Predictions of Finite Unified Theories

Sven Heinemeyer
Ernest Ma
Myriam Mondragón
George Zoupanos

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- What happens as we approach the Planck scale?
- How do we go from a fundamental theory to field theory as we know it?
- How are the gauge, Yukawa and Higgs sectors related at a more fundamental level?
- How do particles get their very different masses?
- What is the nature of the Higgs?

Search for understanding relations between parameters

addition of symmetries.

$N = 1$ SUSY GUTs.

Complementary approach: look for RGI relations among couplings at GUT scale $\longrightarrow$ Planck scale

$\Rightarrow$ **reduction of couplings**

$\Rightarrow$ **FINITENESS**

resulting theory: less free parameters $\therefore$ more predictive

scale invariant .

Dimensionless sector of all-loop finite $SU(5)$ model

prediction for M_{top}, large $\tan\beta$

Can be extended to Soft Supersymmetry Breaking (SSB) sector expressed only in terms of

- g (gauge coupling) and
- M (unified gaugino mass)

too restrictive

Constraint can be relaxed

- sum-rule for soft scalars
- better phenomenology

Confronting with low energy precision data

- Discriminate among different models
- $\Rightarrow$ **Prediction for Higgs mass and s-spectra**

Reduction of Couplings

A RGI relation among couplings $\Phi(g_1, \ldots, g_N) = 0$ satisfies

$$\mu\, d\Phi/d\mu \;=\; \sum_{i=1}^{N} \beta_i\, \partial\Phi/\partial g_i \;=\; 0.$$

$g_i =$ coupling, β_i its β function

Finding the $(N-1)$ independent Φ's is equivalent to solve the reduction equations (RE)

$$\beta_g\, (dg_i/dg) = \beta_i \ ,$$

$i = 1, \cdots, N$

- completely reduced theory contains only one independent coupling and its β function
- complete reduction: power series solution of RE
- uniqueness of the solution can be investigated at one-loop

Zimmermann, Oehme, Sibold

- The complete reduction might be too restrictive, one may use fewer Φ's as RGI constraints
- Reduction of couplings is essential for finiteness

finiteness: absence of ∞ renormalizations

$$\Rightarrow \quad \beta^N = 0$$

- In SUSY no-renormalization theorems
  - $\Rightarrow$ only study one and two-loops
  - guarantee that is gauge and reparameterization invariant at all loops

Finiteness

A chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g has a superpotential

$$W = \frac{1}{2}\, m^{ij}\, \Phi_i\, \Phi_j + \frac{1}{6}\, C^{ijk}\, \Phi_i\, \Phi_j\, \Phi_k \ ,$$

Requiring one-loop finiteness $\beta_g^{(1)} = 0 = \gamma_i^{j(1)}$ gives the following conditions:

$$\sum_i T(R_i) = 3 C_2(G) \,, \qquad \frac{1}{2} C_{ipq} C^{jpq} = 2\delta_i^j g^2 C_2(R_i) \,.$$

$C_2(G)$ = quadratic Casimir invariant, C_{ijk} = Yukawa coup., $T(R_i)$ Dynkin index of R_i.

- **restricts the particle content of the models**
- **relates the gauge and Yukawa sectors**

- One-loop finiteness $\Rightarrow$ two-loop finiteness

Jack, Jones, Mezincescu and Yao

- One-loop finiteness restricts the choice of irreps R_i, as well as the Yukawa couplings
- Cannot be applied to the susy Standard Model (SSM): $C_2[U(1)] = 0$
- The finiteness conditions allow only SSB terms

It is possible to achieve all-loop finiteness $\beta^n = 0$:

Lucchesi, Piguet, Sibold

1. One-loop finiteness conditions must be satisfied
2. The Yukawa couplings must be a formal power series in g, which is solution (isolated and non-degenerate) to the reduction equations

RGI in the Soft Supersymmetry Breaking Sector

Supersymmetry is essential. It has to be broken, though…

$$-\mathcal{L}_{\rm SB} = \frac{1}{6}\, h^{ijk}\, \phi_i\phi_j\phi_k + \frac{1}{2}\, b^{ij}\, \phi_i\phi_j + \frac{1}{2}\,(m^2)^j_i\, \phi^{*\,i}\phi_j + \frac{1}{2}\, M\, \lambda\lambda + \text{H.c.}$$

The RGI method has been extended to the SSB of these theories.

- One- and two-loop finiteness conditions for SSB have been known for some time

  Jack, Jones, et al.

- It is also possible to have all-loop RGI relations in the finite and non-finite cases

  Kazakov; Jack, Jones, Pickering

SSB terms depend only on g and the unified gaugino mass M
universality conditions

$$h = -MC, \qquad m^2 \propto M^2, \qquad b \propto M\mu$$

Very appealing! But too restrictive;
it leads to phenomenological problems:

- The lightest susy particle (LSP) is charged. Yoshioka; Kobayashi et al
- It is incompatible with radiative electroweak breaking.

Brignole, Ibáñez, Muñoz

Possible to relax the universality condition to a sum-rule for the soft scalar masses

$\Rightarrow$ better phenomenology.

Kobayashi, Kubo, Mondragón, Zoupanos

Soft scalar sum-rule for the finite case

Finiteness implies

$$C^{ijk} = g \sum_{n=0} \rho^{ijk}_{(n)} g^{2n} \ ,$$

The one- and two-loop finiteness for h gives

$$h^{ijk} = -MC^{ijk} + \cdots = -M\rho^{ijk}_{(0)}\, g + O(g^5) \ .$$

Assume that lowest order coefficients $\rho^{ijk}_{(0)}$ and $(m^2)^i_j$ satisfy diagonality relations

$$\rho_{ipq(0)}\rho^{jpq}_{(0)} \propto \delta^j_i \ , \qquad (m^2)^i_j = m^2_j \delta^i_j$$

for all p and q.

We find the the following soft scalar-mass sum rule

$$(\ m^2_i + m^2_j + m^2_k\ )/MM^\dagger = 1 + \frac{g^2}{16\pi^2}\,\Delta^{(1)} + O(g^4)$$

for $i,\ j,\ k$ with $\rho^{ijk}_{(0)} \neq 0$, where $\Delta^{(1)}$ is the two-loop correction,

$$\Delta^{(1)} = -2\sum_l [(m^2_l/MM^\dagger) - (1/3)]\ T(R_l) \ ,$$

which vanishes for the universal choice.

All-loop sum rule

One can generalize the sum rule for finite and non-finite cases to all-loops!!

Possible thanks to renormalization properties of $N = 1$ susy gauge theories. Kazakov et al; Jack, Jones et al; Yamada; Hisano, Shifman

The sum-rule in the NSVZ scheme is Kobayashi, Kubo, Zoupanos

$$
\begin{aligned}
m_i^2 + m_j^2 + m_k^2 \quad &= \quad |M|^2 \{ \frac{1}{1 - g^2 C(G)/(8\pi^2)} \frac{d \ln C^{ijk}}{d \ln g} + \frac{1}{2} \frac{d^2 \ln C^{ijk}}{d (\ln g)^2} \} \\
&+ \sum_l \frac{m_l^2 T(R_l)}{C(G) - 8\pi^2/g^2} \frac{d \ln C^{ijk}}{d \ln g} .
\end{aligned}
$$

Interesting: Finite sum rule satisfied also in certain certain class of orbifold models in which the massive states are organized into $N = 4$ supermultiples, if $d \ln C^{ijk} / d \ln g = 1$.

Several aspects of Finite Models have been studied

- $SU(5)$ Finite Models studied extensively

  Rabi et al; Kazakov et al; López-Mercader, Quirós et al; M.M, Kapetanakis, Zoupanos; etc
- One of the above coincides with a non-standard Calabi-Yau $SU(5) \times E_8$ Greene et al; Kapetanakis, M.M., Zoupanos
- Finite theory from compactified string model also exists (albeit not good phenomenology) Ibáñez
- Criteria for getting finite theories from branes exist

  Hanany, Strassler, Uranga
- Realistic models involving all generations exist

  Babu, Eckbahrt, Gogoladze
- Some models with $SU(N)^k$ **finite** $\Longleftrightarrow$ **3 generations,** good phenomenology with $SU(3)^3$ Ma, M.M, Zoupanos
- Relation between commutative field theories and finiteness studied Jack and Jones
- Proof of conformal invariance in finite theories Kazakov

$SU(5)$ Finite Models

We study two models with $SU(5)$ gauge group. The matter content is

$$3\,\overline{\mathbf{5}} + 3\,\mathbf{10} + 4\,\{\mathbf{5} + \overline{\mathbf{5}}\} + \mathbf{24}$$

The models are finite to all-loops in the dimensionful and dimensionless sector. In addition:

- The soft scalar masses obey a sum rule
- At the M_{GUT} scale the gauge symmetry is broken and we are left with the MSSM
- At the same time finiteness is broken
- The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs $\{\mathbf{5} + \overline{\mathbf{5}}\}$ which couple to the third generation

The difference between the two models is the way the Higgses couple to the **24**

Kapetanakis, Mondragón, Zoupanos; Kazakov et al.

The superpotential which describes the two models takes the form

$$W = \sum_{i=1}^{3} [\, \frac{1}{2} g_i^u \,\mathbf{10}_i \mathbf{10}_i H_i + g_i^d \,\mathbf{10}_i \overline{\mathbf{5}}_i \,\overline{H}_i \,] + g_{23}^u \,\mathbf{10}_2 \mathbf{10}_3 H_4$$
$$+ g_{23}^d \,\mathbf{10}_2 \overline{\mathbf{5}}_3 \,\overline{H}_4 + g_{32}^d \,\mathbf{10}_3 \overline{\mathbf{5}}_2 \,\overline{H}_4 + \sum_{a=1}^{4} g_a^f \,H_a \,\mathbf{24}\,\overline{H}_a + \frac{g^\lambda}{3} \,(\mathbf{24})^3$$

find isolated and non-degenerate solution to the finiteness conditions

The unique solution implies discrete symmetries

For Model A the superpotetial has the discrete symmetry

$$Z_7 \times Z_3 \times Z_2$$

	$\bar{\mathbf{5}}_1$	$\bar{\mathbf{5}}_2$	$\bar{\mathbf{5}}_3$	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	H_1	H_2	H_3	H_4	$\overline{H}_1$	$\overline{H}_2$	$\overline{H}_3$	$\overline{H}_4$	**24**
Z_7	4	1	2	1	2	4	5	3	6	-5	-3	-6	0	0	0
Z_3	0	0	0	1	2	0	1	2	0	-1	-2	0	0	0	0
Z_2	1	1	1	1	1	1	0	0	0	0	0	0	0	0	0

Whereas for Model B

$$Z_4 \times Z_4 \times Z_4$$

	$\bar{\mathbf{5}}_1$	$\bar{\mathbf{5}}_2$	$\bar{\mathbf{5}}_3$	$\mathbf{10}_1$	$\mathbf{10}_2$	$\mathbf{10}_3$	H_1	H_2	H_3	H_4	$\overline{H}_1$	$\overline{H}_2$	$\overline{H}_3$	$\overline{H}_4$	**24**
Z_4	1	0	0	1	0	0	2	0	0	0	-2	0	0	0	0
Z_4	0	1	0	0	1	0	0	2	0	3	0	-2	0	-3	0
Z_4	0	0	1	0	0	1	0	0	2	3	0	0	-2	-3	0

The finiteness relations give at the M_{GUT} scale

Model A

- $g_t^2 = \frac{8}{5}\, g^2$
- $g_{b,\tau}^2 = \frac{6}{5}\, g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 + m_{\mathbf{\bar{5}}}^2 + m_{\mathbf{10}}^2 = M^2$

- **3 free parameters:**
  M, $m_{\mathbf{\bar{5}}}^2$ and $m_{\mathbf{10}}^2$

Model B

- $g_t^2 = \frac{4}{5}\, g^2$
- $g_{b,\tau}^2 = \frac{3}{5}\, g^2$
- $m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2$
- $m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3}$
- $m_{\mathbf{\bar{5}}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3}$

- **2 free parameters:**
  M, $m_{\mathbf{\bar{5}}}^2$

Phenomenology

The gauge symmetry is broken below M_{GUT}, and what remains are boundary conditions of the form $C_i = \kappa_i g$, $h = -MC$ and the sum rule at M_{GUT}, below that is the MSSM.

- We assume a unique susy breaking scale
- The LSP is neutral
- The solutions should be compatible with radiative electroweak breaking
- No fast proton decay

We also

- Allow 5% variation of the Yukawa couplings at GUT scale due to threshold corrections
- Include radiative corrections to bottom and tau, plus resummation (very important!)
- Estimate theoretical uncertainties

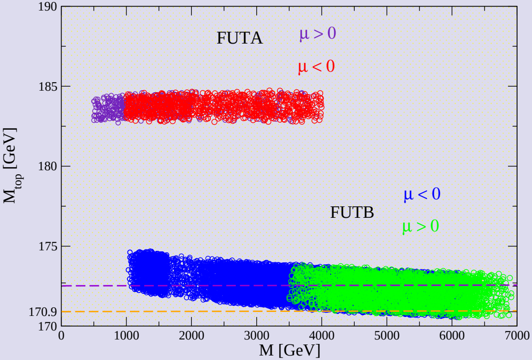
We look for the solutions that satisfy the following constraints:

- Right masses for top and bottom
  fact of life — FeynHiggs
- The decay $b \to s\gamma$ — MicroOmegas
  fact of life
- The branching ratio $B_s \to \mu^+\mu^-$ — MicroOmegas
  fact of life
- Cold dark matter density $\Omega_{CDM}h^2$ — MicroOmegas
  loose constraint
- The anomalous magnetic moment of the muon $g - 2$
  see what we get

The lightest MSSM Higgs boson mass
The SUSY spectrum

FeynHiggs, Suspect, FUT

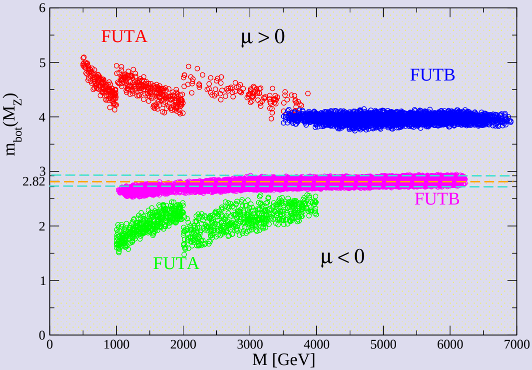
TOP MASS



FUTA: $M_{top} \sim 183\ GeV$
FUTB: $M_{top} \sim 172\ GeV$

Theoretical uncertainties $\sim$ 4%

BOTTOM MASS

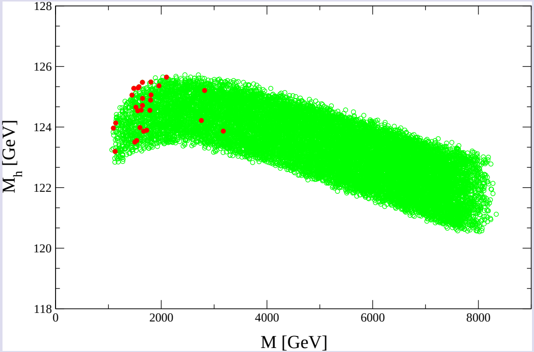


Δb and Δtau included, resummation done

FUTB $\mu < 0$ favoured

uncertainties $\sim 8\%$

Higgs mass

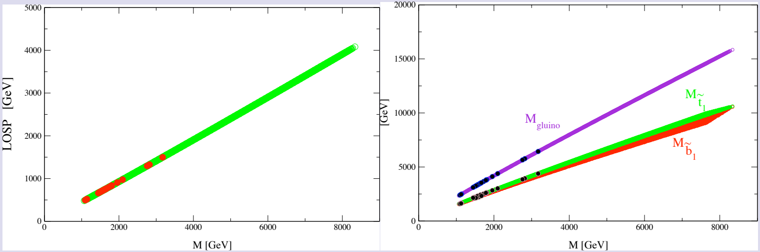


FUTB: $M_{Higgs} = 122 \sim 126\ GeV$

Uncertainties $\pm 3\ GeV$ (FeynHiggs)

$\Omega_{\mathrm{CDM}} h^2 < 0.3$

LOSP and Coloured Particles



LOSP and coloured particles that satisfy B physics and loose CDM constraint

Challenging for LHC

Results

When confronted with low-energy precision data

only FUTB $\mu < 0$ survives

No solution for g-2, very constrained from dark matter

- $M_{top} \sim 172\ GeV$ 4%
- $m_{bot}(M_Z) \sim 2.8 GeV$ 8 %
- $M_{Higgs} \sim 122 - 126\ GeV$ 3 GeV
- $\tan\beta \sim 44 - 46$

Extension to 3 fams on its way with discrete flavour symmetry; with $\not{R} \Rightarrow$ neutrino masses

in this case dark matter candidate is not LSP, results may change

Finite $SU(N)^k$ Unification

Consider $N = 1$ supersymmetric gauge theories based on the group

$$SU(N)_1 \times SU(N)_2 \times ... \times SU(N)_k$$

with matter content

$$(N, N^*, 1, ..., 1) + (1, N, N^*, ..., 1) + ... + (N^*, 1, 1, ..., N)$$

with β-function coefficient in the renormalization-group equation of each $SU(N)$ gauge given by

$$b = \left(\frac{-11}{3} + \frac{2}{3}\right) N + n_f \left(\frac{2}{3} + \frac{1}{3}\right) \left(\frac{1}{2}\right) 2N = -3N + n_f N.$$

$\mathbf{n_f = 3 \Leftrightarrow b = 0}$, FINITE
independently of the values of N and k

Ma, M.M., Zoupanos

Possible Models

Minimum requirements:

- leads to the SM or the MSSM at low energies
- it predicts correctly $sin^2\theta_W$.

MODELS:

- $SU(3)_C \times SU(3)_L \times SU(3)_R$ ✔
- $SU(3)^4 \to SU(3)_C$ predicted value of α_s be too small. ✘
- $SU(4)^4$ non-susy unification at scale of 4×10^{11} GeV. ✘
- $SU(4)^3$ either $\sin^2\theta_W$ wrong or an unbroken $U(1)$ coupled to everything. ✘

Lots of interest lately in these finite or reduced theories, since they could provide a bridge between strings or branes and ordinary GUTs

Ibáñez; Kachru and Silverstein

Finite $SU(3)^3$

Invariant is $(N, N^*, 1)(1, N, N^*)(N^*, 1, N)$

Could come from the compactification of $E_8 \to E_6$ over a Calabi-Yau manifold, or via coset space dimensional reduction, with a Wilson line

$$E_8 \to E_6 \to SU(3)^3 \to MSSM \to SM$$

We consider the $SU(3)^3$ between M_{GUT} and M_{Planck}, below MSSM

Unification of couplings $\Rightarrow$ cyclic symmetry Z_3

$$q \to \lambda \to q^c \to q$$

Now we have $\beta_g = 0$, search for unique solutions.

$SU(3)_C \times SU(3)_L \times SU(3)_R$ with quarks transforming as

De Rújula, Georgi, and Glashow; Lazarides, Panagiotakopoulos, and Shafi

$$q = \begin{pmatrix} d & u & h \\ d & u & h \\ d & u & h \end{pmatrix} \sim (3, 3^*, 1), \quad q^c = \begin{pmatrix} d^c & d^c & d^c \\ u^c & u^c & u^c \\ h^c & h^c & h^c \end{pmatrix} \sim (3^*, 1, 3)$$

and leptons transforming as

$$\lambda = \begin{pmatrix} N & E^c & \nu \\ E & N^c & e \\ \nu^c & e^c & S \end{pmatrix} \sim (1, 3, 3^*)$$

The breaking down of

$$SU(3)^3 \to SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{Y_L+Y_R}$$

is achieved with the (3,3) entry of λ, and the further breaking of $SU(2)_R \times U(1)_{Y_L+Y_R}$ to $U(1)_Y$ with the (3,1) entry.

The superpotential is

$$f\ Tr(\lambda q^c q) + \frac{1}{6} f'\ \epsilon_{ijk}\epsilon_{abc}(\lambda_{ia}\lambda_{jb}\lambda_{kc} + q^c_{ia} q^c_{jb} q^c_{kc} + q_{ia} q_{jb} q_{kc})$$

Most general superpotential $\Rightarrow$ $11 f$ and $10 f'$ couplings, subject to 9 conditions, due to the vanishing of the anomalous dimensions of each superfield:

$$\sum_{j,k} f_{ijk}(f_{ljk})^* + \frac{2}{3}\sum_{j,k} f'_{ijk}(f'_{ljk})^* = \frac{16}{9} g^2 \delta_{il},$$

where $f_{ijk} = f_{jki} = f_{kij}$; $f'_{ijk} = f'_{jki} = f'_{kij} = f'_{ikj} = f'_{kji} = f'_{jik}$

Quarks and leptons receive masses when the scalar part of the superfields $\tilde{N}_{1,2,3}$ and $\tilde{N}^c_{1,2,3}$ obtain vevs

$$(\mathcal{M}_d)_{ij} = \sum_k f_{kij}\langle \tilde{N}_k \rangle, \quad (\mathcal{M}_u)_{ij} = \sum_k f_{kij}\langle \tilde{N}^c_k \rangle,$$

$$(\mathcal{M}_e)_{ij} = \sum_k f'_{kij}\langle \tilde{N}_k \rangle, \quad (\mathcal{M}_\nu)_{ij} = \sum_k f'_{kij}\langle \tilde{N}^c_k \rangle.$$

Since we have MSSM $\Rightarrow$ two Higgs doublets
we choose the linear combinations coupled to the third generation

$$\tilde{N}^c = \sum_i a_i \tilde{N}_i^c$$

and

$$\tilde{N} = \sum_i b_i \tilde{N}_i$$

this can be done by choosing appropriately the masses in the superpotential, León et al

- Then these two Higgs doublets couple to the three families differently providing the freedom to understand their different masses and mixings
- Solutions give all-loop or two-loop finite models, with Universal soft terms or with the sum rule
- We need to fulfill the second (and most difficult) finiteness requirement for all-loop finite theories

Phenomenology

Phenomenology of the models was analyzed for an all-loop finite and a two- finite case.
Best results (so far) for the two-loop finite model:

$$m_{top} \sim 169{-}172\ GeV \qquad \tan\beta \sim 551 \qquad M_{Higgs} \sim 120{-}125\ GeV,$$

Notice: it involves three generations $\Rightarrow$ requires a discrete symmetry.

A more thorough analysis is under way.

Heinemeyer, Ma, M.M., Zoupanos

Conclusions

- Finiteness: powerful, interesting and intriguing principle $\Rightarrow$ **reduces greatly the number of free parameters**
- completely finite theories
  i.e. including the SSB terms, that satisfy the sum rule.
- Confronting the $SU(5)$ models with low-energy precision data does distinguish among models:
  - FUTB $\mu < 0$ survives (remarkably)
  - large $\tan\beta$
  - s-spectrum starts above ~ 400 GeV
  - a prediction for the Higgs $M_h \sim 122 - 126$ GeV
  - no solution for $g - 2$, constrained from dark matter
- Extension to three fams with $\not{R}$ on its way
- Detailed study of finite $SU(3)^3 \iff$ 3 generations in progress