Discrete2008, Valencia, Spain, *12.December 2008*

Cosmology of Non-Hermitian (C)PT-invariant scalar matter

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Highlights from:
AAA, F.Cannata and A.Y.Kamenshchik, Phys.Rev.D72:043531,2005;
Int.J.Mod.Phys.D15:1299-1310,2006 J.Phys.A39:9975-9982,2006
ACK and D.Regoli, JCAP02:015,2008; JCAP 0810:019,2008; arXiv:0810.5076 [gr-qc]

Scalar field description of the Universe evolution

↓

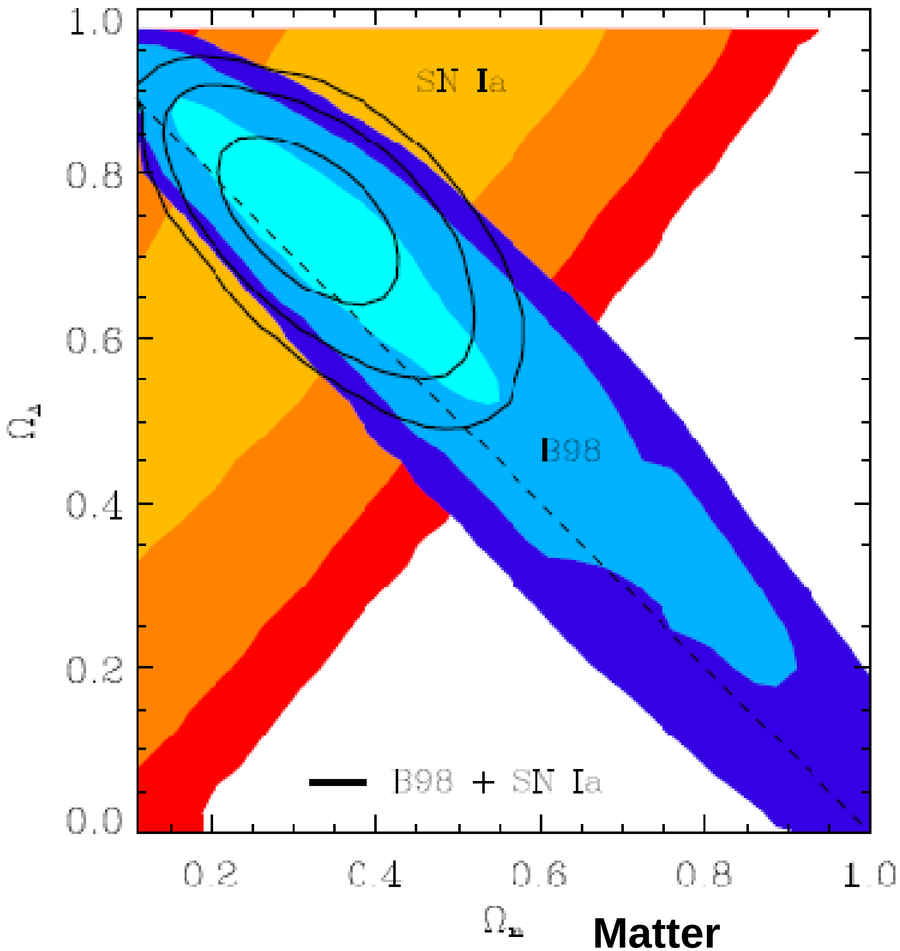
Modern insights to the equation of state:
around cosmological constant but with *hints* on its evolution

↓

Phantom component of the large-scale Universe:
if not a myth then how to understand such a QFT?

Non-Hermitian Lagrangians!?

Dark energy



Matter

Figure 9. Limits on Ω_M and Ω_Λ from the CMB and from Type Ia supernovae. The two data sets together strongly favor a flat universe (CMB) with a cosmological constant (SNIa). [24]

P. de Bernardis *et al.*, Astrophys. J. **564**, 559 (2002), astro-ph/0105296.

Flat isotropic, homogeneous space: $\Omega = \frac{\rho}{\rho_{crit}}$ □ 1 $\qquad \rho_c \equiv \frac{3H_0^2}{8\pi G},$

ρ **is a large-scale (100 Mpc) energy density of the Universe**

FRW cosmology for flat space

$$ds^2 = dt^2 - a^2(t)d\mathbf{x}^2 \qquad g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -a(t) & 0 & 0 \\ 0 & 0 & -a(t) & 0 \\ 0 & 0 & 0 & -a(t) \end{pmatrix}$$

What is driving the Universe evolution?
Cosmological constant vs. matter+energy density?

Consider our Universe filled by a barotropic fluid with an equation of state

$$\boldsymbol{p = \mathrm{w}\, \rho}$$

$$T^{\mu}{}_{\nu} = \begin{pmatrix} \rho & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & -p & 0 \\ 0 & 0 & 0 & -p \end{pmatrix}$$

where ρ and p are energy density and pressure of the fluid,.

Einstein-Friedmann eqs. in units $G_{Newton} = \frac{3}{8\pi}$

$$h^2 = \rho,$$

$h \equiv \frac{\dot{a}}{a}$ **Hubble variable**

Energy conservation $\dot{\rho} = -3h(\rho + p)$

Pressure $p = -\frac{2}{3}\dot{h} - h^2.$

$$w = -1 - \frac{2}{3}\frac{\dot{h}}{h^2}$$

Dark energy is characterized by a negative pressure with $w < -\frac{1}{3}$

Quintessence fields

It is well-known that for a given cosmological evolution $h(t)$ satisfying some simple conditions one can find a minimally coupled scalar field cosmological model with Lagrangian

$$L = \frac{\dot{\phi}^2}{2} - V(\phi), \tag{10}$$

which contains this evolution as a particular solution,

Einstein-Friedmann equations

Gauge transformation: $t \rightarrow \tau; \quad dt = \beta(\tau)\, d\tau$

$$h^2 \equiv \boxed{\frac{\dot{a}^2}{a^2} = \frac{1}{2}\dot{\varphi}^2 + V(\varphi)} \equiv \rho \qquad \text{density}$$

$$\delta a: \qquad \boxed{-h^2 - \frac{2}{3}\dot{h} = \frac{1}{2}\dot{\varphi}^2 - V(\varphi)} \equiv p \qquad \text{pressure}$$

$$\delta\varphi: \qquad \boxed{\ddot{\varphi} + 3\,h\,\varphi + V'(\varphi) = 0}$$

Eq. of state $\quad w = \frac{p}{\rho} \geq -1$ **!!!**

If quintessence fields are slowly rolling $\quad \dot{\varphi}^2 \ll V(\varphi)$

then

$$w \approx -1$$

What about superaccelerated evolution $\quad w < -1$??

It can be designed with “phantom” fields $\quad L = -\frac{\dot{\phi}^2}{2} - V(\phi).$

Evidently phantom fields have an energy unbounded from below

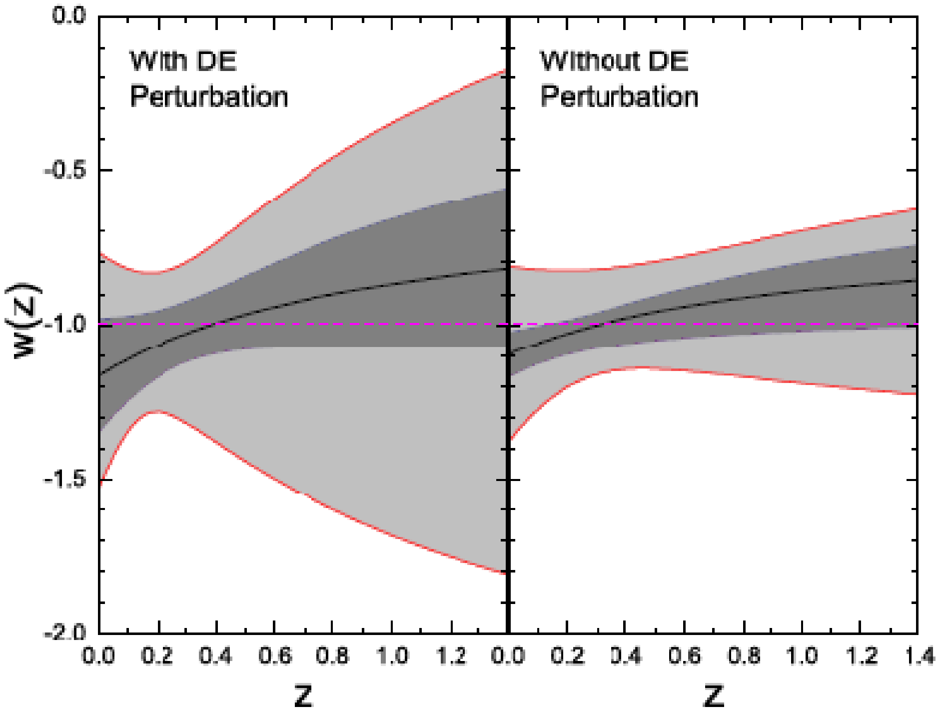
$$L_{phantom} \Rightarrow H_{phantom} = -\pi^2 + V(\phi)$$

Classically not! Because

$$\rho \text{ is real } \Rightarrow -\dot{\varphi}^2 + V(\varphi) > 0 \text{ during classical evolution}$$

BUT catastrophe against fluctuations?

Dark energy (DE) change (=evolution) with red shift z



(moderately optimistic!)

FIG. 4: Constrains on w(z) using WMAP + 157 "gold" SNIa data + SDSS with/without DE perturbation. Median(central line), 68%(inner, dark grey) and 95%(outer, light grey) intervals of w(z) using 2 parameter expansion of the EOS in (4).

$$w(z) = w_0 + w_1 \frac{z}{1+z}.$$

Jun-Qing Xia et al, astro-ph/0511625

Wilkinson Microwave Anisotropy Probe (WMAP)
Type Ia supernova (SNIa)
Sloan Digital Sky Survey (SDSS)

Thus w = w(t) is a dynamical variable and one crosses the divide line w = -1 .
One needs a multicomponent scalar QFT to justify such a behavior .
Quintom, hessence, hybride → scalar QFT with two components.

Caldwell R. R., A Phantom menace ? , Phys. Lett. B 545, (2002) 23

No ! Classical phantom fields may well be originated from a CPT invariant non-Hermitian but *crypto*-Hermitian QFT !

Andrey Smilga

Instructive exercise:

PT-symmetric QM oscillators $H = p^2 + x^2 (ix)^{\varepsilon},\ 0 \le \varepsilon \le 2$

(rigorously proven to possess a real energy spectrum bounded from below)

Dorey,Dunning,Tateo, 2001 $(\varepsilon = 1)$ Andrianov, 1982 $(\varepsilon = 2)$

Classics: $H = p^2 + ix^3 \to \ddot{x}_{cl} = 3ix_{cl}^2 \xrightarrow{\text{a solution}} \mathrm{Re}\, x = 0,\ x_{cl} = i\, f(t)$

$$S_{class} = \int dt\ \left(-\ (\dot{f}(t))^2 + f^3(t)\right) \quad \text{real!} \quad \text{but “phantom”}$$

“phantom” oscillator

Cosmology: ρ and p are real $\Rightarrow$ $\dot{x}_{cl}^2$ and $V(x_{cl})$ are real !

Quasiclassics: $x(t) = i\, f(t) + \delta x(t);\quad S^{(2)} = \int dt\ \delta x(t)\ (-\ \partial_t^2 + 6\, f(t)\ )\ \delta x(t)$ real!

quantum fluctuations

PT symmetric Quantum Mechanics (⟶ QFT?)

Parity transformation $\mathcal{P}\hat{x}\mathcal{P} = -\hat{x} \quad \text{and} \quad \mathcal{P}\hat{p}\mathcal{P} = -\hat{p}.$

Time reversal $\mathcal{T}\hat{x}\mathcal{T} = \hat{x} \quad \text{and} \quad \mathcal{T}\hat{p}\mathcal{T} = -\hat{p}. \qquad \mathcal{T}i\mathcal{T} = -i.$

PT symmetry $H(\mathcal{PT}) - (\mathcal{PT})H = 0.$

A $\mathcal{PT}$-symmetric Hamiltonian need not be Hermitian ! **!!**

C.Bender et al 1998,…,2008

But if it has real spectrum it must be crypto-Hermitian !

Theorem: $\exists C$, $H\,C = C\,H^{+}$; positive operator $C = C^{+} = \vartheta^{2}$; $\vartheta = \vartheta^{+}$

$$\Rightarrow \vartheta^{-1} H \vartheta \equiv h\,;\quad h = h^{+}$$ **(Mostafazadeh, 2003)**

This operator is essentially non-local ! Thus one has two options:
either to work with a local but non-Hermitian QM
or with a non-local (non-Lagrangian!) one but Hermitian

What do you like?

The model based on
1-dim non-Hermitian QM with real energy spectrum

(proof of real spectrum in QM: Curtright, Mezincescu, 2007)

$$L = \frac{\dot{\phi}^2}{2} + \frac{\dot{\chi}^2}{2} - Ae^{\alpha\phi} + Be^{i\beta\chi}$$

where A and B are real, positive constants

$$\ddot{\phi} + 3h\dot{\phi} + A\alpha e^{\alpha\phi} = 0,$$

$$\ddot{\chi} + 3h\dot{\chi} - iB\beta e^{i\beta\chi} = 0,$$

$$\chi = i\xi, \quad \xi \text{ real},$$

ρ and p are real $\Rightarrow$ $\dot{\varphi}^2 + \dot{\chi}^2$ and V(φ, χ) are real separately!

$$h^2 = \frac{\dot{\phi}^2}{2} - \frac{\dot{\xi}^2}{2} + Ae^{\alpha\phi} - Be^{-\beta\xi}.$$

$$L_{eff} = \frac{1}{2}\delta\dot{\chi}^2 - B\beta^2 e^{i\beta\chi_0}(\delta\chi)^2,$$

where χ_0 is a homogeneous purely imaginary solution of the dynamical system

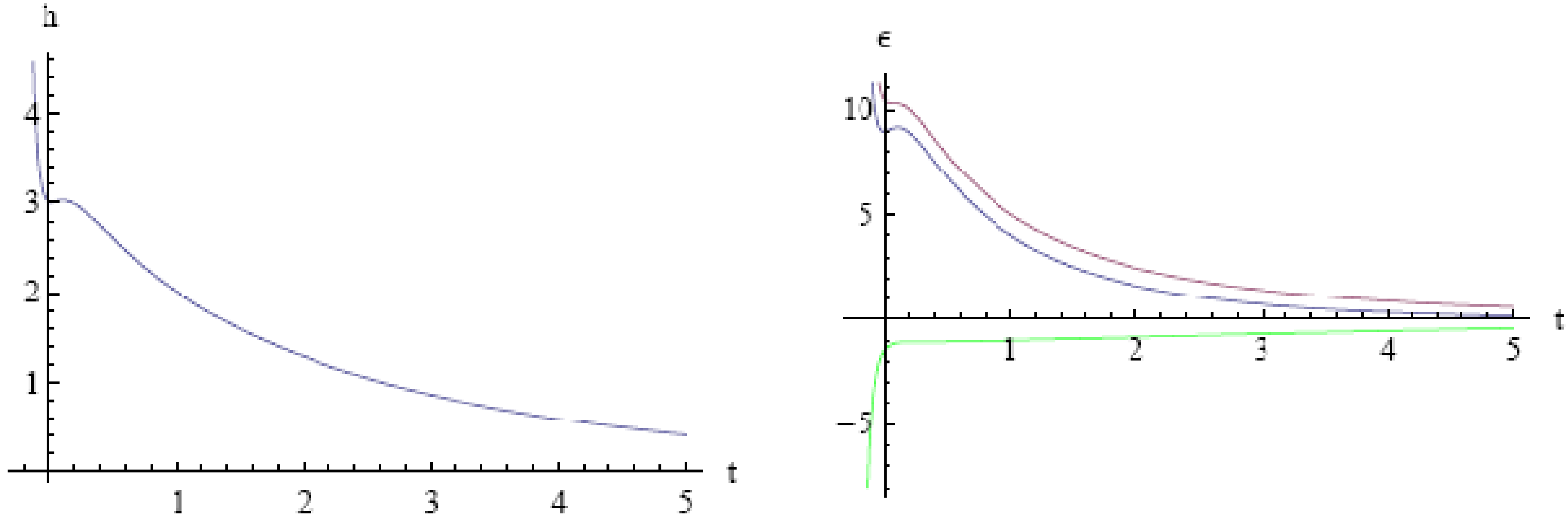


Figure 1. (left) Plot of the Hubble parameter representing the cosmological evolution. The evolution starts from a Big Bang-type singularity and goes through a transient phase of superaccelerated expansion ("phantom era"), which lies between two crossings of PDL (when the derivative of h crosses zero). Then the universe expands infinitely. (right) Plots of the total energy density (blue), and of the energy density of the normal field (purple) and of the phantom one (green).

For comparison a model for evolution with quintom = quintessence and phantom fields

the Hubble variable:

$$h(t) = \frac{A}{t\,(t_R - t)}.$$

The evolution begins at $t = 0$, which represents a standard initial big bang cosmological singularity, and comes to an end in the big rip type singularity at $t = t_R$.

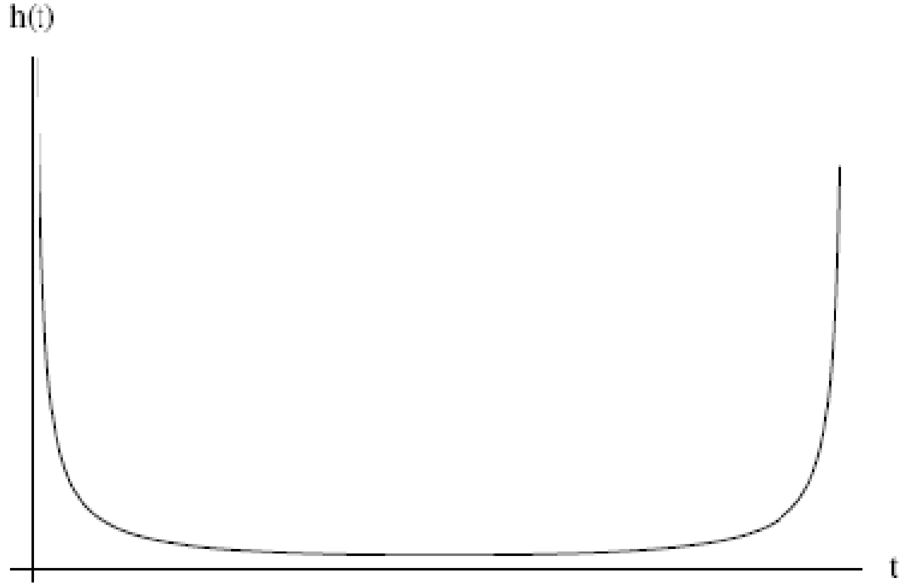


FIG. 2. $h(t)$ dependence in the model, describing the big rip.

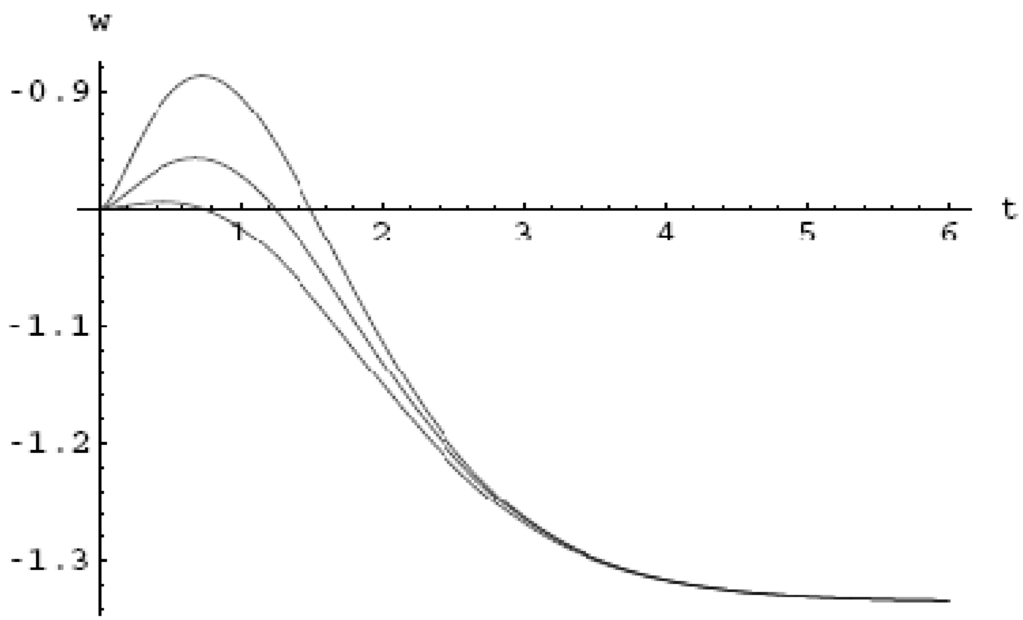


Figure 1: The evolution of the effective equation of state of the phantom and normal scalar fields with $V(\phi,\sigma) = V_{\phi 0}\, e^{-\lambda_\phi \kappa \phi} + V_{\sigma 0}\, e^{-\lambda_\sigma \kappa \sigma}$ for the case $\lambda_\phi = 1$.

From: Guo Z. K. et al, Phys. Lett. B 608, (2005) 177

Non-Hermitian (C)PT symmetric scalar QFT

Let us consider a non-Hermitian (complex) Lagrangian of a scalar field

$$L = \tfrac{1}{2}\partial_\mu\phi\partial^\mu\phi^* - V(\phi,\phi^*),$$

with the corresponding action,

$$S(\phi,\phi^*,g) = \int \mathrm{d}^4x\sqrt{-\|g\|}\left(L + \frac{1}{6}R(g)\right),$$

In units $G_{Newton} = \dfrac{3}{8\pi}$

where $\|g\|$ stands for the determinant of a metric $g^{\mu\nu}$ and $R(g)$ is the scalar curvature term

We employ potentials $V(\phi,\phi^*)$ satisfying the invariance condition

$$(V(\phi,\phi^*))^* = V(\phi^*,\phi),$$

while the condition

$$(V(\phi,\phi^*))^* = V(\phi,\phi^*)$$

is not satisfied. For example, such a potential can have a form

$$V(\phi,\phi^*) = V_1(\phi+\phi^*)V_2(\phi-\phi^*),$$

where V_1 and V_2 are real functions of their arguments. If one defines

$$\phi_1 \equiv \tfrac{1}{2}(\phi+\phi^*),$$

and

$$\phi_2 \equiv \frac{1}{2\mathrm{i}}(\phi-\phi^*),$$

one can consider potentials of the form

$$V(\phi,\phi^*) = V_0(\phi_1)\exp(\mathrm{i}\alpha\phi_2),$$

Resume′

1) The present-day knowledge of dark energy evolution leaves a room for eq. of state with $w < -1$.
2) It poses the problem of existing of an unusual scalar matter with negative kinetic energy.
3) Such a classical FT can be derived from a (C)PT invariant non-Hermitian QFT, quantizable in quasiclassics and possessing a real energy spectrum.
4) From QM to QFT: are there crypto-Hermitian QFT? It has not yet been proven rigorously.

Backside slides

Important Cosmological Parameters ***(nowadays)***

i. The present value of the Hubble parameter (known as Hubble constant) $H_0 \equiv H(t_0) = 100\ h$ km sec^{-1} Mpc^{-1} ($h = 0.72 \pm 0.07$ from the Hubble space telescope [23]).

$H(t) = \dot{a}/a$ □ const

ii. The fraction $\Omega = \rho/\rho_c$, where ρ_c is the critical energy density corresponding to a flat universe. From Eq. (7), $\rho_c = 3H^2/8\pi G$ and $\Omega = 1 + k/a^2H^2$. $\Omega = 1, > 1$, or < 1 corresponds to flat, closed or open universe.

iii. The deceleration parameter

$$q = -\frac{(\ddot{a}/\dot{a})}{(\dot{a}/a)} = \frac{\rho + 3p}{2\rho_c}. \tag{12}$$

Measurements of type Ia supernovae [28] indicate that the universe is speeding up ($q_0 < 0$). This requires that, at present, $p < 0$ as can be seen from Eq. (12). Negative pressure can only be attributed to the dark energy since matter is pressureless. Equation (12) gives $q_0 = (\Omega_0 + 3w_X\Omega_X)/2$, where $\Omega_X = \rho_X/\rho_c$ and $w_X = p_X/\rho_X$ with ρ_X and p_X being the dark energy density and pressure. Observations prefer $w_X = -1$. Actually, the 95% confidence level limit $w_X < -0.6$ from the Ia supernovae data combined with constraints from large-scale structure (see Ref. [29]) is now improved, after the WMAP three year results [6] combined with the supernova legacy survey data [30], to $w_X < -0.83$ for a flat universe.

(An)other scalar field(s) describe(s) the dark energy in the hot Friedmann universe!

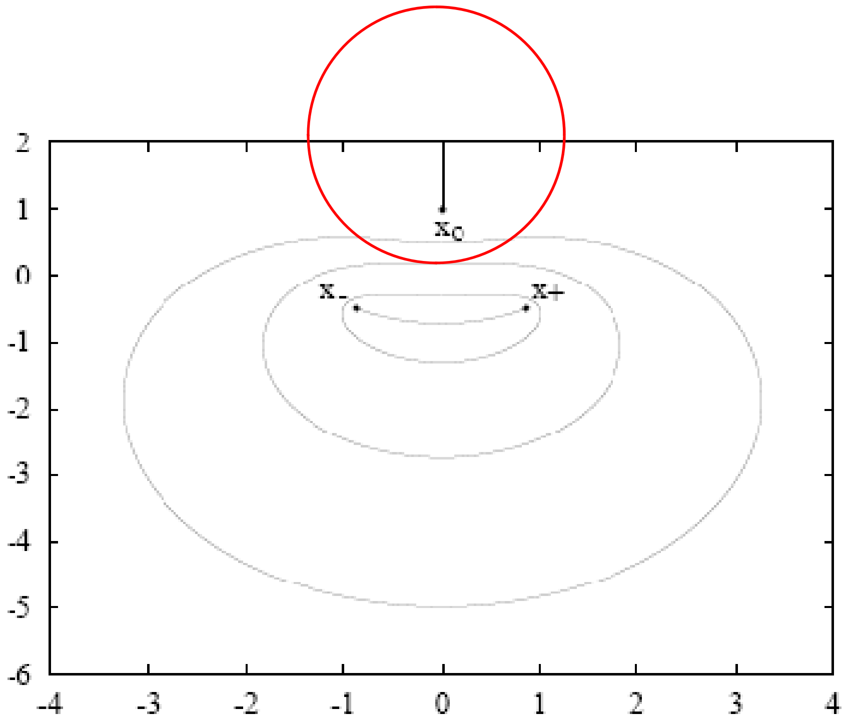


Figure 11. Classical trajectories in the complex-x plane for a particle of energy $E = 1$ described by the Hamiltonian $H = p^2 + ix^3$. An oscillatory trajectory connects the turning points $x_\pm$. This trajectory is enclosed by a set of closed, nested paths that fill the finite complex-x plane except for points on the imaginary axis at or above the turning point $x_0 = i$. Trajectories that originate on the imaginary axis above $x = i$ either move off to $i\infty$ or else approach x_0, stop, turn around, and then move up the imaginary axis to $i\infty$.

ρ **and p are real** $\Rightarrow$ $\dot{\mathrm{q}}^2_{classical}$ **and** $V(\dot{\mathrm{q}}_{classical})$ **are real !**