Lepton flavour violating processes in an S_3-invariant extension of the standard model

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Irreducible representations of S_3

The group of permutations of three objects S_3 has two one-dimensional irreps (singlets) and one two-dimensional irrep (doublet)

- one dimensional: $\mathbf{1}_A$ antisymmetric singlet, $\mathbf{1}_s$ symmetric singlet
- Two - dimensional: 2 doublet

Direct product of irreps of S_3

$$\mathbf{1}_s \otimes \mathbf{1}_s = \mathbf{1}_s, \quad \mathbf{1}_s \otimes \mathbf{1}_A = \mathbf{1}_A, \quad \mathbf{1}_A \otimes \mathbf{1}_A = \mathbf{1}_s, \quad \mathbf{1}_s \otimes 2 = 2, \quad \mathbf{1}_A \otimes 2 = 2$$

$$\boxed{2 \otimes 2 = \mathbf{1}_s \oplus \mathbf{1}_A \oplus 2}$$

the direct (tensor) product of two doublets

$$\mathbf{p_D} = \begin{pmatrix} p_{D1} \\ p_{D2} \end{pmatrix} \qquad \text{and} \qquad \mathbf{q}_D = \begin{pmatrix} q_{D1} \\ q_{D2} \end{pmatrix}$$

has two singlets, r_s and r_A, and one doublet r_D^T

$$r_s = p_{D1}q_{D1} + p_{D2}q_{D2} \quad \text{is invariant,} \qquad r_A = p_{D1}q_{D2} - p_{D2}q_{D1} \quad \text{is not invariant}$$

$$r_D^T = \begin{pmatrix} p_{D1}q_{D2} + p_{D2}q_{D1} \\ p_{D1}q_{D1} - p_{D2}q_{D2} \end{pmatrix}$$

A Minimal S_3 invariant extension of the SM

The Higgs sector is extended,

$$\Phi \rightarrow H = (\Phi_1, \Phi_2, \Phi_3)^T$$

H is a reducible $\mathbf{1_s} \oplus 2$ rep. of S_3

$$H_s = \frac{1}{\sqrt{3}}\Big(\Phi_1 + \Phi_2 + \Phi_3\Big)$$

$$H_D = \begin{pmatrix} \frac{1}{\sqrt{2}}(\Phi_1 - \Phi_2) \\ \\ \frac{1}{\sqrt{6}}(\Phi_1 + \Phi_2 - 2\Phi_3) \end{pmatrix}$$

Quark, lepton and Higgs fields are

$$Q^T = (u_L, d_L), u_R, d_R, \qquad L^\dagger = (\nu_L, e_L), e_R, \nu_R, \qquad H$$

All these fields have three species (flavours) and belong to a reducible $\mathbf{1} \oplus 2$ rep. of S_3

Leptons' Yukawa interactions

Leptons

$$\begin{aligned}\mathcal{L}_{Y_E} &= -Y_1^e \overline{L}_I H_S e_{IR} - Y_3^e \overline{L}_3 H_S e_{3R} - Y_2^e [\, \overline{L}_I \kappa_{IJ} H_1 e_{JR} + \overline{L}_I \eta_{IJ} H_2 e_{JR} \,] \\ &- Y_4^e \overline{L}_3 H_I e_{IR} - Y_5^e \overline{L}_I H_I e_{3R} + h.c.,\end{aligned}$$

$$\begin{aligned}\mathcal{L}_{Y_\nu} &= -Y_1^\nu \overline{L}_I (i\sigma_2) H_S^* \nu_{IR} - Y_3^\nu \overline{L}_3 (i\sigma_2) H_S^* \nu_{3R} \\ &- Y_2^\nu [\, \overline{L}_I \kappa_{IJ} (i\sigma_2) H_1^* \nu_{JR} + \overline{L}_I \eta_{IJ} (i\sigma_2) H_2^* \nu_{JR} \,] \\ &- Y_4^\nu \overline{L}_3 (i\sigma_2) H_I^* \nu_{IR} - Y_5^\nu \overline{L}_I (i\sigma_2) H_I^* \nu_{3R} + h.c.\end{aligned}$$

$$\kappa = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}; \qquad \eta = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \qquad I, J = 1, 2$$

Furthermore, the Majorana mass terms for the right handed neutrinos are

$$\mathcal{L}_M = -M_1 \nu_{IR}^T C \nu_{IR} - M_3 \nu_{3R}^T C \nu_{3R},$$

C is the charge conjugation matrix.

Masses in the leptonic sector

To achieve a further reduction of the number of parameters an additional discrete Z_2 symmetry is introduced in the leptonic sector

$-$	$+$
$H_I,\ \nu_{3R}$	$H_S,\ L_3,\ L_I,\ e_{3R},\ e_{IR},\ \nu_{IR}$

then,

$$Y_1^e = Y_3^e = Y_1^\nu = Y_5^\nu = 0$$

Then, the Yukawa interactions yield leptonic mass matrices of the form

$$M_e = \begin{pmatrix} \mu_2^e & \mu_2^e & \mu_5^e \\ \mu_2^e & -\mu_2^e & \mu_5^e \\ \mu_4^e & \mu_4^e & 0 \end{pmatrix} \qquad \text{and} \qquad M_{\nu D} = \begin{pmatrix} \mu_2^\nu & \mu_2^\nu & 0 \\ \mu_2^\nu & -\mu_2^\nu & 0 \\ \mu_4^\nu & \mu_4^\nu & \mu_3^\nu \end{pmatrix}$$

The Majorana masses for ν_L are obtained from the see-saw mechanism

$$M_\nu = M_{\nu D} \tilde{\mathsf{M}}^{-1} (M_{\nu D})^T \qquad \text{with} \qquad \tilde{\mathsf{M}} = \mathrm{diag}(M_1, M_1, M_3)$$

The Mass Matrix of the charged leptons as function of its eigenvalues

The mass matrix of the charged leptons is

$$M_e \approx m_\tau \begin{pmatrix} \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & -\frac{1}{\sqrt{2}}\frac{\tilde{m}_\mu}{\sqrt{1+x^2}} & \frac{1}{\sqrt{2}}\sqrt{\frac{1+x^2-\tilde{m}_\mu^2}{1+x^2}} \\ \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & \frac{\tilde{m}_e(1+x^2)}{\sqrt{1+x^2-\tilde{m}_\mu^2}}e^{i\delta_e} & 0 \end{pmatrix}.$$

$x = m_e/m_\mu$, $\tilde{m}_\mu = m_\mu/m_\tau$ and $\tilde{m}_e = m_e/m_\tau$

This expression is accurate to order 10^{-9} in units of the τ mass

There are no free parameters in $\mathbf{M}_e$ other than the Dirac Phase δ!!

The Unitary Matrix U_{eL}

The unitary matrix U_{eL} is calculated from

$$U_{eL}^{\dagger} M_e M_{eL}^{\dagger} U_{eL} = diag\big(m_e^2, m_\mu^2, m_\tau^2\big)$$

We find

$$U_{eL} = \Phi_{eL} O_{eL}, \qquad \Phi_{eL} = diag\big[1, 1, e^{i\delta_D}\big]$$

and

$$\mathbf{O}_{eL} \approx \begin{pmatrix} \frac{1}{\sqrt{2}} x \frac{(1+2\tilde{m}_\mu^2+4x^2+\tilde{m}_\mu^4+2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -\frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{1}{\sqrt{2}} x \frac{(1+4x^2-\tilde{m}_\mu^4-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & \frac{1}{\sqrt{2}} \frac{(1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4)}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{1}{\sqrt{2}} \\ -\frac{\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}(1+\tilde{m}_\mu^2+x^2-2\tilde{m}_e^2)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4-\tilde{m}_\mu^6+\tilde{m}_e^2+12x^4}} & -x \frac{(1+x^2-\tilde{m}_\mu^2-2\tilde{m}_e^2)\sqrt{1+2x^2-\tilde{m}_\mu^2-\tilde{m}_e^2}}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4-4\tilde{m}_\mu^6-5\tilde{m}_e^2}} & \frac{\sqrt{1+x^2}\tilde{m}_e\tilde{m}_\mu}{\sqrt{1+x^2-\tilde{m}_\mu^2}} \end{pmatrix},$$

$x = m_e/m_\mu$, $\tilde{m}_\mu = m_\mu/m_\tau$ and $\tilde{m}_e = m_e/m_\tau$

The neutrino mass matrix

The neutrino mass matrix is obtained from the see-saw mechanism

$$\mathbf{M}_\nu = \mathbf{M}_{\nu_\mathbf{D}} \tilde{\mathbf{M}}^{-1} (\mathbf{M}_{\nu_\mathbf{D}})^T = \begin{pmatrix} 2(\rho_2^\nu)^2 & 0 & 2\rho_2^\nu \rho_4^\nu \\ 0 & 2(\rho_2^\nu)^2 & 0 \\ 2\rho_2^\nu \rho_4^\nu & 0 & 2(\rho_4^\nu)^2 + (\rho_3^\nu)^2 \end{pmatrix}$$

M_ν is reparametrized in terms of its eigenvalues

$$M_\nu = \left(\begin{array}{ccc} m_{\nu_3} & 0 & \sqrt{(m_{\nu_3} - m_{\nu_1})(m_{\nu_2} - m_{\nu_3})} e^{-i\delta_\nu} \\ 0 & m_{\nu_3} & 0 \\ \sqrt{(m_{\nu_3} - m_{\nu_1})(m_{\nu_2} - m_{\nu_3})} e^{-i\delta_\nu} & 0 & (m_{\nu_1} + m_{\nu_2} - m_{\nu_3}) e^{-2i\delta_\nu} \end{array} \right).$$

$$\mathbf{U}_\nu^T \mathbf{M}_\nu \mathbf{U}_\nu = \mathrm{diag}(|m_{\nu_1}| e^{i\phi_1 - i\phi_\nu}, |m_{\nu_2}| e^{i\phi_2 - i\phi_\nu}, |m_{\nu_3}|)$$

$$U_\nu = \left(\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & e^{i\delta_\nu} \end{array} \right) \left(\begin{array}{ccc} \sqrt{\frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_2} - m_{\nu_1}}} & \sqrt{\frac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_1}}} & 0 \\ 0 & 0 & 1 \\ -\sqrt{\frac{m_{\nu_3} - m_{\nu_1}}{m_{\nu_2} - m_{\nu_1}}} & \sqrt{\frac{m_{\nu_2} - m_{\nu_3}}{m_{\nu_2} - m_{\nu_1}}} & 0 \end{array} \right)$$

The neutrino mixing matrix I

$$V^{th}_{PMNS} = U^\dagger_{eL} U_\nu$$

The theoretical mixing matrix V^{th}_{PMNS} is

$$V^{th}_{PMNS} = \begin{pmatrix} O_{11}\cos\eta + O_{31}\sin\eta e^{i\delta} & O_{11}\sin\eta - O_{31}\cos\eta e^{i\delta} & -O_{21} \\ -O_{12}\cos\eta + O_{32}\sin\eta e^{i\delta} & -O_{12}\sin\eta - O_{32}\cos\eta e^{i\delta} & O_{22} \\ O_{13}\cos\eta - O_{33}\sin\eta e^{i\delta} & O_{13}\sin\eta + O_{33}\cos\eta e^{i\delta} & O_{23} \end{pmatrix}$$

$$\times \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

where O_{ij} are the absolute values of the elements of $\mathbf{O}_e$

$$\mathbf{V}^{PDG}_{PMNS} = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & e^{i\alpha} & 0 \\ 0 & 0 & e^{i\beta} \end{pmatrix}$$

Neutrino Mixing Angles

From a comparison of $\mathbf{V}^{th}_{PMNS}$ with $\mathbf{V}^{PDG}_{PMNS}$, we obtain the neutrino mixing angles as functions of the lepton masses

The mixing angles θ_{13} and θ_{23} depend only on the charged lepton masses

Theoretical Prediction	Experimental	Theoretical
$\sin\theta_{13} \approx \frac{1}{\sqrt{2}} x \frac{(1+4x^2-\tilde{m}_\mu^4)}{\sqrt{1+\tilde{m}_\mu^2+5x^2-\tilde{m}_\mu^4}}$	$(\sin^2\theta_{13})^{exp} = 0.01^{+0.016}_{-0.011}$	$\sin^2\theta_{13} = 1.1 \times 10^{-5}$
$\sin\theta_{23} \approx \frac{1}{\sqrt{2}} \frac{1-2\tilde{m}_\mu^2+\tilde{m}_\mu^4}{\sqrt{1-4\tilde{m}_\mu^2+x^2+6\tilde{m}_\mu^4}}$	$(\sin^2\theta_{23})^{exp} = 0.5^{+0.07}_{-0.06}$	$\sin^2\theta_{23} = 0.5$

$x = m_e/m_\mu$ and $\tilde{m}_\mu = m_\mu/m_\tau$

The solar angle θ_{12} is strongly dependent on the neutrino masses but depends only very weakly on the charged lepton masses

$$\tan\theta_{12}^2 = \frac{(\Delta m_{12}^2 + \Delta m_{13}^2 + |m_{\nu_3}|^2\cos^2\phi_\nu)^{1/2} - |m_{\nu_3}||\cos\phi_\nu|}{(\Delta m_{13}^2 + |m_{\nu_3}|^2\cos^2\phi_\nu)^{1/2} + |m_{\nu_3}||\cos\phi_\nu|}.$$

The neutrino mass spectrum I

In the present model, the experimental restriction

$$|\Delta m_{21}^2| < |\Delta m_{23}^2|$$

implies an inverted neutrino mass spectrum $m_{\nu_3} < m_{\nu_1}, m_{\nu_2}$

From our previous expressions for $\tan\theta_{12}$

$$|m_{\nu_3}| = \frac{\sqrt{\Delta m_{13}^2}}{2\cos\phi_\nu \tan\theta_{12}} \frac{1 - \tan^4\theta_{12} + r^2}{\sqrt{1+\tan^2\theta_{12}}\sqrt{1+\tan^2\theta_{12}+r^2}},$$

where $r = \Delta m_{21}^2 / \Delta m_{23}^2$.

The mass $|m_{\nu_3}|$ gets its minimal value when $\sin\phi_\nu = 0$,

$$|m_{\nu_3}| \approx \frac{1}{2} \frac{\sqrt{\Delta m_{13}^2}}{\tan\theta_{12}} (1 - \tan^2\theta_{12})$$

Neutrino mass spectrum II

- We wrote the neutrino mass differences, $m_{\nu_i} - m_{\nu_j}$, in terms of the differences of the squared masses $\Delta_{ij}^2 = m_{\nu_i}^2 - m_{\nu_j}^2$ and one of the neutrino masses, say m_{ν_3}.
- The mass m_{ν_2} was taken as a free parameter in the fitting of our formula for $\tan\theta_{12}$ to the experimental value
- with

$$\Delta m_{21}^2 = 7.65 \times 10^{-5} eV^2 \qquad \Delta m_{13}^2 = 2.4 \times 10^{-3} eV^2$$

and

$$\tan\theta_{12} = 0.661$$

we get

$$|m_{\nu_3}| \approx 0.021\ eV \implies |m_{\nu_2}| \approx 0.054\ eV \quad \text{and} \quad |m_{\nu_1}| \approx 0.053\ eV$$

- The neutrino mass spectrum has an inverted hierarchy of masses

S_3 symmetry and tribimaximal form of the mixing matrix

WE ASSUME $\delta = \delta_\nu - \delta_e = \pi/2$

$$\mathbf{V}^{th}_{PMNS} = \mathbf{V}^{tri}_{PMNS} + \mathbf{\Delta V}^{tri}_{PMNS}$$

where

$$\mathbf{V}^{tri}_{PMNS} = \begin{pmatrix} \sqrt{\frac{2}{3}} & \sqrt{\frac{1}{3}} & 0 \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & -\sqrt{\frac{1}{2}} \\ -\sqrt{\frac{1}{6}} & \sqrt{\frac{1}{3}} & \sqrt{\frac{1}{2}} \end{pmatrix}$$

$$\mathbf{\Delta V}^{tri}_{PMNS} = \mathbf{\Delta V_e} + \delta t_{12} \frac{(\sqrt{2} + \delta t_{12})}{1 + \frac{2}{3}\delta t_{12}(\sqrt{2} + \delta t_{12})} \widetilde{\mathbf{\Delta V}}_\nu$$

$$\mathbf{\Delta V_e} \sim m_e/m_\mu = 4.8 \times 10^{-3}$$

$$\delta t_{12} = \frac{\sqrt{2}}{2} - t_{12}$$

$$\delta t_{12}^{(\mathrm{exp})} = \frac{\sqrt{2}}{2} - 0.661 \approx 0.046$$

V^{th}_{PMNS} is nearly tri-bimaximal!!!

FCNC I

In the Standard Model the FCNC at tree level are suppressed by the GIM mechanism.

Models with more than one Higgs $SU(2)$ doublet have tree level FCNC due to the exchange of scalar fields. The mass matrix written in terms of the Yukawa couplings is

$$\mathcal{M}^e_Y = Y^{E1}_w H^0_1 + Y^{E2}_w H^0_2,$$

FCNC processes:

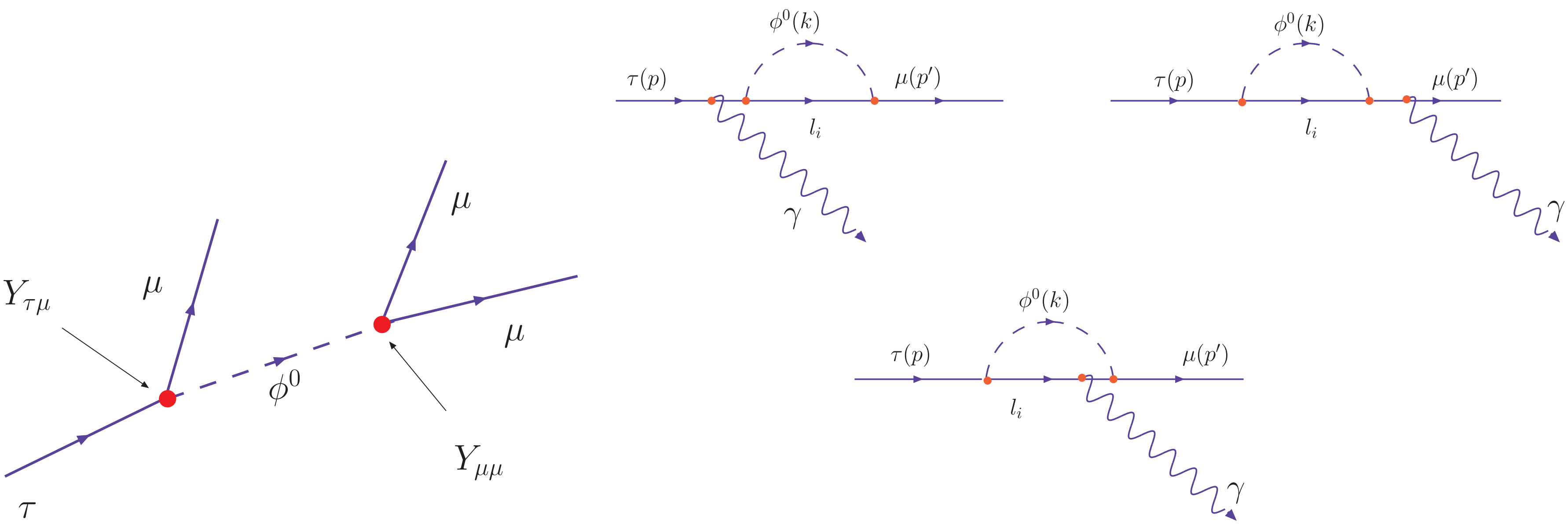


Figure 1: The diagram in the left contributes to the process $\tau^- \longrightarrow 3\mu$. The three diagrams in the right contribute to the process $\tau \longrightarrow \mu\gamma$.

Yukawa matrices in the mass representation

The Yukawa matrices in the mass basis defined by

$$\tilde{Y}_m^{EI} = U_{eL}^{\dagger} Y_w^{EI} U_{eR}$$

$$\tilde{Y}_m^{E1} \approx \frac{m_\tau}{v_1} \begin{pmatrix} 2\tilde{m}_e & -\frac{1}{2}\tilde{m}_e & \frac{1}{2}x \\ -\tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & -\frac{1}{2} \\ \frac{1}{2}\tilde{m}_\mu x^2 & -\frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m ,$$

and

$$\tilde{Y}_m^{E2} \approx \frac{m_\tau}{v_2} \begin{pmatrix} -\tilde{m}_e & \frac{1}{2}\tilde{m}_e & -\frac{1}{2}x \\ \tilde{m}_\mu & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \\ -\frac{1}{2}\tilde{m}_\mu x^2 & \frac{1}{2}\tilde{m}_\mu & \frac{1}{2} \end{pmatrix}_m ,$$

all off diagonal terms give rise to FCNC processes!!

Branching ratios

We define the partial branching ratios (only leptonic decays) as

$$Br(\tau \to \mu e^+ e^-) = \frac{\Gamma(\tau \to \mu e^+ e^-)}{\Gamma(\tau \to e\nu\bar{\nu}) + \Gamma(\tau \to \mu\nu\bar{\nu})}, \quad \Gamma(\tau \to \mu e^+ e^-) \approx \frac{m_\tau^5}{3 \times 2^{10}\pi^3} \frac{(Y_{\tau\mu}^{1,2} Y_{ee'}^{1,2})^2}{M_{H_{1,2}}^4}$$

thus

$$Br(\tau \to \mu e^+ e^-) \approx \frac{9}{4} \left(\frac{m_e m_\mu}{m_\tau^2}\right)^2 \left(\frac{m_\tau}{M_{H_{1,2}}}\right)^4,$$

Similar computations lead to

$$Br(\tau \to e\gamma) \approx \frac{3\alpha}{8\pi} \left(\frac{m_\mu}{M_H}\right)^4,$$

$$Br(\tau \to \mu\gamma) \approx \frac{3\alpha}{128\pi} \left(\frac{m_\mu}{m_\tau}\right)^2 \left(\frac{m_\tau}{M_H}\right)^4,$$

$$Br(\tau \to 3\mu) \approx \frac{9}{64} \left(\frac{m_\mu}{M_H}\right)^4,$$

$$Br(\mu \to 3e) \approx 18 \left(\frac{m_e m_\mu}{m_\tau^2}\right)^2 \left(\frac{m_\tau}{M_H}\right)^4,$$

$$Br(\mu \to e\gamma) \approx \frac{27\alpha}{64\pi} \left(\frac{m_e}{m_\mu}\right)^4 \left(\frac{m_\tau}{M_H}\right)^4.$$

FCNC: Numerical results

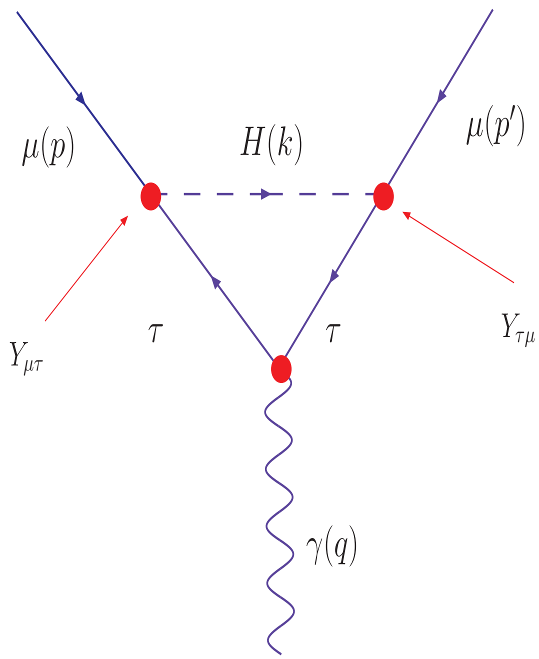
Table 1: Leptonic processes via FCNC

FCNC processes	Theoretical BR	Experimental upper bound BR	References
$\tau \longrightarrow 3\mu$	8.43×10^{-14}	5.3×10^{-8}	B. Aubert *et. al.* (2007)
$\tau \longrightarrow \mu e^+ e^-$	3.15×10^{-17}	8×10^{-8}	B. Aubert *et. al.* (2007)
$\tau \longrightarrow \mu\gamma$	9.24×10^{-15}	6.8×10^{-8}	B. Aubert *et. al.*(2005)
$\tau \longrightarrow e\gamma$	5.22×10^{-16}	1.1×10^{-11}	B. Aubert *et. al.* (2006)
$\mu \longrightarrow 3e$	2.53×10^{-16}	1×10^{-12}	U. Bellgardt *et al.* (1998)
$\mu \longrightarrow e\gamma$	2.42×10^{-20}	1.2×10^{-11}	M. L. Brooks *et al.* (1999)

Small FCNC processes mediating non-standard quark-neutrino interactions could be important in the theoretical description of the **gravitational core collapse and shock generation** in the explosion stage of a supernova

Muon Anomalous Magnetic Moment

The anomalous magnetic moment of the muon is related to the gyroscopic ratio by



$$a_\mu = \frac{\mu_\mu}{\mu_B} - 1 = \frac{1}{2}(g_\mu - 2)$$

In models with more than one Higgs $SU(2)$ doublet, the exchange of flavour changing neutral scalars also contribute to the anomalous magnetic moment of the muon

$$\delta a_\mu^{(H)} = \frac{Y_{\mu\tau}Y_{\tau\mu}}{16\pi^2}\frac{m_\mu m_\tau}{M_H^2}\left(log\left(\frac{M_H^2}{m_\tau^2}\right) - \frac{3}{2}\right)$$

From our results: $Y_{\mu\tau}Y_{\tau\mu} = \frac{m_\mu m_\tau}{4v_1v_2}$

$$\delta a_\mu^{(H)} = \frac{m_\tau^2}{(246\ GeV)^2}\frac{(2+\tan^2\beta)}{32\pi^2}\frac{m_\mu^2}{M_H^2}\left(log\left(\frac{M_H^2}{m_\tau^2}\right) - \frac{3}{2}\right), \quad \tan\beta = \frac{v_s}{v_1}$$

From the experimental upper bound on $(\mu \to 3e)$, we get $\tan\beta \leq 14$, Hence

$$\delta a_\mu = 1.7 \times 10^{-10}$$

Contribution to the anomaly of the muon's magnetic moment

The difference between the experimental value and the Standard Model prediction for the anomaly is

$$\Delta a_\mu = a_\mu^{exp} - a_\mu^{SM} = (28.7 \pm 9.1) \times 10^{-10}$$

$\Delta a_\mu \sim 3\sigma$ (three standard deviations) !!

But, the uncertainty in the computation of higher order hadronic effects is large

$$\delta a_\mu^{LBL}(3, had) \approx 1.59 \times 10^{-9}; \quad \delta a_\mu^{VP}(3, had) \approx -1.82 \times 10^{-9}$$

$$\frac{\delta a_\mu^{(H)}}{\Delta a_\mu} \approx \frac{1.7}{28} \approx 6\% \quad \text{and} \quad \delta a_\mu^{(H)} < \delta a_\mu(3, had)$$

The contribution of the exchange of flavour changing scalars to the anomaly of the muon's magnetic moment, $\delta a_\mu^{(H)}$, is small but not negligible, and it is compatible with the best, state of the art, measurements and theoretical predictions.

Summary

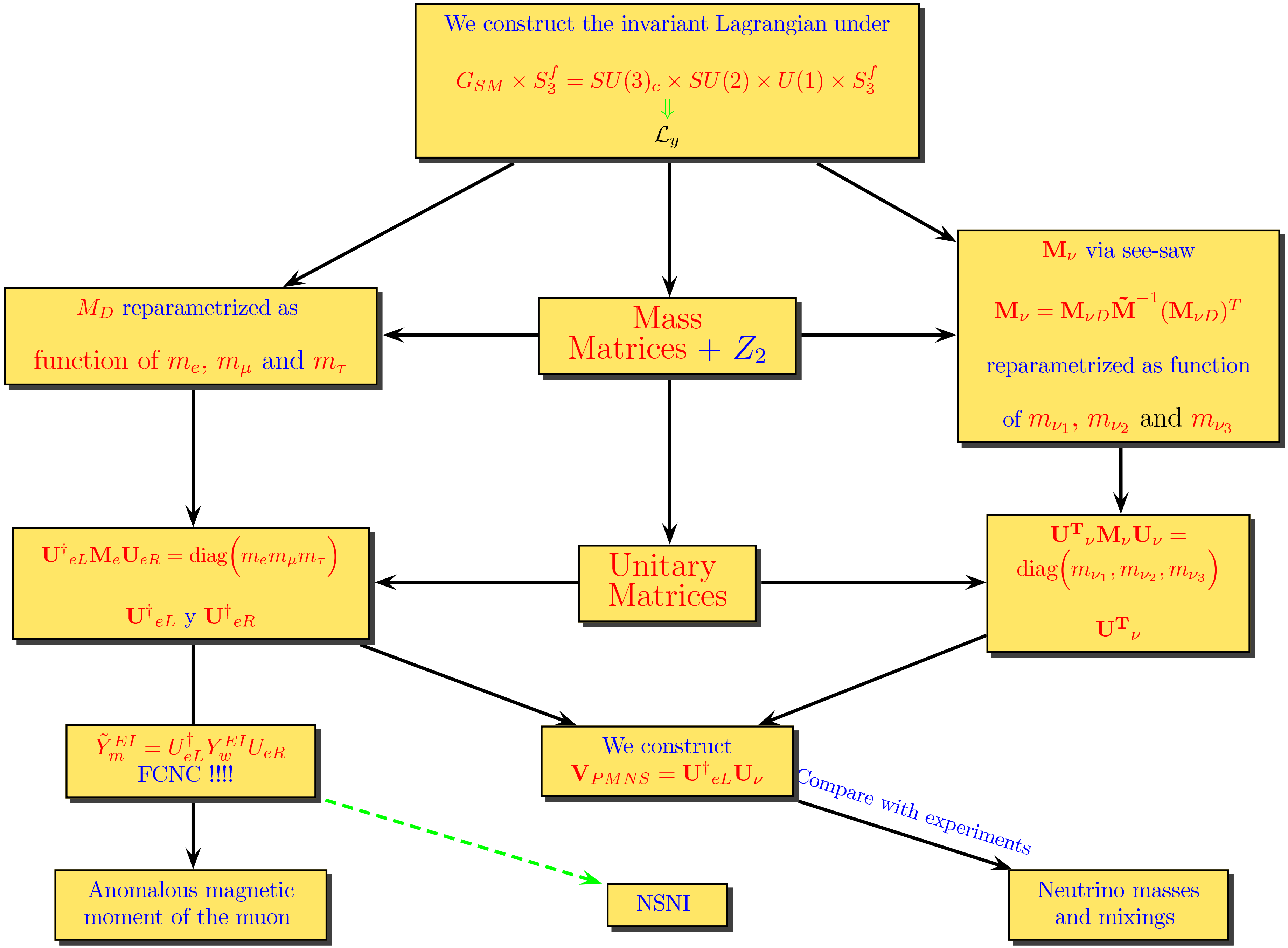
- By introducing in the theory three $SU(2)_L$ Higgs doublet fields, we extended the concept of flavour and generations to the Higgs sector and formulated a minimal S_3-invariant Extension of the Standard Model
- A further reduction of free parameters is achieved in the leptonic sector by introducing a Z_2 symmetry
- The magnitudes of the three mixing angles θ_{12}, θ_{23} and θ_{13}, are determined by an interplay of the $S_3 \times Z_2$ symmetry, the see-saw mechanism and the lepton mass hierarchy
- The mixing angles, θ_{23} and θ_{13}, depend only on the masses of the charged leptons and are in excellent agreement with the best experimental values
- The solar mixing angle, θ_{12}, fixes the scale and origin of the neutrino mass spectrum which has an inverted mass hierarchy with values

$$|m_{\nu_2}| \approx 0.054 \; eV, \qquad |m_{\nu_1}| \approx 0.053 \; eV, \qquad |m_{\nu_3}| \approx 0.021 \; eV$$

- The branching ratios of all flavour changing neutral current processes in the leptonic sector are strongly suppressed by the $S_3 \times Z_2$ symmetry and powers of the small mass ratios m_e/m_τ, m_μ/m_τ, and $\left(m_\tau/M_{H_{1,2}}\right)^4$, but could be important in astrophysical processes
- The anomalous magnetic moment of the muon gets a small but non-negligible contribution from the exchange of flavour changing scalar fields

Conclusions

- The Minimal S_3-Invariant Extension of the Standard Model describes successfully masses and mixings in the quark (not shown here) and leptonic sectors with a small number of free parameters.
- It predicts the numerical values of θ_{13} and θ_{23} neutrino mixing angles.
- It predicts a small deviation of the neutrino mixing matrix from the tri-bimaximal form proportional to the small ratio m_e/m_μ, in excellent agreement with the experimental values of the neutrino mixing angles.
- It also predicts values for the branching ratios of the the lepton flavour violating processes well below the experimental upper bounds.
- It gives a small but non-negligible contribution to the anomaly of the magnetic moment of the muon.



We construct the invariant Lagrangian under
$G_{SM} \times S_3^f = SU(3)_c \times SU(2) \times U(1) \times S_3^f$
$\Downarrow$
$\mathcal{L}_y$
M_D reparametrized as
function of m_e, m_μ and m_τ
Mass Matrices + Z_2
$\mathbf{M}_\nu$ via see-saw
$\mathbf{M}_\nu = \mathbf{M}_{\nu D}\tilde{\mathbf{M}}^{-1}(\mathbf{M}_{\nu D})^T$
reparametrized as function
of m_{ν_1}, m_{ν_2} and m_{ν_3}
$\mathbf{U}^\dagger{}_{eL}\mathbf{M}_e\mathbf{U}_{eR} = \mathrm{diag}\left(m_e m_\mu m_\tau\right)$
$\mathbf{U}^\dagger{}_{eL}$ y $\mathbf{U}^\dagger{}_{eR}$
Unitary Matrices
$\mathbf{U^T}{}_\nu\mathbf{M}_\nu\mathbf{U}_\nu =$
$\mathrm{diag}\left(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}\right)$
$\mathbf{U^T}{}_\nu$
$\tilde{Y}_m^{EI} = U_{eL}^\dagger Y_w^{EI} U_{eR}$
FCNC !!!!
We construct
$\mathbf{V}_{PMNS} = \mathbf{U}^\dagger{}_{eL}\mathbf{U}_\nu$
Compare with experiments
Anomalous magnetic moment of the muon
NSNI
Neutrino masses and mixings