Spontaneous parity breaking in dense nuclear matter

A.A. Andrianov and D. Espriu

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Phys.Lett.B663:450 (2008).

We conjecture it to occur at zero temperature but large baryon number density due to condensation of parity-odd mesons (pions, kaons,… and their radial excitations)

How large?

<u>Beyond the range of validity</u> of pion-nucleon effective Lagrangian but <u>not large enough</u> for quark percolation, *i.e.* in the hadronic phase with heavy meson excitations playing an essential role

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We conjecture it to occur at zero temperature but large baryon number density due to condensation of parity-odd mesons (pions, kaons,… and their radial excitations)

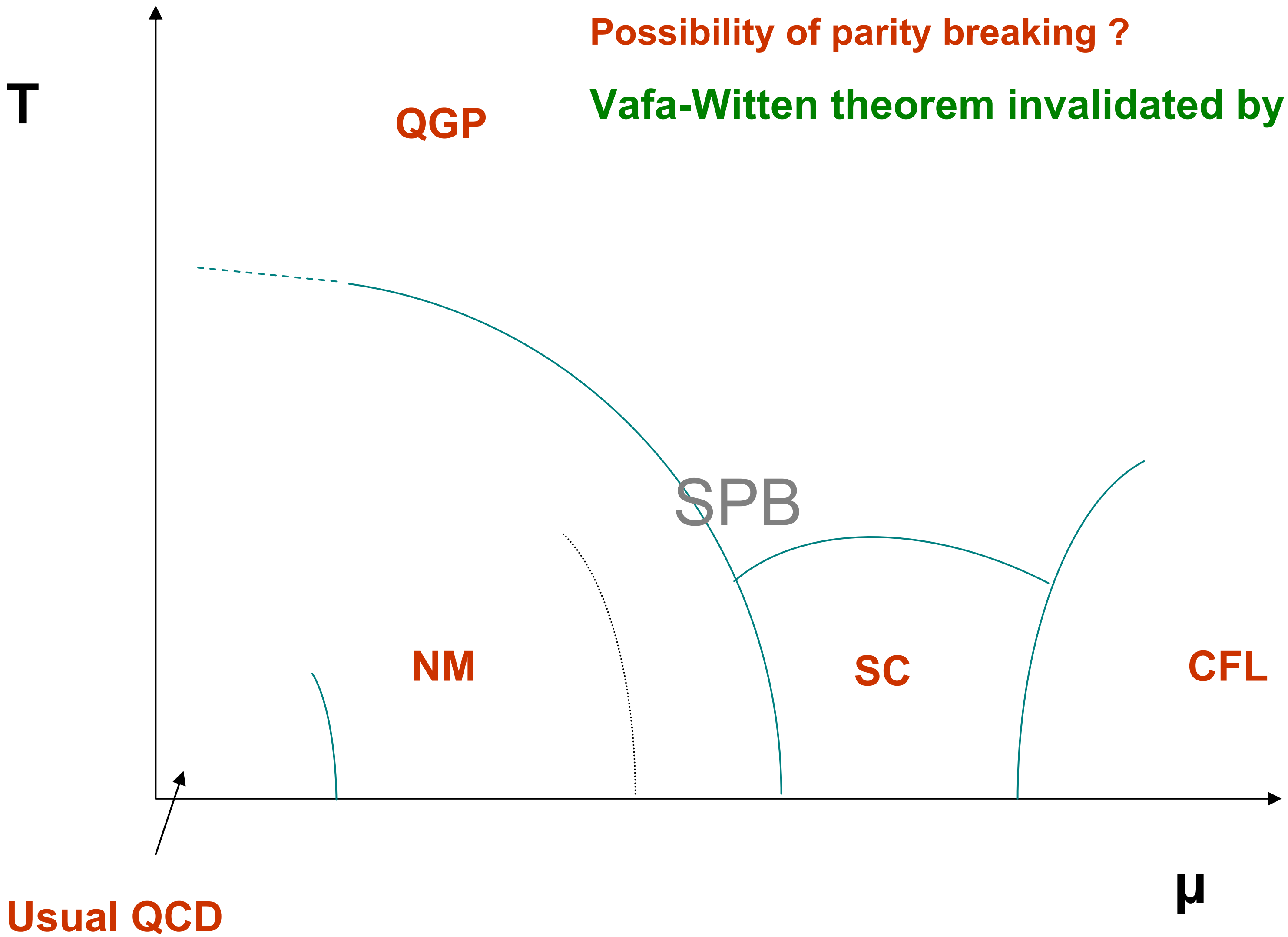
$$\rho_B \sim (3 - 10)\,\rho_N \qquad \rho_N \sim 0.17\ \mathrm{fm}^{-3} = (0.18\ \mathrm{fm})^{-3}$$

<u>Beyond the range of validity</u> of pion-nucleon effective Lagrangian but <u>not large enough</u> for quark percolation, *i.e.* in the hadronic phase with heavy meson excitations playing an essential role

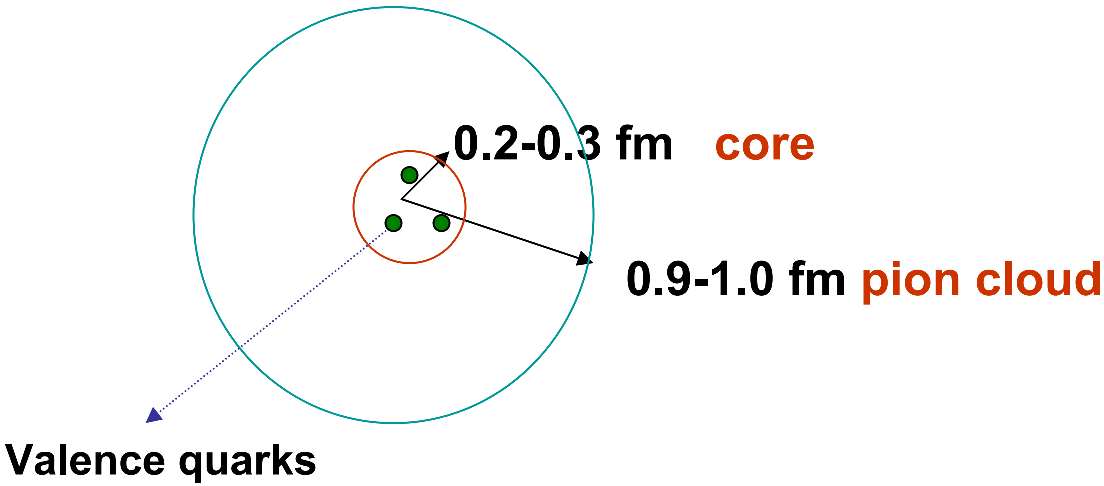
At very high temperatures the theory is effectively 3 dimensional $L = \int d^3x\, AdA + \ldots$

Possibility of parity breaking ?

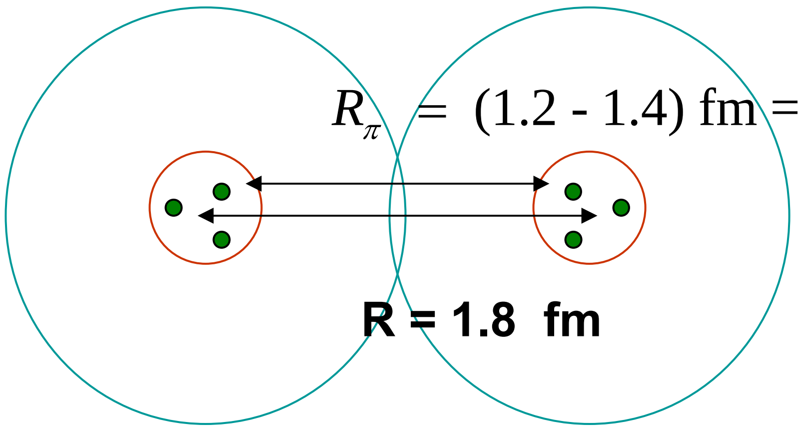
Vafa-Witten theorem invalidated by $\mu > 0$



Nucleons



Normal nuclear matter



$$R_\pi = (1.2 - 1.4)\ \mathrm{fm} = (140\text{-}160\ \mathrm{MeV})^{-1} \approx (m_\pi)^{-1}$$

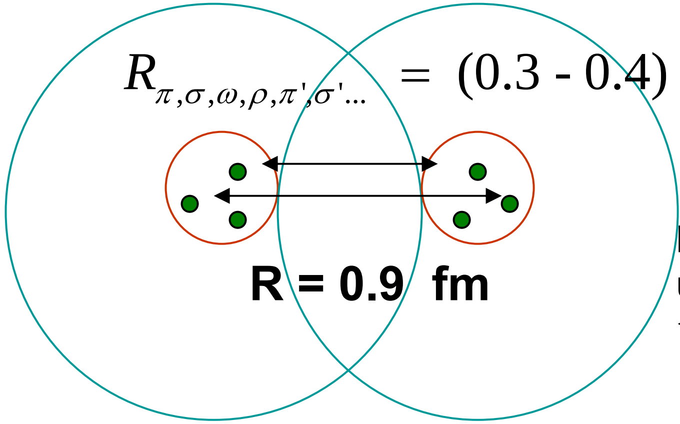
Pion-nucleon realm

Dense nuclear matter

$$R_{\pi,\sigma,\omega,\rho,\pi',\sigma'...} = (0.3 - 0.4)\ \text{fm} = (500\text{-}700\ \text{MeV})^{-1}$$

$$\varrho_B = 8\varrho_N \simeq (0.9 fm)^{-3}$$

Neutron stars

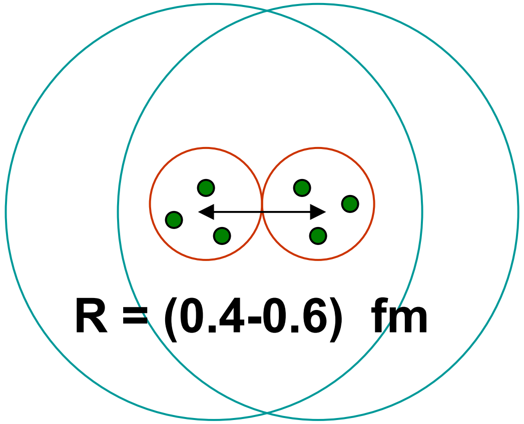


Higher-mass mesons + undressed nucleons
$\longleftrightarrow$ dressed quarks

Superdense nuclear matter

$$\begin{aligned} \varrho_B &= (25 \div 100)\varrho_N \\ &\simeq (0.4 \div 0.6 fm)^{-3} \end{aligned}$$

Quark stars?



Quark percolation
2SC, CFL …

This picture emerges as a consensus of several models: nuclear potentials, meson-nucleon effective Lagrangians, extended Skyrme models, chiral bag models … The NJL ones give larger core sizes due to lack of confinement.

A bit of history:
pion condensation in symmetric nuclear matter $\rho_p = \rho_n$

A. Migdal (1971)

$$\omega^2 = 1 + k^2 - 4\pi n F(k) \qquad \hbar = c = m_\pi = 1$$

where n is the nucleon density and $F(k)$ is the forward pion-nucleon scattering amplitude which for both π^+ and π^- mesons, has the sign corresponding to attraction ($F > 0$), and therefore at sufficient-density the frequency can vanish, meaning instability of the pion field. However, $F(k)$ is small at small k and instability sets in at $k = k_0$, which corresponds to the minimal value of $k^2 - 4\pi n F(k)$. The instability condition is $\omega^2 = 0$ or

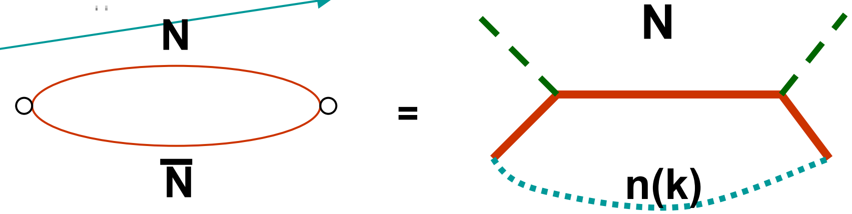
$$1 + k_0^2 = 4\pi n F(k_0)$$

In this approach a pion condensate is spatially inhomogeneous!

A more precise calculation includes the particle-hole excitations of the nuclear medium

$$\omega^2 = 1 + k^2 + \Pi(k, \omega)$$

Polarization operator =



Off-shell amplitudes!

Polarization operator in detail

$$\Pi^{(1)\pi^-}(\omega, k) = -2\int \frac{d^3p}{(2\pi)^3}\left[D_{\pi^-n}(\omega, k)n_n(p) + D_{\pi^-p}(\omega, k)n_p(p)\right],$$

where D_{π^-n} and D_{π^-p} are the spin-averaged forward scattering amplitudes, $n_n(p)$ and $n_p(p)$ are the occupation functions of the neutrons and the protons respectively,

$$\Pi^{(1)\pi^-}(\omega, k; \rho) = \Pi_N^{(1)\pi^-} + \Pi_\Delta^{(1)\pi^-} + \Pi_D^{(1)\pi^-} + \Pi_\sigma^{(1)\pi^-}$$

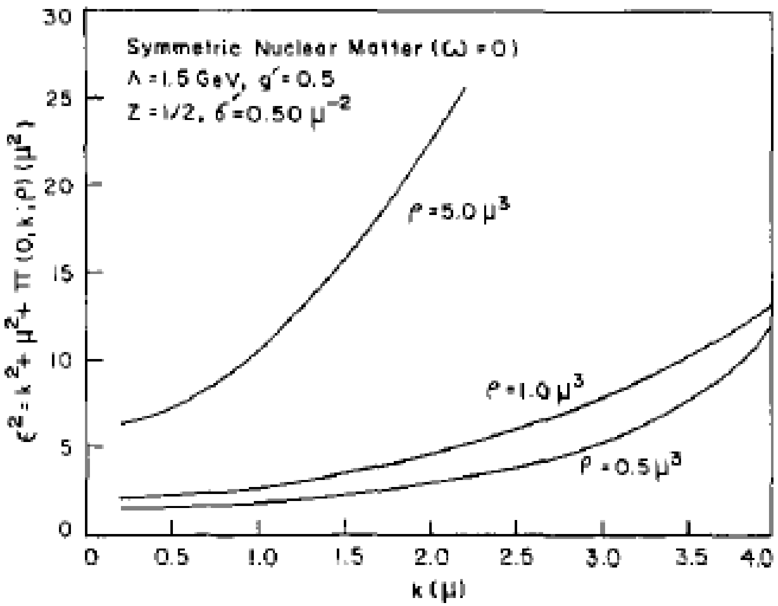
Off-shell amplitudes from an effective lagrangian

where the subscripts N, Δ, D and σ refer to contributions from nucleon exchange, delta exchange, direct pion–nucleon scattering and the pion–nucleon σ term,

T Shamsunnahar et al.
J. Phys. G. 17 (1991) 887

No pion condensate

Simplest sigma model is not rich enough to provide SPB – only one vacuum condensate that must necessarily align with the (real) fermion condensate to fulfill current algebra



Extended linear sigma model

with two multiplets of scalar and pseudoscalar mesons
(as close to QCD as possible)

$$H_j = \sigma_j \mathbf{I} + i\hat{\pi}_j, \quad j = 1,2; \quad H_j H_j^\dagger = (\sigma_j^2 + (\pi_j^a)^2)\mathbf{I}, \qquad \hat{\pi}_j \equiv \pi_j^a \tau^a$$

Chiral limit $\longrightarrow$ $SU_L(2) \times SU_R(2)$ **symmetry**

$$\begin{aligned} V_{\text{eff}} \;=\; & \frac{1}{2}\text{tr}\Big\{ -\sum_{j,k=1}^{2} H_j^\dagger \Delta_{jk} H_k \\ & +\lambda_1 (H_1^\dagger H_1)^2 + \lambda_2 (H_2^\dagger H_2)^2 + \lambda_3 H_1^\dagger H_1 H_2^\dagger H_2 \\ & +\frac{1}{2}\lambda_4 (H_1^\dagger H_2 H_1^\dagger H_2 + H_2^\dagger H_1 H_2^\dagger H_1) + \frac{1}{2}\lambda_5 (H_1^\dagger H_2 + H_2^\dagger H_1) H_1^\dagger H_1 \\ & +\frac{1}{2}\lambda_6 (H_1^\dagger H_2 + H_2^\dagger H_1) H_2^\dagger H_2 \Big\} + \mathcal{O}\Big(\frac{|H|^6}{\Lambda^2}\Big), \end{aligned}$$

9 real constants

$$\Delta_{jk} \sim \lambda_A \sim N_c$$

Chiral expansion in $1/\Lambda$
in hadron phase of QCD

$$\Lambda \;;\; 4\pi F_\pi \;:\; M_{\text{dyn}}$$

Chirally symmetric parametrization

$$H_1(x) = \sigma_1(x)U(x) = \sigma_1(x)\xi^2(x); \qquad \langle H_1 \rangle = \langle \sigma_1 \rangle > 0$$

$$H_2(x) = \xi(x)\big(\sigma_2(x) + i\hat{\pi}_2(x)\big)\xi(x) = \sigma_2(x)U(x) + i\xi(x)\hat{\pi}_2(x)\xi(x)$$

Effective potential

$$\begin{aligned} V_{\rm eff} \;\; &= \;\; -\sum_{j,k=1}^{2} \sigma_j \Delta_{jk} \sigma_k - \Delta_{22}(\pi_2^a)^2 \\ &+\lambda_1\sigma_1^4 + \lambda_2\sigma_2^4 + (\lambda_3+\lambda_4)\sigma_1^2\sigma_2^2 + \lambda_5\sigma_1^3\sigma_2 + \lambda_6\sigma_1\sigma_2^3 \\ &+(\pi_2^a)^2\Big((\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big) + \lambda_2\Big((\pi_2^a)^2\Big)^2 \end{aligned}$$

Minimal nontrivial choice admitting SPB

$\lambda_5 = \lambda_6 = 0 \;\Rightarrow\; \Delta_{12} = 0$ **from consistency**

Such an effective potential is symmetric under $Z_2 \times Z_2$

$$H_1 \to -H_1 \quad \text{or} \quad H_2 \to -H_2$$

Vacuum state

$$\boxed{\pi_2^a = \delta^{a0}\rho}$$

Neutral pseudoscalar condensate breaking P-parity?

No in QCD at zero quark density (Vafa-Witten theorem)

$$\rho = 0$$

Gap equations

$$\begin{aligned}
2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) &= 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3 \\
&\quad +\rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big), \\
2(\Delta_{12}\sigma_1 + \Delta_{22}\sigma_2) &= \lambda_5\sigma_1^3 + 2(\lambda_3+\lambda_4)\sigma_1^2\sigma_2 + 3\lambda_6\sigma_1\sigma_2^2 + 4\lambda_2\sigma_2^3 \\
&\quad +\rho^2\Big(\lambda_6\sigma_1 + 4\lambda_2\sigma_2\Big), \\
0 &= 2\pi_2^a\Big(-\Delta_{22} + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 + 2\lambda_2\rho^2\Big)
\end{aligned}$$

Sufficient condition to avoid P-parity breaking in normal QCD vacuum (μ = 0)

$$\boxed{(\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22}}$$

Second variation for $\rho = 0$

$$\frac{1}{2}V^{(2)}_{11} = -\Delta_{11} + 6\lambda_1\sigma_1^2 + 3\lambda_5\sigma_1\sigma_2 + (\lambda_3+\lambda_4)\sigma_2^2,$$
$$V^{(2)}_{12} = -2\Delta_{12} + 3\lambda_5\sigma_1^2 + 4(\lambda_3+\lambda_4)\sigma_1\sigma_2 + 3\lambda_6\sigma_2^2,$$
$$\frac{1}{2}V^{(2)}_{22} = -\Delta_{22} + (\lambda_3+\lambda_4)\sigma_1^2 + 3\lambda_6\sigma_1\sigma_2 + 6\lambda_2\sigma_2^2$$
$$\frac{1}{2}V^{(2)\pi}_{ab} = \delta_{ab}\Big(-\Delta_{22} + (\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2\Big) > 0$$

Mass of π_2 if $\rho=0$ → **necessary** condition

$$\boxed{(\lambda_3-\lambda_4)\sigma_1^2 + \lambda_6\sigma_1\sigma_2 + 2\lambda_2\sigma_2^2 > \Delta_{22}}$$

Necessary condition to have a minimum for non-zero v.e.v. (CSB)

$$\mathrm{Det}\Delta > 0,\ \mathrm{tr}\{\Delta\} > 0 \qquad \text{or} \qquad \mathrm{Det}\Delta < 0$$

Sufficient conditions for non-trivial CSB minimum

$$\mathrm{tr}\left\{\hat{V}^{(2)}\right\} > 0 \qquad \mathrm{Det}\hat{V}^{(2)} > 0$$

Where are the baryons ?

Embedding a chemical potential

$$\int dx\, \mu \left(\bar{q}\gamma_0 q(x) - \rho_B \right)$$

Local coupling to quarks

$$\mathcal{L}_{int} = -(\bar{q}_R H_1 q_L + \bar{q}_L H_1^\dagger q_R)$$

Superposition of physical meson states (quark matter=nuclear matter)

From a quark model

$$\begin{aligned}\Delta V_{\rm eff}(\mu) \;=\; & \frac{\mathcal{N}}{2}\Theta(\mu-|H_1|)\left[\mu|H_1|^2\sqrt{\mu^2-|H_1|^2}-\frac{2\mu}{3}(\mu^2-|H_1|^2)^{3/2}-\right.\\ & \left.|H_1|^4\ln\frac{\mu+\sqrt{\mu^2-|H_1|^2}}{|H_1|}\right]\left(1+O\left(\frac{\mu^2}{\Lambda^2}\right)\right)\end{aligned}$$

$$\boxed{\mathcal{N}\equiv\frac{N_cN_f}{4\pi^2}}$$

Density and Fermi momentum

$$\varrho_B=-\frac{1}{3}\partial_\mu\Delta V_{\rm eff}(\mu)=\frac{N_cN_f}{9\pi^2}p_F^3=\frac{N_cN_f}{9\pi^2}(\mu^2-|\langle H_1\rangle|^2)^{3/2}$$

The calculation can be easily extended to finite T

$$\boxed{\Delta V_{\rm eff}(\mu) \longrightarrow \Delta V_{\rm eff}(T,\mu)}$$

How dense matter modifies the minimum of the effective potential

Only the first equation for stationary points is modified

$$2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3$$
$$+\rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big) + 2\mathcal{N}\Theta(\mu-\sigma_1)\left[\mu\sigma_1\sqrt{\mu^2-\sigma_1^2} - \sigma_1^3\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\right]$$

Second variation matrix depends on μ only through its first element

$$\begin{aligned}\frac{1}{2}V_{11}^{(2)\sigma} \;&=\; -\Delta_{11} + 6\lambda_1\sigma_1^2 + 3\lambda_5\sigma_1\sigma_2 + (\lambda_3+\lambda_4)\sigma_2^2 + (\lambda_3-\lambda_4)\rho^2\\ &\quad +\mathcal{N}\Theta(\mu-\sigma_1)\left[\mu\sqrt{\mu^2-\sigma_1^2} - 3\sigma_1^2\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\right]\end{aligned}$$

Extension to hot and dense matter

$$2(\Delta_{11}\sigma_1 + \Delta_{12}\sigma_2) = 4\lambda_1\sigma_1^3 + 3\lambda_5\sigma_1^2\sigma_2 + 2(\lambda_3+\lambda_4)\sigma_1\sigma_2^2 + \lambda_6\sigma_2^3$$
$$+\rho^2\Big(2(\lambda_3-\lambda_4)\sigma_1 + \lambda_6\sigma_2\Big) + \underline{2\mathcal{N}\Theta(\mu-\sigma_1)\left[\mu\sigma_1\sqrt{\mu^2-\sigma_1^2} - \sigma_1^3\ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1}\right]}$$

replace

$$\mathcal{A}(\sigma_1,\mu,\beta) = \int_{\sigma_1}^{\infty} dE\ \sqrt{E^2-\sigma_1^2}\,\frac{\cosh(\beta\mu)+\exp(-\beta E)}{\cosh(\beta\mu)+\cosh(\beta E)}$$

one quark loop
no thermal loops for mesons
(OK in large N-limit)

Necessary condition to approach to P-breaking phase

Recall $$\boxed{(\lambda_3-\lambda_4)\sigma_1^2+\lambda_6\sigma_1\sigma_2+2\lambda_2\sigma_2^2>\Delta_{22}}$$

$$\partial_\mu\left[(\lambda_3-\lambda_4)\sigma_1^2+\lambda_6\sigma_1\sigma_2+2\lambda_2\sigma_2^2\right]<0$$

Above a critical point $\mu>\mu_{crit}$ **we *expect***

$$\boxed{(\lambda_3-\lambda_4)\sigma_1^2+\lambda_6\sigma_1\sigma_2+2\lambda_2\left(\sigma_2^2+\rho^2\right)=\Delta_{22}}$$

Necessary condition to have SPB

From eqs. for extremum. . . .

$$\lambda_5\sigma_1^2+4\lambda_4\sigma_1\sigma_2+\lambda_6\left(\sigma_2^2+\rho^2\right)=2\Delta_{12}$$

If $\lambda_2=0$ **and/or** $\lambda_6=0$ **these equations completely fix** σ_1 and σ_2

The two v.e.v. are rigidly fixed in the SPB phase

Otherwise, for $\lambda_2\lambda_6\neq 0$ **we get a relation between the v.e.v.**

$$\sigma_2=A\sigma_1+\frac{B}{\sigma_1}\qquad A\equiv\frac{2\lambda_5\lambda_2+\lambda_6(\lambda_4-\lambda_3)}{\lambda_6^2-8\lambda_2\lambda_4}\qquad B\equiv\frac{\lambda_6\Delta_{22}-4\lambda_2\Delta_{12}}{\lambda_6^2-8\lambda_2\lambda_4}$$

All these relations do not depend on the parity breaking v.e.v. ρ **and on μ**

Critical points μ_c vhere $\rho(\mu_c) = 0$

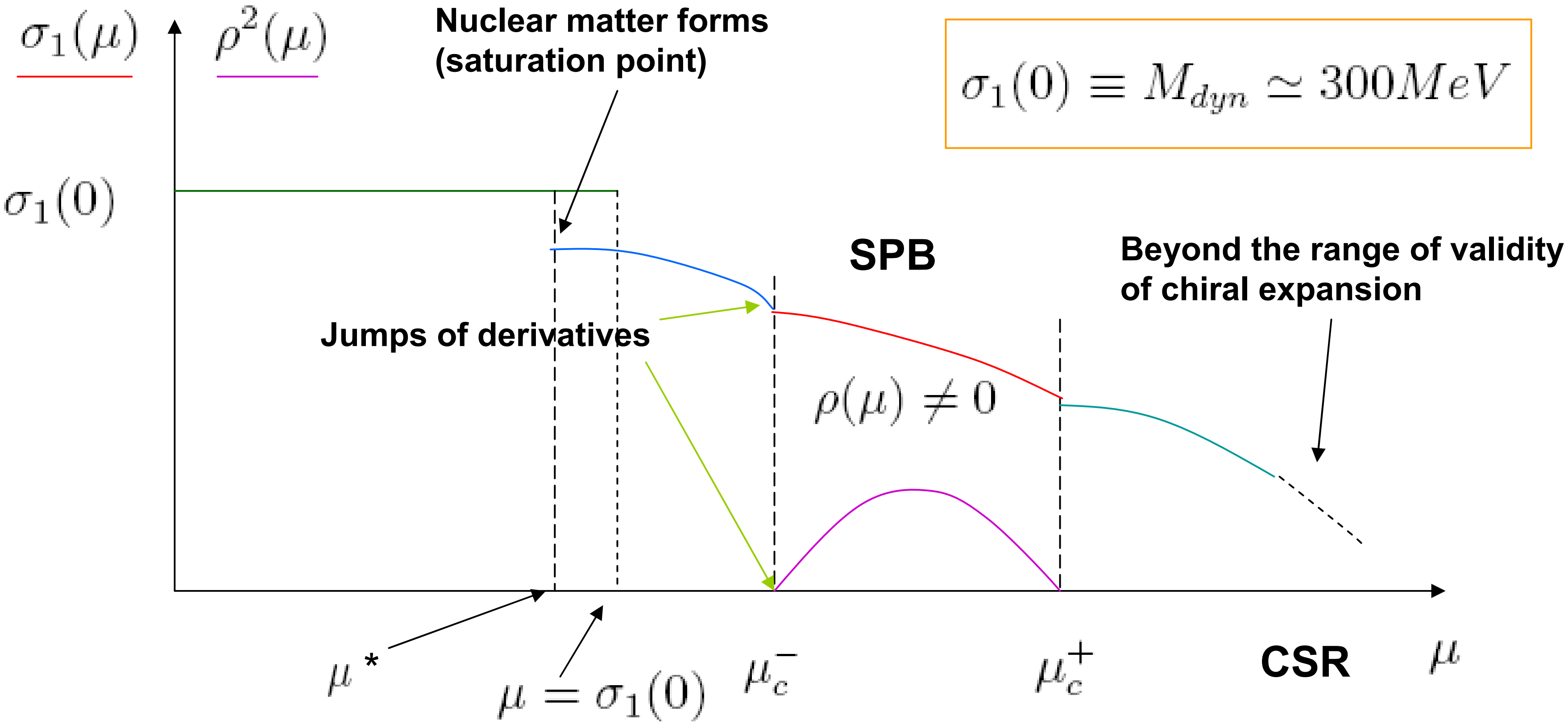
$$\boxed{(4\lambda_2\Delta_{12} - \lambda_6\Delta_{22})r^2 + (2\lambda_6\Delta_{12} - 4\lambda_4\Delta_{22})r + 2(\lambda_3 - \lambda_4)\Delta_{12} - \lambda_5\Delta_{22} = 0}$$

with $r \equiv \dfrac{\sigma_2}{\sigma_1}$

There are in general two solutions $\mu_c^- < \mu_c^+$

But for $4\lambda_2\Delta_{12} = \lambda_6\Delta_{22}$ **only one solution exits and P-breaking phase may be left in the high μ regime via 1st order phase transition, possibly beyond the range of validity of our effective theory**

Spontaneous P-parity breaking (2nd order phase transition)



With increasing μ one enters SPB phase and leaves it before (?) encountering any new phase (CSR, CFL …)

Could a 1st order phase transition also exist?

However . . .

This model is not realistic from the point of view of nuclear physics.

In order to get stable nuclear matter and describe the saturation point one has to modify slightly the model.

The modification requires the introduction of the iso-singlet ω, whose zeroth component ω^0 acquires a v.e.v and interacts with the chemical potential μ.

This has no further consequences for the rest of the discussion.

This model has no problems with 'chiral collapse'

Meson spectrum in SPB phase

Neutral pi-prime condensate breaks vector SU(2) to U(1)
and two charged pi-prime mesons become massless

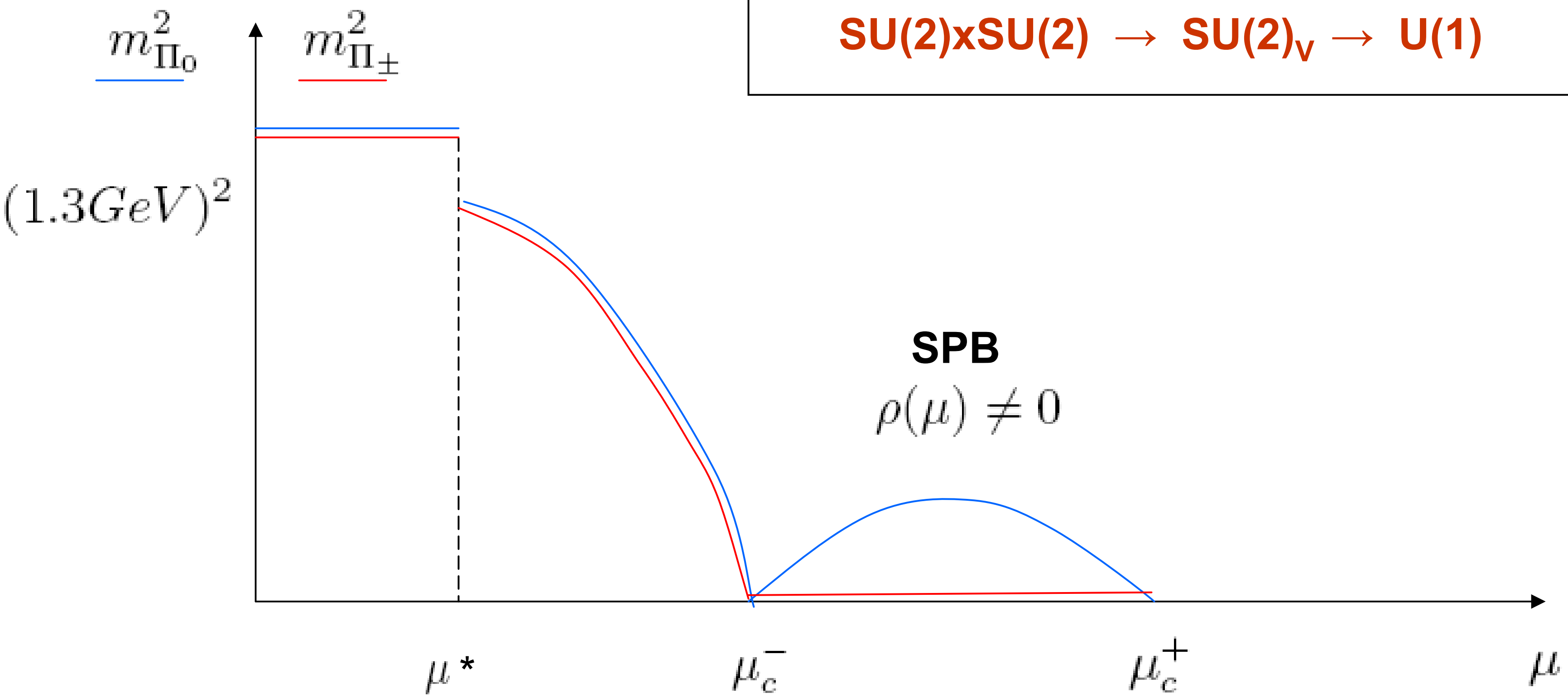
$$
\begin{aligned}
\frac{1}{2}V_{11}^{(2)\sigma} &= 4\lambda_1\sigma_1^2 + 2\lambda_5\sigma_1\sigma_2 + 2\lambda_4\sigma_2^2 - 2\mathcal{N}\sigma_1^2 \ln\frac{\mu+\sqrt{\mu^2-\sigma_1^2}}{\sigma_1} \\
V_{12}^{(2)\sigma} &= 2\lambda_5\sigma_1^2 + 4\lambda_3\sigma_1\sigma_2 + 2\lambda_6\sigma_2^2 \\
\frac{1}{2}V_{22}^{(2)\sigma} &= 2\lambda_4\sigma_1^2 + 2\lambda_6\sigma_1\sigma_2 + 4\lambda_2\sigma_2^2 \\
V_{10}^{(2)\sigma\pi} &= \Big(4(\lambda_3-\lambda_4)\sigma_1 + 2\lambda_6\sigma_2\Big)\rho \\
V_{20}^{(2)\sigma\pi} &= \Big(2\lambda_6\sigma_1 + 8\lambda_2\sigma_2\Big)\rho \\
\frac{1}{2}V_{00}^{(2)\pi} &= 4\lambda_2\rho^2 \qquad \boxed{\frac{1}{2}V_{\pm\mp}^{(2)\pi} = 0}
\end{aligned}
$$

Mixture of massive scalar and neutral pseudoscalar states

Mass spectrum of “pseudoscalar” states

(parity no longer a good quantum number in strong interactions !)

SU(2)xSU(2) $\rightarrow$ SU(2)$_V$ $\rightarrow$ U(1)



pseudoscalar mesons

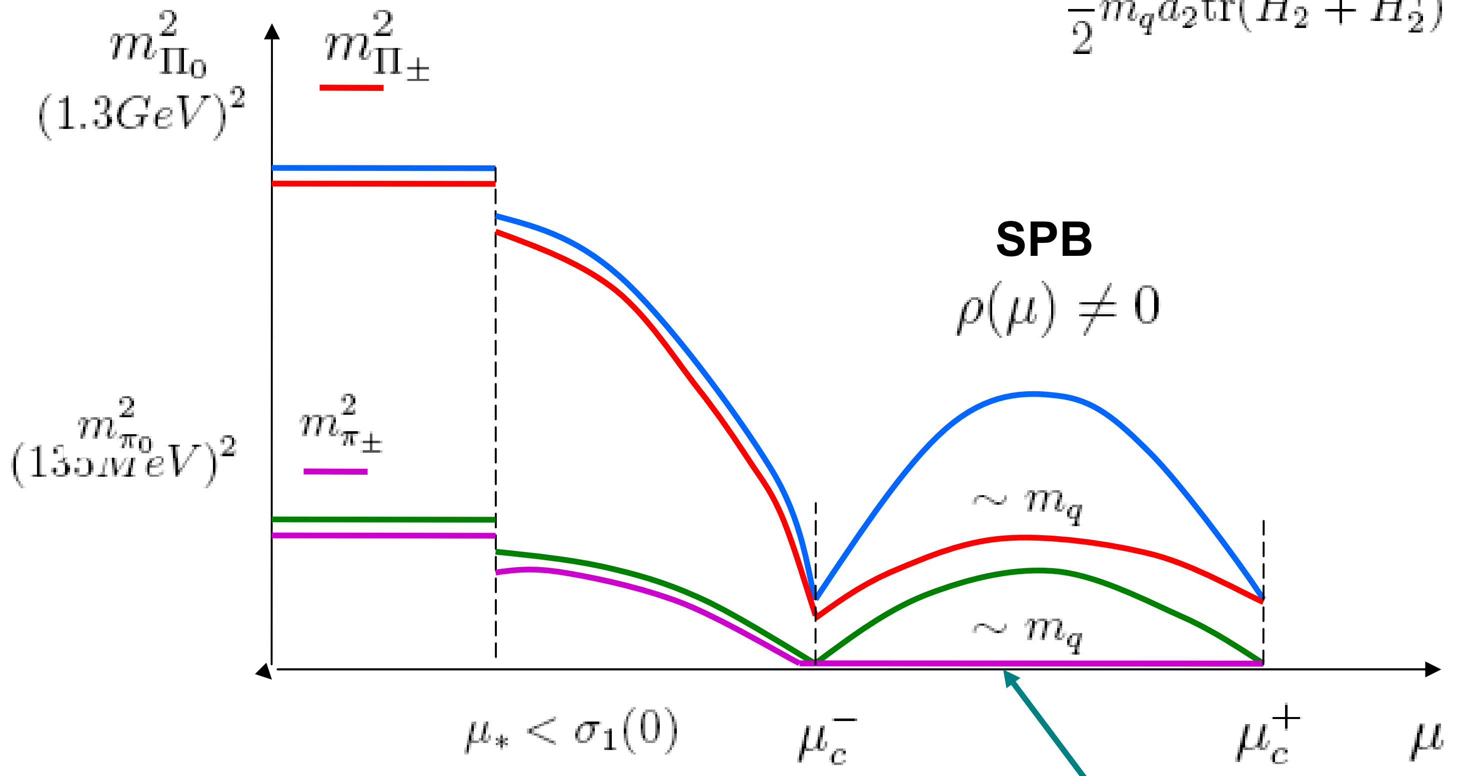
$\pi_0 \quad \pi_\pm \quad \Pi_\pm$

Mass spectrum of “pseudoscalar” states
(beyond chiral limit)

two new low-dimensional operators

$$\frac{1}{2} m_q d_1 \mathrm{tr}(H_1 + H_1^\dagger)$$

$$\frac{1}{2} m_q d_2 \mathrm{tr}(H_2 + H_2^\dagger)$$



In P-breaking phase there are 2 massless ‘pseudoscalar’ mesons

$\tilde{\pi}_\pm$

Kinetic terms

Symmetric under $SU_L(2) \times SU_R(2)$

$$\mathcal{L}_{kin} = \frac{1}{4} \sum_{j,k=1}^{2} A_{jk} \mathrm{tr} \left\{ \partial_\mu H_j^\dagger \partial^\mu H_k \right\}$$

Chirally symmetric parametrization

$$H_1(x) = \sigma_1(x) U(x) = \sigma_1(x) \xi^2(x)$$

$$H_2(x) = \xi(x)(\sigma_2(x) + i\hat{\pi}_2)(x)\xi(x) = \sigma_2(x) U(x) + i\xi(x)\hat{\pi}_2(x)\xi(x)$$

Expanding around a vacuum configuration…

$$U = 1 + i\hat{\pi}/F_0 + \cdots \qquad \xi = 1 + i\hat{\pi}/2F_0 + \cdots$$

$$\sigma_j \equiv \bar{\sigma}_j + \Sigma_j \qquad \hat{\pi} = \tau_3 \rho + \hat{\Pi}$$

Kinetic part quadratic in meson fields

$$\mathcal{L}^{(2)}_{kin} = \frac{1}{2}\sum_{j,k=1}^{2} A_{jk}\partial_\mu \Sigma_j \partial^\mu \Sigma_k + \frac{\rho^2}{2F_0^2} A_{22}\partial_\mu \pi^0 \partial^\mu \pi^0 - \frac{\rho}{F_0}\sum_{j=1}^{2} A_{j2}\partial_\mu \Sigma_j \partial^\mu \pi^0$$

$$+ \sum_{j,k=1}^{2}\left[\frac{1}{2F_0^2} A_{jk}\bar{\sigma}_j\bar{\sigma}_k \partial_\mu \pi^a \partial^\mu \pi^a + \frac{1}{F_0} A_{j2}\bar{\sigma}_j \partial_\mu \pi^a \partial^\mu \Pi^a\right] + \frac{1}{2} A_{22}\partial_\mu \Pi^a \partial^\mu \Pi^a$$

$$F_0^2 = \sum_{j,k=1}^{2} A_{jk}\bar{\sigma}_j\bar{\sigma}_k \simeq (90\mathrm{MeV})^2, \qquad \zeta \equiv \frac{1}{F_0}\sum_{j=1}^{2} A_{j2}\bar{\sigma}_j$$

Pion weak decay constant

Mixing !

P-breaking phase

Mixing with massless pions is different for neutral and charged ones because vector isospin symmetry is broken

$$\tilde{\pi}^{\pm} = \pi^{\pm} + \zeta \Pi^{\pm}, \qquad \tilde{\pi}^{0} = \pi^{0} + \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\left(\zeta \Pi^0 - \frac{\rho}{F_0}\sum_{j=1}^{2} A_{j2}\partial_\mu \Sigma_j\right).$$

Partially diagonalized kinetic term

$$\begin{aligned}
\mathcal{L}_{kin}^{(2)} &= \partial_\mu \tilde{\pi}^{\pm}\partial^\mu \tilde{\pi}^{\mp} + (A_{22} - \zeta^2)\partial_\mu \Pi^{\pm}\partial^\mu \Pi^{\mp} + \frac{1}{2}\left(1 + \frac{A_{22}\rho^2}{F_0^2}\right)\partial_\mu \tilde{\pi}^0 \partial^\mu \tilde{\pi}^0 \\
&+ \frac{1}{2}\left(A_{22} - \frac{F_0^2}{F_0^2 + A_{22}\rho^2}\zeta^2\right)\partial_\mu \Pi^0 \partial^\mu \Pi^0 + \frac{1}{2}\sum_{j,k=1}^{2}\frac{A_{jk}F_0^2 + \rho^2 \det A\, \delta_{1j}\delta_{1k}}{F_0^2 + A_{22}\rho^2}\partial_\mu \Sigma_j \partial^\mu \Sigma_k \\
&- \frac{F_0\rho}{F_0^2 + A_{22}\rho^2}\zeta \partial_\mu \Pi^0 \sum_{j=1}^{2} A_{j2}\partial^\mu \Sigma_j
\end{aligned}$$

Isospin breaking $SU_V(2) \to U(1)$

Further diagonalization $\Pi^0 \quad \Sigma_1 \quad \Sigma_2 \quad \Longrightarrow \quad \tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2$

mixes neutral pseudoscalar and scalar states

Therefore genuine mass states don't possess a definite parity in decays

$$\tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2 \longrightarrow \pi_0\,\pi_0\,\pi_0 \qquad \pi_0\,\pi_+\,\pi_- \qquad \pi_0\,\Pi_+\,\Pi_- \ldots$$

$$\tilde{\Pi}^0 \quad \tilde{\Sigma}_1 \quad \tilde{\Sigma}_2 \longrightarrow \pi_0\,\pi_0 \qquad \pi_+\,\pi_- \qquad \Pi_+\,\Pi_- \ldots$$

Minimal model admitting SPB

$$\boxed{\lambda_5 = \lambda_6 = 0 \Rightarrow \Delta_{12} = 0}$$ **from consistency**

Such an effective Lagrangian is symmetric under $Z_2 \times Z_2$

$$H_1 \to -H_1 \quad \text{or} \quad H_2 \to -H_2$$

Fit on hadron phenomenology

$$A_{11} = \frac{1}{9} = A_{22}, \quad F_0 = 100MeV, \quad \sigma_1 = 300MeV = 3F_0,$$

$$m_{s,1} = 0.7GeV = 7F_0, \quad m_p = 1.3GeV = 13F_0, \quad m_{s,2} = 1.5GeV = 15F_0,$$

$$\Delta_{11} \simeq 2.7F_0^2, \quad \lambda_1 \simeq 0.15, \quad \lambda_4 \simeq 0.35.$$

For $\boxed{\varrho_{B,crit} \simeq 3\varrho_{B,nuclear}} \longrightarrow \boxed{\mu_c \simeq 4.3F_0} \longleftrightarrow \boxed{\sigma_{1,crit} \simeq 1.8F_0}$

$$\lambda_3 \simeq 3.6 \qquad \Delta_{22} \simeq 11F_0^2$$

$$\boxed{m_{s,1} \simeq 1.7F_0, \;\; m_{s,2} \simeq 4.5F_0}$$

Going back to polarization operator in Migdal's approach …

$$\omega^2 = \mathbf{k}^2 + \Pi(\omega, \mathbf{k}, \mu)$$

What happens for two pseudoscalar states π, π' ?

Take masses $m_\pi^2(\mu) = 0 \qquad m_{\pi'}^2(\mu)\Big|_{\mu\to\mu_{\mathrm{crit}}} \to 0$

and wave function normalizations $Z_\pi \approx Z_{\pi'} \approx 1$

$$\Pi(\omega, \mathbf{k}, \mu) = -\frac{\left(\omega^2 - \mathbf{k}^2\right)^2}{m_{\pi'}^2(\mu) - 2\left(\omega^2 - \mathbf{k}^2\right)}$$

Has a pole in the narrow resonance approach and may change sign

Possible experimental signatures of P-parity breaking

a) Decays of higher-mass meson resonances (radial excitations) into pions. Resonances do not have a definite parity and the same heavy resonance can decay both in two and three pions . It will look like doubling of states.

b) At the very point of the phase transition leading to parity breaking one has six massless pion-like states. Approaching to it one finds an abnormally light and long-living resonance!
After phase transition the massless charged 'pseudoscalar' states remain as Goldstone bosons enhancing charged pion production, whereas the neutral scalar states become heavy.

c) One can search for enhancement of long-range correlations in the scalar channel in lattice simulations.

d) Modifications in the equation of state.

e) F_{Π} and extended PCAC: it is modified for massless charged pions giving an enhancement of electroweak decays.

f) Additional isospin breaking: $f_{\pi_0} \neq f_{\pi_\pm}$

Program for GSI SIS 200 ?
Compressed Baryon Matter (CBM)? NICA proposal (Dubna)

Schematic cross-section of neutron star

G. Baym / Nuclear Physics A590 (1995) 233c-248c