The Omega Effect as a Discriminant of Space-Time Foam

Sarben Sarkar

King's College London, Dept. of Physics

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Outline

- Theories with Broken CPT? Various philosophies
- Systems to search for CPT violation
- Orders of magnitude
- Suitable formalism? String based/ thermal bath based/ Lindblad based
- No one measure of breaking: subtle phenomenology of decoherence effects
  - Entanglement
  - Modified entanglement ω effect
  - Entanglement generated by evolution

Issues in CPT symmetry

- Meaning of CPT symmetry
  - Theoretical foundations
- How can CPT be violated?
  - Theoretical models, ideas and order of magnitude of effects
    - Models of quantum gravity violating quantum coherence
- CPT violation tests involving coherence:
  - Entangled states of neutral K and B mesons
  - Neutrino oscillations

CPT theorem

- Θ =C(harge)-P(arity)-T(ime) symmetry
  - is a symmetry of a local, unitary. Lorentz invariant quantum field theory in flat space-time with lagrangian L
  - Proof based on covariance properties of Wightman functions under Lorentz transformations and the unitarity of the latter
- For quantum gravity QG
  - no Lorentz invariance
  - no unitarity due to inacessibility of states within horizons
    - lacking QG, arguments based on semi-classical intuition
  - Breakdown of Θ invariance

$$\Theta\, \mathrm{L}(\mathrm{x})\,\Theta^{\dagger} = \mathrm{L}(\text{-x})$$

$$L = L^{\dagger}$$

$$\langle 0|\Phi(x_1)\Phi(x_2)\ldots\Phi(x_n)|0\rangle$$

off shell correlators in quantum field theory

(cf O Greenberg, hep-ph)

Highly curved space-time backgrounds, such as the black hole horizon type, leading to space-time foams arising in models of quantum gravity

Decoherence vs CPT Violation in Quantum Mechanics

- Distinguish 2 types of CPT violation CPTV
  - (I) CPTV within QM

$$\delta M = m_{K^0} - m_{\overline{K}^0}, \delta\Gamma = \cdots$$

  - This could be due to spontaneous violation of Lorentz symmetry, extensions of the standard model
  - (ii) CPTV through decoherence (entanglement with QG environment) e.g. through recoil parameters in a D particle model of the environment or other parameters in the Lindblad formalism
- Experimentally they can be disentangled e.g. through the study of the ratios

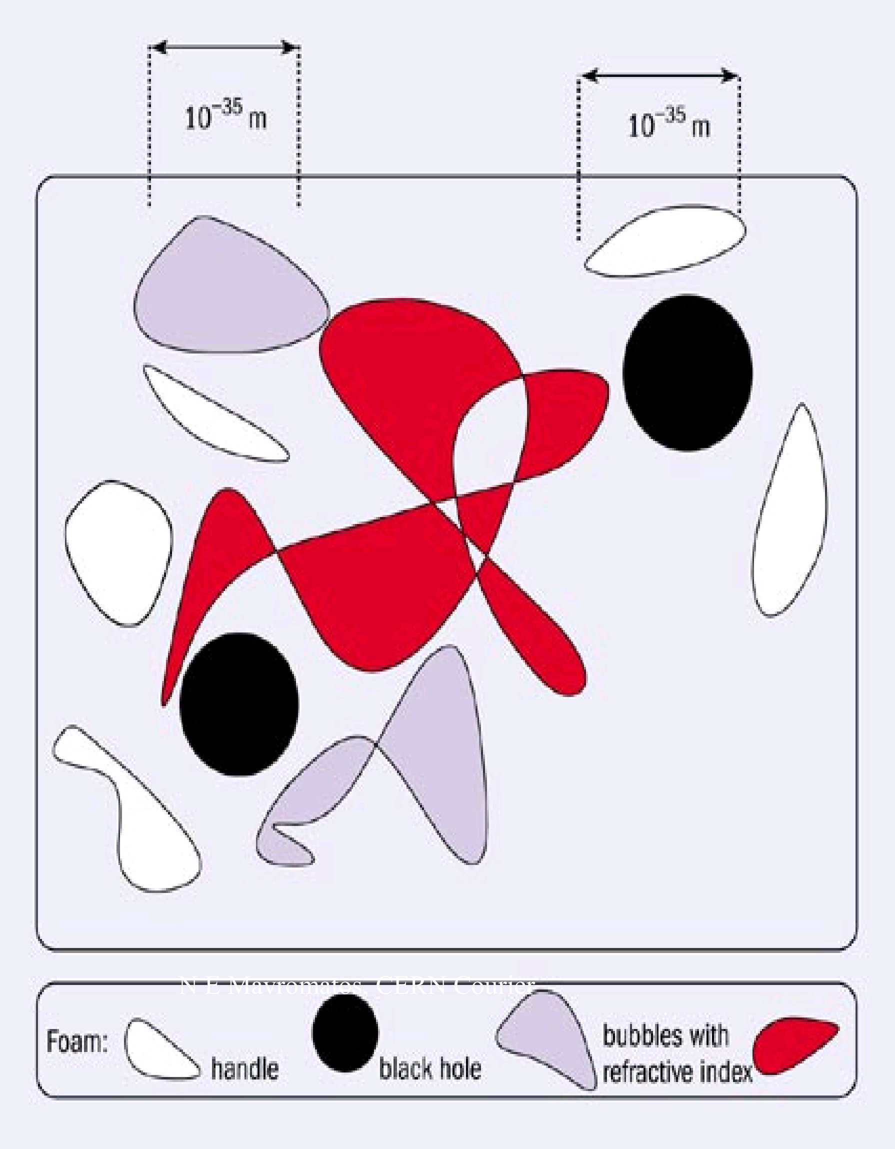
$$A(t) = \frac{R\left(\overline{K}^0_{t=0} \to \overline{f}\right) - R\left(K^0_{t=0} \to f\right)}{R\left(\overline{K}^0_{t=0} \to \overline{f}\right) + R\left(K^0_{t=0} \to f\right)}$$

where ***R*** represents decay rate into final state ***f***

- **Here we shall discuss another quantity involving entanglement**

Discrete space-time

- At (Planck scale) 10^{-35}m discrete Lorentz violation?
- Microscopic black holes: inaccessible degrees of freedom (certainly at low energy)
  - Other types of space-time defects in string theories in terms of D-branes
  - Collectively space-time foam



N E Mavromatos, CERN Courier

Violation of unitarity (e.g .R Wald, 1979, D N Page 1980)

- Hilbert spaces H_i
  - H_1 space of initial states
  - H_2 space of states of hidden hypersurfaces of micro black holes
  - H_3 space of final states
  - Pure state

  Mixed state

  \$ is not invertible

  i.e. lack of unitarity
- Differing views with unitarity:
  - Holography for strings in anti-de Sitter space-time (Maldacena, Witten); Euclidean approach and superposition of space-times (Hawking)

$$Let\,|\mathrm{X}\rangle \in \mathrm{H}_1, |\mathrm{Y}\rangle \in \mathrm{H}_2, |\mathrm{Z}\rangle \in \mathrm{H}_3$$

$$Let\,|\bar{\mathrm{X}}\rangle = \Theta|\mathrm{X}\rangle, |\bar{\mathrm{Y}}\rangle = \Theta|\mathrm{Y}\rangle, |\bar{\mathrm{Z}}\rangle = \Theta|\mathrm{Z}\rangle$$

Evolution of initial state: $x_{\mathrm{A}}|\mathrm{X}\rangle_A \to S_{Abc}x_A|\bar{\mathrm{Y}}\rangle_b|\bar{\mathrm{Z}}\rangle_c$

$$|X\rangle_A{}^A\langle X| \to \sum_{c,c'} \$_{Ac}{}^{Ac'}|\bar{Z}\rangle_c{}^{c'}\langle\bar{Z}| = mixed\ state$$

$$where\ \$_{Ac}{}^{Ac'} = \sum_{b,b'} S_{Abc}S^{*Ab'c'}$$

$$\Rightarrow \$ \neq UU^{\dagger}\ with\ U = e^{iHt}$$

However these arguments depend heavily on supersymmetry

Continuation back from Euclidean may be problematic

CPT and non-unitarity

- For $\$ \neq UU^{\dagger}$ Θ not conserved
  - since if Θ is conserved $\$^{-1}$ exists
  - Proof:

$$\rho'_{out} = \$\rho'_{in},\ \Theta\rho_{in} = \rho'_{out},\ \Theta^{-1}\rho_{out} = \rho'_{in}$$

$$\Rightarrow \Theta\rho_{in} = \$\rho'_{in} = \$\Theta^{-1}\rho_{out} = \$\Theta^{-1}\$\rho_{in}$$

  - Hence $1 = \Theta^{-1}\$\Theta^{-1}\$$

$$\begin{tikzcd}
\rho'_{in} \arrow[r, "\$"] & \rho'_{out} \arrow[d, "\Theta^{-1}"] \\
\rho_{out} \arrow[u, "\Theta^{-1}"] & \rho_{in} \arrow[l, "\$"]
\end{tikzcd}$$

and so \$ is invertible

- Decoherence from space-time foam can lead to a lack of an inverse for \$ and *so* non-conservation of Θ

Order of magnitude of CPTV

- Although QG not solved, estimates can be given for orders of magnitude at comparatively low energies
  - Since $G_N \sim \frac{1}{M_P{}^2}$ and $M_P = 10^{19}\,GeV$ effective lagrangian approach in terms of expansion in powers of $\frac{E}{M_P}$ with

    E being typical low energy scale of probe
    - Gives leading order quantum correction $O\left(\left[\frac{E}{M_P}\right]^2\right)$
      - Energy change $\sim \frac{E^3}{M_P{}^2}$
        - Neutrinos at Ice Cube may be sensitive
        - Neutral mesons insensitive

Other non-perturbative approaches: loop gravity, stringy

QG can lead to larger detectable $O\left(\frac{E^2}{M_P}\right)$ effects

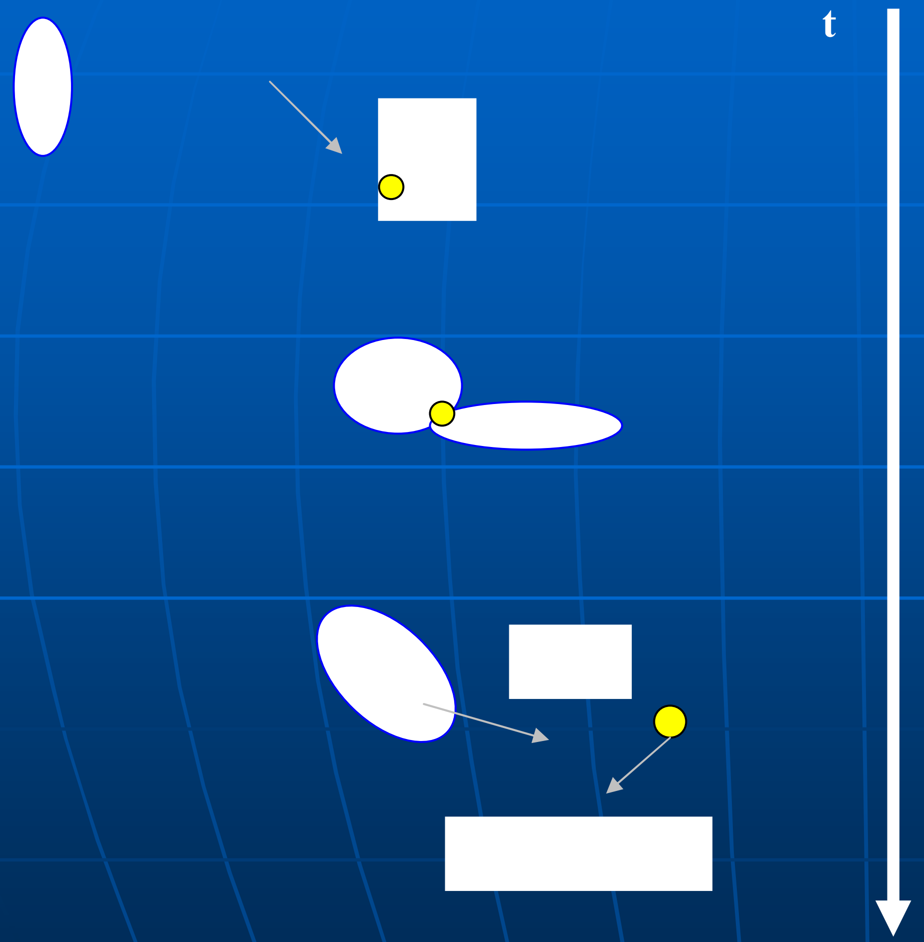
New EPR entanglement from broken CPT (Bernabeu, Mavromatos and Sarkar, PRD)

- Θ operator does not exist
  - K^0 and $\bar{K}^0$ can be distinguished
- EPR pair correlations produced in decay $\Phi \to K^0 \bar{K}^0$
  - Φ has $J^{PC} = 1^{--}$, particle-antiparticle symmetry,

$$|i\rangle = \frac{1}{\sqrt{2}}\left(|K^0(-\vec{q})\rangle|\bar{K}^0(\vec{q})\rangle - |\bar{K}^0(-\vec{q})\rangle|K^0(\vec{q})\rangle\right)$$

  - conservation of parity and strangeness in strong interaction
- Relaxing $CP = +$ gives ω effect: decay product has additional piece $\frac{\omega}{\sqrt{2}}\left(|K^0(-\vec{q})\rangle|\bar{K}^0(\vec{q})\rangle + |\bar{K}^0(-\vec{q})\rangle|K^0(\vec{q})\rangle\right)$
  - QG origin for such an initial state? String picture?
  - Is such entanglement generated during evolution in space-time foam?

D-particle foam and thermal bath



- Closed string scatters off a D-particle
- D-particle recoils on scattering
  - Weak non-conformality described by logarithmic conformal field theory
- In brane worlds D-particles in the bulk cross the brane and interact with matter
- Foam has also been modelled by a thermal bath (Garay)
- Lindblad phenomenology

Stringy Master Equation

- **Master equation for stringy low-energy matter**

$$\frac{\partial}{\partial t}\rho = i\left[\rho, H\right] + \alpha'\Omega[g_{MN},\left[g^{MN},\rho\right]]$$

in de Sitter space with α' Regge slope, Ω cosmological constant, *H* the matter Hamiltonian

- For a single scattering event the distortion caused is

$$g_{0i} \propto g_s \frac{\Delta p_i}{M_s}$$

where g_s is the string coupling, M_s is the string mass scale and Δp_i is the momentum transfer in a collision

- The recoil aspects can be incorporated in a phenomenological manner by making g_{0i} flavour changing

Phenomenological 2 Flavour Stringy Decoherence

- **Model momentum transfer operator due to recoil by**

$$\frac{\Delta p}{M_p} \sim \frac{r}{M_p}\hat{p}$$

where r is a gaussian random variable $\langle r \rangle = 0$ and $\langle r^2 \rangle = \Delta$

- **Target space metric state** $\rho_{grav} = \int d^5 r\, f(r_\mu)\,|g(r_\mu)\rangle\langle g(r_\mu)|$

with $\langle r_\mu \rangle = 0, \quad \langle r_\mu r_\nu \rangle = \Delta_\mu \delta_{\mu\nu}, \quad \Delta_\mu = O\left(\frac{E^2}{M_P^2}\right)$

- **Semi-classical picture with $|g(r_\mu)\rangle$ a coherent state**
- **Neutral meson two flavour structure incorporated in metric tensor with components**

$$g^{00} = (-1+r_4)1, \quad g^{01} = g^{10} = r_0 1 + r_1\sigma_1 + r_2\sigma_2 + r_3\sigma_3, \quad g^{11} = (1+r_5)1$$

- **No hair theorem permits the non-conservation of flavour**
- **For neutral mesons flavour denotes particle/antiparticle or the different mass eigenstates**

Klein-Gordon equation with recoil fluctuations

- **In mass eigenstate basis the Klein-Gordon equation is**

$$\left(g^{\alpha\beta}D_\alpha D_\beta - m^2\right)\Phi = 0$$

with $\Phi = \begin{pmatrix}\varphi_1\\ \varphi_2\end{pmatrix}$

- **Associated Hamiltonian** $\widehat{H}$ **is**

$$\widehat{H} = g^{01}\left(g^{00}\right)^{-1}\widehat{k} - \left(g^{00}\right)^{-1}\sqrt{\left(g^{01}\right)^2\widehat{k}^2 - g^{00}\left(g^{11}\widehat{k}^2 + \widehat{m}^2\right)}$$

acting on the space of states $|p,\uparrow\rangle$ or $|p,\downarrow\rangle$ **with** $\widehat{k}\left|p,\{\uparrow,\downarrow\}\right\rangle = p\left|p,\{\uparrow,\downarrow\}\right\rangle$ **and**

$$\widehat{m}^2 = \frac{1}{2}\left(m_1^2 + m_2^2\right)1 + \frac{1}{2}\left(m_1^2 - m_2^2\right)\sigma_3$$

- **In terms of mass eigenstates** $|K_S\rangle(=|\downarrow\rangle), |K_L\rangle(=|\uparrow\rangle)$, **the** ω **effect state is**

$$|\Psi\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix}\left|k,\uparrow\right\rangle^{(1)}\left|-k,\downarrow\right\rangle^{(2)} - \left|k,\downarrow\right\rangle^{(1)}\left|-k,\uparrow\right\rangle^{(2)}\\ +\omega\left(\left|k,\uparrow\right\rangle^{(1)}\left|-k,\uparrow\right\rangle^{(2)} - \left|k,\downarrow\right\rangle^{(1)}\left|-k,\downarrow\right\rangle^{(2)}\right)\end{pmatrix}$$

- **Strictly space-time foam is entangled with the 2-meson state and so would lead to a density matrix description**

Gravitational dressing of states

- To lowest order in Δ, $\widehat{H}_I = -\left(r_1\sigma_1 + r_2\sigma_2\right)\widehat{k}$
- The gravitational dressing $\left|k^{(i)},\downarrow\right\rangle_{QG}^{(i)}$ of $\left|k,\downarrow\right\rangle^{(i)}$ is

$$\left|k^{(i)},\downarrow\right\rangle_{QG}^{(i)} = \left|k^{(i)},\downarrow\right\rangle^{(i)} + \alpha^{(i)}\left|k^{(i)},\uparrow\right\rangle^{(i)}$$

where $\alpha^{(i)} = \dfrac{{}^{(i)}\left\langle\uparrow,k^{(i)}\right|\widehat{H}_I\left|k^{(i)},\downarrow\right\rangle^{(i)}}{E_2 - E_1}$ and $E_i = \left(m_i^{\,2} + k^2\right)^{\frac{1}{2}}$

- Similarly $\left|k^{(i)},\downarrow\right\rangle_{QG}^{(i)}$ becomes $\left|k^{(i)},\uparrow\right\rangle_{QG}^{(i)} = \left|k^{(i)},\uparrow\right\rangle^{(i)} + \beta^{(i)}\left|k^{(i)},\downarrow\right\rangle^{(i)}$ with

$$\beta^{(i)} = \frac{{}^{(i)}\left\langle\downarrow,k^{(i)}\right|\widehat{H}_I\left|k^{(i)},\uparrow\right\rangle^{(i)}}{E_1 - E_2}$$

- Dressed antisymmetric state

$$\left|\Psi\right\rangle_{QG} = \left|k,\uparrow\right\rangle_{QG}^{(1)}\left|-k,\downarrow\right\rangle_{QG}^{(2)} - \left|k,\downarrow\right\rangle_{QG}^{(1)}\left|-k,\uparrow\right\rangle_{QG}^{(2)}$$

Generation of ω effect

$$\left|\Psi\right\rangle_{QG} = \left|k,\uparrow\right\rangle^{(1)}\left|-k,\downarrow\right\rangle^{(2)} - \left|k,\downarrow\right\rangle^{(1)}\left|-k,\uparrow\right\rangle^{(2)} + \left(\beta^{(1)} - \beta^{(2)}\right)\left|k,\downarrow\right\rangle^{(1)}\left|-k,\downarrow\right\rangle^{(2)} +$$

$$\left(\alpha^{(2)} - \alpha^{(1)}\right)\left|k,\uparrow\right\rangle^{(1)}\left|-k,\uparrow\right\rangle^{(2)} + \beta^{(1)}\alpha^{(2)}\left|k,\downarrow\right\rangle^{(1)}\left|-k,\uparrow\right\rangle^{(2)} - \alpha^{(1)}\beta^{(2)}\left|k,\uparrow\right\rangle^{(1)}\left|-k,\downarrow\right\rangle^{(2)}$$

- **For** (a) $r_i \propto \delta_{i1} \Rightarrow \alpha^{(i)} = -\beta^{(i)}$ **no** ω **effect**
- **So** (b) $\boxed{r_i \propto \delta_{i2} \Rightarrow \alpha^{(i)} = \beta^{(i)},}$ ω **effect**
- **(a) corresponds to non-strangeness conservation in** ϕ **decay**
- **(b) corresponds to a strangeness conserving** ϕ **decay**

- Averaging density matrix **over** r_i **→ terms of** $O\left(|\omega|^2\right)$

$$|\omega|^2 = O\left(\frac{1}{\left(E_1-E_2\right)^2}\left\langle\downarrow,k^{(i)}\right|\hat{H}_I\left|k^{(i)},\uparrow\right\rangle^2\right) = O\left(\frac{\Delta_2 k^2}{\left(E_1-E_2\right)^2}\right) \sim \frac{\Delta_2 k^2}{\left(m_1-m_2\right)^2}$$

$$\Delta_2 \sim \frac{\varsigma^2 k^2}{M_P{}^2} \Rightarrow |\omega|^2 \sim \frac{\varsigma^2 k^4}{M_P^2\left(m_1-m_2\right)^2}$$

- **D particle recoil picture**
  - **For neutral kaons with momenta of order the rest energies** $|\omega| \sim 10^{-4}\zeta$
  - **For B mesons** $|\omega| \sim 10^{-6}\zeta$
  - **For** $1 > \zeta \geq 10^{-2}$ **not far from current sensitivities**

r_i

ω effect from evolution

- **Evolution with stochastic recoil hamiltonian from CPT invariant state can lead to** $O\left(\omega(t)\right) = \varpi_0 \sin\left(2\left|\lambda^{(1)}\right|t\right)$

With $\varpi_0 = \dfrac{\Delta_1^{1/2} k}{\left(k^2+m_1^2\right)^{1/2} - \left(k^2+m_2^2\right)^{1/2}}$

- **On taking** $\Delta_1^{1/2} = |\varsigma| \dfrac{k}{M_P}$ **similar magnitude to initial state** ω **possible**
- **How robust?**
  - **Lindblad (popular phenomenology in particle physics, Markovian, positive probabilities)**

$$\frac{d\rho}{dt} = i[\rho, H] - \frac{1}{2}\sum_k \left(L^\dagger{}_k L_k \rho + \rho L^\dagger{}_k L_k - 2L_k \rho L^\dagger{}_k\right)$$

- **Based on the idea of dynamical semigroup**
- **Although** $|K_L\rangle|K_L\rangle$ **and** $|K_S\rangle|K_S\rangle$ **are generated the relative weights for** ω **effect cannot (N Mavromatos et al.)**

(Thermal) bath model and ω effect

- Thermal bath has minimum information since only mean energy $\hbar \bar{n} \nu$ is known and

$$\rho_{bath} = \sum_{n=0}^{\infty} \frac{\bar{n}^{n}}{\left(1+\bar{n}\right)^{n+1}} \left|n\right\rangle \left\langle n\right|$$

$$\text{but more generally } \rho = \sum_{n,m=0}^{\infty} \rho_{nm} \left|n\right\rangle \left\langle m\right|$$

Arguments have been given (Garay) why such models may be relevant for modelling space-time foam

- The hamiltonian for the total system (neutral mesons + bath) is given by the Jaynes-Cummings model

$$H = \hbar \nu a^{\dagger} a + \frac{1}{2} \hbar \Omega \sigma_{3}^{(1)} + \frac{1}{2} \hbar \Omega \sigma_{3}^{(2)} + \hbar \gamma \sum_{i=1}^{2} \left(a \sigma_{+}^{(i)} + a^{\dagger} \sigma_{-}^{(i)} \right)$$

- Both dressed states and evolving the total system and tracing over the bath do not lead to an ω effect

Conclusions

- The ω effect is a sensitive test for discriminating against different models of quantum decoherence
- A non-conventional approach motivated by D-particles is needed
- Clearly other defects are allowed within string theory and robustness of the effect needs investigation