Introduction to Accelerator Physics

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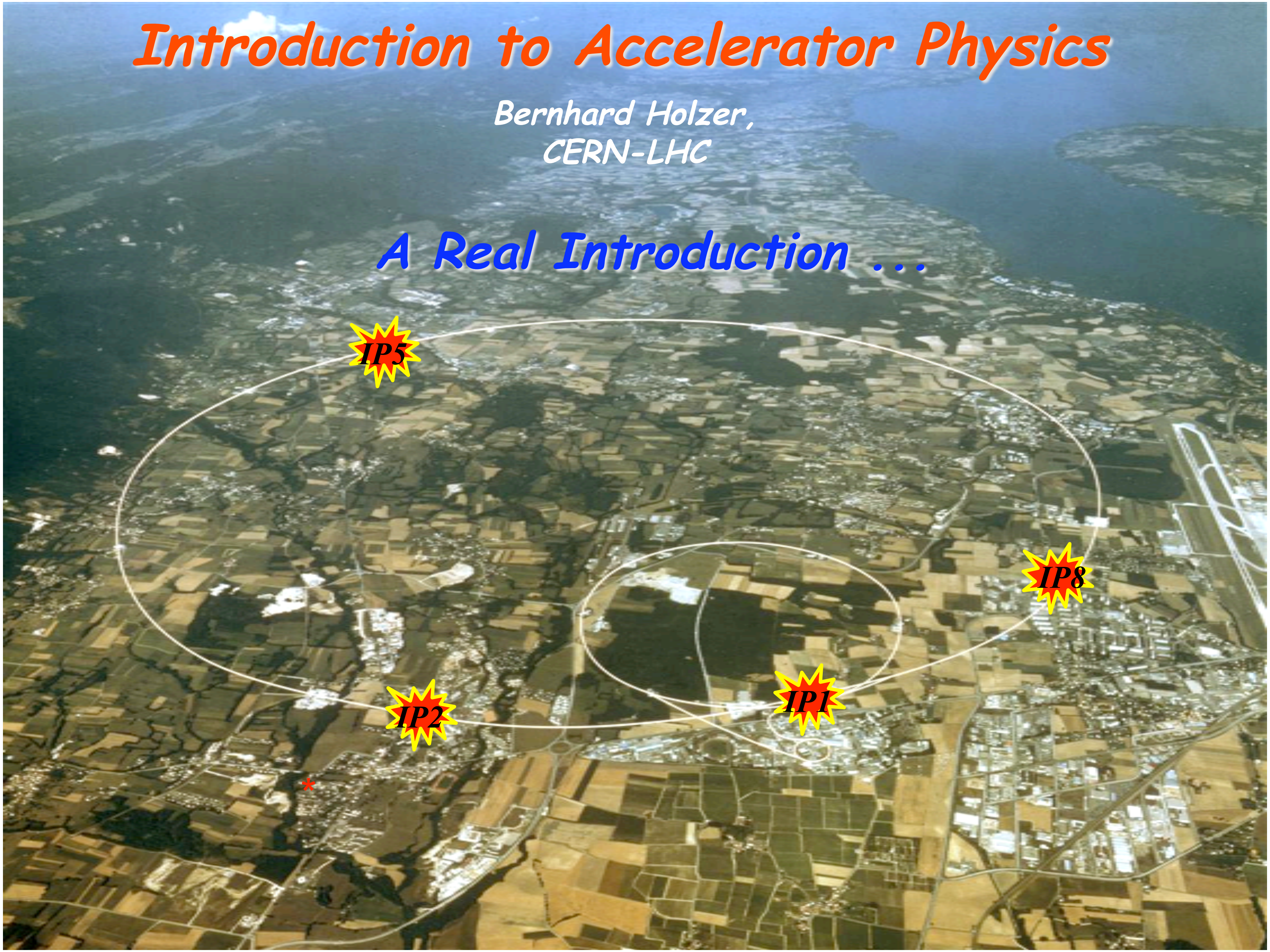
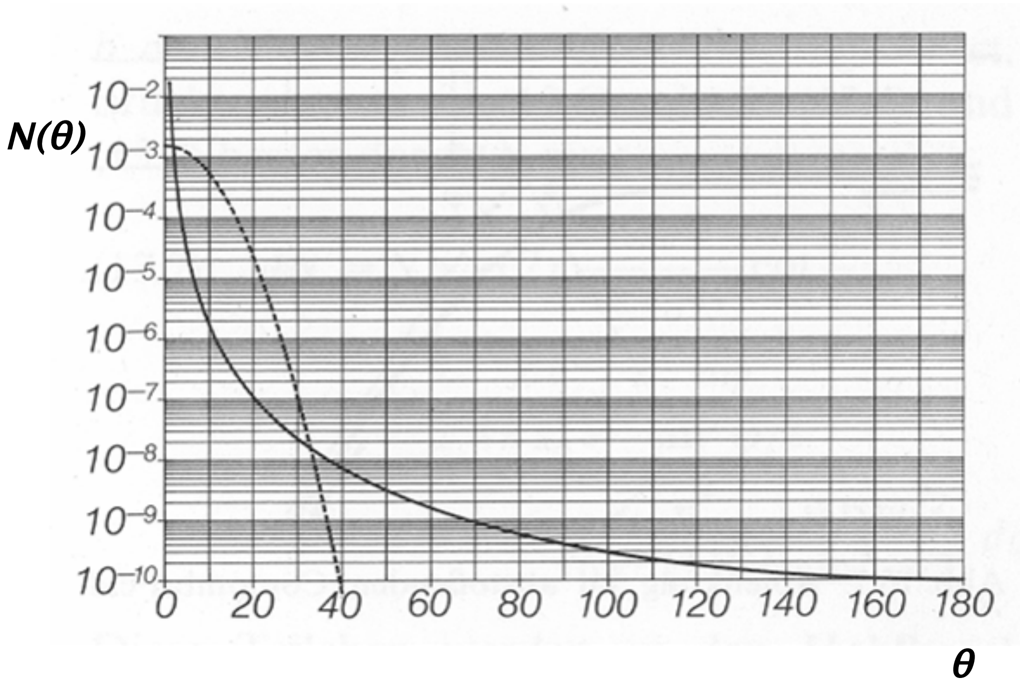
CERN-LHC

A Real Introduction ...



I.) A Bit of History

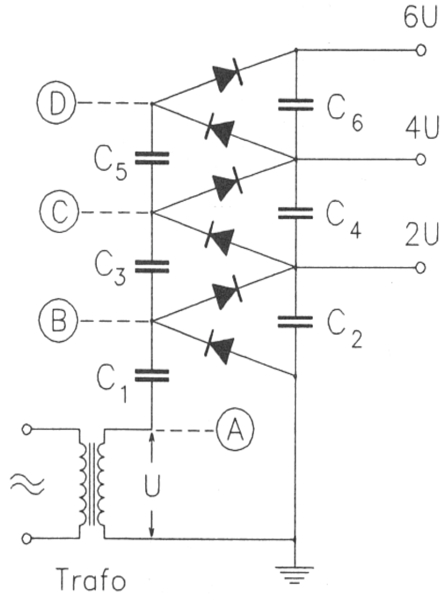
$$N(\theta) = \frac{N_i n t Z^2 e^4}{(8\pi\varepsilon_0)^2 r^2 K^2} * \frac{1}{\sin^4(\theta/2)}$$



Rutherford Scattering, 1906

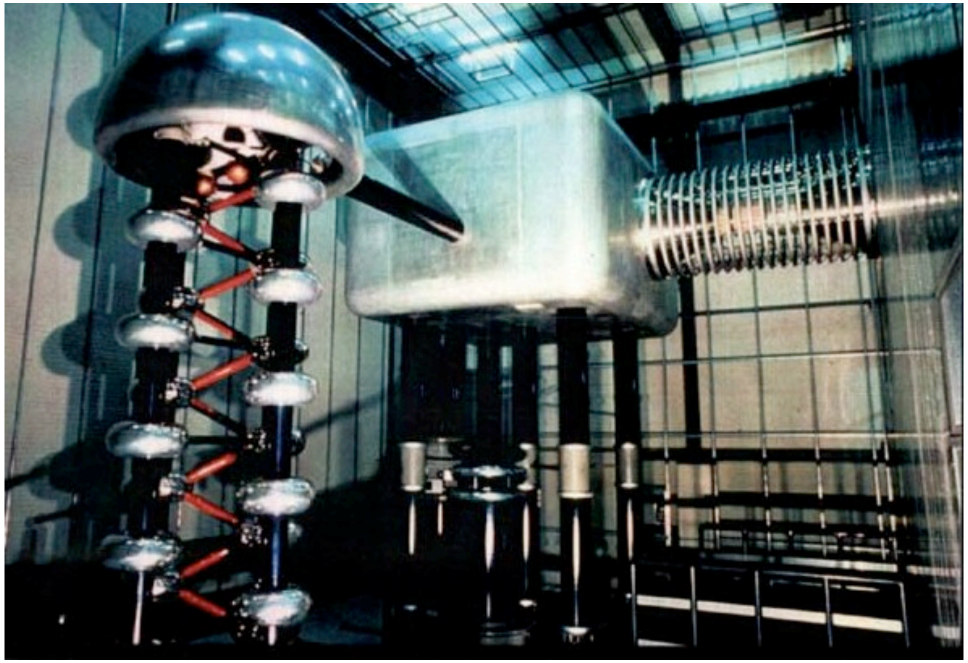
Using radioactive particle sources:
α-particles of some MeV energy

1.) Electrostatic Machines:
The Cockcroft-Walton Generator



1928: ***Encouraged by Rutherford Cockcroft and Walton start the design & construction of a high voltage generator to accelerate a proton beam***

1932: ***First particle beam (protons) produced for nuclear reactions: splitting of Li-nuclei with a proton beam of 400 keV***



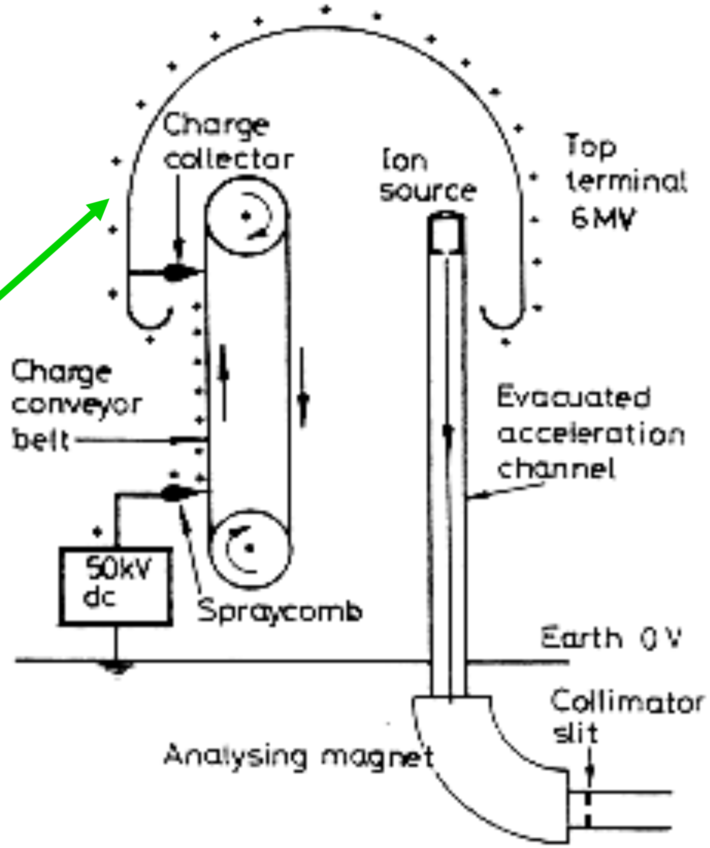
Particle source: ***Hydrogen discharge tube on 400 kV level***
Accelerator: ***evacuated glas tube***
Target: ***Li-Foil on earth potential***

Technically: rectifier circuit, built of capacitors and diodes (Greinacher)

robust, simple, on-knob machines
largely used in history as pre-accelerators for proton and ion beams
recently replaced by modern structures (RFQ)

2.) Electrostatic Machines:
(Tandem –) van de Graaff Accelerator (1930 ...)

creating high voltages by mechanical transport of charges



** Terminal Potential: U ≈ 12 ...28 MV*
using high pressure gas to suppress discharge (SF_6)

*Problems: * Particle energy limited by high voltage discharges*
** high voltage can only be applied once per particle ...*
... or twice ?

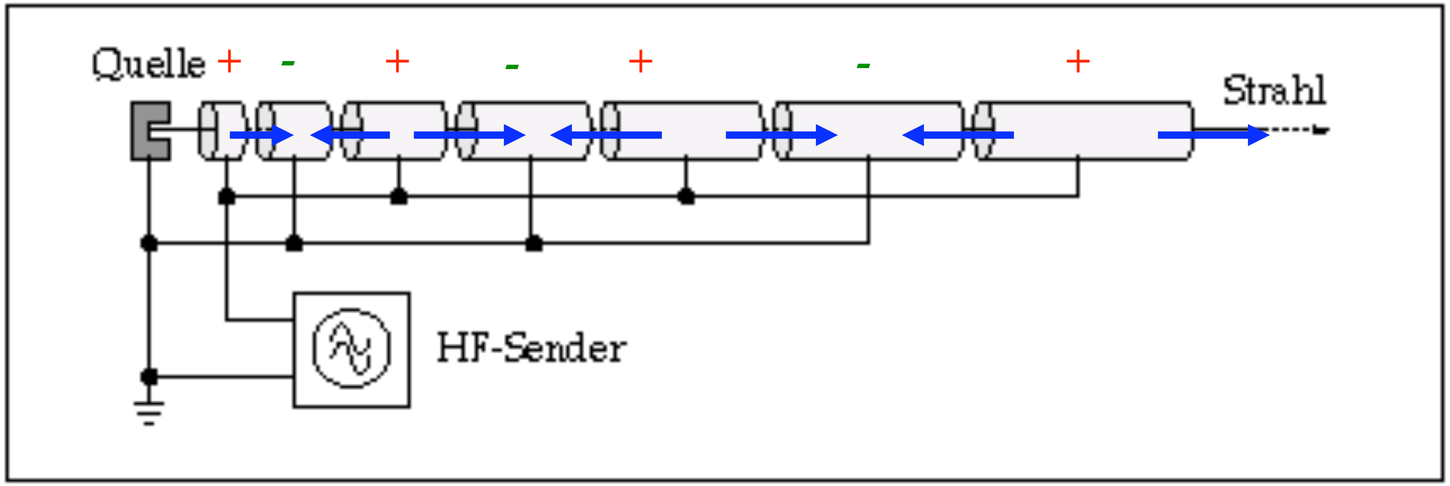
The „Tandem principle“: Apply the accelerating voltage twice ...
... by working with negative ions (e.g. H^-) and stripping the electrons in the centre of the structure

Example for such a „steam engine“: 12 MV-Tandem van de Graaff Accelerator at MPI Heidelberg

3.) The first RF-Accelerator: „Linac"

1928, Wideroe: ***how can the acceleration voltage be applied several times to the particle beam***

schematic Layout:



Energy gained after n acceleration gaps

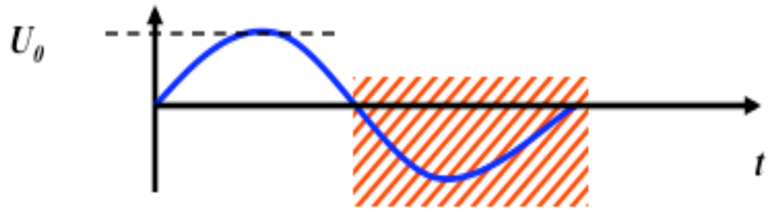
$$E_n = n * q * U_0 * \sin\psi_s$$

n *number of gaps between the drift tubes*
q *charge of the particle*
U_0 *Peak voltage of the RF System*
Ψ_S *synchronous phase of the particle*

**** acceleration of the proton in the first gap***
**** voltage has to be „flipped" to get the right sign in the second gap → RF voltage***
→ shield the particle in drift tubes during the negative half wave of the RF voltage

Wideroe-Structure: the drift tubes

shielding of the particles during the negative half wave of the RF



Time span of the negative half wave: $\tau_{RF}/2$

Length of the Drift Tube: $l_i = v_i * \frac{\tau_{rf}}{2}$

Kinetic Energy of the Particles $E_i = \frac{1}{2} m v^2$

$$\rightarrow v_i = \sqrt{2E_i/m}$$

$$l_i = \frac{1}{\nu_{rf}} * \sqrt{\frac{i * q * U_{0 * \sin\psi_s}}{2m}}$$

valid for non relativistic particles ...

Alvarez-Structure: 1946, surround the whole structure by a rf vessel

Energy: ≈ 20 MeV per Nucleon $\beta \approx 0.04 \ldots 0.6$, Particles: Protons/Ions

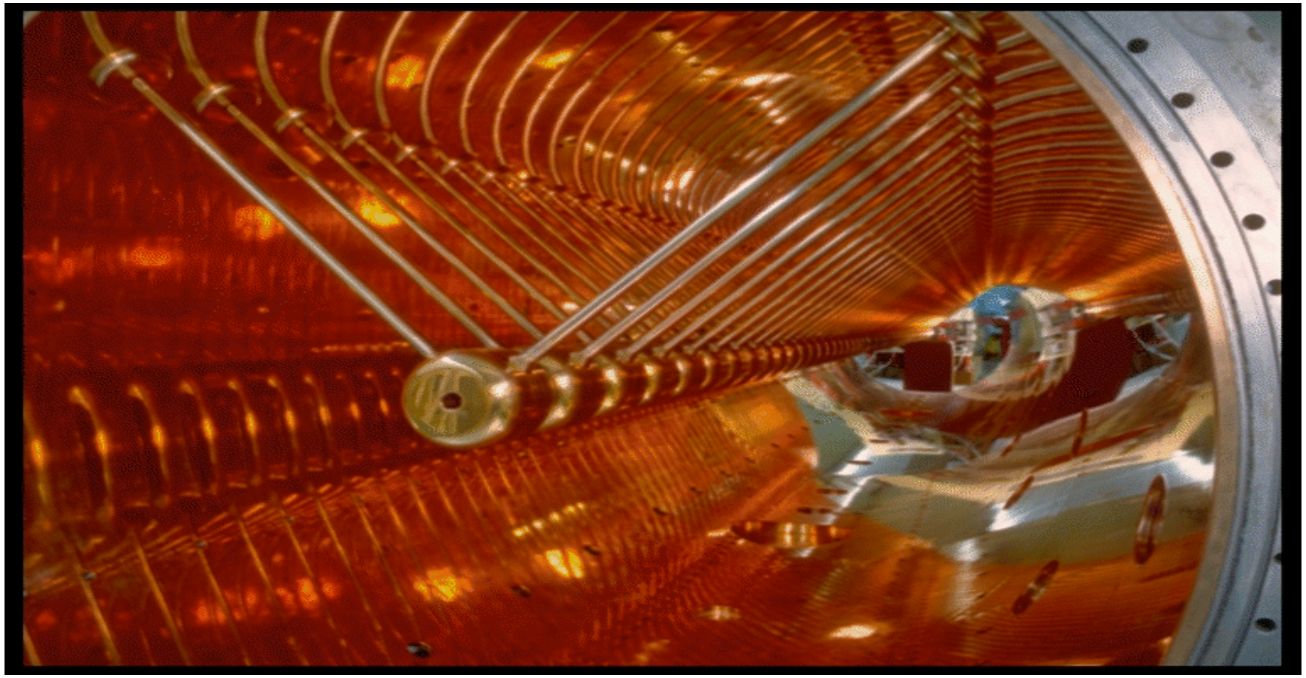
Accelerating structure of a Proton Linac (DESY Linac III)

$E_{total} = 988\ MeV$

$m_0c^2 = 938\ MeV$

$p = 310\ MeV\ /\ c$

$E_{kin} = 50\ MeV$



Beam energies

Energy Gain per „Gap": $W = q\,U_0 \sin \omega_{RF} t$

1.) reminder of some relativistic formula

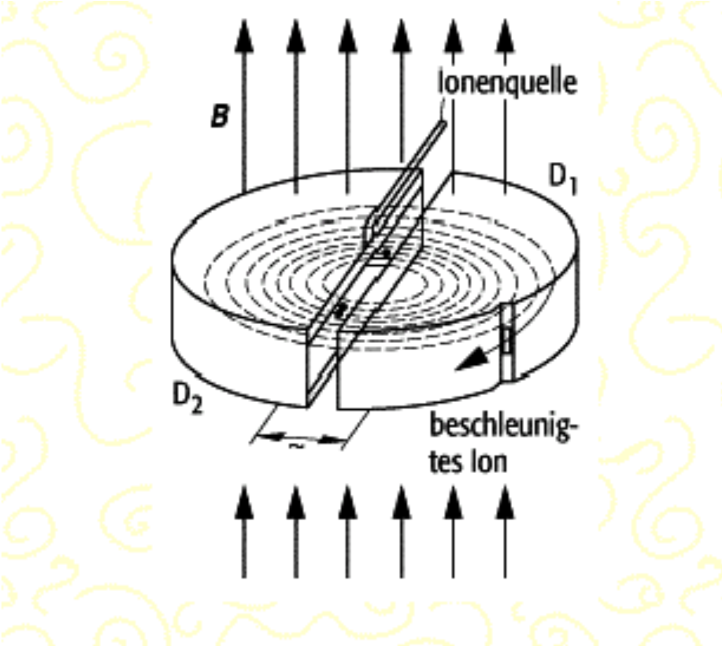
rest energy $E_0 = m_0c^2$

total energy $E = \gamma * E_0 = \gamma * m_0 c^2$

momentum $E^2 = c^2p^2 + m_0^2c^4$

kinetic energy $E_{kin} = E_{total} - m_0c^2$

4.) The Cyclotron: (Livingston / Lawrence ~1930)



Idea: Bend a Linac on a Spiral
Application of a constant magnetic field
keep B = const, RF = const

➔ ***Lorentzforce***

$$\vec{F} = q * (\vec{v} \times \vec{B}) = q * v * B$$

circular orbit

$$q * v * B = \frac{m * v^2}{R} \quad \longrightarrow \quad B * R = p / q$$

increasing radius for
increasing momentum
➔ ***Spiral Trajectory***

revolution frequency

$$\omega_z = \frac{q}{m} * B_z$$

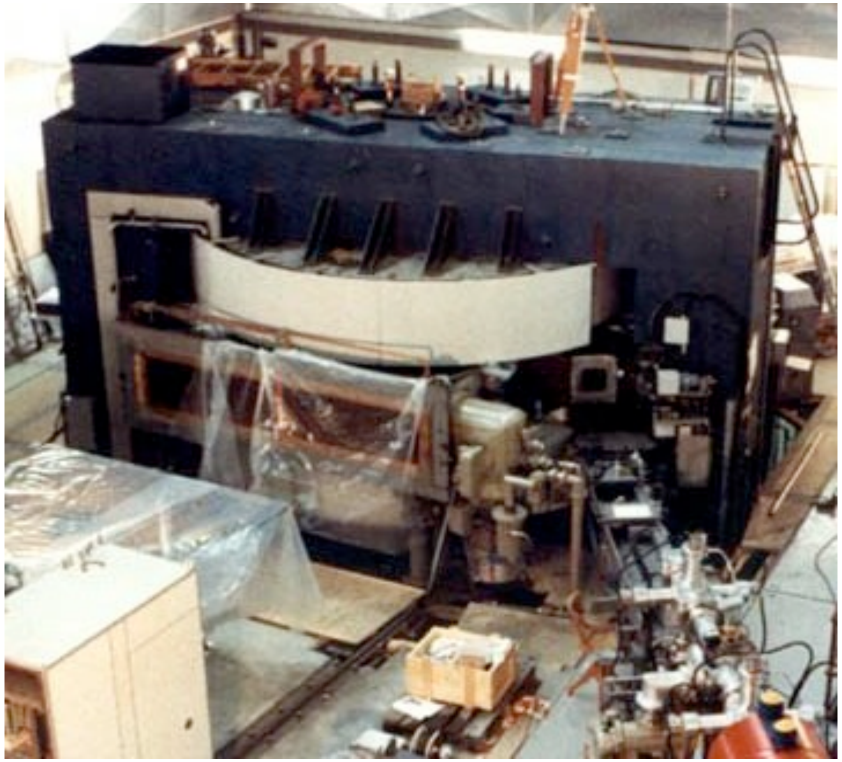
the cyclotron (rf-) frequency
is independent of the momentum

Cyclotron:

*! **ω is constant** for a given q & B*

*!! B*R = p/q*
large momentum → huge magnet

*!!!! **ω ~ 1/m ≠ const** works properly only for non relativistic particles*

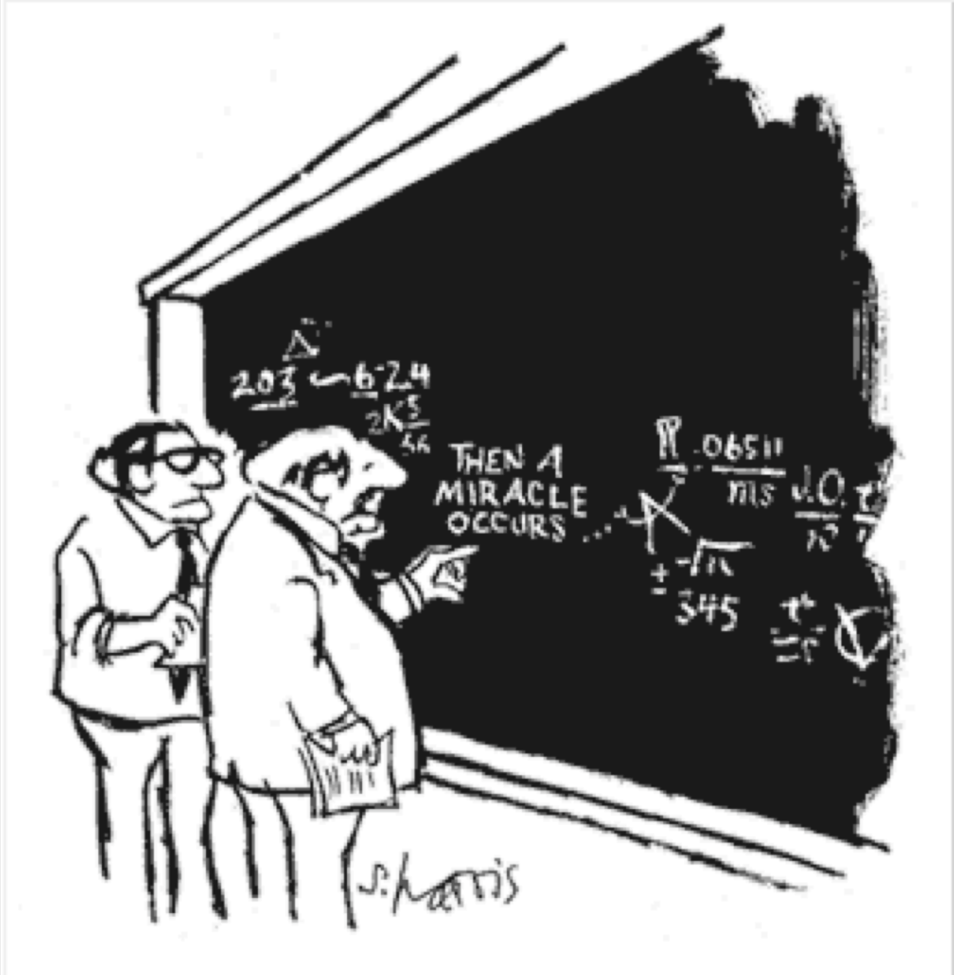


PSI Zurich

Application:
Work horses for medium energy protons
Proton / Ion Acceleration up to ≈ 60 MeV (proton energy)
nuclear physics
radio isotope production, proton / ion therapy

II.) A Bit of Theory

die grossen Speicherringe: „Synchrotrons“



"I THINK YOU SHOULD BE MORE EXPLICIT HERE IN STEP TWO."

1.) Introduction and Basic Ideas

„ ... *in the end and after all it should be a kind of circular machine*"

→ *need transverse deflecting force*

Lorentz force $\vec{F} = q * (\vec{E} + \vec{v} \times \vec{B})$

typical velocity in high energy machines: $v \approx c \approx 3*10^8 \, m/s$

Example:

$$B = 1\,T \quad \rightarrow \quad F = q * 3 * 10^8 \frac{m}{s} * 1 \frac{Vs}{m^2}$$

$$F = q * 300 \frac{MV}{m}$$

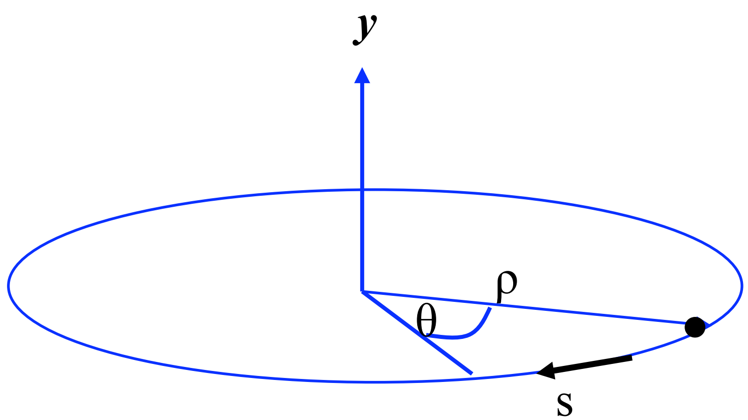
equivalent electrical field: E

technical limit for el. field:

$$E \leq 1 \frac{MV}{m}$$

old greek dictum of wisdom:
if you are clever, you use magnetic fields in an accelerator wherever it is possible.

The ideal circular orbit



circular coordinate system

condition for circular orbit:

Lorentz force $F_L = e\,v\,B$

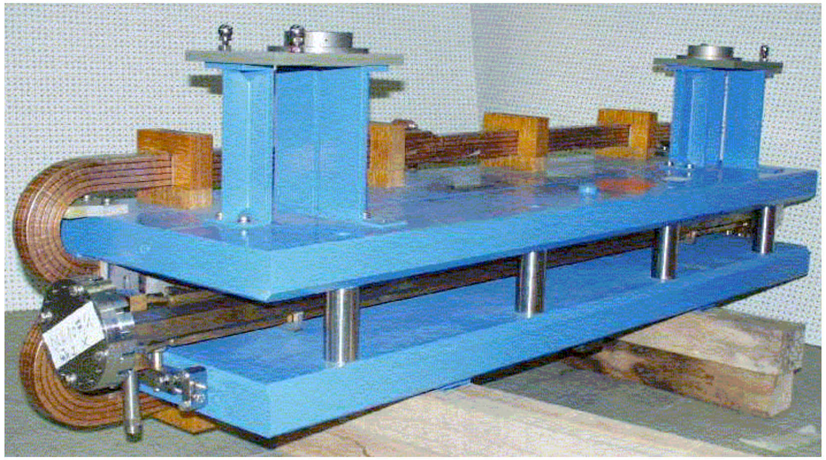
centrifugal force $F_{centr} = \frac{\gamma\, m_0\, v^2}{\rho}$

$$\frac{\gamma\, m_0\, v^2}{\rho} = e\,v\,B$$

$$\frac{p}{e} = B\,\rho$$

$B\rho$ = "beam rigidity"

2.) The Magnetic Guide Field



Dipole Magnets:

define the ideal orbit
homogeneous field created
by two flat pole shoes

$$B = \frac{\mu_0 \, n \, I}{h}$$

Normalise magnetic field to momentum:

$$\frac{p}{e} = B\rho \quad \longrightarrow \quad \frac{1}{\rho} = \frac{eB}{p}$$

convenient units:

$$B = [T] = \left[\frac{Vs}{m^2}\right] \qquad p = \left[\frac{GeV}{c}\right]$$

Example LHC:

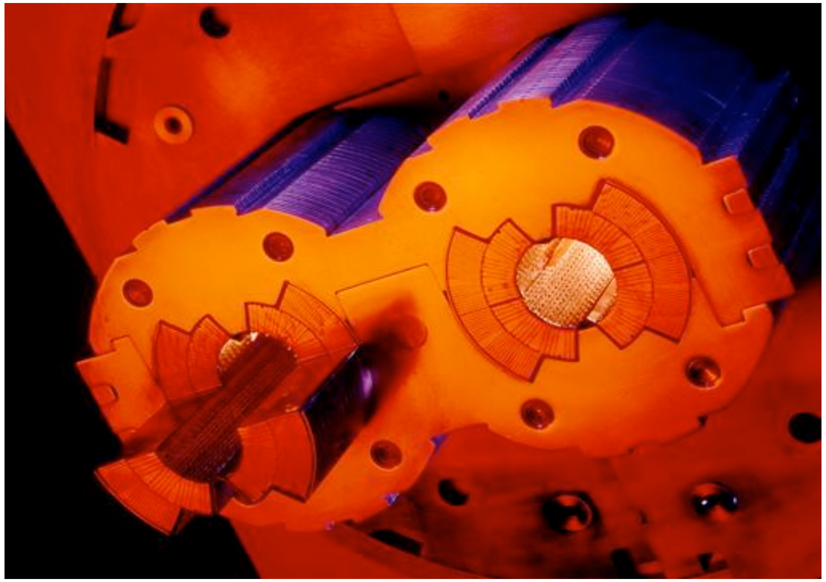
$$B = 8.3\,T$$

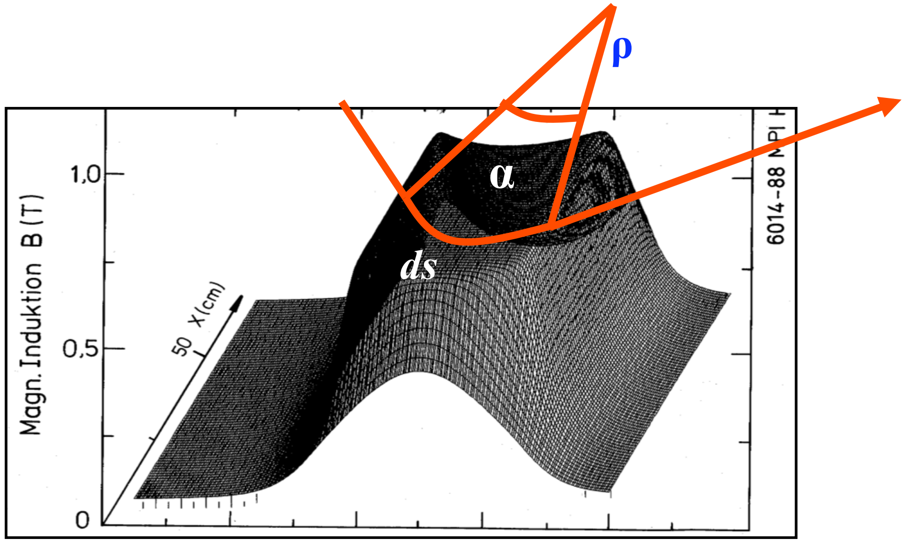
$$p = 7000\,\frac{GeV}{c}$$

$$\frac{1}{\rho} = e\,\frac{8.3\ Vs/m^2}{7000 * 10^9\ eV/c} = \frac{8.3\ s * 3 * 10^8\ m/s}{7000 * 10^9\ m^2}$$

$$\frac{1}{\rho} = 0.333\,\frac{8.3}{7000}\ 1/m$$

The Magnetic Guide Field





field map of a storage ring dipole magnet

$\rho = 2.53\ km$ → $2\pi\rho = 17.6\ km$

$\approx 66\%$

$B \approx 1 \dots 8\ T$

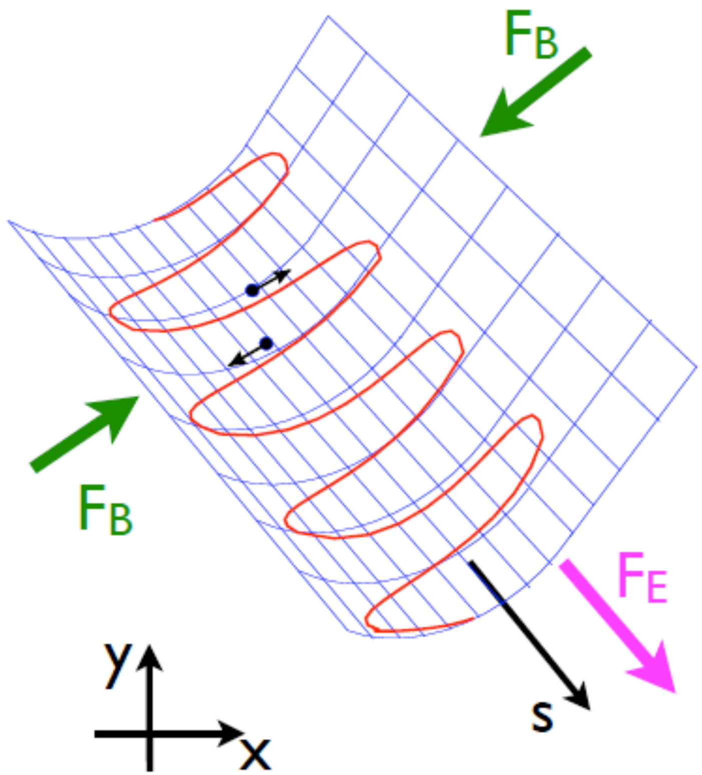
rule of thumb: $\frac{1}{\rho} \approx 0.3 \frac{B[T]}{p[GeV/c]}$

„normalised bending strength“

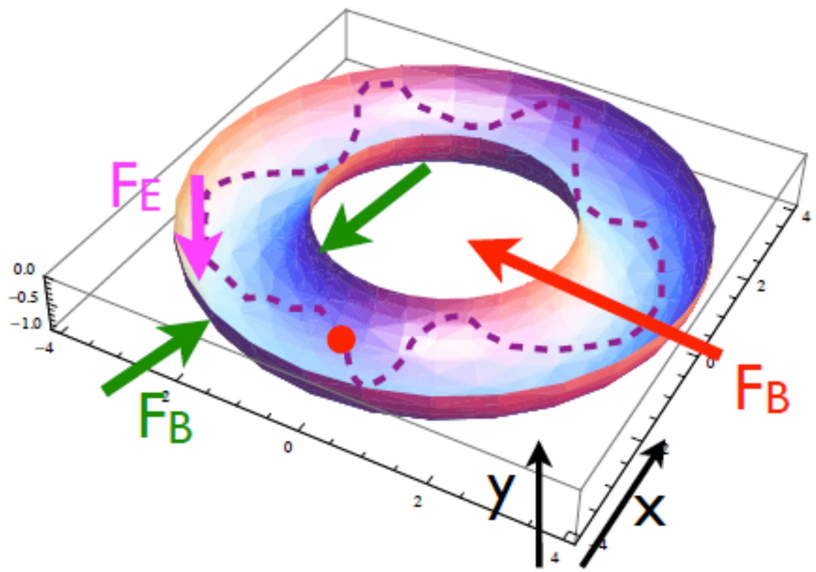
2.) Focusing Properties – Transverse Beam Optics

$$\overline{F(t)} = \underbrace{q\,\Big(\overline{E(t)}}_{F_E} + \underbrace{\overline{v(t)} \otimes \overline{B(t)}}_{F_B}\Big)$$

Linear Accelerator

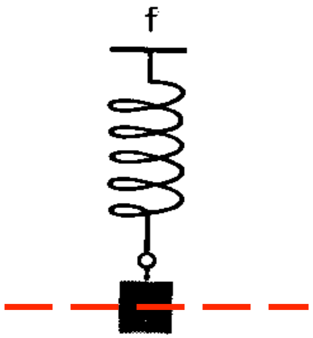


Circular Accelerator



2.) Focusing Properties – Transverse Beam Optics

classical mechanics:
pendulum



there is a restoring force, proportional to the elongation x:

$$m * \frac{d^2 x}{dt^2} = -c * x$$

general solution: free harmonic oszillation

$$x(t) = A * \cos(\omega t + \varphi)$$

Storage Ring:** we need a **Lorentz force** that rises as a function of the **distance to .... ?

.... the design orbit

$$F(x) = q * v * B(x)$$

Quadrupole Magnets:

required: *focusing forces to keep trajectories in vicinity of the ideal orbit*

linear increasing Lorentz force

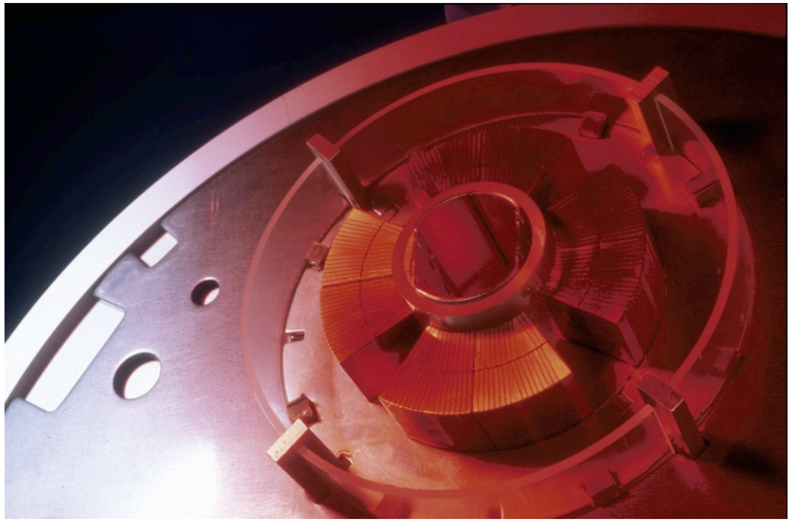
linear increasing magnetic field $B_y = g\,x \qquad B_x = g\,y$

normalised quadrupole field:

$$\longrightarrow \qquad k = \frac{g}{p/e}$$

simple rule:

$$k = 0.3\,\frac{g(T/m)}{p(GeV/c)}$$



LHC main quadrupole magnet

$$g \approx 25 \ldots 220 \;\; T/m$$

what about the vertical plane:
... Maxwell

$$\vec{\nabla} \times \vec{B} = \vec{j} + \frac{\partial \vec{E}}{\partial t} = 0 \qquad \Rightarrow \qquad \frac{\partial B_y}{\partial x} = \frac{\partial B_x}{\partial y} = g$$

Focusing forces and particle trajectories:

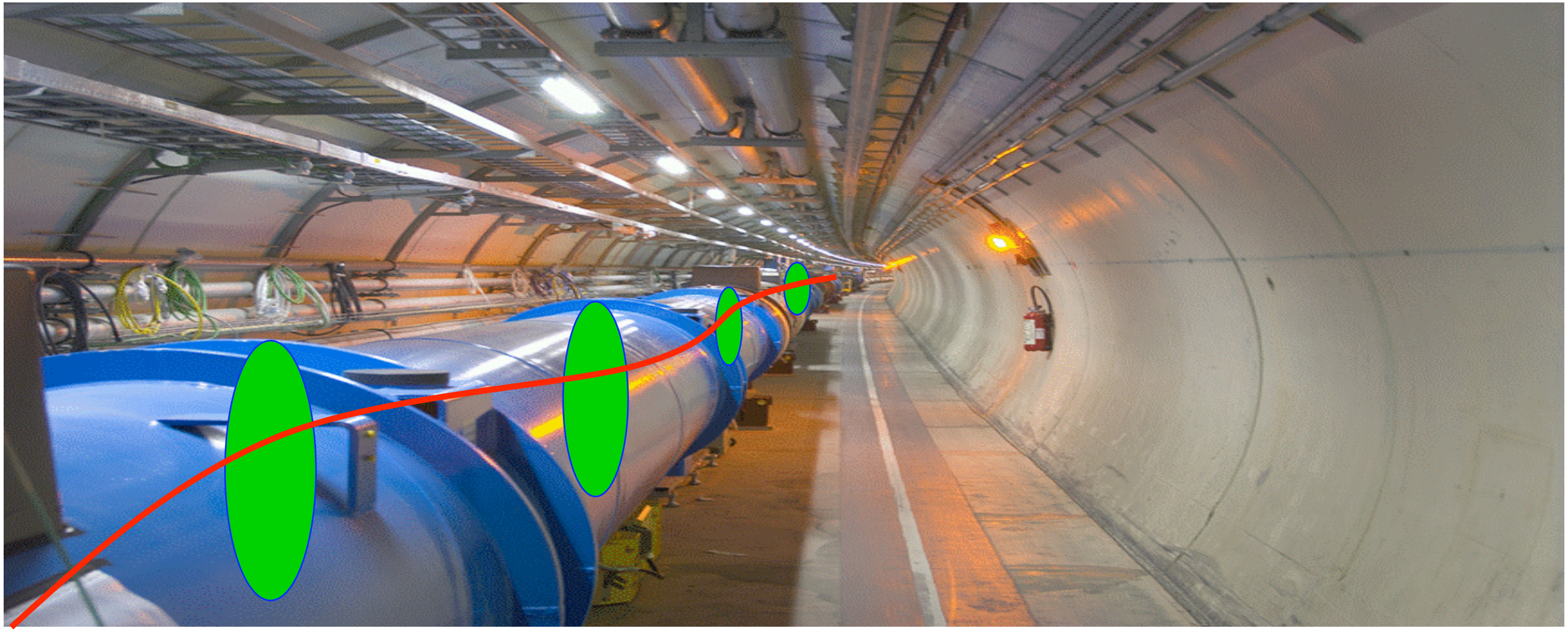
normalise magnet fields to momentum
(remember: $B*\rho = p/q$ *)*

Dipole Magnet

$$\frac{B}{p/q} = \frac{B}{B\rho} = \frac{1}{\rho}$$

Quadrupole Magnet

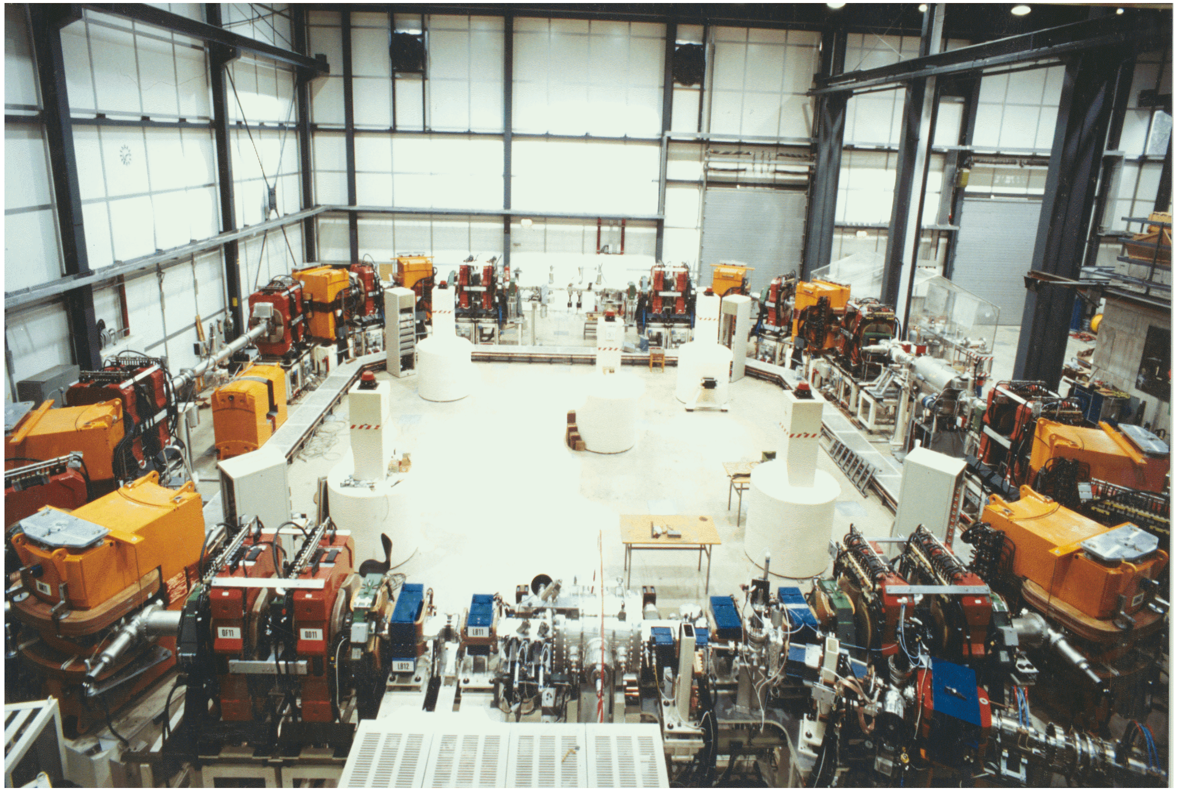
$$k := \frac{g}{p/q}$$



3.) The Equation of Motion:

$$\frac{B(x)}{p/e} = \frac{1}{\rho} + k\,x + \frac{1}{2!}m\,x^2 + \frac{1}{3!}n\,x^3 + \ldots$$

only terms linear in x, y taken into account *dipole fields*
quadrupole fields



Separate Function Machines:

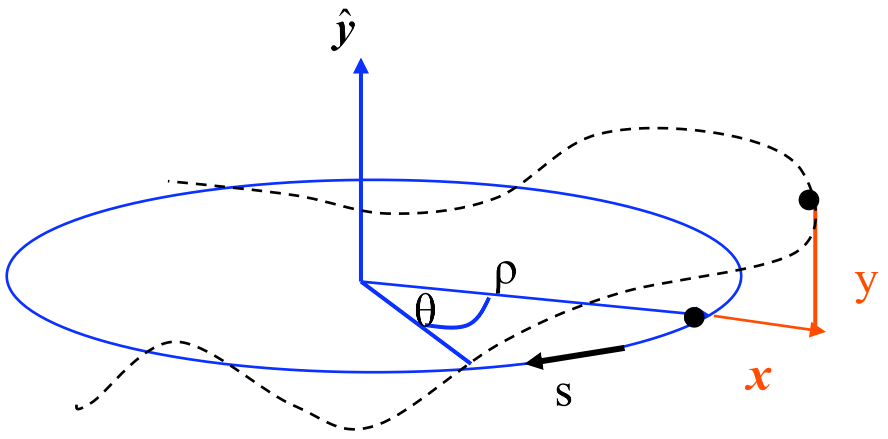
Split the magnets and optimise them according to their job:

bending, focusing etc

Example:
heavy ion storage ring TSR

* *man sieht nur dipole und quads → linear*

The Equation of Motion:



* *Equation for the horizontal motion:*

$$x'' + x\,(\frac{1}{\rho^2} + k) = 0$$

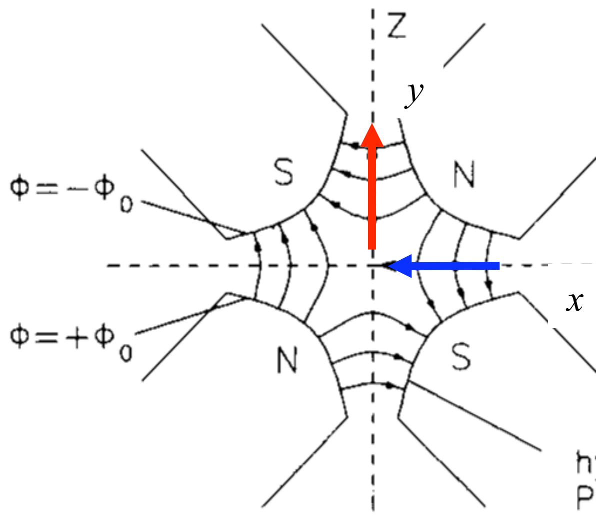
x = particle amplitude
x' = angle of particle trajectory (wrt ideal path line)

* *Equation for the vertical motion:*

$\frac{1}{\rho^2} = 0$ *no dipoles … in general …*

$k \leftrightarrow -k$ *quadrupole field changes sign*

$$y'' - k\,y = 0$$



4.) Solution of Trajectory Equations

Define … hor. plane: K= 1/ρ² + k

… vert. Plane: K = - k

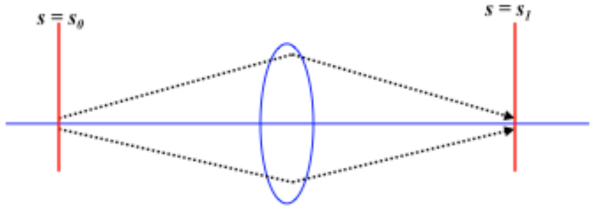
$$x'' + K\,x = 0$$

Differential Equation of harmonic oscillator … with spring constant K

Ansatz: *Hor. Focusing Quadrupole K > 0:*

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}s) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \cdot \sin(\sqrt{|K|}s) + x_0' \cdot \cos(\sqrt{|K|}s)$$



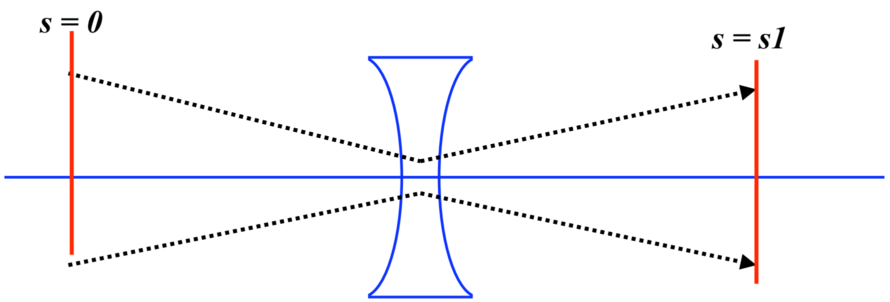
For convenience expressed in matrix formalism:

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s1} = M_{foc} * \begin{pmatrix} x \\ x' \end{pmatrix}_{s0}$$

$$M_{foc} = \begin{pmatrix} \cos\left(\sqrt{|K|}\,l\right) & \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}\,l\right) \\ -\sqrt{|K|}\sin\left(\sqrt{|K|}\,l\right) & \cos\left(\sqrt{|K|}\,l\right) \end{pmatrix}$$

hor. defocusing quadrupole:

$$x'' - K\,x = 0$$



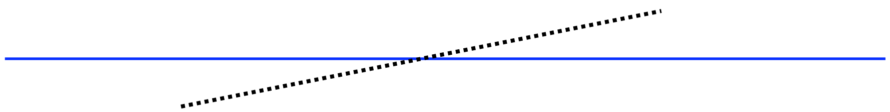
Ansatz: ***Remember from school***

$$x(s) = a_1 \cdot \cosh(\omega s) + a_2 \cdot \sinh(\omega s)$$

$$M_{defoc} = \begin{pmatrix} \cosh\sqrt{|K|}l & \frac{1}{\sqrt{|K|}}\sinh\sqrt{|K|}l \\ \sqrt{|K|}\sinh\sqrt{|K|}l & \cosh\sqrt{|K|}l \end{pmatrix}$$

drift space:

$$K = 0$$



$$x(s) = x'_0 * s$$

$$M_{drift} = \begin{pmatrix} 1 & l \\ 0 & 1 \end{pmatrix}$$

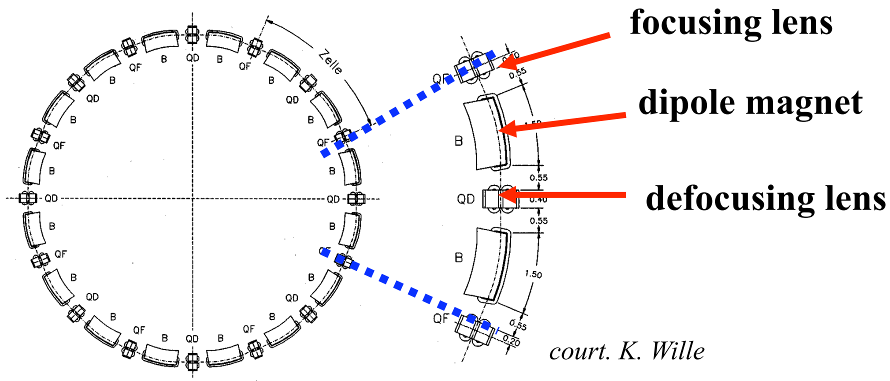
! with the assumptions made, the motion in the horizontal and vertical planes are independent „ ... the particle motion in x & y is uncoupled“

Transformation through a system of lattice elements

combine the single element solutions by multiplication of the matrices

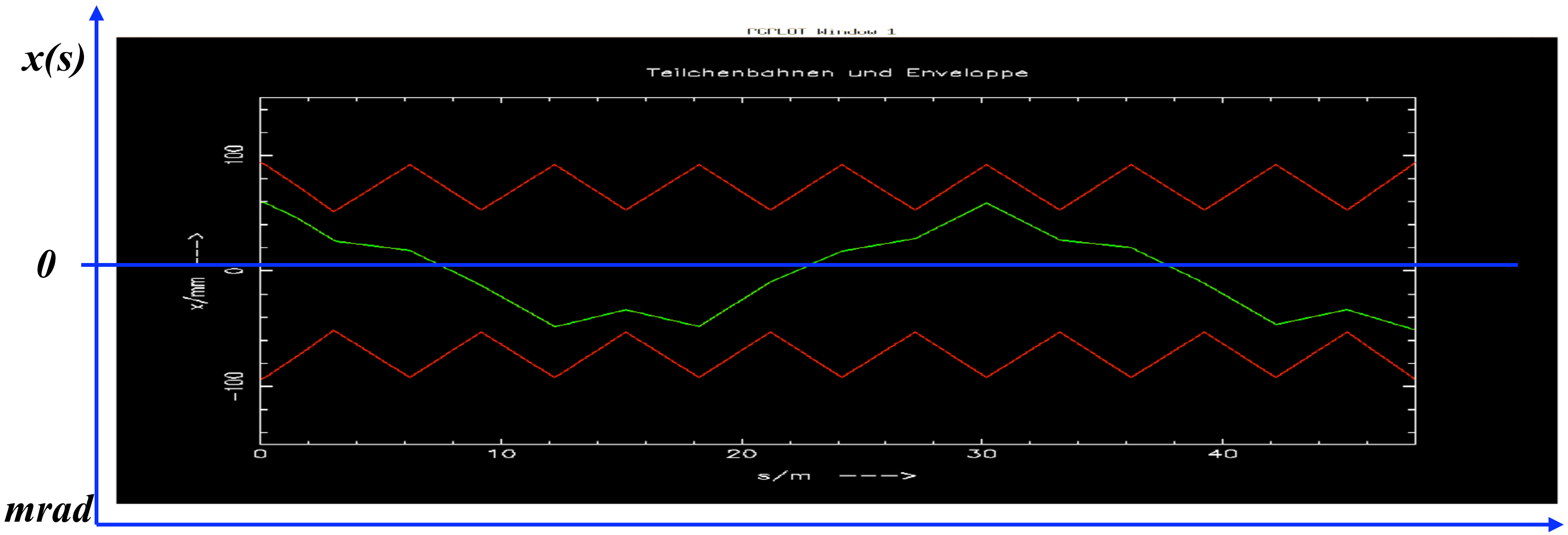
$$M_{total} = M_{QF} * M_D * M_{QD} * M_{Bend} * M_{D*} .....$$

$$\begin{pmatrix} x \\ x' \end{pmatrix}_{s2} = M(s_2, s_1) * \begin{pmatrix} x \\ x' \end{pmatrix}_{s1}$$



court. K. Wille

in each accelerator element the particle trajectory corresponds to the movement of a harmonic oscillator „



typical values in a strong foc. machine:

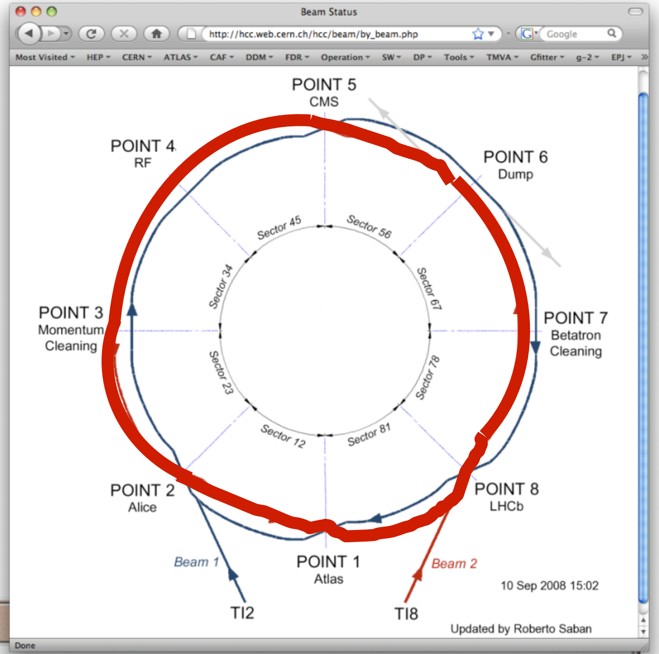
$x \approx mm$, $x' \leq mrad$

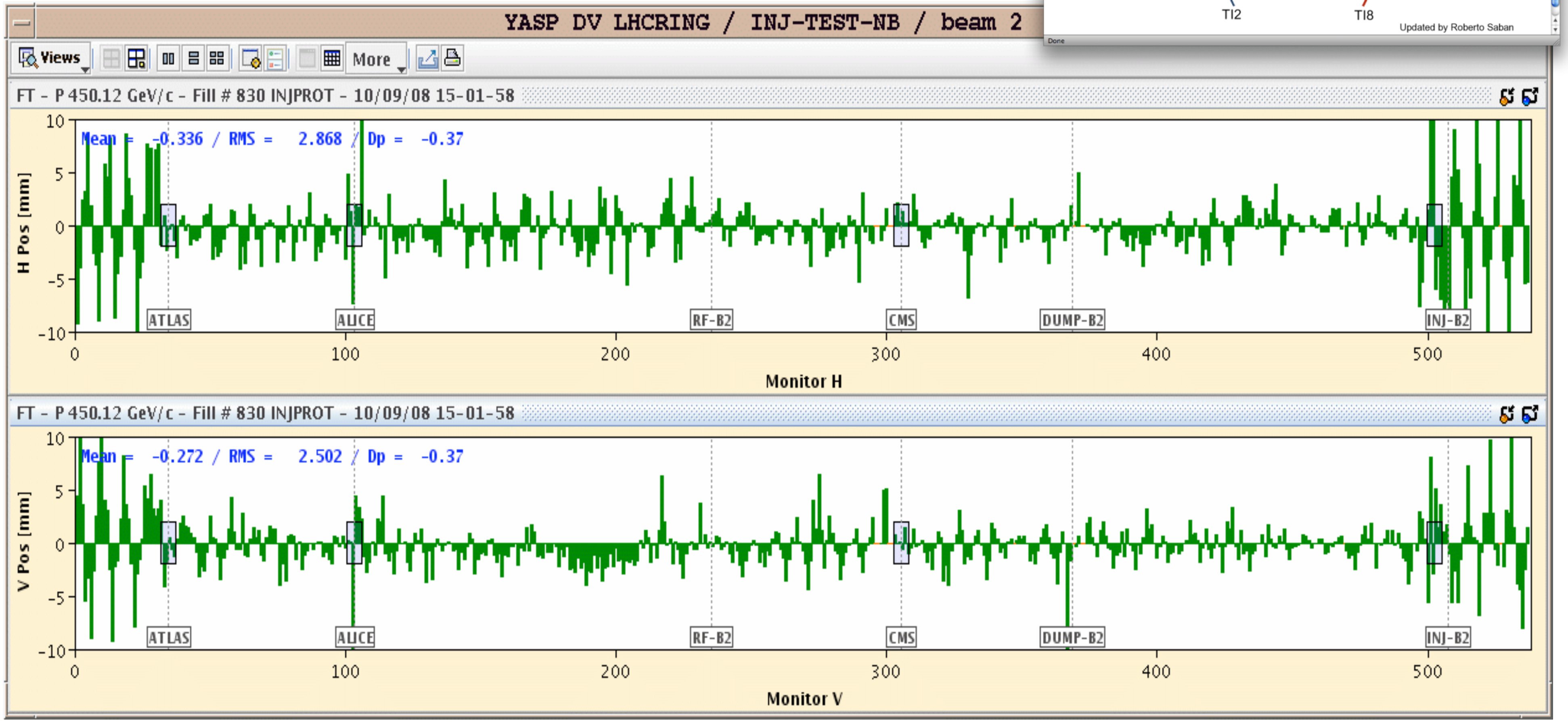
LHC Operation: Beam Commissioning

First turn steering "by sector:"

- One beam at the time
- Beam through 1 sector (1/8 ring),

correct trajectory, open collimator and move on.

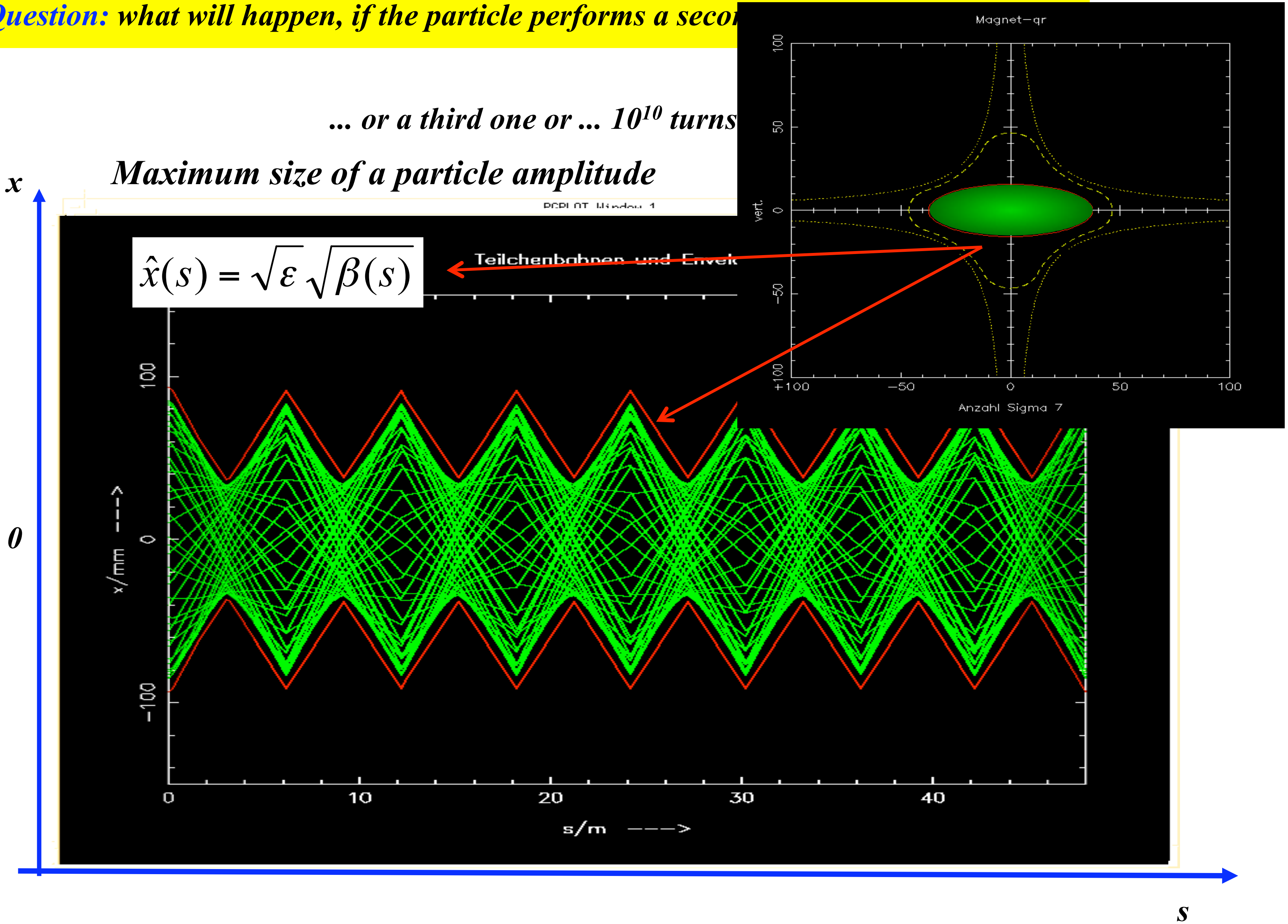




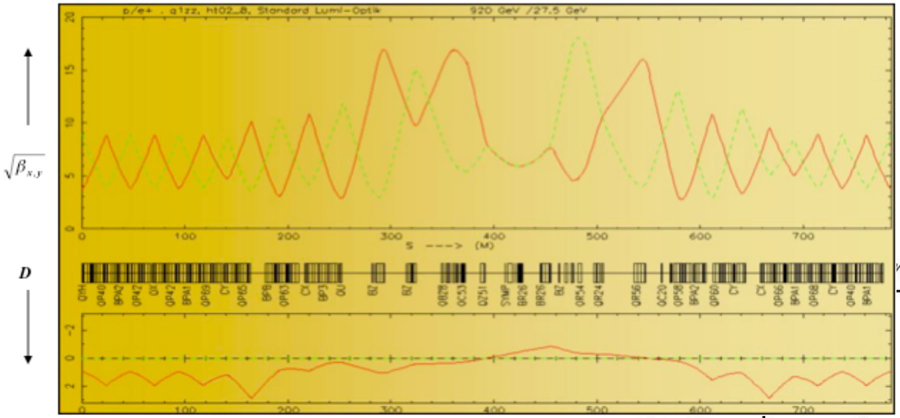
Question: ***what will happen, if the particle performs a seco***

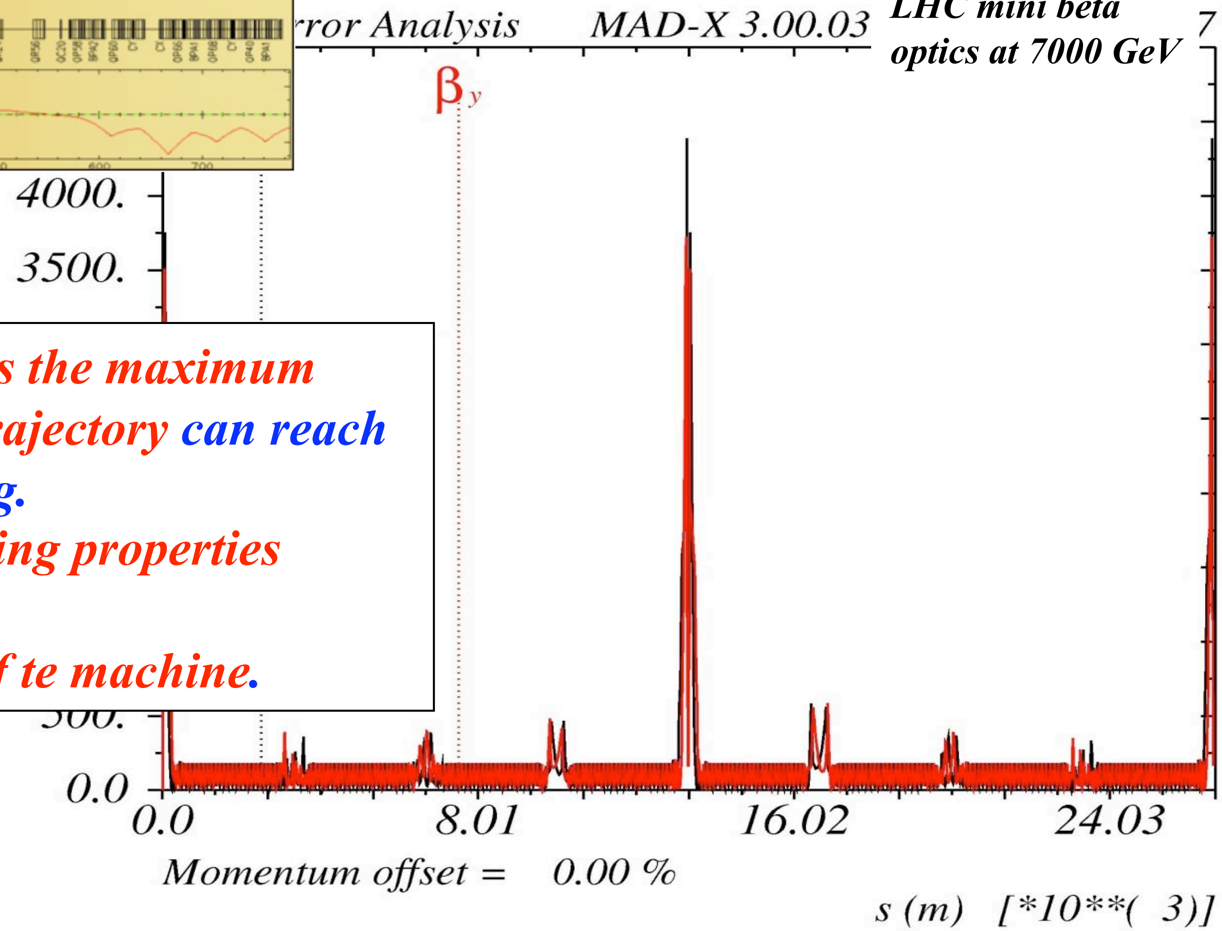
... or a third one or ... 10^{10} turns

Maximum size of a particle amplitude



The Beta Function: Lattice Design & Beam Optics



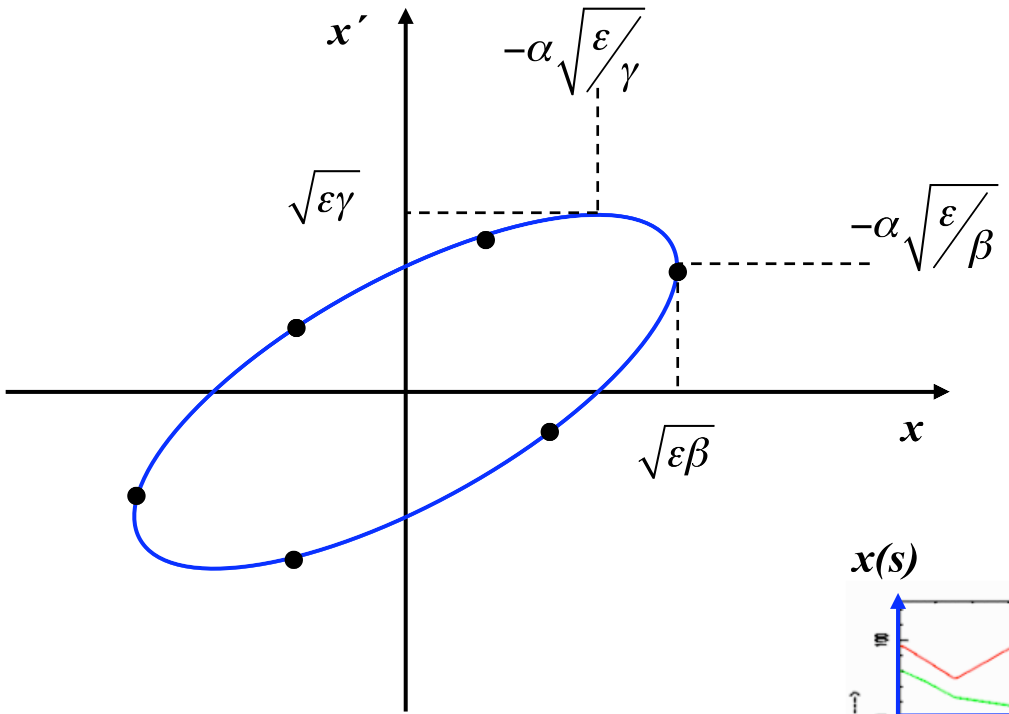


LHC mini beta optics at 7000 GeV

The beta function determines the maximum amplitude a single particle trajectory can reach at a given position in the ring.
It is determined by the focusing properties of the latttice
and follows the periodicity of te machine.

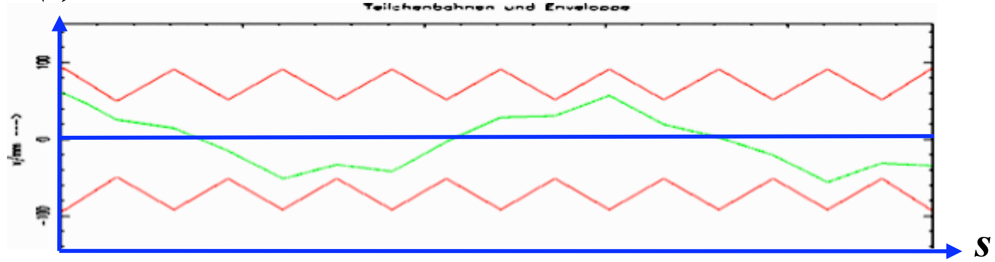
Beam Emittance and Phase Space Ellipse

$$\varepsilon = \gamma(s) * x^2(s) + 2\alpha(s)x(s)x'(s) + \beta(s)x'(s)^2$$



Liouville: ***in reasonable storage rings area in phase space is constant.***

A = π*ε=const

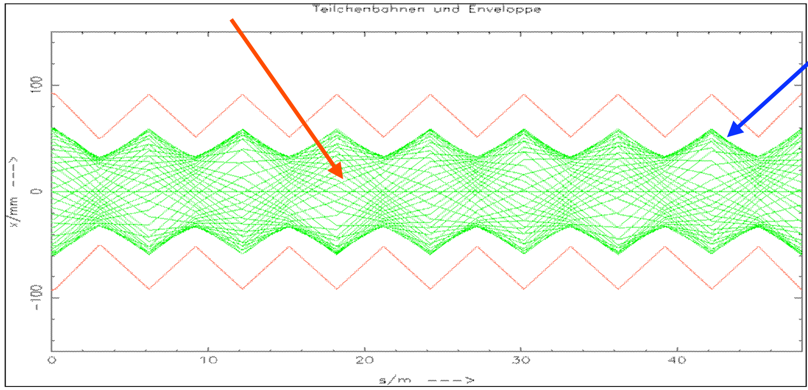


ε beam emittance = ***woozilycity*** ***of the particle ensemble,*** ***intrinsic beam parameter,***
cannot be changed by the foc. properties.

Scientifiquely spoken: ***area covered*** ***in transverse x, x´ phase space*** ***… and it is constant !!!***

Emittance of the Particle Ensemble:

$$x(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)} \cdot \cos(\Psi(s) + \phi) \qquad \hat{x}(s) = \sqrt{\varepsilon}\,\sqrt{\beta(s)}$$



single particle trajectories, N ≈ 10 [11] per bunch

Gauß Particle Distribution:

$$\rho(x) = \frac{N \cdot e}{\sqrt{2\pi}\,\sigma_x} \cdot e^{-\frac{1}{2}\frac{x^2}{\sigma_x^2}}$$

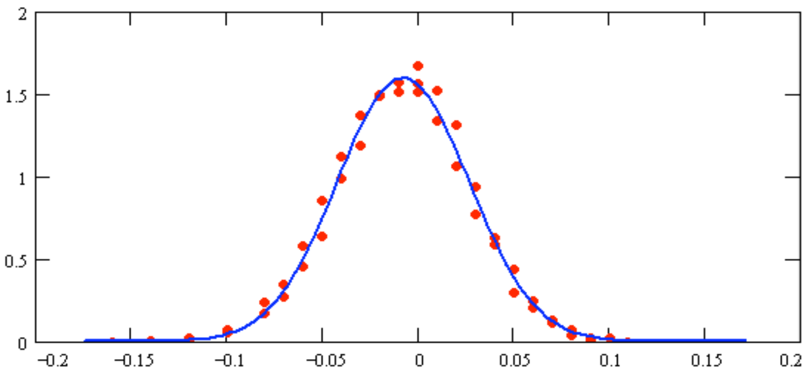
particle at distance 1 σ from centre
↔ 68.3 % of all beam particles

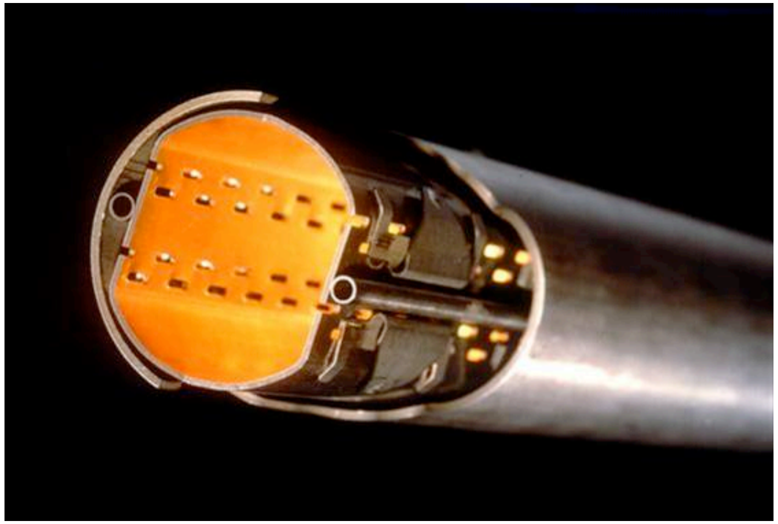
LHC:

$$\beta = 180\,m$$

$$\varepsilon = 5 * 10^{-10}\,m\,rad$$

$$\sigma = \sqrt{\varepsilon * \beta} = \sqrt{5 * 10^{-10}\,m * 180\,m} = 0.3\,mm$$

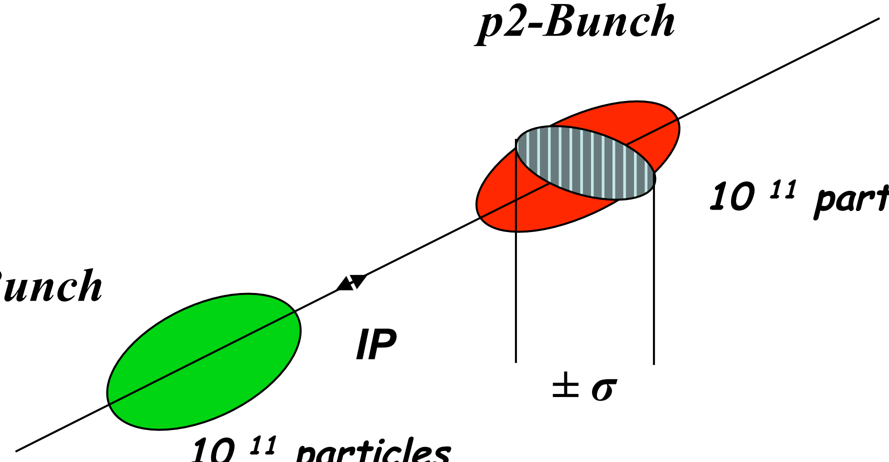




aperture requirements: $r_0 = 17 * \sigma$

10.) Luminosity

$$R = L * \Sigma_{react}$$



Example: Luminosity run at LHC

$$\beta_{x,y} = 0.55\ m \qquad f_0 = 11.245\ kHz$$

$$\varepsilon_{x,y} = 5 * 10^{-10}\ rad\ m \qquad n_b = 2808$$

$$\sigma_{x,y} = 17\ \mu m$$

$$L = \frac{1}{4\pi e^2 f_0 n_b} * \frac{I_{p1} I_{p2}}{\sigma_x \sigma_y}$$

$$I_p = 584\ mA$$

$$L = 1.0 * 10^{34}\ \frac{1}{cm^2 s}$$

Luminosity optimization

$$L = \frac{N_1 N_2 f_{rev} N_b}{2\pi\sqrt{\sigma_{1x}^2+\sigma_{2x}^2}\sqrt{\sigma_{1y}^2+\sigma_{2y}^2}} F \cdot W$$

N_i = number of protons/bunch
N_b = number of bunches
f_{rev} = revolution frequency
σ_{ix} = beam size along x for beam i
σ_{iy} = beam size along y for beam i

F is a pure **crossing angle (*Φ*) contribution**:

$$F = \frac{1}{\sqrt{1+2\frac{\sigma_s^2}{\sigma_{1x}^2+\sigma_{2x}^2}\tan^2\frac{\phi}{2}}}$$

F_{LHC} = 0.836

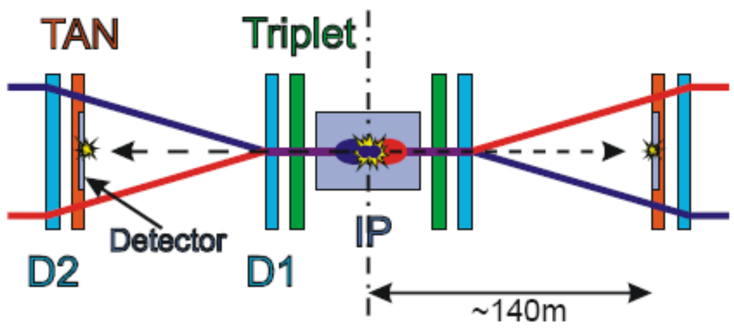
... cannot be avoided

25 ns

W is a **pure beam offset contribution**.

... can be avoided by careful tuning

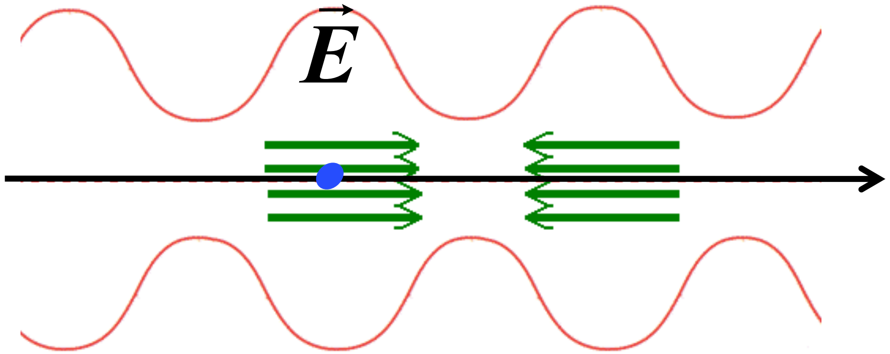
$$W = e^{-\frac{(d_2-d_1)^2}{2(\sigma_{x1}^2+\sigma_{x2}^2)}}$$

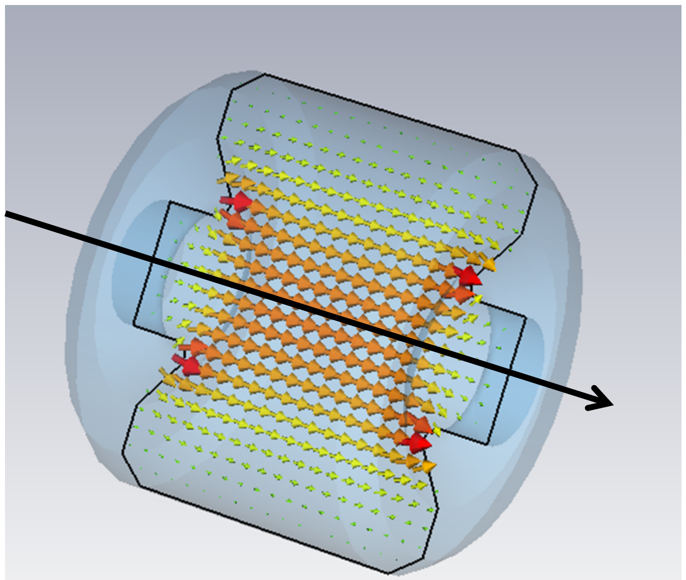


13.) The Acceleration

Where is the acceleration?

Install an RF accelerating structure in the ring:





B. Salvant
N. Biancacci

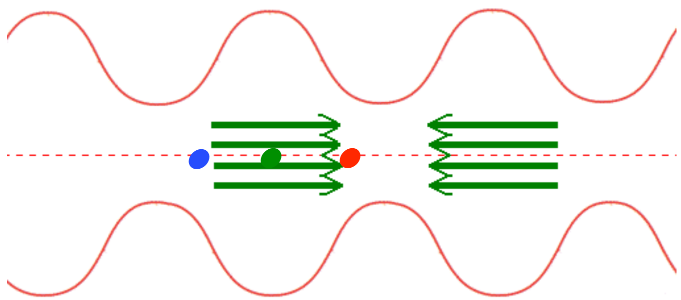
14.) The Acceleration for Δp/p≠0

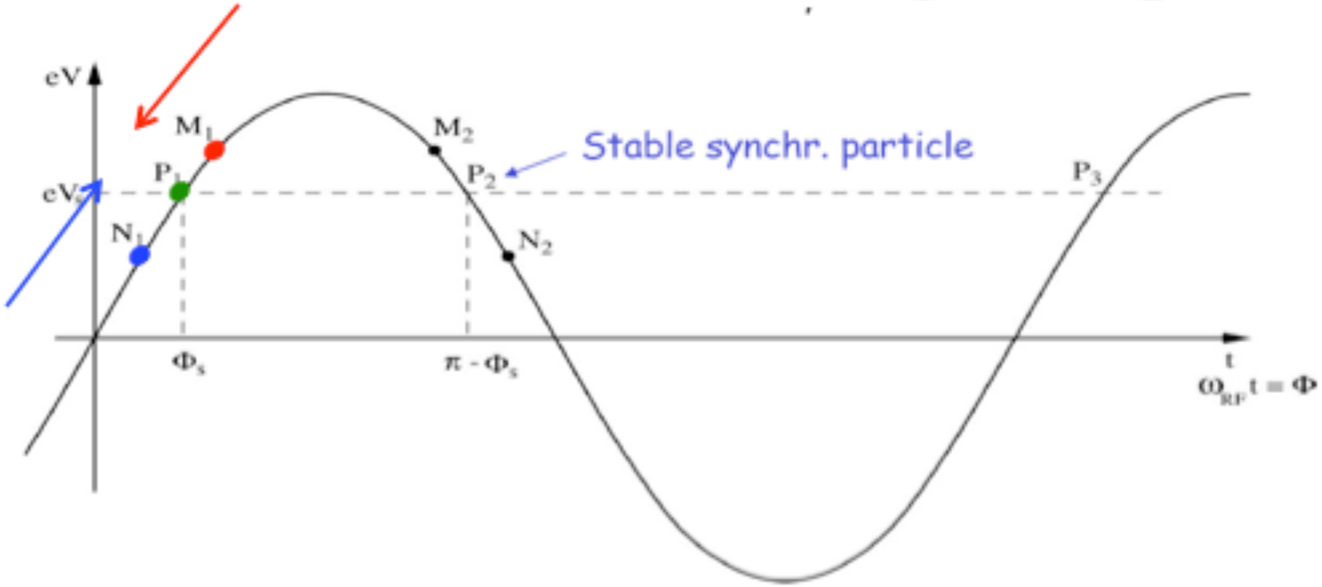
"Phase Focusing" below transition

ideal particle ●

particle with Δp/p > 0 ● *faster*

particle with Δp/p < 0 ● *slower*



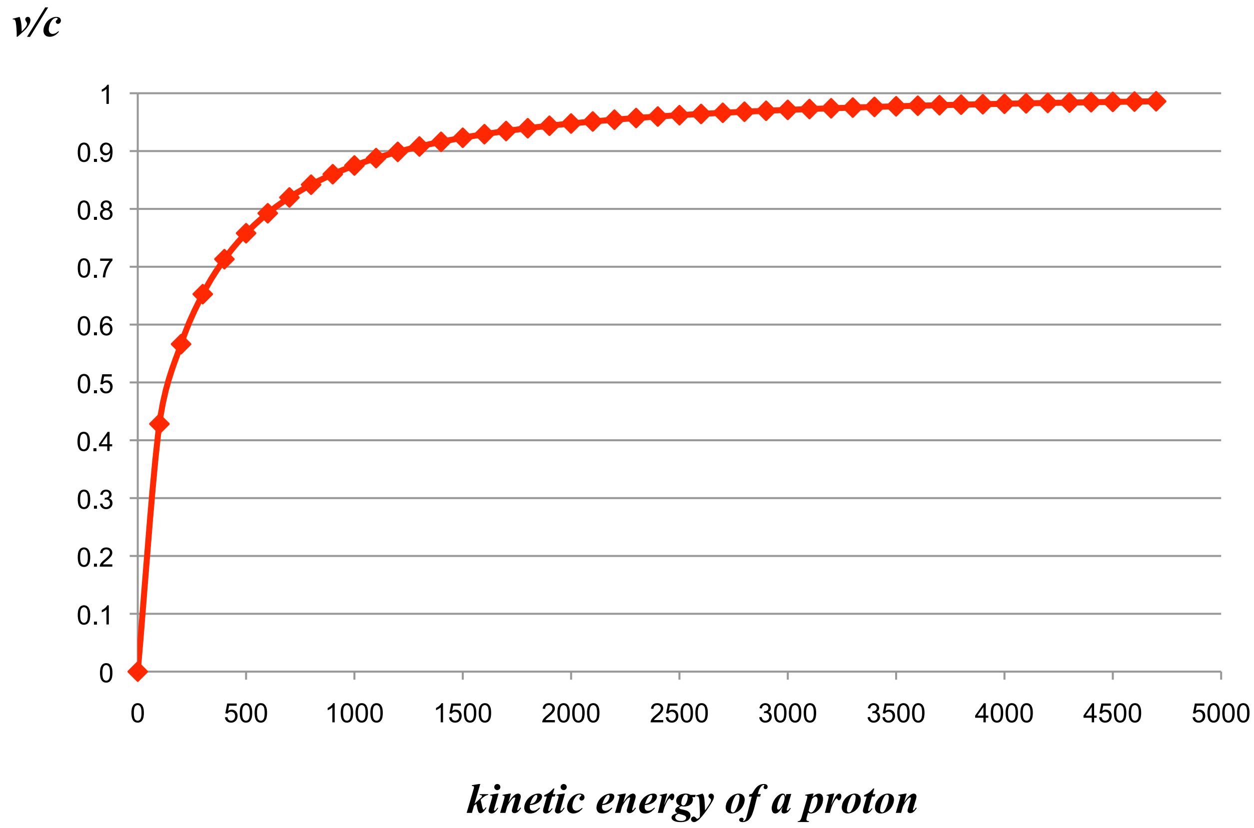


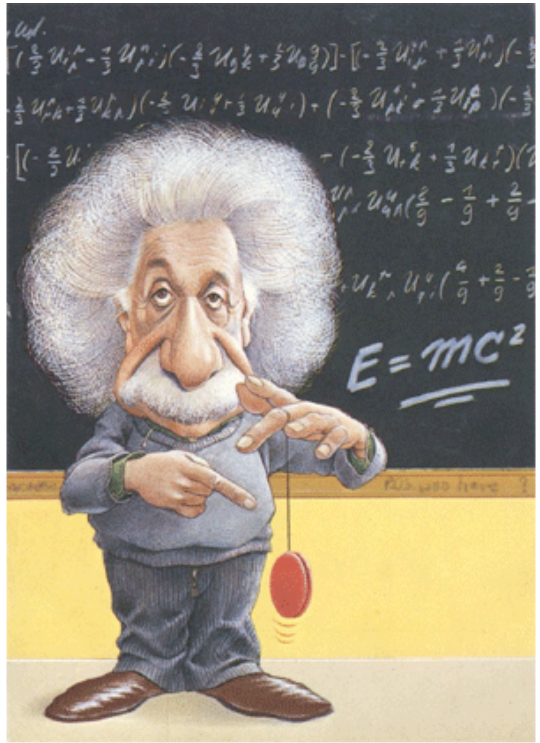
Focussing effect in the longitudinal direction keeping the particles close together ... forming a "bunch"

oscillation frequency: $f_s = f_{rev}\sqrt{-\frac{h\alpha_s}{2\pi} * \frac{qU_0\cos\phi_s}{E_s}}$ *≈ some Hz*

... so sorry, here we need help from Albert:

$$\gamma = \frac{E_{total}}{mc^2} = \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \longrightarrow \frac{v}{c} = \sqrt{1-\frac{mc^2}{E^2}}$$





... some when the particles do not get faster anymore

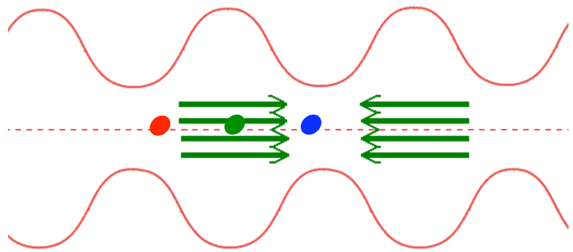
.... but heavier !

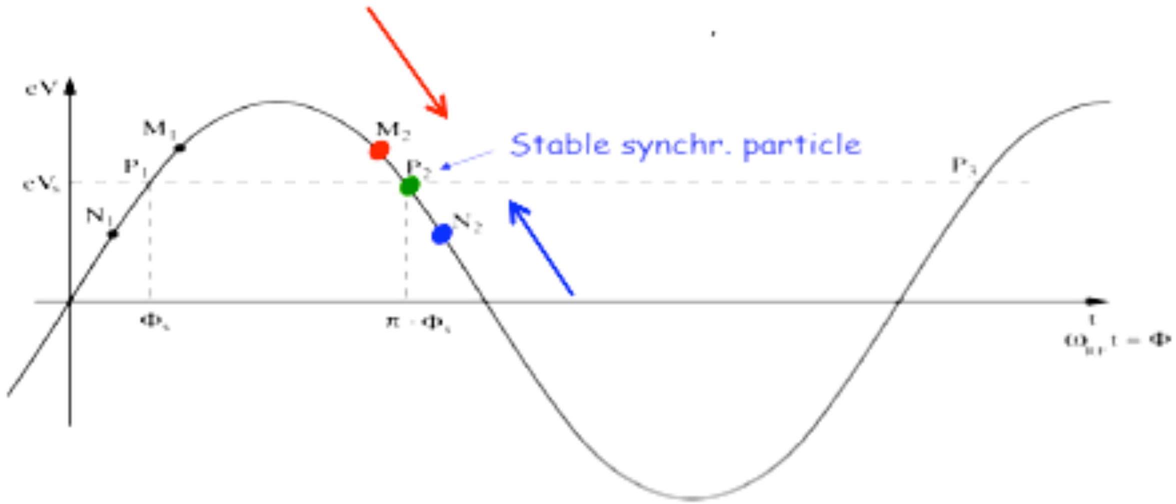
15.) The Acceleration for Δp/p≠0
"Phase Focusing" above transition

ideal particle •

particle with Δp/p > 0 • *heavier*

particle with Δp/p < 0 • *lighter*



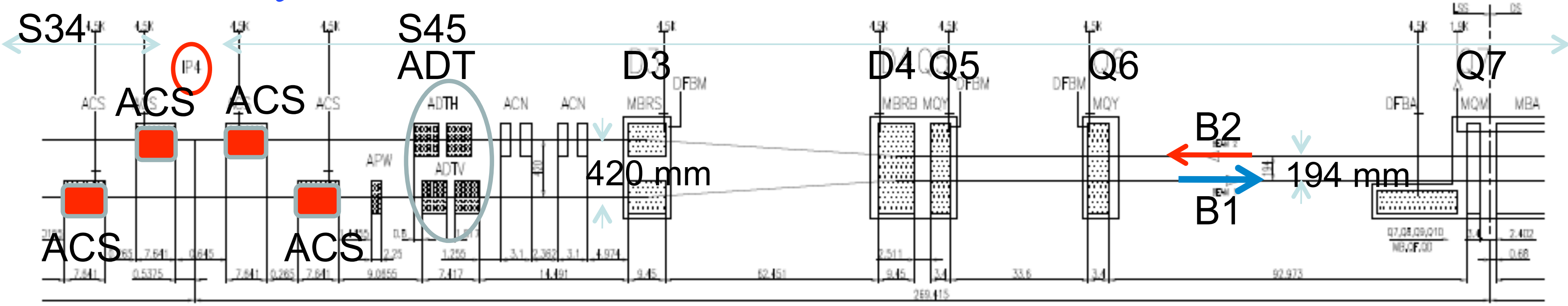


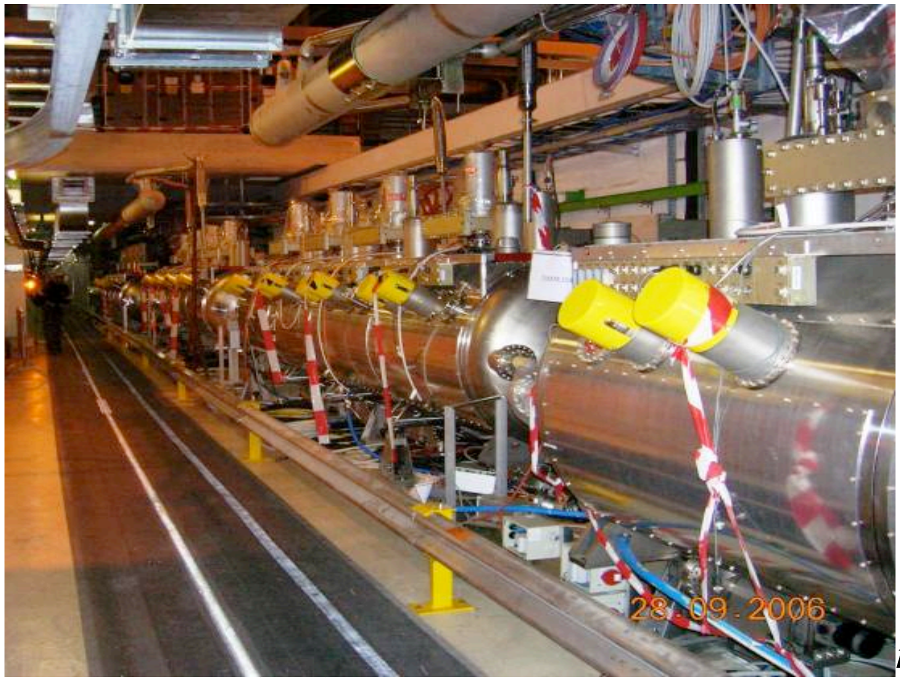
Focussing effect in the longitudinal direction
keeping the particles close together ... forming a "bunch"

... and how do we accelerate now ???

with the dipole magnets !

The RF system: IR4



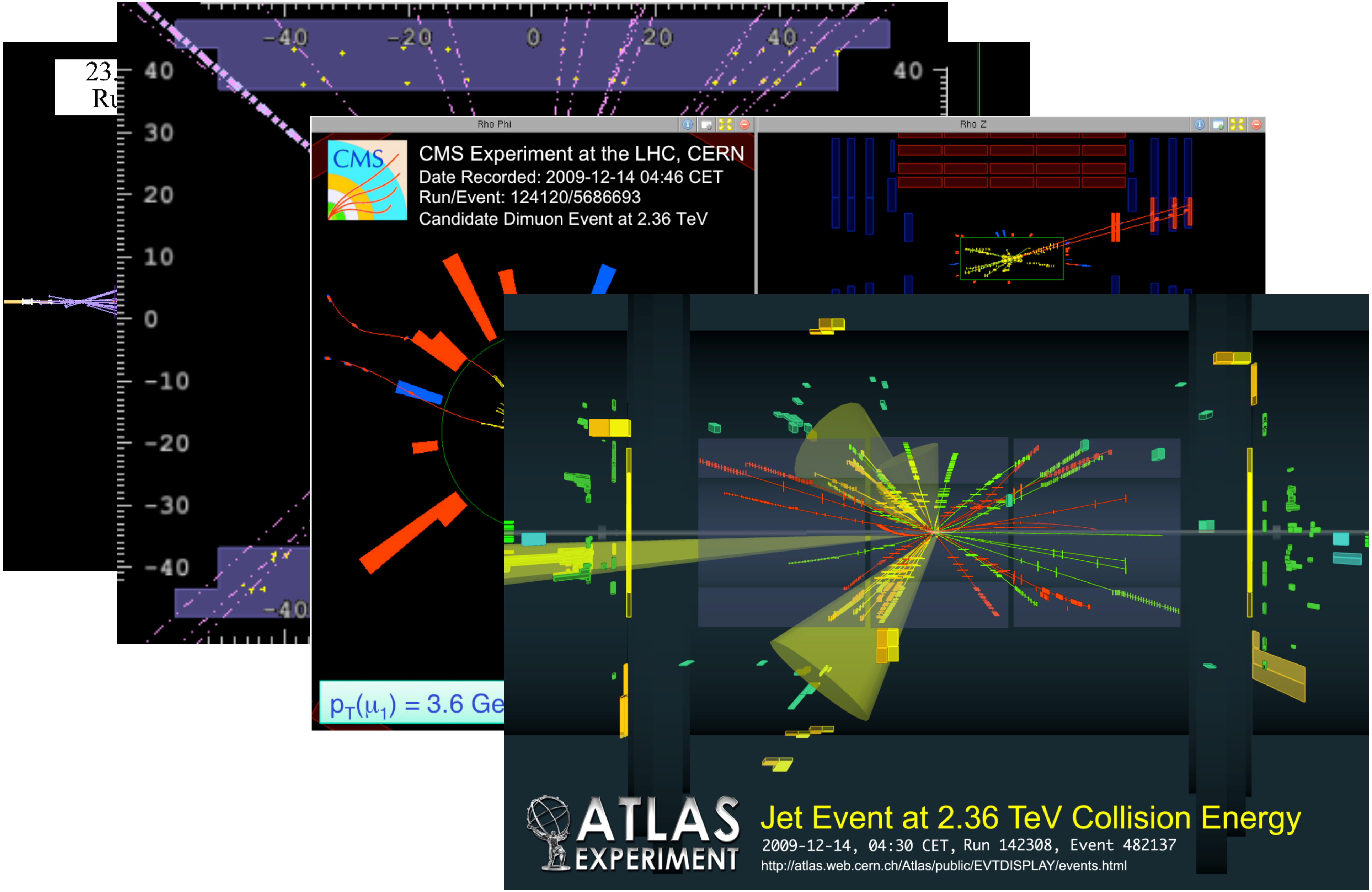


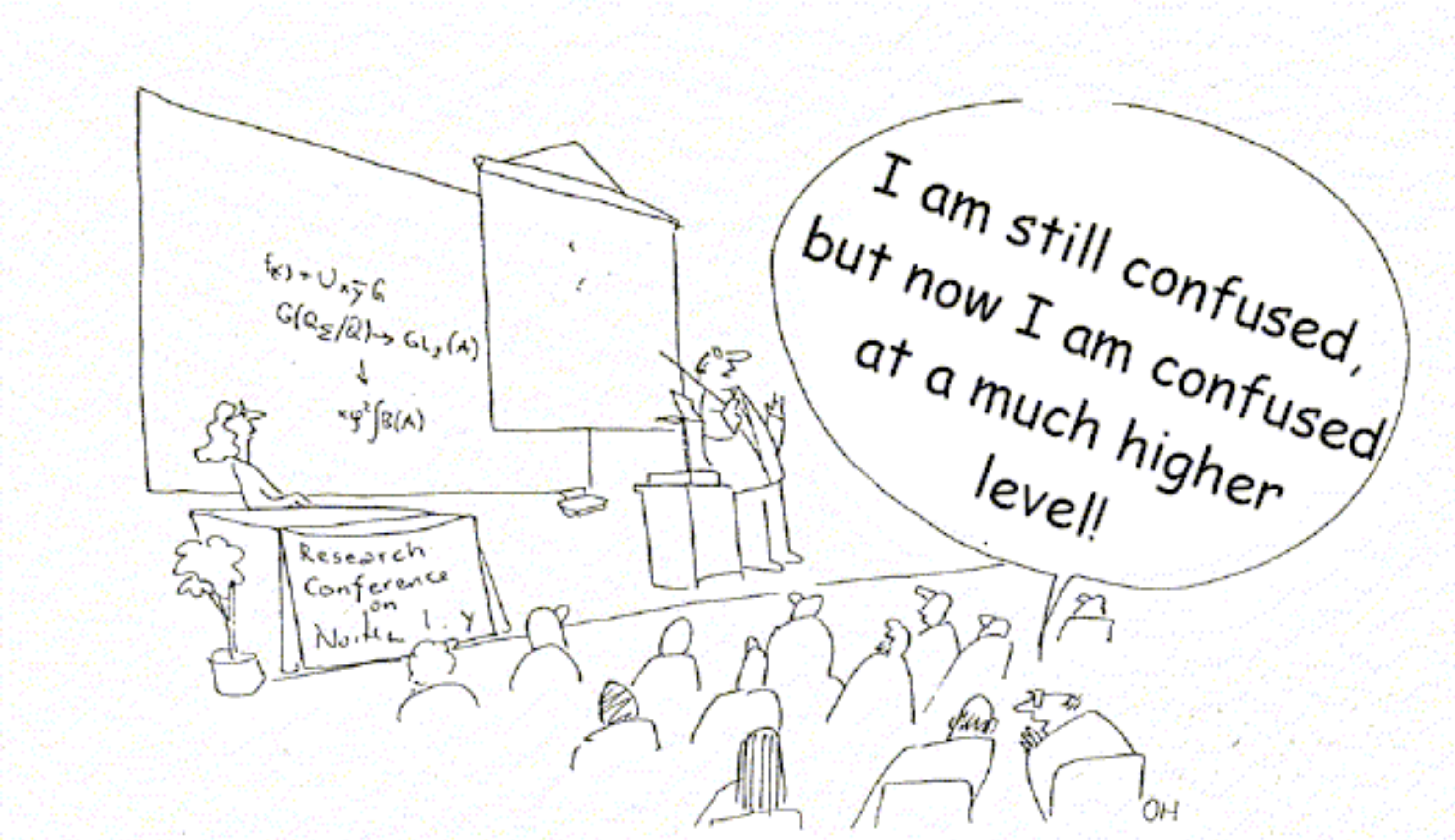
m

Nb on Cu cavities @4.5 K (=LEP2)
Beam pipe diam.=300mm

Bunch length (4σ)	*ns*	*1.06*
Energy spread (2σ)	10^{-3}	*0.22*
Synchr. rad. loss/turn	*keV*	*7*
Synchr. rad. power	*kW*	*3.6*
RF frequency	*M Hz*	*400*
Harmonic number		*35640*
RF voltage/beam	*MV*	*16*
Energy gain/turn	*keV*	*485*
Synchrotron frequency	*Hz*	*23.0*

LHC Operation: Collisions at 3.5 TeV per beam




I am still confused, but now I am confused at a much higher level!
Research Conference on
OH