

# SUSY contributions to $\phi_{B_s}$ from hierarchical sfermions

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# SUSY flavour breaking sources

- New sources of flavour breaking (squarks only)

$$\begin{aligned} & \tilde{q}_L^\dagger \tilde{m}_{q_L}^2 \tilde{q}_L + \tilde{d}_R^\dagger \tilde{m}_{d_R}^2 \tilde{d}_R + \tilde{u}_R^\dagger \tilde{m}_{u_R}^2 \tilde{u}_R \\ & + \left( \tilde{d}_R^\dagger Y_D A_D \tilde{q}_L h_D + \tilde{u}_R^\dagger Y_U A_U \tilde{q}_L h_U + \text{h.c.} \right) \end{aligned}$$

- D-squark mass matrix (in the super-CKM basis)

$$\mathcal{M}_D^2 = \begin{pmatrix} LL & LR \\ RL & RR \end{pmatrix} = \mathcal{W}_D \mathcal{M}_D^{2 \text{diag}} \mathcal{W}_D^\dagger \quad (\tilde{d}_L^\dagger, \tilde{d}_R^\dagger) \mathcal{M}_D^2 \begin{pmatrix} \tilde{d}_L \\ \tilde{d}_R \end{pmatrix}$$

$$LL = \tilde{m}_{q_L}^2 + M_D^\dagger M_D + M_Z^2 z_D c_{2\beta} \mathbf{1}$$

$$RL = -M_D (A_D + \mu \tan \beta)$$

$$M_D = \text{down quark mass matrix} = M_D^{\text{diag}}$$

# Comments

- About 100 physical real parameters
- Large FCNC (and CPV) processes in most of the parameter space, (SUSY flavour problem)
- For  $\tilde{m} < \text{TeV}$ : need **small mixing** +  $\tilde{m}_d \approx \tilde{m}_s$  (barring alignment)
- Note: fermion masses also have peculiar structure:  
small mixing +  $m_d, m_s \ll m_b$ ; moreover, both  $m_d, m_s \ll m_b$  and  $\tilde{m}_d \approx \tilde{m}_s$  correspond to an approximate U(2) symmetry
- U(3) badly broken by  $Y_\dagger = O(1) \rightarrow$  expect  $\tilde{m}_b \neq \tilde{m}_{d,s}$  How much?  
Two opposite (complementary) limits:
  - **Degeneracy**:  $\tilde{m}_b \approx \tilde{m}_s \approx \tilde{m}_d$
  - **Hierarchy**:  $\tilde{m}_b \ll \tilde{m}_{s,d}$  (not incompatible with naturalness, see below)

# Model-independent analysis

- Expand in small off-diagonal elements of squark mass matrix

$$\mathcal{M}^2 = \mathcal{M}_0^2 + \mathcal{M}_1^2$$

- **Degeneracy:**  $\mathcal{M}_0^2 = \tilde{m}^2 \mathbf{1}$  [MIA, Gabbiani Gabrielli Masiero Silvestrini '96]

$$A(\Delta F = 1)_{ij} = x f^{(1)}(x) \delta_{ij}$$
$$A(\Delta F = 2)_{ij} = \frac{x^2}{3!} g^{(3)}(x) \delta_{ij}^2$$
$$\delta_{ij} \equiv \frac{(\mathcal{M}_1^2)_{ij}}{\tilde{m}^2}, \quad x \equiv \frac{\tilde{m}^2}{M^2}$$

- $\delta_{ij}$ : source of flavour violation, process independent
- $f, g$ : loop functions (process dependent, flavour conserving)
- # of derivatives and factorial  $\leftrightarrow$  # of identical propagators (-1)
- OK in case of short RGE running, in any case useful recipe up to (harmless? see below)  $O(1)$  differences between  $f$  and  $f'$

# Model-independent analysis

- Expand in small off-diagonal elements of squark mass matrix

$$\mathcal{M}^2 = \mathcal{M}_0^2 + \mathcal{M}_1^2$$

- Hierarchy:

[Cohen Kaplan Lepeintre Nelson '97]

$$\mathcal{M}_0^2 = \left( \begin{array}{c|c} \text{heavy} & \\ \hline & \tilde{m}^2 \\ \hline & \\ \text{heavy} & \\ & \tilde{m}^2 \end{array} \right) \quad \begin{aligned} A(\Delta F = 1)_{ij} &= f(x) \hat{\delta}_{ij} \\ A(\Delta F = 2)_{ij} &= g^{(1)}(x) \hat{\delta}_{ij}^2 \end{aligned}$$

- $\hat{\delta}_{ij} \equiv \mathcal{W}_{i3} \mathcal{W}_{3j}^\dagger \quad x = \frac{\tilde{m}^2}{M^2} \quad \hat{\delta}_{a3} = \frac{\mathcal{M}_{a3}^2}{\tilde{m}_a^2} \quad (a = 1, 2)$

- $\hat{\delta}_{ab} \approx \hat{\delta}_{a3} \hat{\delta}_{3b} \quad (a, b, = 1, 2)$

- $\hat{\delta}_{a3}^{LR} \approx \hat{\delta}_{a3}^{LL} \hat{\delta}_{33}^{LR} \quad \hat{\delta}_{33}^{LR} = -\frac{m_b (A_{33}^D + \mu \tan \beta)}{\tilde{m}^2} \quad (\text{under assumptions})$

# Theoretical considerations on the hierarchical case

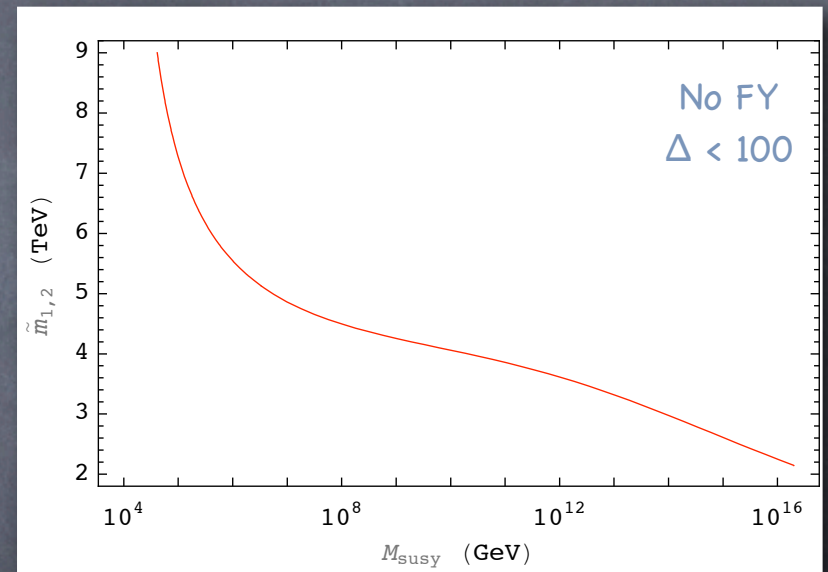
- The naturalness bound on  $\tilde{m}_{1,2}$  is milder than the one on  $\tilde{m}_3$  [Dimopoulos Giudice '95]

- Effective supersymmetry

[Cohen Kaplan Lepeintre Nelson '97]

- Alleviates the SUSY flavour problem:  $\hat{\delta} \lesssim \tilde{m}/\tilde{m}_{1,2}$

- Related to physical quantities: squark masses, CKM angles



# Degeneracy vs Hierarchy

- Different correlation between  $\Delta F = 1$  and  $\Delta F = 2$  in the two cases

$$\left. \frac{A(\Delta F = 2)}{[A(\Delta F = 1)]^2} \right|_{\text{degeneracy}} = \frac{1}{6} \frac{g^{(3)}}{g^{(1)}} \left( \frac{f}{f^{(1)}} \right)^2 \left. \frac{A(\Delta F = 2)}{[A(\Delta F = 1)]^2} \right|_{\text{hierarchy}}.$$

E.g.:  $A(\Delta m_{B_s})$  vs  $A(b \rightarrow s\gamma)$   
LL insertion  $\left. \frac{g^{(3)}}{g^{(1)}} \left( \frac{f}{f^{(1)}} \right)^2 \right|_{x=1} \approx \frac{25}{27}$

# Bounds on $\delta$ 's

Hierarchy

Degeneracy

$D_0 - \bar{D}_0$  mixing

$$\left| \hat{\delta}_{ut}^{LL} \hat{\delta}_{ct}^{LL*} \right| < 8.0 \times 10^{-3} \left( \frac{m_{\bar{t}}}{350 \text{ GeV}} \right) \quad \left| \delta_{uc}^{LL} \right| < 3.4 \times 10^{-2} \left( \frac{m_{\bar{q}}}{350 \text{ GeV}} \right)$$

$B \rightarrow X_s \gamma$

$$\begin{array}{|l} \left| \text{Re}(\hat{\delta}_{sb}^{LL}) \right| < 2.2 \times 10^{-2} \left( \frac{m_{\bar{b}}}{350 \text{ GeV}} \right)^2 \left( \frac{10}{\tan \beta} \right) \\ \left| \text{Im}(\hat{\delta}_{sb}^{LL}) \right| < 6.7 \times 10^{-2} \left( \frac{m_{\bar{b}}}{350 \text{ GeV}} \right)^2 \left( \frac{10}{\tan \beta} \right) \end{array} \quad \begin{array}{|l} \left| \text{Re}(\delta_{sb}^{LL}) \right| < 3.8 \times 10^{-2} \left( \frac{m_{\bar{q}}}{350 \text{ GeV}} \right)^2 \left( \frac{10}{\tan \beta} \right) \\ \left| \text{Im}(\delta_{sb}^{LL}) \right| < 1.1 \times 10^{-1} \left( \frac{m_{\bar{q}}}{350 \text{ GeV}} \right)^2 \left( \frac{10}{\tan \beta} \right) \end{array}$$

$$|\hat{\delta}_{sb}| \gtrsim 4 \cdot 10^{-2}$$

$\Delta m_{B_s}$

$$\begin{array}{|l} \left| \text{Re}(\hat{\delta}_{sb}^{LL}) \right| < 9.4 \times 10^{-2} \left( \frac{m_{\bar{b}}}{350 \text{ GeV}} \right) \\ \left| \text{Im}(\hat{\delta}_{sb}^{LL}) \right| < 7.2 \times 10^{-2} \left( \frac{m_{\bar{b}}}{350 \text{ GeV}} \right) \end{array} \quad \begin{array}{|l} \left| \text{Re}(\delta_{sb}^{LL}) \right| < 4.0 \times 10^{-1} \left( \frac{m_{\bar{q}}}{350 \text{ GeV}} \right) \\ \left| \text{Im}(\delta_{sb}^{LL}) \right| < 3.1 \times 10^{-1} \left( \frac{m_{\bar{q}}}{350 \text{ GeV}} \right) \end{array}$$

$B_d^0 - \bar{B}_d^0$  mixing

$$\begin{array}{|l} \left| \text{Re}(\hat{\delta}_{db}^{LL}) \right| < 4.3 \times 10^{-3} \left( \frac{m_{\bar{b}}}{350 \text{ GeV}} \right) \\ \left| \text{Im}(\hat{\delta}_{db}^{LL}) \right| < 7.3 \times 10^{-3} \left( \frac{m_{\bar{b}}}{350 \text{ GeV}} \right) \end{array} \quad \begin{array}{|l} \left| \text{Re}(\delta_{db}^{LL}) \right| < 1.8 \times 10^{-2} \left( \frac{m_{\bar{q}}}{350 \text{ GeV}} \right) \\ \left| \text{Im}(\delta_{db}^{LL}) \right| < 3.1 \times 10^{-2} \left( \frac{m_{\bar{q}}}{350 \text{ GeV}} \right) \end{array}$$

$$|\hat{\delta}_{db}| \gtrsim 0.8 \cdot 10^{-2}$$

$\Delta m_K$

$$\sqrt{\left| \text{Re}(\hat{\delta}_{db}^{LL} \hat{\delta}_{sb}^{LL*})^2 \right|} < 1.0 \times 10^{-2} \left( \frac{m_{\bar{b}}}{350 \text{ GeV}} \right) \quad \sqrt{\left| \text{Re}(\delta_{ds}^{LL})^2 \right|} < 4.2 \times 10^{-2} \left( \frac{m_{\bar{q}}}{350 \text{ GeV}} \right)$$

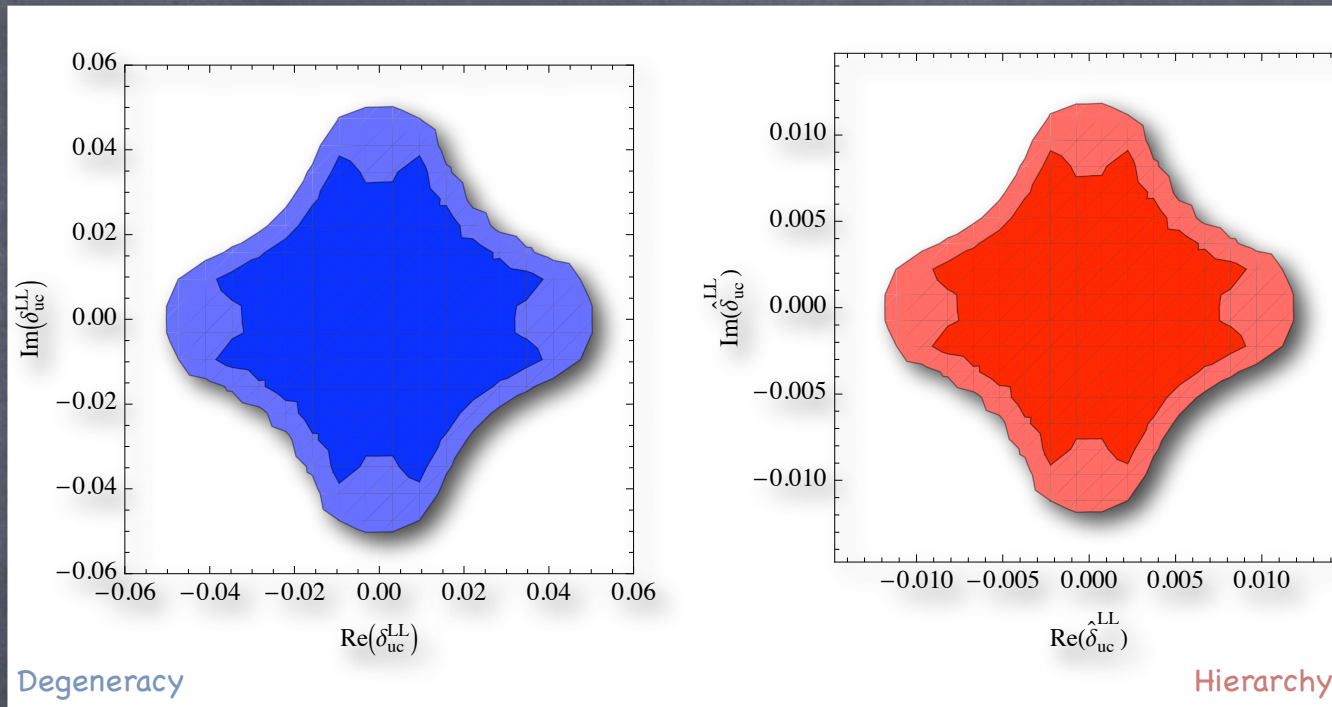
$$|\hat{\delta}_{ds}| \gtrsim 3 \cdot 10^{-4}$$

$\epsilon_K$

$$\sqrt{\left| \text{Im}(\hat{\delta}_{db}^{LL} \hat{\delta}_{sb}^{LL*})^2 \right|} < 4.4 \times 10^{-4} \left( \frac{m_{\bar{b}}}{350 \text{ GeV}} \right) \quad \sqrt{\left| \text{Im}(\delta_{ds}^{LL})^2 \right|} < 1.8 \times 10^{-3} \left( \frac{m_{\bar{q}}}{350 \text{ GeV}} \right)$$



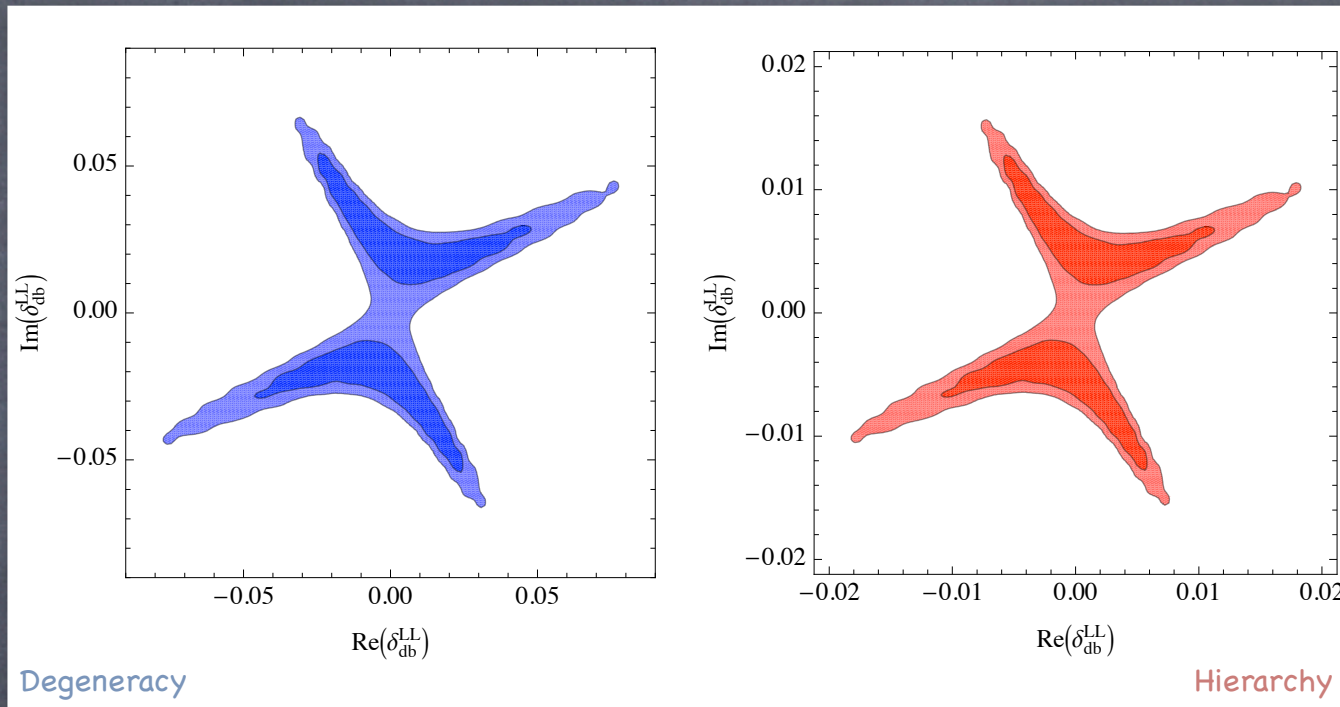
# $u \leftrightarrow c$ transitions ( $D^0 - \bar{D}^0$ )



68%, 95% C.L.

$$\tilde{m} = M_3 = 350 \text{ GeV}$$

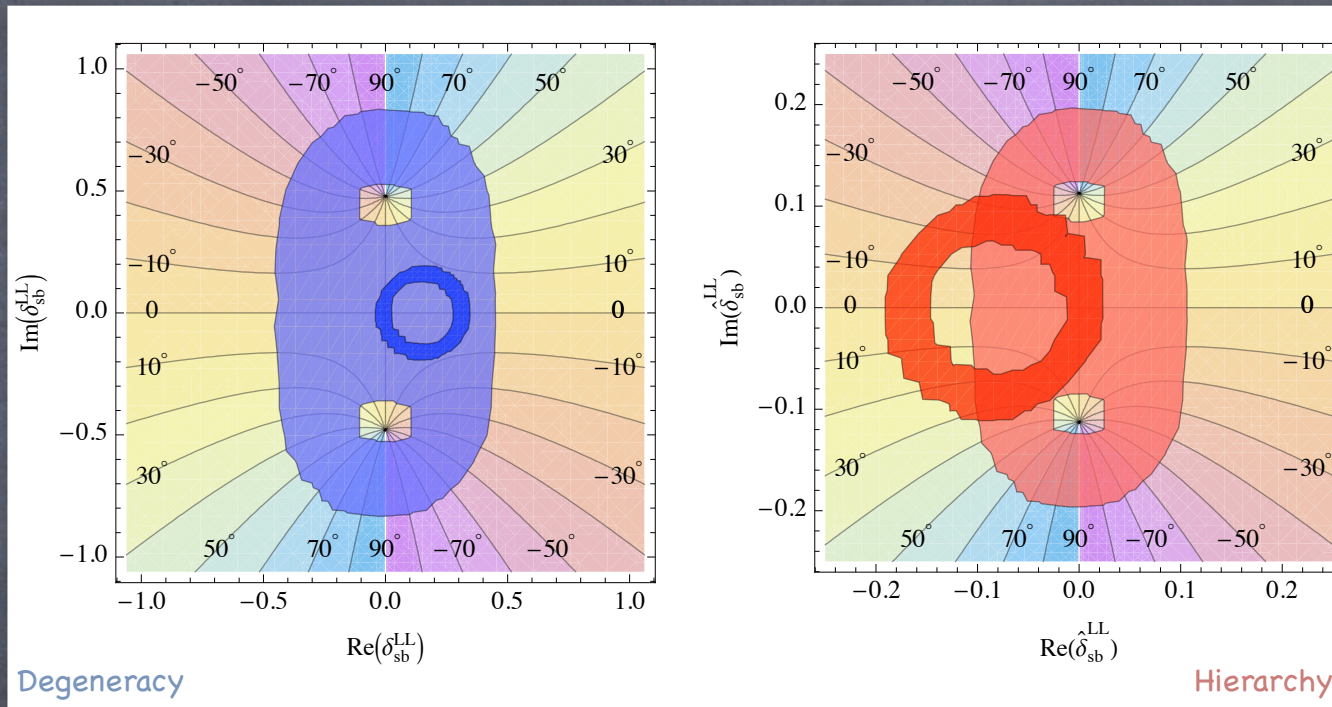
# $b \leftrightarrow d$ transitions ( $\Delta m_{Bd}, \sin 2\beta_{\text{eff}}$ )



68%, 95% C.L.

$\tilde{m} = M_3 = 350 \text{ GeV}$

# $b \leftrightarrow s$ transitions ( $\Delta m_{B_s}$ , $b \rightarrow s\gamma$ , $\phi_{B_s}$ )



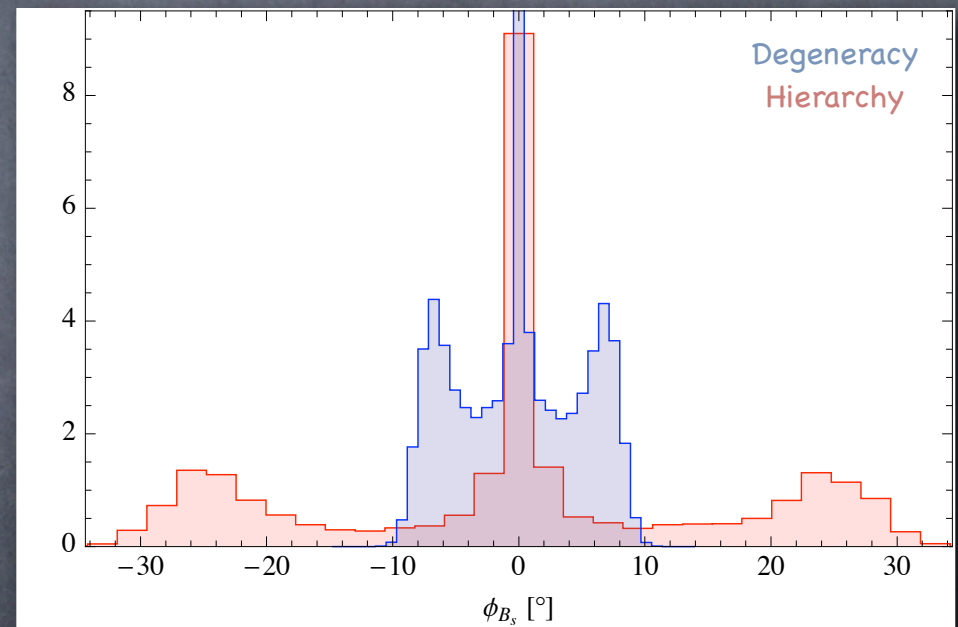
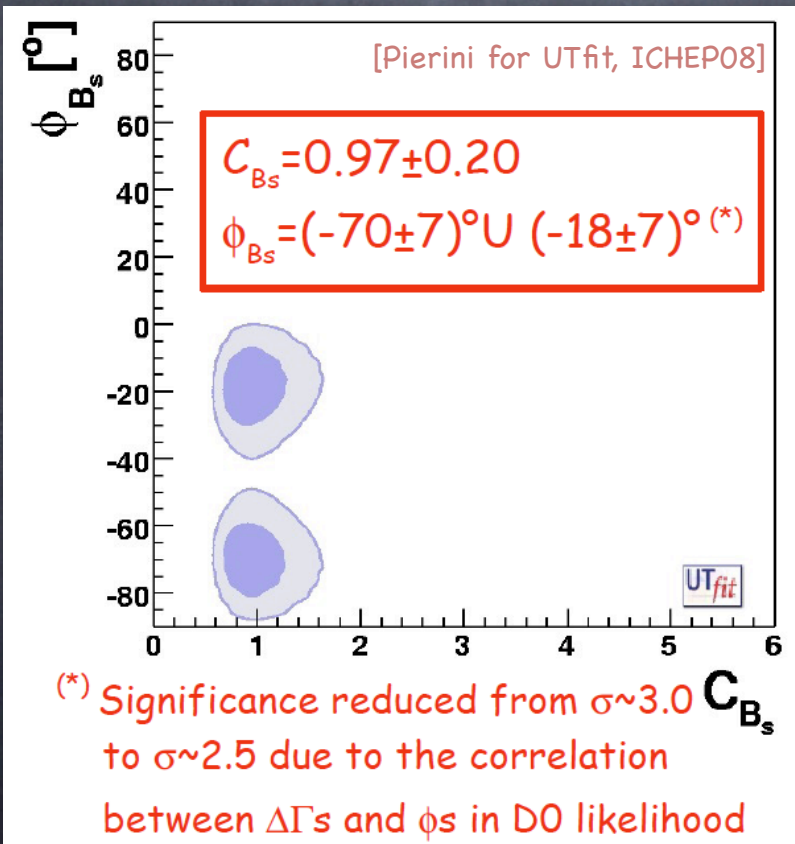
95% C.L.

$$\tilde{m} = M_3 = \mu = 350 \text{ GeV}, \tan\beta = 10, A = 0$$

# $\phi_{B_s}$

$$\langle B_s | H_{\text{eff}}^{\text{full}} | \bar{B}_s \rangle = C_{B_s} e^{2i\phi_{B_s}} \langle B_s | H_{\text{eff}}^{\text{SM}} | \bar{B}_s \rangle$$

$$\langle B_s | H_{\text{eff}}^{\text{SM}} | \bar{B}_s \rangle = A_s^{\text{SM}} e^{-2i\beta_s} \quad \beta_s = \arg(-(V_{ts}V_{tb}^*)/(V_{cs}V_{cb}^*)) = 0.018 \pm 0.001$$



# Summary

- $\tilde{m}_3 \ll \tilde{m}_{1,2}$  not incompatible with naturalness
- Welcome to alleviate the SUSY flavour problem
- Useful to describe scenarios where the mass separation is moderate but sufficient to make the degeneracy assumption a poor starting point. Which is somewhat expected because of  $U(3)$  breaking by  $Y_+$
- Complementary to the degeneracy assumption in a general study of SUSY flavour effects, with predictions different by factors  $O(1-10)$
- Predictions
  - Less parameters than in the case of degeneracy:  $d \leftrightarrow s$  via  $d \leftrightarrow b \leftrightarrow s$   
 $s_L \leftrightarrow b_R$  via  $s_L \leftrightarrow b_L \leftrightarrow b_R$  (under assumptions)
  - Specific, distinct correlation  $\Delta F = 1$  vs  $\Delta F = 2$ , with larger effects in  $\Delta F = 2$  allowed

# Inputs

Parameter	Value	Gaussian ( $\sigma$ )	Uniform ( $\frac{\Delta}{2}$ )	Reference
$ \varepsilon_K $	$2.229 \times 10^{-3}$	$0.012 \times 10^{-3}$	—	[19]
$\Delta m_K$ (ps $^{-1}$ )	$5.292 \times 10^{-3}$	$0.009 \times 10^{-3}$	—	[19]
BR( $B \rightarrow X_s \gamma$ )	$3.55 \times 10^{-4}$	$0.26 \times 10^{-4}$	—	[20]
$\Delta m_{B_s}$ (ps $^{-1}$ )	17.77	0.12	—	[19]
$\Delta m_{B_d}$ (ps $^{-1}$ )	0.507	0.005	—	[19]
$ M_{12}^D $ (ps $^{-1}$ )	$7.7 \times 10^{-3}$	$2.5 \times 10^{-3}$	—	[21]
$\bar{\rho}$	0.167	0.051	—	[22]
$\bar{\eta}$	0.386	0.035	—	[22]
$\lambda$	0.2255	0.010	—	[19]
$ V_{cb} $	$41.2 \times 10^{-3}$	$1.1 \times 10^{-3}$	—	[19]
$F_K$ (GeV)	0.160	—	—	[19]
$F_{B_d}$ (MeV)	189	27	—	[23]
$F_{B_s} \sqrt{B_s}$ (MeV)	262	35	—	[23]
$F_D$ (MeV)	201	3	17	[24]
$\hat{B}_K$	0.79	0.04	0.08	[24]
$B_1^B$	0.88	0.04	0.10	[24]
$\eta_{cc}$	0.47	0.04	—	[25]
$\eta_{ct}$	0.5765	0.0065	—	[25]
$\eta_{tt}$	1.43	0.23	—	[25]
$\bar{m}_t$ (GeV)	161.2	1.7	—	[24]
$\bar{m}_b$ (GeV)	4.21	0.08	—	[24]
$\bar{m}_c$ (GeV)	1.224	0.057	—	[26]