Symmetry aspects in emergent quantum mechanics

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$\Rightarrow$ fundamental puzzles

QM = needs external time, t

$i\partial_t \psi = \hat{H}\psi$

$\hookrightarrow \sum_{\text{paths}} e^{iS(t_2,t_1)/\hbar}$

GR = forbids external time, $\cancel{t}$

general coordinate invariance

$\hookrightarrow t \to f(t)$

$\hookrightarrow \hat{H}\psi = 0$, constraint

QM = what is a measurement?

$\hookrightarrow$ wave fct. collapse / reduction

GR = no local gauge invar. observables

$\Rightarrow$ QM an emergent phenomenon?

't Hooft, Adler, Smolin, ...

distinguished from reformulations / alternative interpretations of QM

Bohm, Parisi & Wu, ...

(somewhere in between Nelson, Rovelli, Hartle)

G. 't Hooft, PASCOS 13 (2007), arXiv: 0707.4568

" For any quantum system there exists at least one deterministic model that reproduces all its dynamics after prequantization."

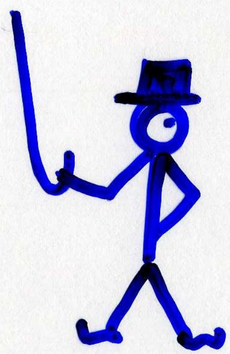
H.-T. Elze, J. Phys. A (2008), in press, arXiv: 0710.2765

$$\frac{d\psi}{dt} = -i\hat{H}\psi \ , \quad \hat{H} : d \times d \text{ matrix}$$

atomistic spacetime $\longrightarrow$ **quantum mechanics**

$l_P \approx 10^{-35}\,m$ $\qquad$ $\frac{1}{10\,TeV} \approx 2 \cdot 10^{-20}\,m$

↪ "atomistic" = set of elements, "atoms" & "relations"



and its

dynamics, changing number of elements •

, changing (causal) relations •

... heuristic abstractions

(A) QM description: objects are identifiable

electrons, molecules, tables, ..., galaxies, ...

(B) if spacetime "atomistic" ⟹ increasingly more "bits" are needed to characterize situation of object w.r.t. it

⟹ 1) oblivious of "atomistic" spacetime

$\overset{\sim}{\Leftrightarrow}_{(B)}$ information loss about situation of objects

2) (A): objects don't get lost

$\overset{\sim}{\Leftrightarrow}$ conservation of probability

A Model

(classical ensemble theory)

$$\underbrace{-\partial_t f = \{H, f\}}_{\text{Liouville eq.}} + \underbrace{i\,\delta H f}_{\substack{\text{information}\\ \text{loss}}} + \underbrace{g[f]}_{\substack{\text{probability}\\ \text{Conservation}}}$$

"PART II" $\Rightarrow$ useful reformulation of Hamilton's mechanics

$$\mathcal{H}(x,p) := \frac{1}{2}p^2 + V(x) \quad \bullet$$

$\hookrightarrow$ consider ensemble, distrib. fct. f

$$\text{prob.} \sim f(x,p;t)\,dx\,dp$$

$\hookrightarrow$ conserv. of probability, Liouville eq.:

$$-\partial_t f = \frac{\partial \mathcal{H}}{\partial p}\frac{\partial f}{\partial x} - \frac{\partial \mathcal{H}}{\partial x}\frac{\partial f}{\partial p} = p\,\partial_x f - V'(x)\,\partial_p f \quad \bullet$$

- Fourier transf., $f(x,p;t) =: \int dy\, e^{-ipy} f(x,y;t)$
- "Wigner rotation", $Q := x + \frac{y}{2}$, $q := x - \frac{y}{2}$

$$\Rightarrow \quad i\partial_t f = \left\{ \hat{\mathcal{H}}_Q - \hat{\mathcal{H}}_q + \Delta(Q,q) \right\} f \quad \bullet$$

$$\hat{\mathcal{H}}_\chi := -\frac{1}{2}\partial_\chi^2 + V(\chi) \quad , \quad \chi = Q, q$$

$$\Delta(Q,q) := (Q-q)V'\left(\tfrac{Q+q}{2}\right) - V(Q) + V(q) = -\Delta(q,Q)$$

, works for matrix or Grassmann val. vars and fields $\bullet$

$\Rightarrow$ <u>observations</u> =

- looks like von Neumann eq. for dens.-op. $\hat{f}$,

  $f(Q,q;t)$ matrix elems.

  HOWEVER = unusual interaction Δ between bras & kets; Hilbert & dual coupled!

- looks like Schrödinger eq. for two d.o.f.;

  $\hat{H}_Q$ and $\hat{H}_q$ contrib. with opposite sign!

  Kaplan & Sundrum (2006) = a symmetry for Δ.

  since Δ is antisymm. $\Rightarrow$ spectrum symm. w.r.t. zero

  $\Rightarrow$ no ground state.

- free particle and harmonic oscill., $\Delta = 0$.

$\Rightarrow$ interaction / Δ is THE stumbling block for emergent QM.

⇒ pursue "von Neumann aspect":

$$1 = \int \frac{dx\,dp}{2\pi}\, f(x,p;t) = \int dQ\,dq\;\delta(Q-q)\, f(Q,q;t)$$

$$=: \mathrm{Tr}\,\hat{f}(t)$$

$$\langle x \rangle := \int \frac{dx\,dp}{2\pi}\, x\, f(x,p;t) = \mathrm{Tr}\big(\hat{X}\,\hat{f}(t)\big)$$

$p \qquad p \qquad \hat{P}$

$$\mathrm{Tr}\big((\hat{X}\hat{P} + \hat{P}\hat{X})\,\hat{f}(t)\big) = \int \frac{dx\,dp}{2\pi}\,(xp + px)\, f(x,p;t)$$

$$,\; X(q,Q) = \delta(Q-q)\,\frac{Q+q}{2} \;,\; P(q,Q) = -i\Big(\delta(Q-q)\,\overrightarrow{\partial}_Q - \overleftarrow{\partial}_q\,\delta(Q-q)\Big)$$

↪ introduce complete orthonorm. basis: $g_j(x;t) := e^{-iE_j t}\, g_j(x)$

$$\hat{H}_x\, g_j(x) = E_j\, g_j(x)$$

↪ $f(Q,q;t) = \sum_{j,k} f_{jk}(t)\, g_j(Q;t)\, g_k^*(q;t)$

⇒ $1 = \sum_j f_{jj}(t)$; $f(x,p;t)$ real ⇒ $f_{jk} = f_{kj}^*$

$\Rightarrow$ dissipative dynamics and vacuum stability

aim: $i\partial_t \hat{f} = [\hat{H}_\chi, \hat{f}] \Longleftrightarrow f_{jk}$'s constant.

we have: $i\partial_t f_{jk} = \sum_{l,m} \Delta_{jklm} f_{lm}$.

, $\Delta_{jklm} = -\Delta^*_{kjml} := \int dQ\,dq\; g^*_j(Q) g_k(q) \Delta(Q,q) g_l(Q) g^*_m(q)$

$\hookrightarrow$ $i\hat{\Delta}$: Hermitian $\to$ Hermitian

$\mathrm{Tr}(\hat{\Delta}\hat{M}) = 0$

$\hookrightarrow$ $\mathrm{Tr}\,\hat{f}(t) = \mathrm{Tr}(e^{-i\hat{\Delta}t}\hat{f}(0)) = \mathrm{Tr}\,\hat{f}(0)$

$\Rightarrow$ What to do next?

better: white noise $\delta H(t)$

$\Leftrightarrow$ minimalist model:

- $i\partial_t \hat{f}(t) = (\hat{\Delta} + \delta H)(\hat{f}(t) - \hat{g}(t))$

$\hookrightarrow$ solve, average: $\boxed{\mathcal{P}(\delta H) \sim e^{-\cancel{t}\,\delta H^2/4\varepsilon}}$

$$\hat{f}(t) = \hat{g}(t) + e^{-i(\hat{\Delta} - i\varepsilon)t}(\hat{f}(0) - \hat{g}(0))$$
$$- \int_0^t ds\, e^{-i(\hat{\Delta} - i\varepsilon)(t-s)}\, \partial_s \hat{g}(s)$$

$\hookrightarrow$ dissip. decay $\sim e^{-\varepsilon t^2}$, eff. elimin. $\hat{\Delta}$

$\hookrightarrow$ $\mathrm{Tr}\,\hat{f}(t) = \mathrm{Tr}\,\hat{f}(0) = 1$, prov. $\mathrm{Tr}\,\hat{g}(t) = 1$

$\Rightarrow t \gg 1/\varepsilon$, $\hat{f}(t) \approx \hat{g}(t) \xrightarrow[\text{fast}]{\text{suff.}} g(\infty)$

and $f_{ij}(t) \longrightarrow g_{ij}(\infty)$

$\Leftrightarrow$ von Neumann eq. valid asympt.

$\Rightarrow$ asymptotically

$$\hat{f}(t) \longrightarrow \hat{g}(\infty) = \hat{U}^{+}\hat{P}\,U \quad , \text{ i.e. constant}$$

$\Longrightarrow$ von Neumann eq. holds, sols. are quantum states

i) $\hat{f}(0)$ diagonal $\leftrightsquigarrow$ $\hat{U} = \hat{U}^{+} = \mathbb{1}$

$$f(Q,q;t) \longrightarrow \sum_{j,k} p_{jk}\, g_j(Q;t)\, g_k^{*}(q;t) = g_{\bar{\ell}}(Q)\, g_{\bar{\ell}}(q)$$

$\Longrightarrow$ stationary state $\leftrightsquigarrow$ large equivalence class of classical distributions

ii) $\hat{f}(0)$ not diagonal $\leftrightsquigarrow$ $\hat{U} \neq \mathbb{1}$

$$f(Q,q;t) \longrightarrow \sum_{j,k} \underbrace{(\hat{U}^{+}\hat{P}\hat{U})_{jk}}_{= U^{+}_{j\bar{\ell}}\, U_{\bar{\ell}k}}\, g_j(Q;t)\, g_k^{*}(q;t)$$

$\Longrightarrow$ pure state $\leftrightsquigarrow$ larger equivalence class

$\Rightarrow$ <u>completion of the model:</u>

- $\tau \partial_t \hat{g}(t) = \left(\hat{f}(0) - \langle \hat{f}(0) \rangle_{\hat{g}(t)} \right) \hat{g}(t)$

  , $\langle \hat{f} \rangle_{\hat{g}} := \mathrm{Tr}(\hat{f}\hat{g}) / \mathrm{Tr}\,\hat{g}$

  solve: $\quad \hat{g}(t) = e^{\hat{f}(0)t/\tau} / \mathrm{Tr}\, e^{\hat{f}(0)t/\tau}$

  $\Rightarrow \mathrm{Tr}\,\hat{g}(t) = 1 \, , \; \hat{g}(0) = \mathbb{1} / \mathrm{Tr}\, \mathbb{1}$

  $\hat{g}$ is diagonalized by $\hat{U}\hat{f}(0)\hat{U}^{+} = \begin{pmatrix} f'_{11}(0) & & 0 \\ & f'_{22}(0) & \\ 0 & & \ddots \end{pmatrix}$

  $f'_{jj}(0) \in [0,1]$

  $\lambda := \max_j f'_{jj}(0)$

  $\Rightarrow \hat{g}_d(t) := \hat{U}\hat{g}(t)\hat{U}^{+} = \dfrac{1}{\sum_j e^{(f'_{jj} - \lambda)t/\tau}} \begin{pmatrix} e^{(f'_{11} - \lambda)t/\tau} & & 0 \\ & \ddots & 1 \\ 0 & & \ddots \end{pmatrix}$

- $\longrightarrow \begin{pmatrix} 0 & & \\ & \ddots & 1 \; 0 \\ & & \ddots \end{pmatrix} =: \hat{P}$, projector

Remark: source $\hat{g}$ deterministically determined by $\hat{f}^{(0)}$

→ modify, in order to describe QM energy measurement on general pure state

$\Longrightarrow$ "wave function collapse" $\leftrightsquigarrow$ reduction of a larger equivalence class of classical distributions to a smaller one •

$\leftrightsquigarrow$ pure state $\longrightarrow$ stationary state

... to be done •

Conclusion:

"atomistic" spacetime structure & dynamics ...

$$-\partial_t f = \{H, f\} + i\,\delta H f + g[f]$$

information loss　　probability conservation

$\Longleftrightarrow$

$$i\partial_t \hat{f} = [\hat{H}, \hat{f}] + (\hat{\Delta} + \delta H)(\hat{f} - \hat{g})$$

↳ asymptotic (fixed point) solutions are quantum states acc. to von Neumann eq.

... might give rise to quantum phenomena