# Conditions for vacuum stability in an $S(3)$ extension of the Standard Model

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# Higgs Boson in the Standard Model

In the Standard Model, one $SU(2)$ doublet Higgs Field is included for the symmetry breaking of the $SU(2) \times U(1)$ gauge groups.

$$V(\Phi) = -\mu^2|\Phi|^2 + \lambda|\Phi|^4 \quad \textit{where} \quad \Phi(X) = \begin{pmatrix} \phi(X)^+ \\ \phi(X)^0 \end{pmatrix}$$

the parameter $\lambda$ must be positive to produce a stable vacuum. the parameter $\mu$ can be either sign. In fact if the sign of the quadratic term is negative namely $\mu^2 > 0$, at the origin the potential has a maximum, hence, the stable vacuum state corresponds to a non-zero value of the $\Phi$ field.

- The states satisfying
  $|\phi(X)^+|^2 + |\phi(X)^0|^2 = \mu^2/2\lambda = v^2/2$
  are degenerate minima of the potential.
- We can choose the vacuum expectation value in the $< \phi^0 >= v/\sqrt{2}$ direction.
- There is one important prediction of this model, one scalar particle appears in the physical spectrum which is called the Higgs boson.
- The mass of the Higgs boson is given by $m_h = \sqrt{2\lambda}v$, the W and Z masses are $m_W = (g/2)\, v$, $m_Z = \left(\sqrt{g^2 + g'^2}/2\right) v$.
- Through the Yukawa couplings, the Higgs gives mass to the quarks and leptons $m_f = Y_f v/\sqrt{2}$.

- Prior to the introduction of the Higgs boson, and mass terms the Lagrangian of the Standard Model is chiral and invariant with respect to any permutations of the left and right quark and lepton fields $\leftrightarrow S(3)$ flavour symmetry.
- If we assume that the $S(3)$ permutational symmetry is not broken, the Higgs in the S. M. is an $S(3)$ singlet and only one fermion can acquire mass.
- Altough the Higgs potential is very simple and sufficient to describe a realistic model of mass generation, we believe that this is not the final form of the theory but rather an effective description of a more fundamental theory.

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The ingredients of the extension of the Standard Model are the following:

- To extend the flavour and family concepts to the Higgs sector
- To asociate each family to an irreducible representation of the flavour group
- To construct a Lagrangian invariant under the action of the $SU(3)_c \times SU(2) \times U(1) \times S_3^f group$

# $S_3$ irreducible representations

The group $S_3$ has two one dimensional irreducible representations (singlets ) and a two dimensional irreducible representation (doublet)

- One dimensional representations: $\mathbf{1}_A$ antisymmetric singlet, $\mathbf{1}_s$ symmetric singlet
- Bi - dimensional: $\mathbf{2}$ doblet

Direct product of an $S_3$ irreducible representations

$$\mathbf{1}_s \otimes \mathbf{1}_s = \mathbf{1}_s, \qquad \mathbf{1}_s \otimes \mathbf{1}_A = \mathbf{1}_A, \qquad \mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_s,$$

$$\mathbf{1}_s \otimes \mathbf{2} = \mathbf{2}, \qquad \mathbf{1}_A \otimes \mathbf{2} = \mathbf{2}$$

$$\boxed{\mathbf{2} \otimes \mathbf{2} = \mathbf{1}_s \oplus \mathbf{1}_A \oplus \mathbf{2}}$$

# Direct product of two $S_3$ doblets

$$\mathbf{p_D} \otimes \mathbf{q}_D = r_s \oplus r_A \oplus \mathbf{r}_D$$

$$\mathbf{p_D} = \begin{pmatrix} p_{D1} \\ p_{D2} \end{pmatrix} \quad \text{and} \quad \mathbf{q}_D = \begin{pmatrix} q_{D1} \\ q_{D2} \end{pmatrix}$$

it has two singlets, $r_s$ and $r_A$, just one doublet $r_D^T$

$$r_s = p_{D1}q_{D1} + p_{D2}q_{D2}$$ which is invariant,

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$$r_D^T = \begin{pmatrix} p_{D1}q_{D2} + p_{D2}q_{D1} \\ p_{D1}q_{D1} - p_{D2}q_{D2} \end{pmatrix}$$

The Higgs sector is modified

$$\Phi \to H \quad = \quad (\Phi_a, \Phi_b, \Phi_c)^T$$

$H$ is a reducible representation to $\mathbf{1_s} \oplus \mathbf{2}$ of $S_3$

$$H_s = \frac{1}{\sqrt{3}}\Big(\Phi_a + \Phi_b + \Phi_c\Big), \qquad \begin{pmatrix} H_1 \\ H_2 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_a - \Phi_b) \\ \frac{1}{\sqrt{6}}(\Phi_a + \Phi_b - 2\Phi_c) \end{pmatrix}$$

The Quark, lepton and Higgs fields are given by

$$Q^T = (u_L, d_L), u_R, d_R, \qquad L^\dagger = (\nu_L, e_L), e_R, \nu_R, \qquad H$$

All the fields have three species (Flavours) and belong to a representation reducible to $\mathbf{1} \oplus \mathbf{2}$ de $S_3$

# $S_3$ invariant Yukawa Lagrangian

Quarks

$$
\begin{aligned}
\mathcal{L}_{Y_D} &= -Y_1^d \overline{Q}_I H_S d_{IR} - Y_3^d \overline{Q}_3 H_S d_{3R} \\
&- Y_2^d [\ \overline{Q}_I \kappa_{IJ} H_1 d_{JR} + \overline{Q}_I \eta_{IJ} H_2 d_{JR}\ ] \\
&- Y_4^d \overline{Q}_3 H_I d_{IR} - Y_5^d \overline{Q}_I H_I d_{3R} + h.c
\end{aligned}
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$$\begin{aligned}\mathcal{L}_{Y_U} &= -Y_1^u \overline{Q}_I (i\sigma_2) H_S^* u_{IR} - Y_3^u \overline{Q}_3 (i\sigma_2) H_S^* u_{3R} \\ &- Y_2^u [\, \overline{Q}_I \kappa_{IJ} (i\sigma_2) H_1^* u_{JR} + \eta \overline{Q}_I \eta_{IJ} (i\sigma_2) H_2^* u_{JR} \,] \\ &- Y_4^u \overline{Q}_3 (i\sigma_2) H_I^* u_{IR} - Y_5^u \overline{Q}_I (i\sigma_2) H_I^* u_{3R} + h.c.,\end{aligned}$$

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Singlets carry the index $s$ or 3, and doublets carry indices $I, J = 1, 2$

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \text{and} \quad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

# $S_3$ invariant Yukawa Lagrangian

## Leptons

$$
\begin{aligned}
\mathcal{L}_{Y_E} &= -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} \\
&- Y_2^e [\, \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} \,] \\
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$$\begin{aligned}\mathcal{L}_{Y_\nu} &= -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\ &- Y_2^\nu [\, \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \,] \\ &- Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + h.c.,\end{aligned}$$

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Furthermore, we add the mass terms for the Majorana neutrinos

$$\mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R},$$

$C$ is the charge matrix.

The Higgs sector

$$\mathcal{L}_\Phi = [D_\mu H_S]^2 + [D_\mu H_1]^2 + [D_\mu H_2]^2 - V(H_1, H_2, H_S)$$
$$\textit{where} \quad D_\mu = \left(\partial_\mu - \frac{i}{2} g \tau \mathbf{A} - \frac{i}{2} g' B_\mu\right)$$

from here we obtain the W and Z mass

$$m_W^2 = \frac{g^2\left(v_1^2 + v_2^2 + v_3^2\right)}{4}, \qquad m_z^2 = \frac{\left(g^2 + g'^2\right)\left(v_1^2 + v_2^2 + v_3^2\right)}{4}$$

# $S_3$ invariant Higgs Potential

The most general Higgs Potential invariant under $SU(3)_C \times SU(2)_L \times U(1)_Y \times S_3$ can be writen as :

$$
\begin{aligned}
V &= \mu_1^2 \left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + \mu_0^2 \left(H_S^\dagger H_S\right) + a \left(H_S^\dagger H_S\right)^2 \\
&+ b \left(H_S^\dagger H_S\right) \left(H_1^\dagger H_1 + H_2^\dagger H_2\right) + c \left(H_1^\dagger H_1 + H_2^\dagger H_2\right)^2 \\
&+ d \left(H_1^\dagger H_2 - H_2^\dagger H_1\right)^2 + e f_{ijk} \left(\left(H_S^\dagger H_i\right) \left(H_j^\dagger H_k\right)\right) \\
&+ f \left\{\left(H_S^\dagger H_1\right) \left(H_1^\dagger H_S\right) + \left(H_S^\dagger H_2\right) \left(H_2^\dagger H_S\right)\right\} \\
&+ g \left\{\left(H_1^\dagger H_1 - H_2^\dagger H_2\right)^2 + \left(H_1^\dagger H_2 + H_2^\dagger H_1\right)\right\} + \\
& \quad h \left\{ \left(H_S^\dagger H_1\right) \left(H_S^\dagger H_1\right) + \left(H_S^\dagger H_2\right) \left(H_S^\dagger H_2\right) + \left(H_1^\dagger H_S\right) \left(H_1^\dagger H_S\right) \right. \\
&+ \left. \left(H_2^\dagger H_S\right) \left(H_2^\dagger H_S\right) \right\} \quad where \quad f_{112} = f_{121} = f_{211} = -f_{222} = 1
\end{aligned} \tag{1}
$$

The $SU(2)_L$ Higgs doublets with a flavour index $1, 2, S$ are

$$H_1 = \begin{pmatrix} \phi_1 + i\phi_2 \\ \phi_7 + i\phi_{10} \end{pmatrix}, \qquad H_2 = \begin{pmatrix} \phi_3 + i\phi_4 \\ \phi_8 + i\phi_{11} \end{pmatrix}, \qquad H_S = \begin{pmatrix} \phi_5 + i\phi_6 \\ \phi_9 + i\phi_{12} \end{pmatrix} \tag{2}$$

Introducing the following notation :

$$\begin{aligned} x_1 &= H_1^\dagger H_1, \quad x_2 = H_2^\dagger H_2, \quad x_3 = H_S^\dagger H_S, \\ x_4 &= Re\left(H_1^\dagger H_2\right), \quad x_5 = Im\left(H_1^\dagger H_2\right), \quad x_6 = Re\left(H_1^\dagger H_S\right), \\ x_7 &= Im\left(H_1^\dagger H_S\right), \quad x_8 = Re\left(H_2^\dagger H_S\right), \quad x_9 = Im\left(H_2^\dagger H_S\right) \end{aligned} \tag{3}$$

.

# The most general Higgs potential invariant under the exact simmetry $SU(2)_L \times U(1)_Y \times S_3$

$$V(\mathbf{X}) = \mathbf{A}^T\mathbf{X} + \frac{1}{2}\mathbf{X}^T\mathbf{B}\mathbf{X} \tag{4}$$

with

$$\mathbf{X}^T = (x_1, x_2, x_3, .........., x_9)\,, \quad \mathbf{A}^T = \left(\mu_1^2, \mu_1^2, \mu_0^2, 0, 0, 0, 0, 0, 0\right) \tag{5}$$

and

$$\mathbf{B} = \begin{pmatrix} 2(c+g) & 2(c-g) & b & 0 & 0 & 0 & 0 & 2e & 0 \\ 2(c-g) & 2(c+g) & b & 0 & 0 & 0 & 0 & -2e & 0 \\ b & b & 2a & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8g & 0 & 4e & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -8d & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 4e & 0 & 2(f+2h) & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2(f-2h) & 0 & 0 \\ 2e & -2e & 0 & 0 & 0 & 0 & 0 & 2(f+2h) & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2(f-2h) \end{pmatrix} \tag{6}$$

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- The normal Minimum with the following field configuration:

$$\phi_7 = v_1, \phi_8 = v_2, \phi_9 = v_3, \phi_i = 0, \quad i \neq 7, 8, 9$$

- The stationary point which breaks the electric charge, here two of the charged fields $\phi$ aquiere a non zero vev's :

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$$\phi_7 = v_1', \phi_8 = v_2', \phi_9 = v_3', \phi_1 = \alpha, \phi_3 = \beta,$$

- The $CP$ breaking minimum, where two imaginary components of the neutral fields $\phi$ acquiere a non zero vev's.

$$\phi_7 = v_1'', \phi_8 = v_2'', \phi_9 = v_3'', \phi_{10} = \delta, \phi_{11} = \gamma,$$

# A. The normal minimum

From the definitions above ,
$x_i = v_i^2$ for $i = 1, 2, 3$ $x_4 = v_1 v_2$, $x_6 = v_1 v_3$, $x_8 = v_2 v_3$, y
$x_5 = x_7 = x_9 = 0$.
We can write the minimization conditions as

$$\frac{\partial V}{\partial v_i} = 0 \leftrightarrow \frac{\partial V}{\partial x_j}\frac{\partial x_j}{\partial v_i} = 0 \quad i = 1, 2, 3 \quad j = 1, 2, .., 9 \qquad (7)$$

this are a set ot three coupled equations. The solution to this equations implies

$$e = 0$$

or

$$v_1^2 = 3v_2^2$$

Let us define the vector $(\mathbf{V}'_N)_i = (\mathbf{V}' \,|\mathbf{x}_N)_i = \frac{\partial V}{\partial x_i}$, evaluated in the minimum. In this notation the minimization conditions gives

$$(\mathbf{V}'_N)_1 = -\frac{(\mathbf{V}'_N)_4}{2v_1 v_2} v_2^2 - \frac{(\mathbf{V}'_N)_6}{2v_1 v_3} v_3^2 \tag{8}$$

$$(\mathbf{V}'_N)_2 = -\frac{(\mathbf{V}'_N)_4}{2v_1 v_2} v_1^2 - \frac{(\mathbf{V}'_N)_8}{2v_2 v_3} v_3^2 \tag{9}$$

$$(\mathbf{V}'_N)_3 = -\frac{(\mathbf{V}'_N)_6}{2v_1 v_2 3} v_1^2 - \frac{(\mathbf{V}'_N)_8}{2v_2 v_3} v_2^2 \tag{10}$$

From here it follows that:

$$\mathbf{V}'_N = \begin{pmatrix} -\left(\frac{\partial V}{\partial x_4} v_2 + \frac{\partial V}{\partial x_6} v_3\right)\frac{1}{2v_1} \\ -\left(\frac{\partial V}{\partial x_4} v_1 + \frac{\partial V}{\partial x_8} v_3\right)\frac{1}{2v_2} \\ -\left(\frac{\partial V}{\partial x_6} v_1 + \frac{\partial V}{\partial x_8} v_2\right)\frac{1}{2v_3} \\ \frac{\partial V}{\partial x_4} \\ 0 \\ \frac{\partial V}{\partial x_6} \\ 0 \\ \frac{\partial V}{\partial x_8} \\ 0 \end{pmatrix} = \begin{pmatrix} \frac{-(\mathbf{V}')_4}{2v_1 v_2} v_2^2 + \frac{-(\mathbf{V}')_6}{2v_1 v_3} v_3^2 \\ \frac{-(\mathbf{V}')_4}{2v_1 v_2} v_1^2 + \frac{-(\mathbf{V}')_8}{2v_2 v_3} v_3^2 \\ \frac{-(\mathbf{V}')_6}{2v_1 v_2 3} v_1^2 + \frac{-(\mathbf{V}')_8}{2v_2 v_3} v_2^2 \\ (\mathbf{V}')_4 \\ 0 \\ (\mathbf{V}')_6 \\ 0 \\ (\mathbf{V}')_8 \\ 0 \end{pmatrix} \tag{11}$$

it is clear from this expression that the first three entries in $\mathbf{V}'_N$ have the same sign if the ratios $\frac{-(\mathbf{V}')_4}{2v_1 v_2}$, $\frac{-(\mathbf{V}')_6}{2v_1 v_3}$, $\frac{-(\mathbf{V}')_8}{2v_2 v_3}$ have equal signs too.

The stationary point is given by the conditions impossed in equation (7) . Analizying the second derivatives of the Higgs potential $V$ we obtain the minimum conditions, these are given by the matrix of the squared scalar Higss masses. Particularly, for the scalar charged Higgs we have the squared masses $m^2_{H^\pm_{1,2}}$ as given in equation (42): $m^2_{H^\pm_{1,2}} = V'_1 + V'_2 + V'_3 \pm$ $\sqrt{(V'_1 + V'_2 + V'_3)^2 - (4V'_1V'_2 + 4V'_1V'_4 + 4V'_2V'_3 + V'^2_4 + V'^2_6 + V'^2_8)}$ The normal minimum exists if $m^2_{H^\pm_{1,2}} > 0$. From equation (11) we can read $(\mathbf{V}'_N)_i$ , its clear from it that the first three entries in $\mathbf{V}'_N$ has the same sign as we mantioned before and the squared masses are possitive if the sign of the ratios $\frac{-(\mathbf{V}')_4}{2v_1v_2}$, $\frac{-(\mathbf{V}')_6}{2v_1v_3}$, $\frac{-(\mathbf{V}')_8}{2v_2v_3}$ are positive too.

The normal minimum exists if $m^2_{H^{\pm}_{1,2}} > 0$.

$$m^2_{H^{\pm}_{1,2}} = \frac{1}{2}\left[Tr\mathbf{M}^2_C \pm \sqrt{(Tr\mathbf{M}^2_C)^2 - 4\chi^2}\right] > 0 \tag{12}$$

from here we have that

$$[Tr\mathbf{M}]^2_C > (Tr\mathbf{M}^2_C) > \tfrac{1}{2}[Tr\mathbf{M}]^2_C \tag{13}$$

In the normal minimuim we get

$$\mathbf{V}'_N = \mathbf{A} + \mathbf{B}\mathbf{X}_N \qquad \mathbf{X}_N^T\mathbf{V}'_N = 0 \tag{14}$$

where $\mathbf{X}_N = \mathbf{X}\,\big|_{normal\ min}$ .
In this notation, the potential evaluated at the normal minimum can be written as:

$$V_N = -\frac{1}{2}\mathbf{X}_N^T\mathbf{B}\mathbf{X}_N = \frac{1}{2}\mathbf{A}^T\mathbf{X}_N \tag{15}$$

# B. Charge breaking minimum

In this case, the $S_3$ CB doublet of the Higgs field: $\phi_7 = v_1'$, $\phi_8 = v_2'$, $\phi_9 = v_3'$ y $\phi_1 = \alpha$, $\phi_3 = \beta$.
The vector $\mathbf{X}_{CB}$ can be written as:

$$\mathbf{X}_{CB} = \begin{pmatrix} \alpha^2 + v_1'^2 \\ \beta^2 + v_2'^2 \\ v_3'^2 \\ \alpha\beta + v_1' v_2' \\ 0 \\ v_1' v_3' \\ 0 \\ v_2' v_3' \\ 0 \end{pmatrix} \tag{16}$$

Direct analizis of the potential gives for this stationary point

$$\mathbf{V}'_{CB} = \mathbf{A} + \mathbf{B}\mathbf{X}_{CB} \tag{17}$$

The potential evaluated at the CB minimum can be written as:

$$V_{CB} = -\frac{1}{2}\mathbf{X}_{CB}^T\mathbf{B}\mathbf{X}_{CB} = \frac{1}{2}\mathbf{A}^T\mathbf{X}_{CB} \tag{18}$$

From this equation and the normal minimum, we can compare the potential evaluated at the two different minima

$$V_{CB} - V_N = \frac{1}{2}\left(\mathbf{X}_{CB}^T\mathbf{V}'_N - \mathbf{X}_N^T\mathbf{V}'_{CB}\right) \tag{19}$$

If the signs of the ratios $\frac{-(\mathbf{V}')_4}{2v_1v_2}$, $\frac{-(\mathbf{V}')_6}{2v_1v_3}$, $\frac{-(\mathbf{V}')_8}{2v_2v_3}$ are positive, the normal minimum exists and the following product is also positive:

$$\begin{aligned} \mathbf{X}_{CB}^T\mathbf{V}'_N &= -\frac{(\mathbf{V}'_N)_6}{2v_1v_3}\left[\alpha^2v_3^2+(v'_1v_3-v'_3v_1)^2\right]-\frac{(\mathbf{V}'_N)_8}{2v_2v_3}\left[\beta^2v_3^2+(v'_2v_3-v'_3v_2)^2\right] \\ &-\frac{(\mathbf{V}'_N)_4}{2v_1v_2}\left[(\alpha v_2-\beta v_1)^2+(v'_1v_2-v'_2v_1)^2\right]. \end{aligned} \tag{20, 21}$$

If the product :

$$\mathbf{X}_N^T\mathbf{V}'_{CB} = \frac{(\mathbf{V}'_{CB})_4}{2\alpha\beta}\left\{(v_2\alpha-v_1\beta)+\frac{v_3}{v'_3}\left(v'_1\beta-v'_2\alpha\right)\right\}^2 \tag{22}$$

vanishes the normal minimum is the deepest one.

# Possible scenarios

First, the two $S(3)$ doublets Higgs fields acquires equal vevs:

$$\frac{v_1}{v_2} = \frac{v_2'}{v_1'} = \frac{\alpha}{\beta} \tag{23}$$

the normal and the electric charge violating minima have an accidental $S(2)$ symmetry $v_1 = v_2$, $v_1' = v_2'$, $\alpha = \beta$. Or

$$v_3 = v_3' \left(\frac{v_2}{v_2'}\right) \left(\frac{\frac{v_1}{v_2} - \frac{\alpha}{\beta}}{\frac{v_1'}{v_2'} - \frac{\alpha}{\beta}}\right) \tag{24}$$

The second case realizes when the $(\mathbf{V'}_{CB})_4$ vanishes, that is

$$g2\left(\alpha\beta + v_1' v_2'\right) + e v_1' v_3' = 0 \tag{25}$$

# C. CP breaking

The CP breaking stationary point is given by the following vevs: $\phi_7 = v_1'$, $\phi_8 = v_2'$, $\phi_9 = v_3'$, $\phi_{10} = \delta$ and $\phi_{11} = \gamma$. Then,

$$\mathbf{X}_{CP} = \begin{pmatrix} \delta^2 + v_1''^2 \\ \gamma^2 + v_2''^2 \\ v_3''^2 \\ \delta\gamma + v_1'' v_2'' \\ v_1''\gamma - v_2''\delta \\ v_2'' v_3'' \\ -\delta v_3'' \\ v_2'' v_3'' \\ -\gamma v_3'' \end{pmatrix} \tag{26}$$

We obtain

$$\mathbf{X}_{CP}^T \mathbf{V'}_{CP} = 0 \tag{27}$$

and

$$V_{CP} = \mathbf{A}^T \mathbf{X}_{CP} + \frac{1}{2}\mathbf{X}_{CP}^T \mathbf{B}_{CP} \mathbf{X}_{CP} \qquad \mathbf{V}'_{CP} = \mathbf{A} + \mathbf{B}_{CP}\mathbf{X}_{CP} \tag{28}$$

In this case, the potential evaluated in the normal minimum and the CP violating minimum can be compared as follows:

$$\mathbf{V}_{CP} - \mathbf{V}_N = \frac{1}{2}\left(\mathbf{X}_{CP}^T \mathbf{V}_N' - \mathbf{X}_N^T \mathbf{V}_{CP}'\right) \tag{29}$$

That is, the normal minimum is the deepest one if $\mathbf{X}_{CP}^T \mathbf{V}'$ is positive. The signs of the ratios $\frac{-(\mathbf{V}')_4}{2v_1 v_2}$, $\frac{-(\mathbf{V}')_6}{2v_1 v_3}$, $\frac{-(\mathbf{V}')_8}{2v_2 v_3}$ are positive,

$$\begin{aligned}\mathbf{x}_{CP}^T \mathbf{V}_N' \quad = \quad & -\frac{(\mathbf{V}_N')_6}{2v_1 v_3}\left[\delta^2 v_3^2 + (v_1'' v_3 - v_3'' v_1)^2\right] - \frac{(\mathbf{V}_N')_8}{2v_2 v_3}\left[\gamma^2 v_3^2 + (v_2'' v_3 - v_3'' v_2)^2\right] \\ & -\frac{(\mathbf{V}_N')_4}{2v_1 v_2}\left[(\delta v_2 - \gamma v_1)^2 + (v_1'' v_2 - v_2'' v_1)^2\right] \end{aligned} \tag{30}$$

and the normal normal minimum is the deepest one if the following vanishes

$$\mathbf{x}_N^T \mathbf{V}_{CP}' \quad = \quad -\frac{(\mathbf{V}_{CP}')_4}{2\gamma\delta}\left[(v_1\gamma - v_2\delta) + \frac{v_3}{v''_3}\left(v''_1\gamma - v''_2\delta\right)\right]^2 \tag{31}$$

that is: $v_1 = v_2 \ \gamma = \delta \ v''_1 = v''_2$.

# Higgs Mass matrix

If we want to know the nature of the stationary points, it is necesary to compute the second derivatives of the Higgs potential. These are given by

$$\frac{\partial^2 V}{\partial\phi_i\partial\phi_j} = \frac{\partial V}{\partial x_l}\frac{\partial^2 x_l}{\partial\phi_i\partial\phi_j} + \frac{\partial^2 V}{\partial x_l\partial x_m}\frac{\partial x_l}{\partial\phi_i}\frac{\partial x_m}{\partial\phi_j} \tag{32}$$

Defining:

$$\left(\mathbf{V}'\right)_i = \frac{\partial V}{\partial X_i} \qquad [\mathbf{B}_{lm}] = \frac{\partial^2 V}{\partial X_l\partial X_m} \qquad l,m = 1,2,...,9$$
$$[\mathbf{C}]_{ij} = \frac{\partial X_i}{\partial\phi_j} \qquad i = 1,2,...,9 \qquad j = 1,2,...,12 \tag{33}$$

For a positive defined matrix **B** we have a minimum, for a negative defined matrix **B** the critical point is a maximum; if the matrix **B** is nither positive or negative defyned we have a saddle point.
If the determinants of the submatrices are all different from zero, but the **B** matrix is nither positive defined or negative defined, the critical point is a saddle point. In the model we found that **B** is positive defined if

$$c > 0, g > 0 \quad , a > \frac{b^2}{4c}, \quad d < 0, f + 2h > \frac{e^2}{g} \quad f > 2h. \tag{34}$$

# Mass matrix

The Higgs mass matrix has the form

$$[\mathbf{M}^2] = \frac{1}{2}\left([\mathbf{M}_l^2] + \mathbf{C}^T\mathbf{B}\mathbf{C}\right) \tag{35}$$

The first term in the right hand side is taken from equation (32) is

$$[\mathbf{M}_l^2]_{ij} = \frac{\partial V}{\partial X_l}\frac{\partial^2 X_l}{\partial\phi_i\partial\phi_j} = V_l'\frac{\partial^2 X_l}{\partial\phi_i\partial\phi_j} \tag{36}$$

In the normal minimum

$$[\mathbf{M}_l^2] = \begin{pmatrix} \mathbf{M}_{11}^2 & 0 \\ 0 & \mathbf{M}_{12}^2 \end{pmatrix} \tag{37}$$

where $\mathbf{M}_{11}^2$ y $\mathbf{M}_{12}^2$ are $6 \times 6$ matrix.

$$[\mathbf{M}_{11}^2] = \begin{pmatrix} 2V_1' & V_4' & V_6' & 0 & 0 & 0 \\ V_6' & V_8' & 2V_3' & 0 & 0 & 0 \\ V_4' & 2V_2' & V_8' & 0 & 0 & 0 \\ 0 & 0 & 0 & 2V_1' & V_4' & V_6' \\ 0 & 0 & 0 & V_6' & V_8' & 2V_3' \\ 0 & 0 & 0 & V_4' & 2V_2' & V_8' \end{pmatrix} \tag{38}$$

$$[\mathbf{M}_{12}^2] = \begin{pmatrix} 2V_1' & V_4' & V_6' & 0 & 0 & 0 \\ V_4' & 2V_2' & 2V_8' & 0 & 0 & 0 \\ V_6' & V_8' & 2V_3' & 0 & 0 & 0 \\ 0 & 0 & 0 & 2V_1' & V_4' & V_6' \\ 0 & 0 & 0 & V_4' & 2V_2' & V_8' \\ 0 & 0 & 0 & V_6' & V_8' & 2V_3' \end{pmatrix} \tag{39}$$

The entries in the matrix **B** are given by the second derivatives of the Higgs potential. Defining the $9 \times 12$ matrix $\mathbf{C}_{ij} = \partial x_i / \partial \phi_j$ as:

$$[\mathbf{C}] = \begin{pmatrix} 2\phi_1 & 0 & 0 & 0 & 0 & 0 & 2\phi_7 & 0 & 0 & 2\phi_{10} & 0 & 0 \\ 0 & 0 & 2\phi_3 & 0 & 0 & 0 & 0 & 2\phi_8 & 0 & 0 & 2\phi_{11} & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2\phi_9 & 0 & 0 & 0 \\ \phi_3 & 0 & \phi_1 & 0 & 0 & 0 & \phi_8 & \phi_7 & 0 & \phi_{11} & \phi_{10} & 0 \\ 0 & -\phi_3 & 0 & \phi_1 & 0 & 0 & \phi_{11} & -\phi_{10} & 0 & -\phi_8 & \phi_7 & 0 \\ 0 & 0 & 0 & 0 & \phi_1 & 0 & \phi_9 & 0 & \phi_7 & 0 & 0 & \phi_{10} \\ 0 & 0 & 0 & 0 & 0 & \phi_1 & 0 & 0 & -\phi_{10} & -\phi_9 & 0 & \phi_7 \\ 0 & 0 & 0 & 0 & \phi_3 & 0 & 0 & \phi_9 & \phi_8 & 0 & 0 & \phi_{11} \\ 0 & 0 & 0 & 0 & 0 & \phi_3 & 0 & 0 & -\phi_{11} & 0 & -\phi_9 & \phi_8 \end{pmatrix} \quad (40)$$

Evaluated at each of the different stationary points, only the fields $\phi_7, \phi_8, \phi_9,\ \phi_1, \phi_3, \phi_5$ and $\phi_{12}$ appear, the remaining fields are zero at the stationary points.

Hence, the mass matrix of the cuadratic masses can be computed from (35)-(44), it takes the following form

$$diag(\mathbf{M}_C^2, \mathbf{M}_C^2, \mathbf{M}_S^2, \mathbf{M}_P^2) \tag{41}$$

$\mathbf{M}_{11}$ is the charged Higgs mass matrix, the mass of the physical charged Higgs can be expresed as

$$\begin{aligned} m^2_{H^\pm_{1,2}} &= V_1' + V_2' + V_3' \\ &\pm \sqrt{(V_1' + V_2' + V_3')^2 - (4V_1'V_2' + 4V_1'V_4' + 4V_2'V_3' + V_4'^2 + V_6'^2 + V_8'^2)} \end{aligned} \tag{42}$$

The mass matrix of the scalar and pseudosclar Higgs fields are given by $\mathbf{M}_S^2$ and $\mathbf{M}_P^2$ respectively these are block diagonal matrix.

In the normal minimum $e = 0$, so, the mass matrix of the scalar and pseudosclar Higgs fields are given by $\mathbf{M}_S^2$ and $\mathbf{M}_P^2$ respectively these are block diagonal matrix.

$$[\mathbf{M}_S] = \begin{pmatrix} 4(c+g)v_1^2 & 4(c+g)v_1 v_2 & 2(b+f+2h)v_1 v_3 \\ 4(c+g)v_1 v_2 & -12ev_2 v_3 + 4(c+g)v_2^2 & 2(b+f+2h)v_2 v_3 \\ 2(b+f+2h)v_1 v_3 & 2(b+f+2h)v_2 v_3 & 4av_3^2 \end{pmatrix} \tag{43}$$

$$[\mathbf{M}_P] = \begin{pmatrix} -8\left\{hv_3^2 + (g+d)v_2^2\right\} & 8(g+d)v_1 v_2 + 4ev_1 v_3 & 4hv_1 v_3 \\ 8(g+d)v_1 v_2 & -8\left\{hv_3^2 + (g+d)v_1^2\right\} & 8hv_2 v_3 \\ 4hv_1 v_3 & 8hv_2 v_3 & -8h(v_1^2 + v_2^2) \end{pmatrix} \tag{44}$$

In the normal minimum $e = 0$, so, the eigenvalues of the scalar mass matrix Higgs fields are given by :

$$\begin{aligned} m_1 &= 0, \\ m_{2,3} &= 2av_3^2 - 2(c+g)(v_1^2+v_2^2) \\ &\pm \Big[4(c+g)^2 v_3^4 - 24a(c+g)(v_1^2+v_2^2)v_3^2 + 4(b+f+2h)(v_1^2+ \end{aligned}$$

# Degrees of freedom

- In the S(3) extended model we have three SU(2) Higgs fields, with four real fields each. There are three massles gauge bosons $W^{\pm}$ and $Z$ with two polarization states each, so the total number of independent fields is 18.
- The normal minimum symmetry breaking is initiated by giving the vacuum expectation values $H_1 = v_1$, $H_2 = v_2$ and $H_S = v_3$ to the neutral Higgs fields in each doublet.
- The result is Nine physical Higgs boson that appear as a real particles and the three massive gauge bosons $W^{\pm}$ and $Z$ with three polarization states each.

# Conclusions

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# Conclusions

- We have analized the most general Higgs potential invariant under the non abelian flavour symmetry $S(3)$ of the extended Standard Model.
- In particular, we study the nature of the critical points in the Higgs potential: The normal one, the Charge violating and the CP breaking one. .
- We have found that the normal minimum is stable and it is the deepest one. We also found an $S(2)$ accidental symmetry at the minimum.
- In the normal minimum Nine Higgs fields aquires mass: two degenerated charge Higgs pairs, two scalar higgs and three seudoscalars higgs fields.