On Quark Lepton Complementarity

Félix González Canales
Instituto de Fisica UNAM, México

December 2008
Discrete 08 Valencia, Spain

Contenido:

1. Mixing Angles
2. Mixing Matrices
3. Quark Lepton Complementarity
4. Conclusions

Mixing Angles experimental values

Quark mixing angles:

$$\theta^q_{12} \approx 13^o \pm 0{,}05^o, \quad \theta^q_{23} \approx 2{,}37^o \pm 0{,}06^o, \quad \theta^q_{13} \approx 0{,}93^o$$

Lepton mixing angles:

$$\Delta m^2_{21} = 7{,}67^{+0{,}67}_{-0{,}21} \times 10^{-5}\mathrm{eV}^2, \qquad \theta^l_{12} = 34{,}5^o \pm 1{,}4^o$$

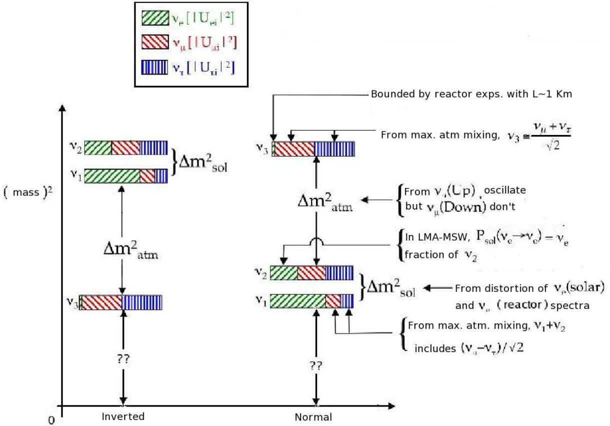
$$\Delta m^2_{32} = \begin{cases} -2{,}37 \pm 0{,}15 \times 10^{-3}\mathrm{eV}^2, & (m_{\nu_2} > m_{\nu_1} > m_{\nu_3}) \\ +2{,}46 \pm 0{,}15 \times 10^{-3}\mathrm{eV}^2, & (m_{\nu_3} > m_{\nu_2} > m_{\nu_1}) \end{cases}$$

$$\theta^l_{23} = \left(42{,}3^{+5{,}1}_{-3{,}3}\right)^o, \qquad \theta^l_{13} < 7{,}9^o$$

M.C. Gonzalez-Gracia and Michele Maltoni, Phys.Rept.460:1-129,2008.

A. Ceccucci, Z. Legeti and Y. Sakai "The CKM quark mixing matrix" Phys.Lett.B667:1,2008.

Possible hierarchies for neutrino masses



B. Kayser arXiv:hep-ph/0506165

Quark Lepton Complementarity

- The solar angle and Cabibbo angle complementarity:

$$\theta^l_{12} + \theta^q_{12} = 45^o + 1{,}5^o \pm 1{,}45^o.$$

- The atmospheric angle and θ^q_{23} angle complementarity:

$$\theta^l_{23} + \theta^q_{23} = \left(44{,}67^{+5{,}15}_{-3{,}35}\right)^o.$$

- The θ^l_{13} angle and θ^q_{13} angle complementarity:

$$\theta^l_{13} + \theta^q_{13} < 8{,}8^o.$$

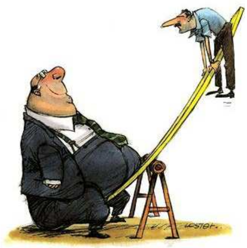
Unified Treatment of Quarks and Leptons

The similarity of quark and charged lepton mass hierarchies suggests the use of similar Fritzsch textures for all Dirac fermion mass matrices in the leptonic sector

$$\mathbf{M^{(F)}} = P^{\dagger}\overline{M}^{(F)}P,$$

$$\mathbf{M^{(F)}} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{-i\phi} & 0 \\ 0 & 0 & e^{-i\phi} \end{pmatrix} \begin{pmatrix} 0 & |a| & 0 \\ |a| & b & c \\ 0 & c & d \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\phi} & 0 \\ 0 & 0 & e^{i\phi} \end{pmatrix}.$$

See-saw Mechanism Type I



$$M_{\nu_L}^{(F)} = M_{\nu_D}^{(F)} (M_{\nu_R}^{(F)})^{-1} M_{\nu_D}^{(F)T}$$

$$M_{\nu_L}^{(F)} = P_D^{\dagger} \bar{M}_{\nu_D}^{(F)} P_D \left(P_R^{\dagger} \bar{M}_{\nu_R}^{(F)} P_R \right)^{-1} \left(P_D^{\dagger} \bar{M}_{\nu_D}^{(F)} P_D \right)^{T},$$

Symmetry of M_{ν_L}, $M_{\nu_L} = M_{\nu_L}^{T}$, fixes the right handed Majorana neutrino phases, $\phi_{\nu_R} = n\pi$.

The right handed Majorana neutrino mass matrix is real and symmetric.

Invariance of Fritzsch Texture

The Fritzsch mass texture is invariant under the operation of see-saw mechanism.

$$M_{\nu_L} = \begin{pmatrix} 0 & |a_{\nu_L}|e^{in\pi} & 0 \\ |a_{\nu_L}|e^{in\pi} & b_{\nu_L} & c_{\nu_L} \\ 0 & c_{\nu_L} & d_{\nu_L} \end{pmatrix},$$

$$a_{\nu_L} = \frac{|a_{\nu_D}|^2}{|a_{\nu_R}|}; \qquad d_{\nu_L} = \frac{d_{\nu_D}^2}{d_{\nu_R}},$$

$$b_{\nu_L} = \frac{c_{\nu_D}^2}{d_{\nu_R}} + \frac{c_{\nu_R}^2 - b_{\nu_R} d_{\nu_R}}{d_{\nu_R}} \frac{|a_{\nu_D}|^2}{|a_{\nu_R}|^2} e^{i2\phi_{\nu_D}} + 2\left(b_{\nu_D} - \frac{c_{\nu_D} c_{\nu_R}}{d_{\nu_R}}\right) \frac{|a_{\nu_D}|}{|a_{\nu_R}|} \cos(\phi_{\nu_D} + n\pi),$$

$$c_{\nu_L} = \frac{c_{\nu_D} d_{\nu_D}}{d_{\nu_R}} + \left(\frac{c_{\nu_D} a_{\nu_D}^*}{|a_{\nu_R}|} - \frac{c_{\nu_R} a_{\nu_D}^* d_{\nu_D}}{|a_{\nu_R}| d_{\nu_R}}\right) e^{in\pi}.$$

In general the left handed Majorana neutrino mass matrix is complex, *i.e.* no hermitian.

Here we study the hermitian case, $M^{(F)} = P^{\dagger}\bar{M}^{(F)}P$

The matrix $\bar{M}^{(F)}$ can be diagonalized by an orthogonal real matrix, $\mathbb{O}$, so that:

$$\mathbb{O}^T \bar{M}^{(F)} \mathbb{O} = \mathrm{diag}\,(m_1, m_2, m_3)\,,$$

where m_i with $i = 1, 2, 3$, are the eigenvalues of $\bar{M}^{(F)}$.
Then,

$$M^{(F)} = P^{\dagger}\bar{M}^{(F)}P = P^{\dagger}\mathbb{O}\;\mathrm{diag}\;(m_1, m_2, m_3)\,\mathbb{O}^T P.$$

Now with the unitary matrix:

$$U \equiv \mathbb{O}^T P,$$

we have,

$$UM^{(F)}U^{\dagger} = \mathrm{diag}\,(m_1, m_2, m_3)\,.$$

In our case, $m_3 > m_2 > m_1$, with $m_2 = -|m_2|$ and $d \equiv 1 - \delta$.

$$\widetilde{M}^{(F)} = \frac{\bar{M}^{(F)}}{m_3} = \begin{pmatrix} 0 & \sqrt{\frac{\widetilde{m}_1 \widetilde{m}_2}{1-\delta}} & 0 \\ \sqrt{\frac{\widetilde{m}_1 \widetilde{m}_2}{1-\delta}} & \widetilde{m}_1 - \widetilde{m}_2 + \delta & \sqrt{\frac{\delta}{(1-\delta)} f_1 f_2} \\ 0 & \sqrt{\frac{\delta}{(1-\delta)} f_1 f_2} & 1 - \delta \end{pmatrix}.$$

where $\widetilde{m}_1 = m_1/m_3$, $\widetilde{m}_2 = m_2/m_3$, $f_1 = (1 - \widetilde{m}_1 - \delta)$, $f_2 = (1 + \widetilde{m}_2 - \delta)$.

$$0 < \delta < 1 - \widetilde{m}_1.$$

Diagonalization of Fritzsch Texture

$$UM^{(F)}U^{\dagger} = \mathbb{O}^{T} P P^{\dagger} \widetilde{M}^{(F)} P P^{\dagger} \mathbb{O} = \mathrm{diag}\left(\widetilde{m}_1, \widetilde{m}_2, 1\right).$$

$$\mathbb{O} = \begin{pmatrix} \left[\frac{\widetilde{m}_2 f_1}{D_1}\right]^{\frac{1}{2}} & -\left[\frac{\widetilde{m}_1 f_2}{D_2}\right]^{\frac{1}{2}} & \left[\frac{\widetilde{m}_1 \widetilde{m}_2 \delta}{D_3}\right]^{\frac{1}{2}} \\ \left[\frac{\widetilde{m}_1 (1-\delta) f_1}{D_1}\right]^{\frac{1}{2}} & \left[\frac{\widetilde{m}_2 (1-\delta) f_2}{D_2}\right]^{\frac{1}{2}} & \left[\frac{(1-\delta)\delta}{D_3}\right]^{\frac{1}{2}} \\ -\left[\frac{\widetilde{m}_1 f_2 \delta}{D_1}\right]^{\frac{1}{2}} & -\left[\frac{\widetilde{m}_2 f_1 \delta}{D_2}\right]^{\frac{1}{2}} & \left[\frac{f_1 f_2}{D_3}\right]^{\frac{1}{2}} \end{pmatrix},$$

$$D_1 = (1-\delta)(\widetilde{m}_1 + \widetilde{m}_2)(1 - \widetilde{m}_1), \quad D_2 = (1-\delta)(\widetilde{m}_1 + \widetilde{m}_2)(1 + \widetilde{m}_2),$$

$$D_3 = (1-\delta)(1 - \widetilde{m}_1)(1 + \widetilde{m}_2).$$

Mixing matrices

$$U^{th}_{CKM(PMNS)} \equiv U^{u(\nu)} U^{d(e)\dagger} = \mathbb{O}^T_{u(\nu)} P^{q(l)} \mathbb{O}_{d(e)},$$

where

$$P^{q(l)} = \mathrm{diag}\left(1, e^{i\phi_{q(l)}}, e^{i\phi_{q(l)}}\right), \quad \phi_{q(l)} = \phi_{u(e)} - \phi_{d(\nu)}.$$

At first order in m_{i_1}/m_{i_2} with $i = u, d, e, \nu$:
The Cabibbo(Solar) angle,

$$\tan^2\theta^{q(l)}_{12} = \frac{\left|U^{th}_{us(e2)}\right|}{\left|U^{th}_{ud(e1)}\right|} \approx \frac{\frac{\widetilde{m}_{d(\nu_1)}}{\widetilde{m}_{s(\nu_2)}} + \frac{\widetilde{m}_{u(e)}}{\widetilde{m}_{c(\mu)}} - 2\sqrt{\frac{\widetilde{m}_{d(\nu_1)}}{\widetilde{m}_{s(\nu_2)}}\frac{\widetilde{m}_{u(e)}}{\widetilde{m}_{c(\mu)}}}\cos\phi^{q(l)}}{1 + \frac{\widetilde{m}_{d(\nu_1)}}{\widetilde{m}_{s(\nu_2)}}\frac{\widetilde{m}_{u(e)}}{\widetilde{m}_{c(\mu)}} + 2\sqrt{\frac{\widetilde{m}_{d(\nu_1)}}{\widetilde{m}_{s(\nu_2)}}\frac{\widetilde{m}_{u(e)}}{\widetilde{m}_{c(\mu)}}}\cos\phi^{q(l)}}.$$

Mixing Angles

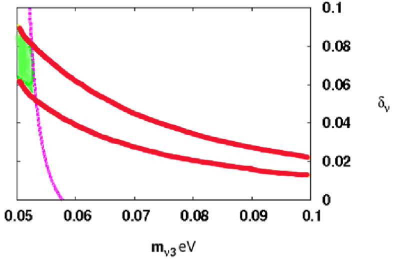
Mixing angle θ^q_{23} (Atmospheric),

$$\tan\theta^{q(l)}_{23} = \frac{\left|U^{th}_{cb(\mu 3)}\right|}{\left|U^{th}_{tb(\tau 3)}\right|} \approx \left(\sqrt{\delta_{d(\nu)}} - \sqrt{\delta_{u(e)}}\right)$$

Mixing angle θ^q_{13} (Reactor),

$$\tan\theta^{q(l)}_{13} = \frac{\left|U^{th}_{ub(e3)}\right|}{\sqrt{1-\left|U^{th}_{ub(e3)}\right|^2}} \approx \sqrt{\frac{m_{u(e)}}{m_{c(\mu)}}}\left(\sqrt{\delta_{d(\nu)}} - \sqrt{\delta_{u(e)}}\right)$$

χ^2 fit for lepton mixing angles



Region of parameter space(green area) in which we can reproduce simultaneously the solar(red line) and the atmospheric(pink line) angles.

$$0{,}05\ \mathrm{eV} \leqslant m_{\nu_3} \leqslant 0{,}058\ \mathrm{eV}\ , \quad 0{,}062 \leqslant \delta_\nu \leqslant 0{,}085$$

With $\phi^{q(l)} = 90^o$, maximal CP violation in both sectors

- Charged lepton masses:

$$m_e = 0{,}5109\ \mathrm{MeV}, \quad m_\mu = 105{,}685\ \mathrm{MeV} \quad m_\tau = 1776{,}99\mathrm{GeV}$$

- Neutrino masses:

$$m_{\nu_1} = 4{,}4\times10^{-3}\mathrm{eV} \quad m_{\nu_2} = 9\times10^{-3}\mathrm{eV} \quad m_{\nu_3} = 5{,}0\times10^{-2}\mathrm{eV}$$

- Theoretical values of mixing angles:

$$\theta^l_{12} \approx 33{,}9^o \quad \theta^l_{23} \approx 41{,}5^o \quad \theta^l_{13} \approx 3{,}58^o.$$

- Quark masses:

$$m_u = 2{,}75\mathrm{MeV} \quad m_c = 1310\mathrm{MeV} \quad m_d = 6{,}0\mathrm{MeV} \quad m_s = 120\ \mathrm{MeV}$$

- Theoretical values of mixing angles:

$$\theta_{12}^q \approx 12{,}8^o \quad \theta_{23}^q \approx 1{,}4^o \quad \theta_{13}^q \approx 0{,}07^o$$

Quark Lepton Complementarity

The solar angle and Cabibbo angle complementarity:

$$\tan\left(\theta_{12}^{q}+\theta_{12}^{l}\right)=1+\Delta_{12}^{\rm th},$$

$$\Delta_{12}^{\rm th}=\frac{\left(\frac{\widetilde{m}_d}{\widetilde{m}_s}+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)^{\frac{1}{2}}\left[\left(\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)^{\frac{1}{2}}+\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)^{\frac{1}{2}}\right]+}{\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)^{\frac{1}{2}}\left(1+\frac{\widetilde{m}_d}{\widetilde{m}_s}\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)^{\frac{1}{2}}-}\times$$

$$\times\frac{+\left(1+\frac{\widetilde{m}_d}{\widetilde{m}_s}\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)^{\frac{1}{2}}\left[\left(\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)^{\frac{1}{2}}-\left(1+\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)^{\frac{1}{2}}\right]}{-\left(\frac{\widetilde{m}_d}{\widetilde{m}_s}+\frac{\widetilde{m}_u}{\widetilde{m}_c}\right)^{\frac{1}{2}}\left(\frac{\widetilde{m}_{\nu 1}}{\widetilde{m}_{\nu 2}}+\frac{\widetilde{m}_e}{\widetilde{m}_\mu}\right)^{\frac{1}{2}}}$$

$$\underline{\theta_{12}^{q}+\theta_{12}^{l}=45^{o}+1{,}7^{o}}$$

The atmospheric angle and θ^q_{23} angle complementarity:

$$\tan\left(\theta^q_{23}+\theta^l_{23}\right)=1+\Delta^{\rm th}_{23},$$

$$\Delta^{\rm th}_{23}=\frac{1-(\sqrt{\delta_\nu}-\sqrt{\delta_e})\left[(\sqrt{\delta_d}-\sqrt{\delta_u})+1\right]-(\sqrt{\delta_d}-\sqrt{\delta_u})}{1-(\sqrt{\delta_\nu}-\sqrt{\delta_e})(\sqrt{\delta_d}-\sqrt{\delta_u})}.$$

$$\underline{\theta^q_{23}+\theta^l_{23}=45^o-0{,}8^o},$$

The θ^l_{13} angle and θ^q_{13} angle complementarity:

$$\tan\left(\theta^q_{13}+\theta^l_{13}\right)=\frac{\sqrt{\frac{\widetilde{m}_u}{\widetilde{m}_c}}(\sqrt{\delta_\nu}-\sqrt{\delta_e})\sqrt{\frac{\widetilde{m}_e}{\widetilde{m}_\mu}}(\sqrt{\delta_d}-\sqrt{\delta_u})}{1-\sqrt{\frac{\widetilde{m}_u}{\widetilde{m}_c}\frac{\widetilde{m}_e}{\widetilde{m}_\mu}}(\sqrt{\delta_\nu}-\sqrt{\delta_e})(\sqrt{\delta_d}-\sqrt{\delta_u})}.$$

$$\underline{\theta^q_{13}+\theta^l_{13}\approx 4^o}.$$

The effective Majorana neutrino mass

$$\langle m_{ee}\rangle_{max} = \left|U_{e1}^2\widetilde{m}_{\nu_1} + U_{e2}^2\widetilde{m}_{\nu_2} + U_{e3}^2\widetilde{m}_{\nu_3}\right|$$

$$\langle m_{ee}\rangle_{max} \approx \frac{4\widetilde{m}_{\nu_1}}{\left(1+\frac{m_e}{m_\mu}\right)^2\left(1+\frac{m_{\nu 1}}{m_{\nu 2}}\right)^2}\left(\widetilde{m}_{\nu_1} + 2\sqrt{\frac{m_e}{m_\mu}\frac{m_{\nu 1}}{m_{\nu 2}}}\left(m_{\nu_1} - m_{\nu_2}\cos\phi_I\right)\right)$$

$$\langle m_{ee}\rangle_{max} \approx 0{,}028\mathrm{eV}$$

Conclusions

1. The strong hierarchy in the mass spectra of the quarks and charged leptons explains the small or very small quark mixing angles, the very small charged lepton mass ratio explain the very small θ^l_{13} which, in our scheme, is independent of the neutrino masses.
2. The see-saw mechanism type I gives very small masses to the left handed Majorana neutrinos with relatively large values of the neutrino mass ratio m_{ν_1}/m_{ν_2} and allows for large θ^l_{12} and θ^l_{23} mixing angles.