamma) \propto \frac{\mu\tan\beta \cdot (\delta^l_{23})_{LL}(M_{\mathrm{GUT}})}{m_S^2}$$

$$A(\mu \to e\gamma) \propto \frac{(\delta^l_{13})_{RR}(M_{\mathrm{GUT}}) \cdot (\delta^l_{33})_{RL}(M_{\mathrm{GUT}}) \cdot (\delta^l_{32})_{LL}(M_{\mathrm{GUT}})}{m_S^2}$$

$$\approx \frac{m_\tau \mu \tan\beta}{m_0^2} \times \frac{(\delta^l_{13})_{RR}(M_{\mathrm{GUT}}) \cdot (\delta^l_{32})_{LL}(M_{\mathrm{GUT}})}{m_S^2}$$

- Monotonically decreasing functions of $m_0^2 = M_{1/2}^2/x$
- Proportional to $\tan\beta$
- $A(\mu \to e\gamma)$ is inversely proportional to $m_0^2 \times m_S^2$

LFV and B mixing as functions of $x \equiv M_{1/2}^2/m_0^2$

- Fix $M_{1/2} = 180\ \mathrm{GeV},\ \tan\beta = 5;$ vary m_0 from $220\ \mathrm{GeV}$ to ∞
- Fix $(\delta_{23}^d)_{RR} = -0.1$ and $(\delta_{ij}^d)_{LL}$ to RG-induced values at M_{GUT}

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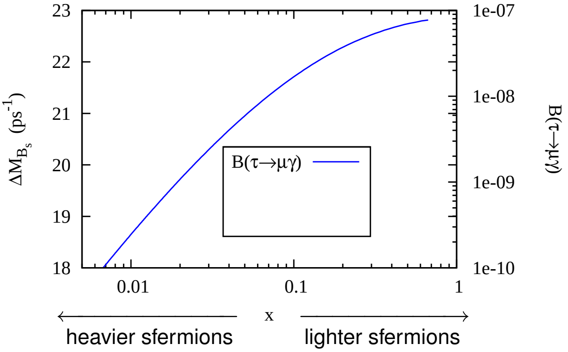

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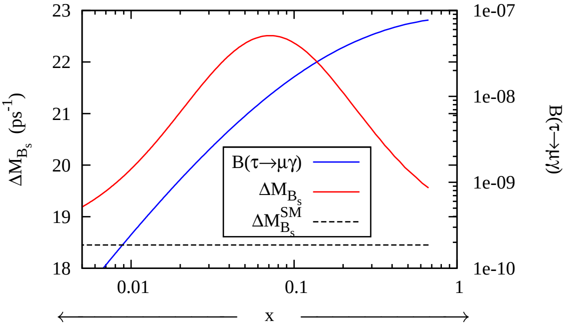

$\longleftarrow$ heavier sfermions x lighter sfermions $\longrightarrow$

LFV and B mixing as functions of $x \equiv M_{1/2}^2/m_0^2$

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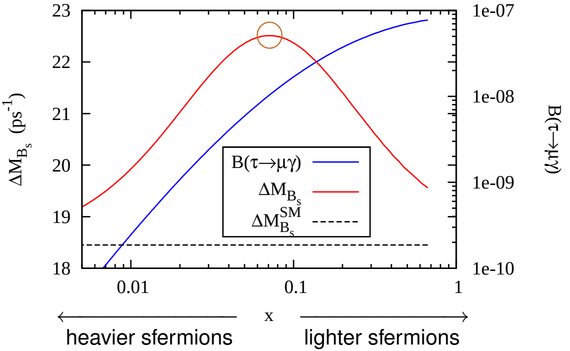

There is a value of x that optimizes the sensitivity of B mixing to GUT scale δ's

$\Longrightarrow$ Important when comparing hadronic and leptonic constraints

RG running of sfermion mass matrix above $M_{\rm GUT}$

- Off-diagonal components of m_T^2 — GUT version of $\Delta[m_Q^2]_{ij}$

$$\Delta m_T^2 \simeq -\frac{6}{(4\pi)^2}[3\lambda_U^*\lambda_U^T + 2\lambda_D^*\lambda_D^T]\ m_0^2\ \ln\frac{M_*}{M_{\rm GUT}}$$

add to $(\delta_{ij}^d)_{LL}$, $\boxed{(\delta_{ij}^l)_{RR}(M_{\rm GUT})}$

$\hookrightarrow$ relates $(\delta_{13}^d)_{RR}$ and $(\delta_{23}^d)_{RR}$ with $\mu \to e\gamma$

- Off-diagonal components of $m_{\overline{F}}^2$ — GUT version of $\Delta[m_l^2]_{ij}$

$$\Delta m_{\overline{F}}^2 \simeq -\frac{6}{(4\pi)^2}[4\lambda_D^\dagger\lambda_D + \lambda_N^\dagger\lambda_N]\ m_0^2\ \ln\frac{M_*}{M_{\rm GUT}}$$

add to $(\delta_{ij}^l)_{LL}$, $\underline{(\delta_{ij}^d)_{RR}(M_{\rm GUT})}$ if λ_N large

$\hookrightarrow$ popular when $S_{CP}^{B\to\phi K}$ showed a discrepancy

Constraints on $(\delta^d_{23})_{RR}$ in SUSY GUT

- Linked to $\tau \rightarrow \mu\gamma$

Hisano, Shimizu, PLB(2003)

Constraints on $(\delta^d_{23})_{RR}$ in SUSY GUT

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Hisano, Shimizu, PLB(2003)

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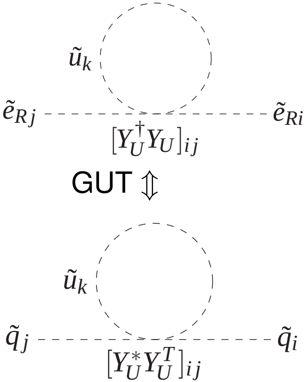

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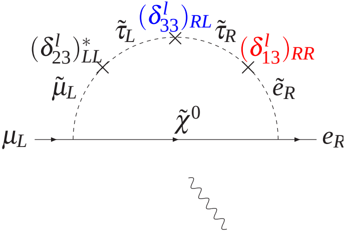
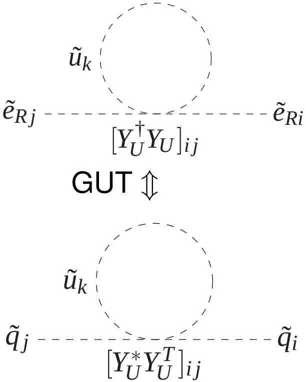
also related to $\mu \to e\gamma$





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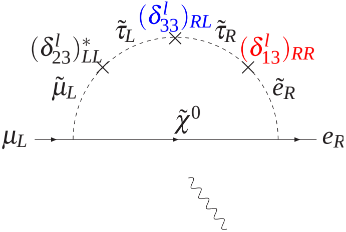
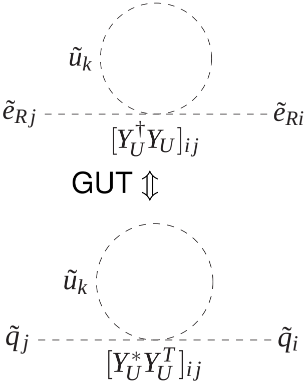


Enhanced by a factor m_τ/m_μ

Hisano, Moroi, Tobe, Yamaguchi, PRD(1996)

Baek, Goto, Okada, Okumura, PRD(2001)



Boundary conditions at M_{GUT}

- Scalar mass matrices

$$m_q^2 = m_0^2 \begin{bmatrix} 1 & (\delta_{12}^d)_{LL} & (\delta_{13}^d)_{LL} \\ (\delta_{12}^d)_{LL}^* & 1 & (\delta_{23}^d)_{LL} \\ (\delta_{13}^d)_{LL}^* & (\delta_{23}^d)_{LL}^* & 1 \end{bmatrix} = m_e^{2*}$$

$$m_d^2 = m_0^2 \begin{bmatrix} 1 & 0 & (\delta_{13}^d)_{RR} \\ 0 & 1 & (\delta_{23}^d)_{RR} \\ (\delta_{13}^d)_{RR}^* & (\delta_{23}^d)_{RR}^* & 1 \end{bmatrix} = m_l^{2*}$$

- Three cases to study

$\lvert(\delta_{12}^d)_{LL}\rvert$	$\lvert(\delta_{13}^d)_{LL}\rvert$	$\lvert(\delta_{23}^d)_{LL}\rvert$	$\lvert(\delta_{23}^d)_{RR}\rvert$
4.8×10^{-5}	1.5×10^{-3}	free	0
4.8×10^{-5}	1.5×10^{-3}	7.4×10^{-3}	free
4.8×10^{-5}	1.5×10^{-3}	free =	free

- 'Default value' of a mass insertion is that from RGE above M_{GUT}, not zero

How high should the cutoff be?

- Suppose low $M_* \sim M_{\mathrm{GUT}}$
- RG running of sfermion masses above M_{GUT} is negligible
  $\Longrightarrow \quad \mu \to e\gamma$ does not constrain $(\delta^d_{23})_{RR}$
- However, uncontrollable non-renormalizable operators spoil quark–lepton flavor relations completely

$$Y_D - Y_E^T = \xi h_2, \quad \xi \approx \frac{M_{\mathrm{GUT}}}{M_*} \sim \mathscr{O}(1)$$

- Nonsense to relate quark and lepton flavors unless $M_* \gg M_{\mathrm{GUT}}$

Hadronic constraints

Observable	Measured value	Imposed constraint
ΔM_s	$17.77 \pm 0.12\ \mathrm{ps}^{-1}$	$17.77\ \mathrm{ps}^{-1} \pm 30\%$
ϕ_s	$-0.76^{+0.37}_{-0.33}$ or $-2.37^{+0.33}_{-0.37}$	$[-1.26, -0.13] \cup$ $[-3.00, -1.88]$ at 90% CL
$B(B \to X_s \gamma)$	$(352 \pm 23 \pm 9) \times 10^{-6}$	$2\ \sigma$
$S_{\phi K}$	0.39 ± 0.17	$2\ \sigma$
$\lvert d_n \rvert$	$< 6.3 \times 10^{-26}\ e\,\mathrm{cm}$	

Leptonic constraints

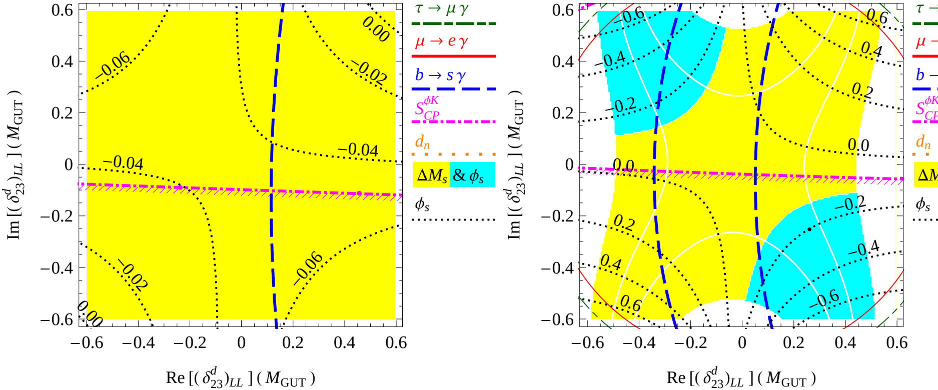
- Upper bounds used in the numerical analysis

Process	Present upper bound		Future upper bound	
$B(\mu \to e\gamma)$	1.2×10^{-11}	MEGA	1×10^{-13}	MEG
$B(\tau \to e\gamma)$	3.1×10^{-7}	BABAR	1×10^{-8}	
$B(\tau \to \mu\gamma)$	6.8×10^{-8}	Belle	1×10^{-8}	

- CDR of super B factory in Italy expects $B(\tau \to e\gamma) < 2 \times 10^{-9}$ and $B(\tau \to \mu\gamma) < 2 \times 10^{-9}$

Bona et al, 0709.0451

LL case

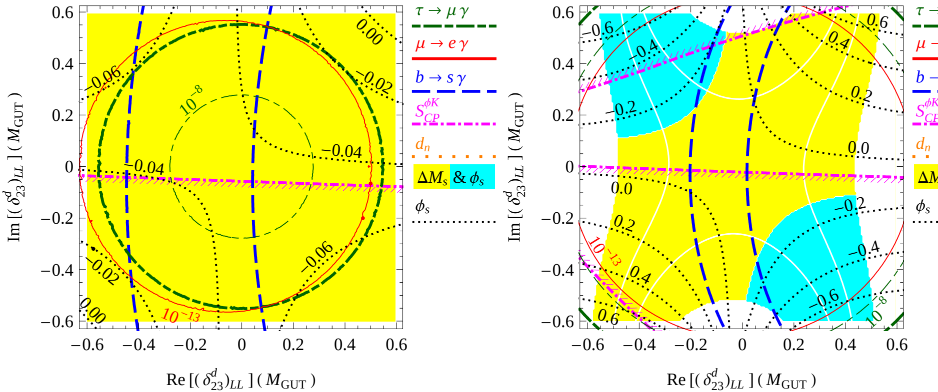


$$m_0 = 220 \text{ GeV} \qquad\qquad m_0 = 600 \text{ GeV}$$

$$M_{1/2} = 180 \text{ GeV}, \quad \tan\beta = 5$$

- For lower m_0, change in ϕ_s too small
- For higher m_0, small corners satisfy ϕ_s, $B \to X_s\gamma$, $S_{CP}^{\phi K}$
- For higher $\tan\beta$, ϕ_s conflicts with tighter $B \to X_s\gamma$ & $S_{CP}^{\phi K}$

LL case



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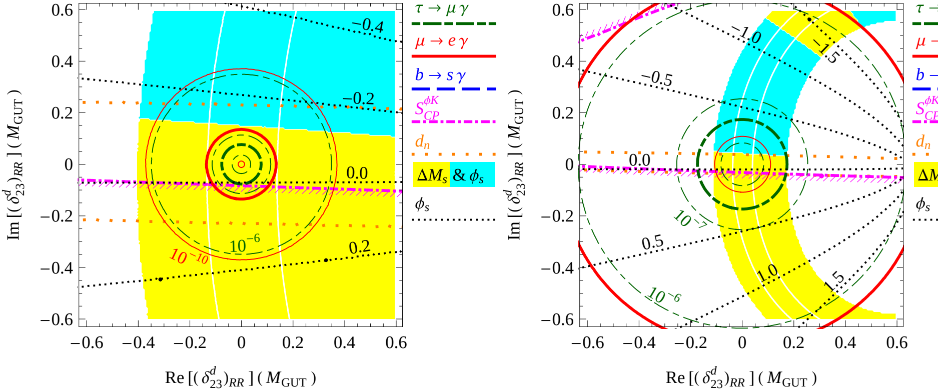
RR case



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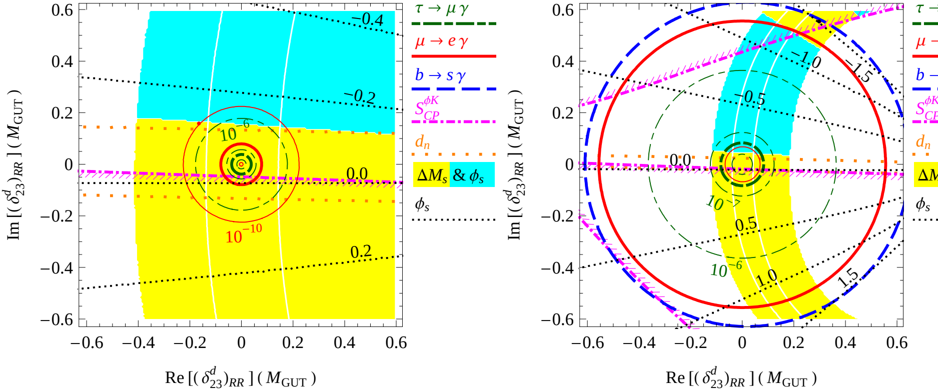
RR case



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Universal boundary condition + large Y_N scenario

- Suppose that soft terms are flavor blind at M_* and RR mixings arise solely from Y_N

  Hisano, Shimizu, 0805.3327

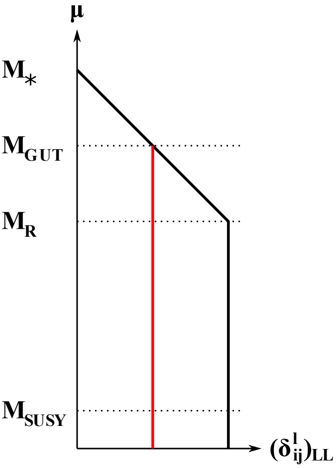
- Can convert a previous RR plot to one for this case, shrinking an LFV circle by $\alpha/(1+\alpha)$ where

$$\alpha \equiv \frac{\ln(M_*/M_{\text{GUT}})}{\ln(M_{\text{GUT}}/M_R)}$$

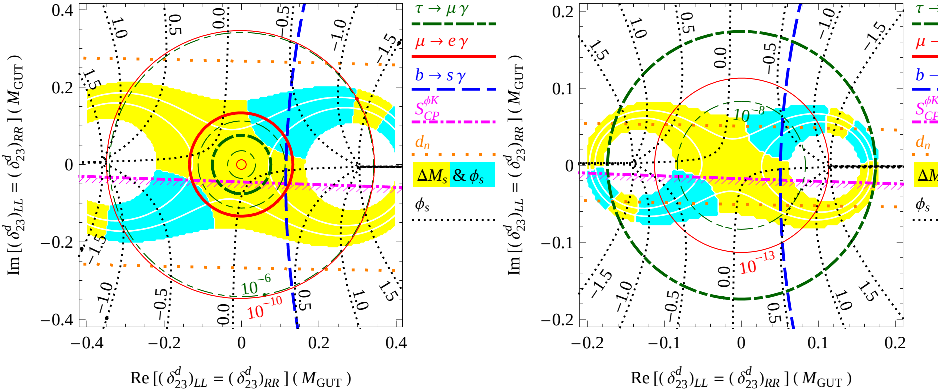
- Recall that

$$(\delta^d_{ij})_{RR}(M_{\text{GUT}}) = (\delta^l_{ij})^*_{LL}(M_{\text{GUT}})$$

$$(\delta^l_{ij})_{LL}(M_{\text{GUT}}) \approx \frac{\alpha}{1+\alpha}(\delta^l_{ij})_{LL}(M_{\text{SUSY}})$$



$LL = RR$ case



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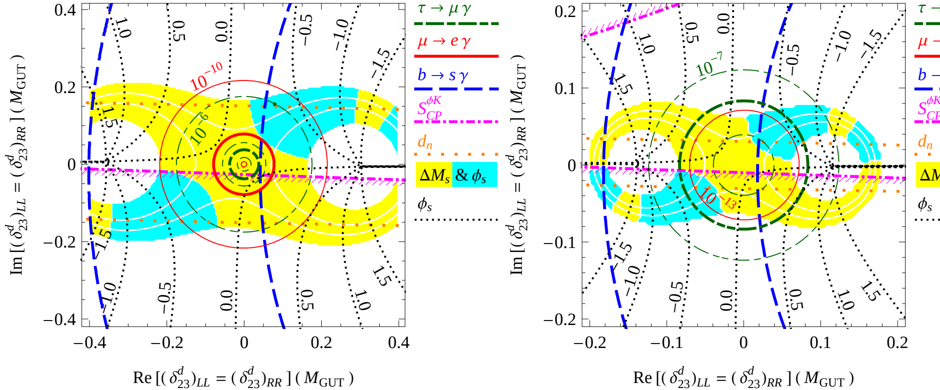
- For higher m_0, there are regions satisfying all the constraints
- Even higher $\tan\beta$ leaves viable corners for higher m_0

$LL = RR$ case



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Summary

SUSY GUT + large ϕ_s = LFV

- Hadronic constraints by themselves disfavor LL-only case
- With nonzero RR insertion, LFV rates are detectable or already excessive

Easy to reconcile ϕ_s with LFV and other constraints if

- $x \equiv M_{1/2}^2/m_0^2 \approx 1/12$
- there are both LL and RR mass insertions
- $\tan\beta$ is low

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Procedure of numerical analysis

1. Run Y_u, Y_d, Y_e from M_Z to $M_{\rm GUT}$
2. Go to the basis where Y_d and Y_e are diagonal
3. Set $m^2_{\tilde{f}}$ and $M_{1/2}$ at $M_{\rm GUT}$ **by hand**;
   Assume $A(M_{\rm GUT}) = 0$
4. Run down to M_R
5. Remove N
6. Run down to M_Z
7. Fix μ from EWSB condition
8. Compute flavor violations