LFV and Dipole Moments in Models with A4 Flavour Symmetry

Luca Merlo
Università di Padova
& INFN Padova

DISCRETE '08 - Symposium on Prospects in the Physics of Discrete Symmetries
11-16 December 2008, IFIC, Valencia

Lepton Mixing & TBM

After *Neutrino 2008* (at 2σ)

	$\sin^2\vartheta_{12}$	ϑ_{12}	$\sin^2\vartheta_{23}$	ϑ_{23}	$\sin^2\vartheta_{13}$
Fogli et al. [0809.2936]	$0.312^{+0.040}_{-0.034}$	$\left(34.0^{+2.4}_{-2.1}\right)^\circ$	$0.47^{+0.14}_{-0.10}$	$\left(43.0^{+7.8}_{-5.8}\right)^\circ$	0.016 ± 0.010

TB mixing:

[Harrison, Perkins & Scott; Zhi-Zhong Xing 2002]

$\vartheta_{12}^{TB}=35.3^\circ$ $\vartheta_{23}^{TB}=45^\circ$ $\vartheta_{13}^{TB}=0^\circ$

$$U_{TB}=\begin{pmatrix}\frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} & 0\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} & \frac{1}{\sqrt{2}}\end{pmatrix}$$

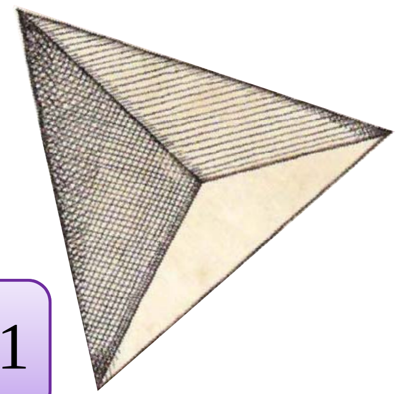
- ϑ_{23} maximal not with an exact symmetry
- ϑ_{13} zero or not?
- ϑ_{12} within $2^\circ\approx0.035$ rad ($<\vartheta_c^2$)

- broken symmetry
- TBM @ LO
- $O(\vartheta_c^2)$ corrections required

[Ma & Rajasekaran 2001;
Ma 2002; Babu, Ma & Valle 2003;
Altarelli & Feruglio 2005;
Altarelli, Feruglio & Lin 2006]

SSB of A_4

A_4 is the group of even permutations of 4 objects isomorphic to the group of the rotations which leave a regular tetrahedron invariant.



A_4 has 4 irreps: 1, 1′, 1″ and 3
A_4 is generated by two elements: T and S

$$S^2 = T^3 = (ST)^3 = 1$$

S generates a subgroup Z_2 of A_4, G_S
T generates a subgroup Z_3 of A_4, G_T

$A_4 \times Z_3 \times U(1)_{FN}$

→ keeps separated the two sectors

→ explains the hierarchy $m_e << m_\mu << m_\tau$

$\langle\varphi\rangle$

$G_T \equiv Z_3$ → $(m_e^+ m_e)$ diagonal

$G_S \equiv Z_2$ → $U_{TB}^T m_\nu U_{TB} = m_\nu^{diag}$

[He, Keum, Volkas 2006;
Lam 2007, 2008]

The Model: SUSY approach

	ℓ	e^c	μ^c	τ^c	$h_{u,d}$	φ_T	φ_S	ξ	θ
A_4	3	1	$1''$	$1'$	1	3	3	1	-1
Z_3	ω	ω^2	ω^2	ω^2	1	1	ω	ω	1
$U(1)_{FN}$	0	$+2$	$+1$	0	0	0	0	0	-1

$$w_\ell = \frac{y_e}{\Lambda}\frac{\theta^2}{\Lambda^2} e^c(\varphi_T \ell) h_d + \frac{y_\mu}{\Lambda}\frac{\theta}{\Lambda}\mu^c(\varphi_T\ell)' h_d + \frac{y_\tau}{\Lambda}\tau^c(\varphi_T\ell)'' h_d +$$

<u>charged leptons</u>

$$+\frac{x_a}{\Lambda^2}\xi(\ell h_u \ell h_u) + \frac{x_b}{\Lambda^2}(\varphi_S \ell h_u \ell h_u) + \text{h.c.} + \ldots$$ (NLO terms)

<u>neutrinos</u>

<u>vacuum alignment</u>

$$\left\langle \frac{\varphi_T}{\Lambda} \right\rangle = (u, 0, 0)$$

$$\left\langle \frac{\varphi_S}{\Lambda} \right\rangle = c_b (u, u, u)$$

$$\left\langle \frac{\xi}{\Lambda} \right\rangle = c_a u$$

$$m_\ell = \begin{pmatrix} y_e t^2 & 0 & 0 \\ 0 & y_\mu t & 0 \\ 0 & 0 & y_\tau \end{pmatrix} v_d u \rightarrow \frac{m_e}{m_\mu} = \frac{m_\mu}{m_\tau} = t \approx 0.05$$

$$m_\nu = \begin{pmatrix} a + \frac{2}{3}b & -\frac{b}{3} & -\frac{b}{3} \\ -\frac{b}{3} & \frac{2}{3}b & a - \frac{b}{3} \\ -\frac{b}{3} & a - \frac{b}{3} & \frac{2}{3}b \end{pmatrix} v_u^2$$

$$y_\tau < 4\pi$$

corrections to TBM $\leq \vartheta_c^2 \approx 0.05$

$$0.001 < u < 0.05$$

Predictions and Comments

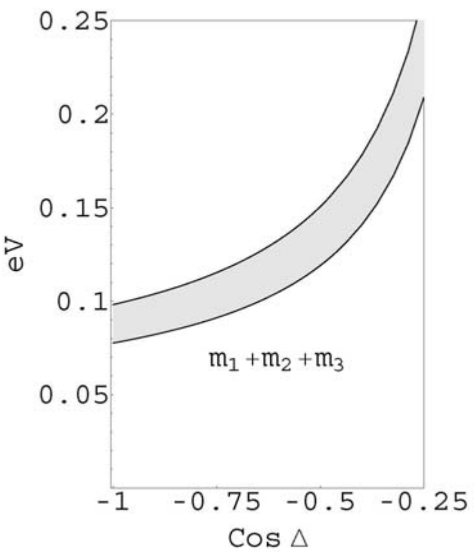
- Angles → $\tan^2\vartheta_{23}=1, \quad \tan^2\vartheta_{12}=0.5, \quad \vartheta_{13}=0$

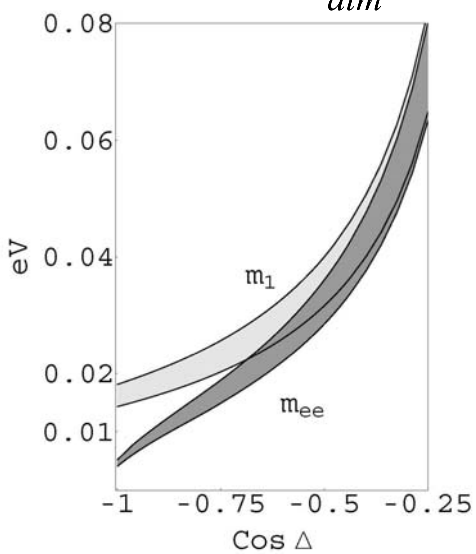
  independently from |a|, |b| and Δ≡arg(a)-arg(b)

  with NLO corrections: $\vartheta_{23}=\frac{\pi}{4}+O(u)$ $\quad \vartheta_{13}=O(u)$

- Spectrum → $m_\nu^{diag}=v_u^2\,\mathrm{diag}(a+b,\,a,\,-a+b)$

  by the request $r\equiv\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\approx\frac{1}{35}$ → rises a moderate fine tuning





NH $\quad m_1\geq 0.017\,\mathrm{eV} \quad \sum m_i \geq 0.09\,\mathrm{eV}$

$$|m_3|^2=|m_{ee}|^2+\frac{10}{9}\Delta m^2_{atm}\left(1-\frac{\Delta m^2_{sol}}{\Delta m^2_{atm}}\right)$$

in a see-saw realization both normal and inverted hierarchies can be accommodated

- Possible extension to the quark sector

[Feruglio, Hagedorn, Lin & M. 2007; Altarelli, Feruglio & Hagedorn 2008; Bazzocchi, Frigerio & Morisi 2008]

A new mass scale

Low Energy Observables:
- ν masses
- ν oscillations

- $(g-2)_\mu$ discrepancy
- dark matter
- *gauge coupling unification*
- hierarchy problem

- GUTs
- flavour symmetries
- ν^c
- superheavy gauge bosons

0 — e.w. scale — M≈(1÷10) TeV — $\langle\varphi\rangle$ — M_{GUT} — Λ — Energy

SM context: without specifying the kind of new physics

After integrating out all the d.o.f. related to Λ and M

$$\mathscr{L}_{eff} = \mathscr{L}_{SM} + \delta\mathscr{L}(m_\nu) + \mathscr{L}_{dip} + \dots$$

Dipole Matrix $\mathcal{M} \equiv \mathcal{M}(\langle\varphi\rangle)$

$$\mathscr{L}_{dip} = i\frac{e}{M^2} e^c H^+ (\sigma\cdot F)\mathcal{M}\ell + [4-\text{fermion}] + \text{h.c.} + \dots$$

$\mu\to eee \quad \tau\to\mu\mu\mu$

$\tau\to eee$

in the basis with charged leptons diagonal:

$$d_i = \frac{e\,v}{\sqrt{2}M^2}\,\mathrm{Im}\,\mathcal{M}_{ii}$$

$$a_i = \frac{2m_i v}{\sqrt{2}M^2}\,\mathrm{Re}\,\mathcal{M}_{ii}$$

$$\underset{(i\neq j)}{R_{ij}} = \frac{BR(\ell_i\to\ell_j\gamma)}{BR(\ell_i\to\ell_j\nu_i\bar{\nu}_j)} = \frac{12\sqrt{2}\pi^3\alpha_{em}}{G_F^3 m_i^2 M^4}\left[|\mathcal{M}_{ij}|^2 + |\mathcal{M}_{ji}|^2\right]$$

The dipole matrix $\mathcal{M}$

the flavour pattern in $\mathscr{L}_{dip}$ is controlled by the same SB parameters of $\mathscr{L}_{SM}$

$$\mathscr{L}_{SM}=\frac{y_e}{\Lambda}\frac{\theta^2}{\Lambda^2}e^c H^+(\varphi_T\ell)+\frac{y_\mu}{\Lambda}\frac{\theta}{\Lambda}\mu^c H^+(\varphi_T\ell)'+\frac{y_\tau}{\Lambda}\tau^c H^+(\varphi_T\ell)''$$

$$\mathscr{L}_{dip}=\frac{ie}{M^2}\left[\frac{\beta_e}{\Lambda}\frac{\theta^2}{\Lambda^2}e^c H^+\sigma\cdot F(\varphi_T\ell)+\frac{\beta_\mu}{\Lambda}\frac{\theta}{\Lambda}\mu^c H^+\sigma\cdot F(\varphi_T\ell)'+\frac{\beta_\tau}{\Lambda}\tau^c H^+\sigma\cdot F(\varphi_T\ell)''\right]$$

$$m_\ell=\begin{pmatrix} y'_e\,t^2u & y_e^{(2)}\,t^2u^2 & y_e^{(3)}\,t^2u^2 \\ y_\mu^{(3)}\,t\,u^2 & y'_\mu\,t\,u & y_\mu^{(2)}\,t\,u^2 \\ y_\tau^{(2)}\,u^2 & y_\tau^{(3)}\,u^2 & y'_\tau\,u \end{pmatrix}v_d \longleftrightarrow \mathcal{M}(\langle\varphi\rangle)=\begin{pmatrix} \beta'_e\,t^2u & \beta_e^{(2)}\,t^2u^2 & \beta_e^{(3)}\,t^2u^2 \\ \beta_\mu^{(3)}\,t\,u^2 & \beta'_\mu\,t\,u & \beta_\mu^{(2)}\,t\,u^2 \\ \beta_\tau^{(2)}\,u^2 & \beta_\tau^{(3)}\,u^2 & \beta'_\tau\,u \end{pmatrix}$$

in the basis in which m_ℓ is diagonal

$$\longrightarrow\quad \mathcal{M}(\langle\varphi\rangle)=\begin{pmatrix} O(t^2u) & O(t^2u^2) & O(t^2u^2) \\ O(t\,u^2) & O(t\,u) & O(t\,u^2) \\ O(u^2) & O(u^2) & O(u) \end{pmatrix}$$

the rotations preserve the structure of the matrix: only the coefficients change

Constraints on M

1) EDMs and MDMs

$d_e < 1.6 \times 10^{-27}\ e\,cm$	$M > 80\ \text{TeV}$
$d_\mu < 2.8 \times 10^{-19}\ e\,cm$	$M > 80\ \text{GeV}$
$\delta a_e < 3.8 \times 10^{-12}$	$M > 350\ \text{GeV}$
$\delta a_\mu \approx 30 \times 10^{-10}$	$M \approx 2.7\ \text{TeV}$

- Strongest constraint from d_e (cancellations in Im[$\mathcal{M}$]: accidental or due to CP-conservation)
- Interesting indication from δa_μ

2) LFV transitions:

- up to O(1) coefficient $R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$ independently from θ_{13} → τ decays are below the expected future sensitivity

Imposing $R_{\mu e} < 1.2 \times 10^{-11} (10^{-13})$

→ $u/M^2 < 1.2 \times 10^{-11} (1.1 \times 10^{-12})$ GeV^{-2}

$|u| \approx 0.001$ → $M > 10\ (30)$ TeV

$|u| \approx 0.05$ → $M > 70\ (200)$ TeV

Probably above the region of interest for $(g-2)_\mu$ and for LHC

SUSY Case

Some operators contributing to $\mathcal{M}_{ij}$ are suppressed and as a result in the basis of diagonal charged leptons, the rotations provide a <u>cancellation</u> in the element below the diagonal

$$\mathcal{M}(\langle\varphi\rangle)=\begin{pmatrix} O(t^2u) & O(t^2u^2) & O(t^2u^2) \\ O(t\,u^2) & O(t\,u) & O(t\,u^2) \\ O(u^2) & O(u^2) & O(u) \end{pmatrix} \longrightarrow \mathcal{M}(\langle\varphi\rangle)=\begin{pmatrix} O(t^2u) & O(t^2u^2) & O(t^2u^2) \\ O(t\,u^3) & O(t\,u) & O(t\,u^2) \\ O(u^3) & O(u^3) & O(u) \end{pmatrix}$$

- The constraints from EDMs and MDMs are the same
- In most of the allowed range for u, $R_{\mu e} \approx R_{\tau\mu} > R_{\tau e}$ → τ decays undetectable

Imposing $R_{\mu e} < 1.2 \times 10^{-11}(10^{-13})$ — $|u| \approx 0.001$ → $M > 0.7\ (2)$ TeV

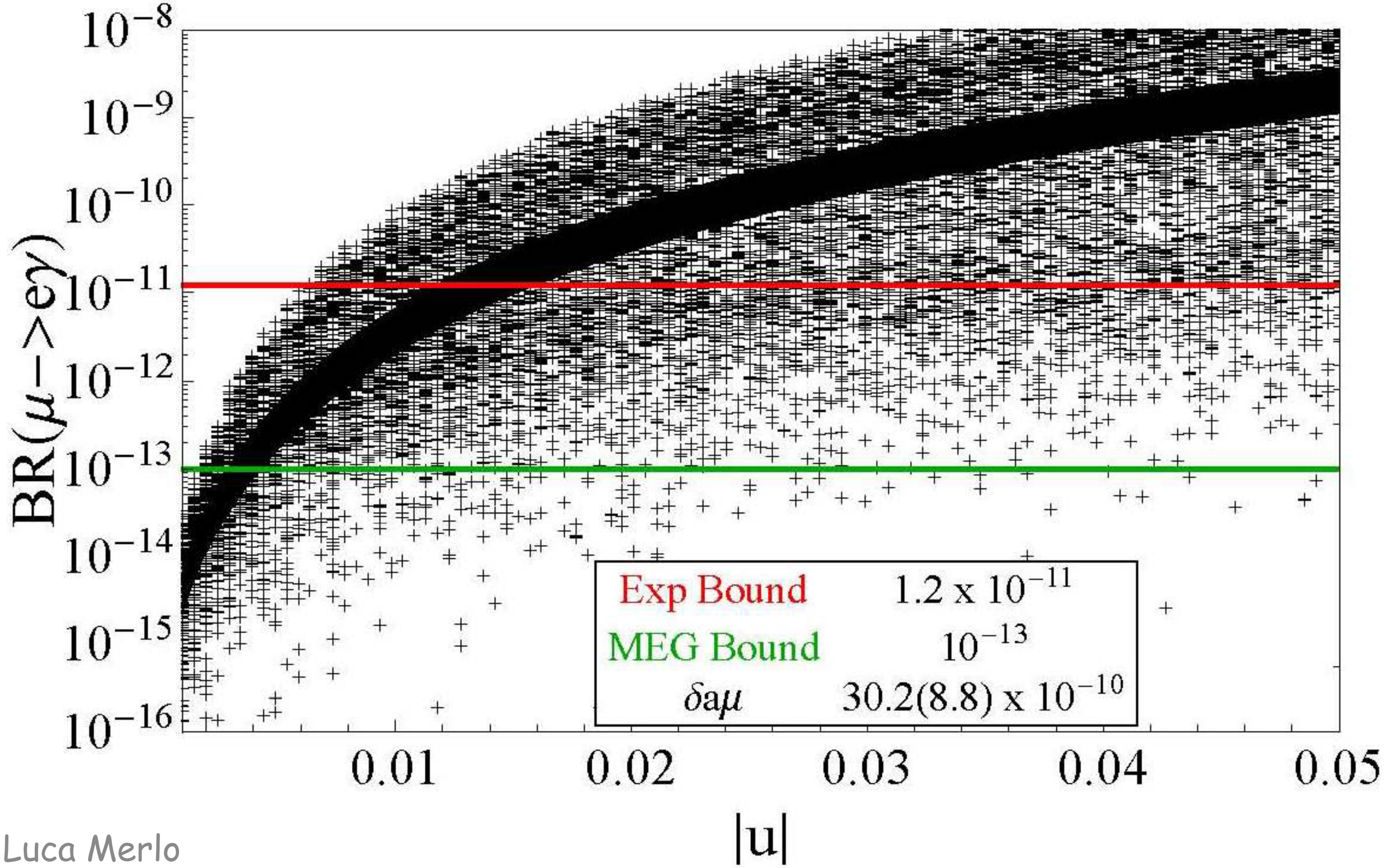
→ $u^2/M^2 < 1.2 \times 10^{-11}(1.1 \times 10^{-12})$ GeV^{-2} — $|u| \approx 0.05$ → $M > 14\ (48)$ TeV

There is a range for |u| in which it is possible to explain at the same time the deviation on $(g-2)_\mu$ and a positive signal for BR($\mu \rightarrow e\gamma$) at MEG

Relation between the observables

$$\delta a_\mu = \frac{2 m_\mu v}{\sqrt{2} M^2} \mathrm{Re}\left[t\, u\right], \qquad R_{\mu e} = \frac{48 \pi^3 \alpha_{em}}{G_F^2 M^4}\left[\left|\gamma^{(1)} u^2\right|^2 + \frac{m_e^2}{m_\mu^2}\left|\gamma^{(2)} u\right|^2\right]$$

$$R_{\mu e} = \frac{12 \pi^3 \alpha_{em}}{G_F^2 m_\mu^4}\left(\delta a_\mu\right)^2 \left[\left|\gamma^{(1)} u^2\right|^2 + \frac{m_e^2}{m_\mu^2}\left|\gamma^{(2)} u\right|^2\right]$$



Considering

$|u| \approx \vartheta_{13}$

not a sharp limit on ϑ_{13}, but only an indication : NOT larger than a few percent.

Conclusions

Additional tests of A4 models from LFV

✓ <u>generic, non-SUSY, case</u>

- ❑ $R_{\mu e} \approx R_{\tau\mu} \approx R_{\tau e}$ independently from ϑ_{13} (cfr MFV)

  ⟶ $\tau \to \mu\gamma$ $\tau \to e\gamma$ below expected future sensitivity

- ❑ $$R_{ij} = \frac{BR(\ell_i \to \ell_j \gamma)}{BR(\ell_i \to \ell_j \nu_i \bar{\nu}_i)} \propto \left(\frac{u}{M^2}\right)^2$$ ⟶ M above 10 TeV

  Not for $(g-2)_\mu$

✓ <u>in the SUSY case</u>

- ❑ $R_{\mu e} \approx R_{\tau\mu} > R_{\tau e}$ independently from ϑ_{13}

- ❑ $$R_{ij} = \frac{BR(\ell_i \to \ell_j \gamma)}{BR(\ell_i \to \ell_j \nu_i \bar{\nu}_i)} \propto \left(\frac{u^2}{M^2}\right)^2$$ ⟶ M can be smaller, in the range of interest for $(g-2)_\mu$

- ❑ The useful cancellation has to be checked: an MSSM+A_4 analysis provides a stricter control on every contributions and it confirms all the previous results. [Feruglio, Hagedorn, Lin & M. to appear]

Thanks

Based on
G.Altarelli & F.Feruglio hep-ph/0504165
G.Altarelli & F.Feruglio hep-ph/0512103
F.Feruglio, C.Hagedorn, Y.Lin & L.M. hep-ph/0702194
F.Feruglio, C.Hagedorn, Y.Lin & L.M. 08073160
F.Feruglio, C.Hagedorn, Y.Lin & L.M. to appear

Luca Merlo
Università di Padova &
INFN Padova