RG-improved predictions for the inclusive Higgs cross section at the LHC

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An Effective Field Theory Assault on the Zeptometer Scale: Exploring the Origins of Flavor and Electroweak Symmetry Breaking

V. Ahrens, T. Becher, M. Neubert, L. L. Yang: 0808.3008 (PRD), 0809.4283 (EPJC), 1008.3162 (PLB)

Here I will discuss SCET-based, RG-resummed predictions for the inclusive Higgs cross section.

Different methods

- based on the same factorization formula
- with same $N^3LL+NNLO$ ($+N^3LO_{partial}$) accuracy

can give fairly (~10%) different cross sections, due to different choices of:

- expansion parameters (power corrections)
- treatment of large virtual corrections
- scale setting prescriptions

Terminal — bash — 70×42

```
becher:~/Documents/Software/RGHiggs-1.1> ./RunHiggs.py 7000 125 MSTW
*************************************************************
  Higgs production at the LHC in NNLO RG-improved QCD
*************************************************************

 Using MSTW PDF sets

 sqrtS =  7000. GeV
   m_H =  125.0 GeV

 Cross sections with scale uncertainties (pb)
                   Fixed order:    13.443    +1.431    -1.373
              Fixed order + EW:    14.135    +1.504    -1.443
      Only threshold resummed:     13.834    +0.703    -0.171
           Only pi^2 resummed:     14.618    +0.549    -0.636
      Threshold+pi^2 resummed:     14.679    +0.415    -0.112
 Threshold+pi^2 resummed + EW:     15.434    +0.436    -0.118

 Cross sections with PDF+alpha_s uncertainties (pb)
                  Fixed order:     13.443    +1.001    -0.968
              Fixed order + EW:    14.135    +1.053    -1.018
     Only threshold resummed:      13.834    +1.051    -1.014
          Only pi^2 resummed:      14.618    +1.166    -1.118
     Threshold+pi^2 resummed:      14.679    +1.172    -1.124
 Threshold+pi^2 resummed + EW:     15.434    +1.232    -1.182

becher:~/Documents/Software/RGHiggs-1.1>
```

PDF uncertainty for MSTW, CT10, CTEQ and NNPDF

run time is ~1.5 min

Available partial N^{3}LO results (hard and soft functions) are currently being implemented!

Results for cross section (N^3LL+NNLO)

(not the most up-to-date results…)

	σ [pb]	scale unc. $\Delta\sigma$ [%]
iHixs	15,37	+9/-8
deFG	15,40	+7/-8
RGHiggs	15,43	+3/-1

(m_H=125 GeV, LHC 7 TeV, m_t=173.1 GeV, m_b=4.2 GeV)

- Based on MSTW08NNLO:
  - ± 8% (PDF + α_s) uncertainty @ 90% CL
  - ± 4% (PDF + α_s) uncertainty @ 68% CL
  - PDF4LHC prescription gives +8/-7% uncertainty
- Numerically, there is excellent agreement for σ !

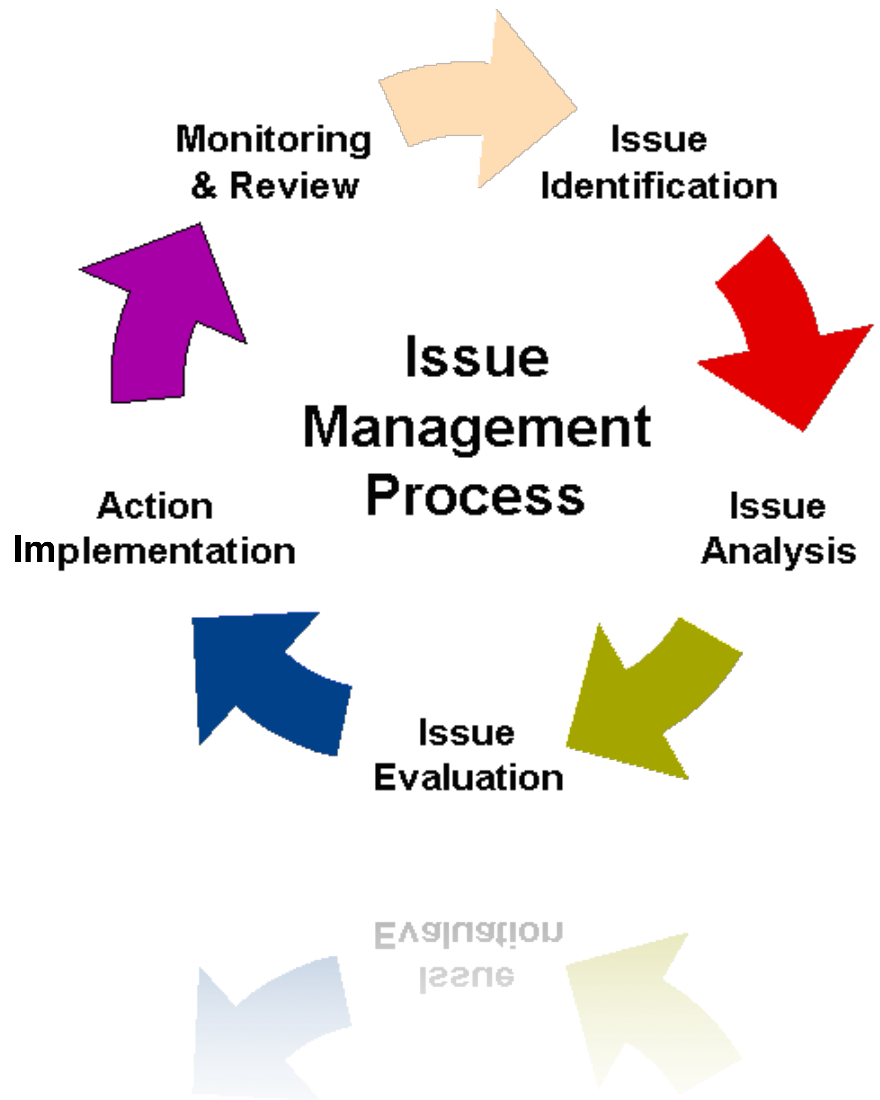
Differences

Several differences in these results are hidden by the excellent numerical agreement:

- We find that soft-gluon resummation alone increases the cross section by 3%, dFG find 8%. This means more than a factor 2 difference in the resummation itself! → power corrections are important!

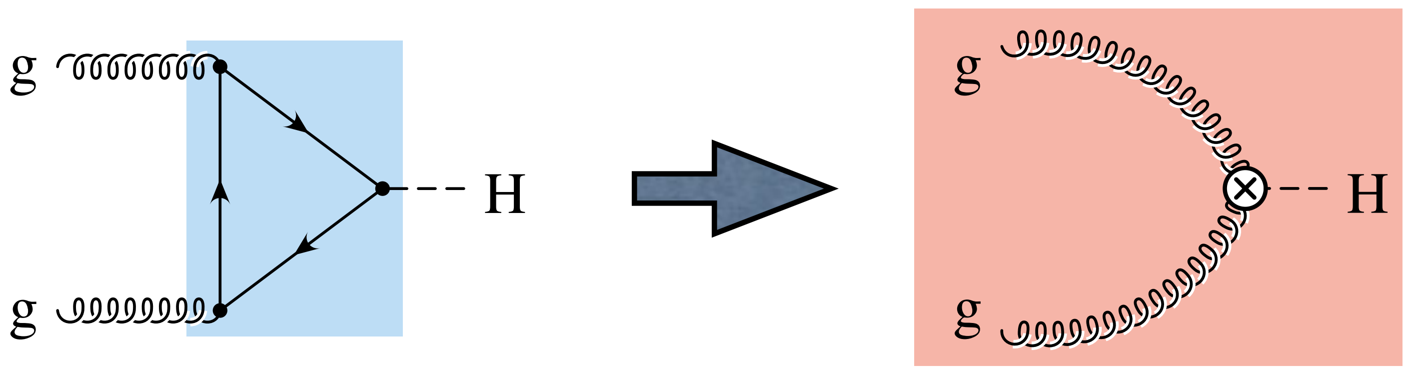
- Different treatment of the hard function in our approach ("π^2 resummation") yields 9% increase. Once this is done, soft-gluon resummation itself becomes a small effect. → might help to reduce remaining ambiguities at N^3LO!

- iHixs uses $\mu = M_H/2$, which enhances σ by 10% compared with $\mu = M_H$.



Some issues
in soft-gluon resummation

Common first step: integrate out the top



$$\mathcal{L}_{\text{eff}} = C_t(m_t^2, \mu^2)\,\frac{H}{v}\,\frac{\alpha_s(\mu^2)}{12\pi}\,G_{\mu\nu,a}\,G_a^{\mu\nu}$$

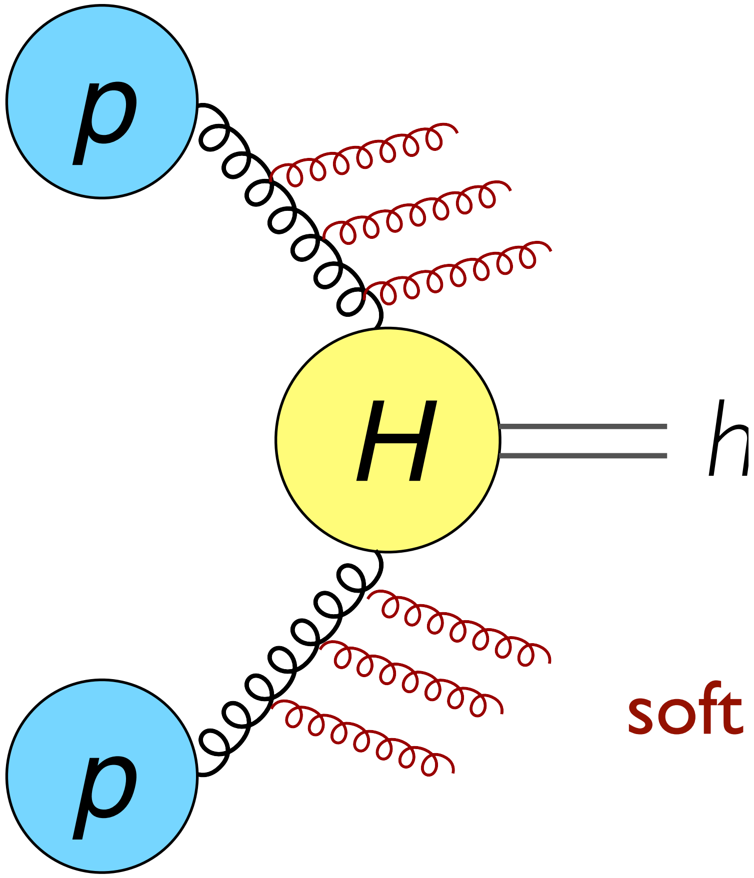
For $m_H \ll 2m_t$ we can integrate out the top quark, i.e. replace the SM by an effective theory with $n_f = 5$.

Calculations in EFT are much simpler (one loop and one scale less). NNLO results are only available in EFT.

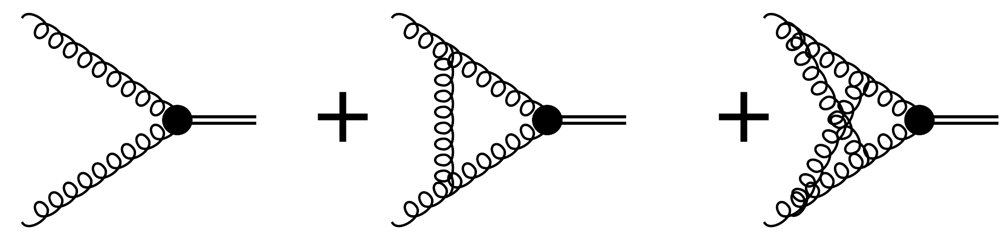
C_t is known to N^3LO and shows excellent convergence. Power corrections $(m_H/m_t)^2$ turn out to be small.

Factorization theorem

Sterman ’87; Catani & Trentadue ’88



soft radiation

$H = \Big|$  $\Big|^2$

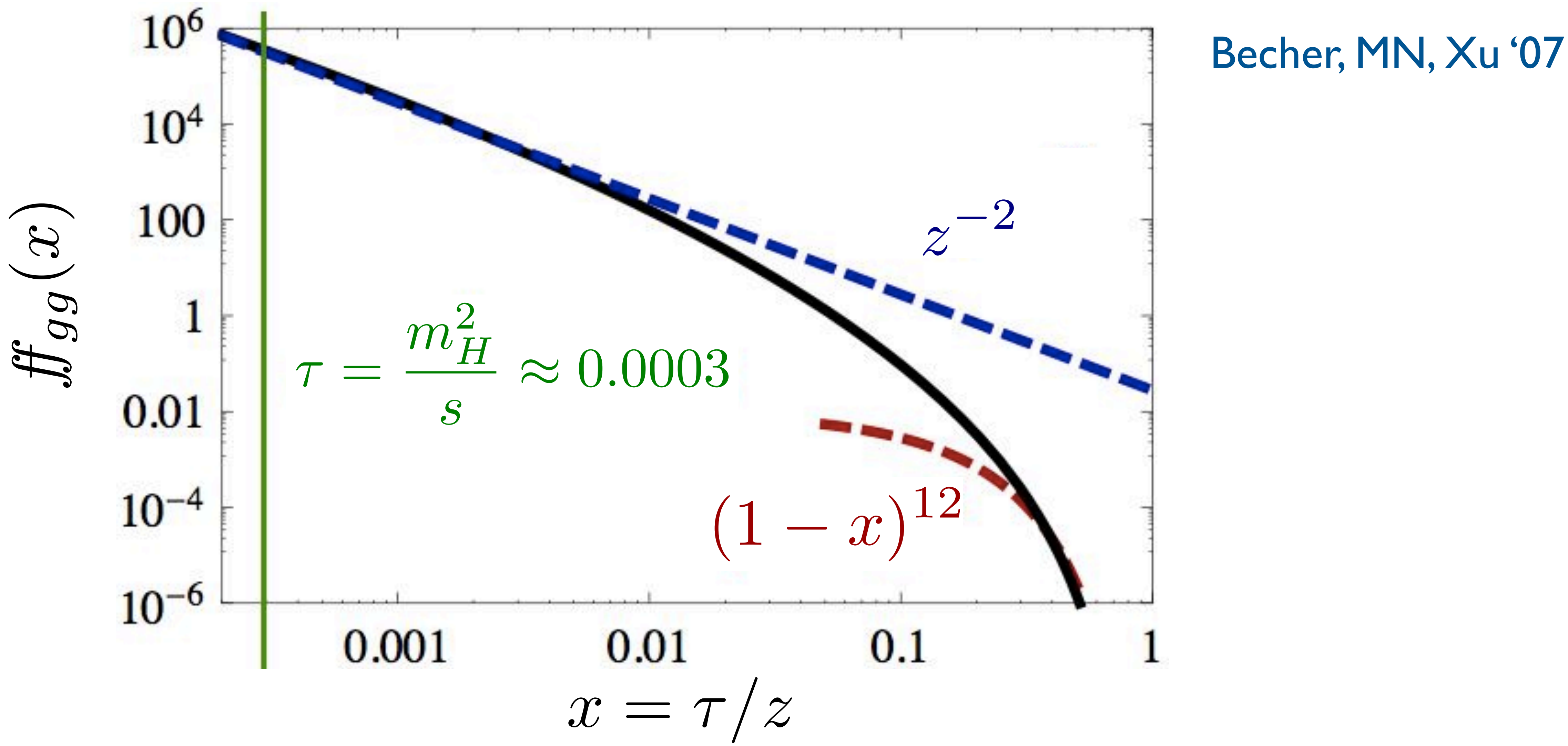
$$\tau = \frac{m_H^2}{s} \approx 0.0003$$

$$H(m_H^2, \mu) \int_\tau^1 \frac{dz}{z}\, S(\sqrt{\hat{s}}(1-z), \mu)\, ff_{gg}(\tau/z, \mu)$$

hard function — soft function — parton luminosity

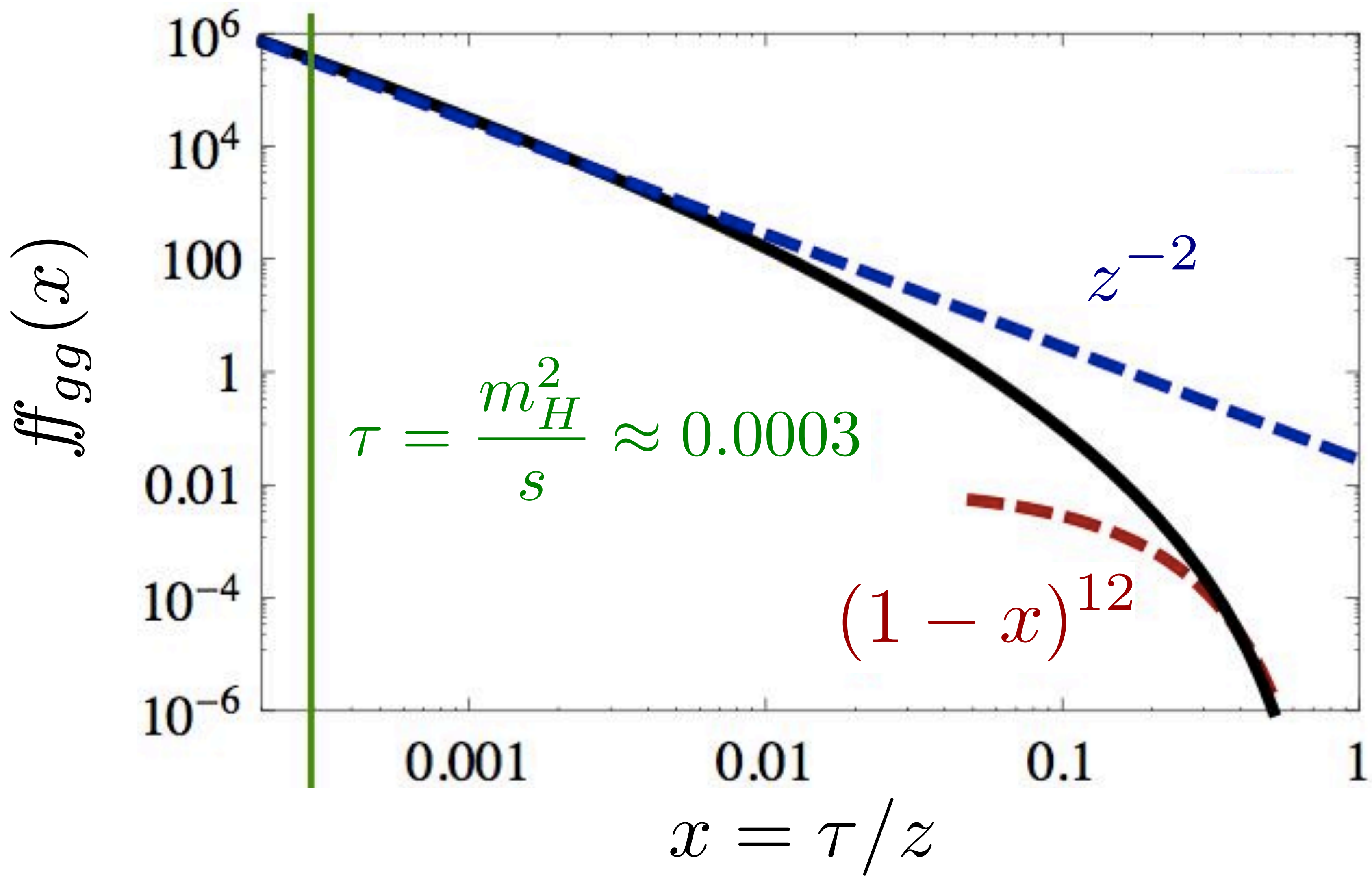
Scale of soft radiation is lower than m_H: large logarithms (?)

Dynamical threshold enhancement ?



Fall-off is not very strong. We find that the typical scale of "soft" radiation is of order $M_H/2$, meaning that there are no parametrically large logarithms!

Dynamical threshold enhancement ?



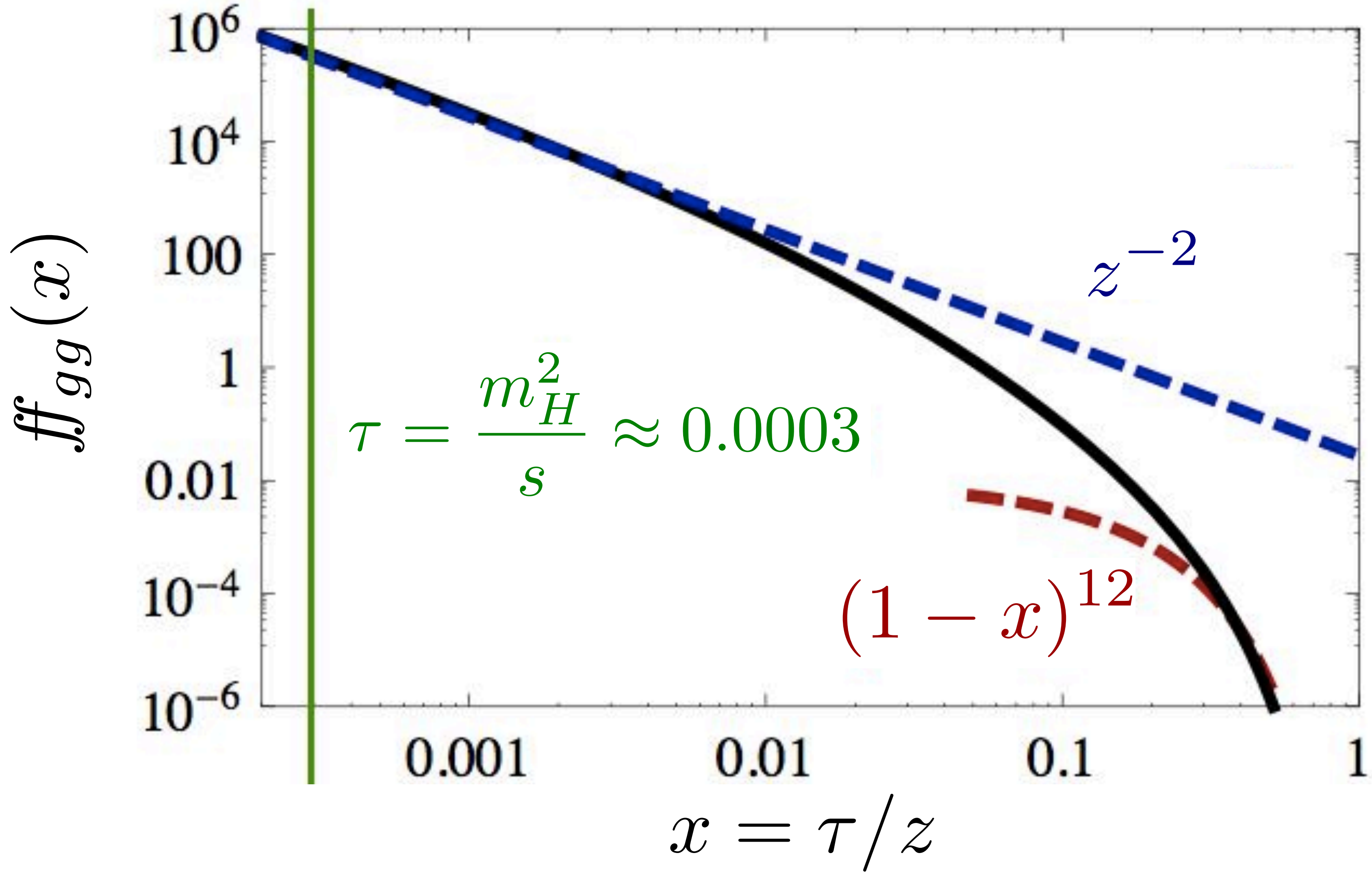
Becher, MN, Xu '07

Find that approximately (with $a - 1 \approx 1.5$):

$$\sigma \approx \sigma_{\text{Born}} \int_0^1 dz\, z^{a-1}\, C(z, m_t, m_H, \mu_f)$$

The threshold region is not strongly enhanced!

Dynamical threshold enhancement ?



Becher, MN, Xu '07

In moment space, dominant contributions arise from $N \sim 2$. Numerical dominance of threshold terms can only be justified *a posteriori* !

It might deteriorate for multiple emissions…

Dynamical threshold enhancement ?

For Higgs production at the LHC, hard emissions are not strongly suppressed by PDFs.

Expansion around the soft limit ($z \to 1$, $N \to \infty$) has an expansion parameter ~1/2, and hence the threshold enhancement cannot be justified parametrically:

- Exact choice of expansion parameter (or space in which the expansion is performed) matters
- Significant scheme dependence from different treatments of “power-suppressed” terms

attention to detail

Three differences

Three differences

1. Integral transform & choice of singular distributions
   - Mellin moments
   - Laplace transform (in E_s)
2. Scale setting for soft emissions
   - on the partonic level
   - on the hadronic level
3. Evaluation of the hard function
   - time-like matching
   - space-like matching & RG evolution (π^2 terms)

BLUE: de Florian, Grazzini

GREEN: Ahrens, Becher, MN, Yang

1. Soft emissions and singular terms

$$\sigma(\tau) = \sigma_0(\mu_f) \int_\tau^1 \frac{dz}{z} \, C_{gg}(z, m_t, m_H, \mu_f) \, ff_{gg}(\tau/z, \mu_f)$$

Soft emissions give rise to singular distributions in partonic cross section C_{gg}, which can be written as:

$$\left[\frac{\ln^n(1-z)}{1-z} \right]_+ \quad \text{or} \quad \left[\frac{1}{1-z} \ln^n \left(\frac{1-z}{\sqrt{z}} \right) \right]_+$$

$$\frac{1-z}{\sqrt{z}} = \frac{2E_s(z)}{M_H}$$

Resummation predicts these singular distributions to all orders in perturbation theory.

1. Soft emissions and singular terms

Second form is particularly natural, since the exact results for the hard-scattering kernels involve precisely these logarithms (at least to NNLO):

$$C_{gg}(z,m_t,m_H,\mu_f) = \delta(1-z) + \frac{\alpha_s}{\pi}\left[\delta(1-z)\left(\frac{11}{2}+2\pi^2\right) + 6\left[\frac{1}{1-z}\ln\frac{m_H^2(1-z)^2}{\mu_f^2 z}\right]_+ + 6\left(\frac{1}{z}-2+z-z^2\right)\ln\frac{m_H^2(1-z)^2}{\mu_f^2 z} - \frac{11}{2}\frac{(1-z)^3}{z}\right]$$

$$C_{gq}(z,m_t,m_H,\mu_f) = \frac{\alpha_s}{\pi}\left[\frac{2}{3}\frac{1+(1-z)^2}{z}\ln\frac{m_H^2(1-z)^2}{\mu_f^2 z} + \frac{2z}{3} - \frac{(1-z)^2}{z}\right]$$

$$C_{q\bar{q}}(z,m_t,m_H,\mu_f) = \frac{\alpha_s}{\pi}\left[\frac{32}{27}\frac{(1-z)^3}{z}\right]$$

→ still true at (α_s^2)!

In this sense, the SCET approach nicely matches onto fixed-order results!

Integral transform

To perform the resummation one takes the Laplace or Mellin moment transform of the cross section.

$$\mathcal{L}_N\big[f(\xi)\big] \equiv \int_0^\infty d\xi\, e^{-\xi N}\, f(\xi)$$

$$\mathcal{M}_N\big[f(\xi)\big] \equiv \int_0^1 d\xi\, (1-\xi)^{N-1}\, f(\xi)$$

- In the SCET approach, we solve the RGEs in Laplace space and then invert analytically.
- Traditionally, resummation is performed in moment space, with a numerical inversion at the end.

Singular terms from Mellin inversion

Threshold limit $z \to 1$ corresponds to expansion around $N \to \infty$. Mellin- and Laplace-space results are the same after $1/N$ expansion.

Difference arises in the inverse transform:

Laplace $$(-\ln N)^{n+1} + \dots \longrightarrow \left[\frac{\ln^n(1-z)}{1-z}\right]_+ + \dots$$

Laplace E_s $$(-\ln N)^{n+1} + \dots \longrightarrow \left[\frac{1}{1-z}\ln^n\left(\frac{1-z}{\sqrt{z}}\right)\right]_+ + \dots$$

Mellin $$(-\ln N)^{n+1} + \dots \longrightarrow \left[\frac{\ln^n(-\ln z)}{-\ln z}\right]_+ + \dots$$

Singular terms from Mellin inversion

$$-\ln z = \frac{1-z}{\sqrt{z}} + \mathcal{O}\left[(1-z)^3\right]$$

The main difference between the two last approaches is a factor of $\sqrt{z}$:

$$\frac{\ln^n(-\ln z)}{-\ln z} \approx \sqrt{z} \times \frac{\ln^n \frac{1-z}{\sqrt{z}}}{1-z}$$

power correction at threshold z=1

such terms do arise in fixed-order computation

On the other hand, the simple Laplace inversion giving rise to $\left[\frac{\ln^n(1-z)}{1-z}\right]_+$ terms is not favored by the structure of fixed-order expressions!

Singular terms up to N^{3}LO

	LO	NLO	NNLO	NNNLO
full	4,69	5,96	3,71	?
Laplace	4,69	4,48	1,68	0,16
Laplace	4,69	5,12	2,87	1,00
Mellin	4,69	5,74	3,71	1,61

Ahrens *et al.* (SCET)

de Florian *et al.* (QCD)

(LHC@7 TeV, $\mu_r = \mu_f = m_H$)

13% of LO

- Large differences between schemes indicate the importance of power corrections!
- The singular pieces in the Mellin approach are very close to the full results, but given that there is no parametric reason this seems to be accidental.

Singular terms up to N^3LO

	LO	NLO	NNLO	NNNLO	
full	13,7	17,6	10,6	?	
Laplace	13,7	12,5	4,36	0,34	
Laplace	13,7	14,6	7,90	2,64	Ahrens *et al.* (SCET)
Mellin	13,7	16,4	10,3	4,32	de Florian *et al.* (QCD)

(LHC@13 TeV, $\mu_r = \mu_f = m_H$)   12% of LO

- Large differences between schemes indicate the importance of power corrections!
- The singular pieces in the Mellin approach are very close to the full results, but given that there is no parametric reason this seems to be accidental.

Singular terms up to N^3LO

	LO	NLO	NNLO	NNNLO	
full	5,74	6,97	3,17	?	
Laplace	5,74	4,19	1,00	-0,028	
Laplace	5,74	5,08	1,89	0,226	Ahrens *et al.* (SCET)
Mellin	5,74	5,42	2,22	0,371	de Florian *et al.* (QCD)

(LHC@7 TeV, $\mu_r = \mu_f = m_H/2$) 2,5% of LO

- For different scale choices, matching corrections at LO and NLO can be larger (and similar) in both approaches.

Singular terms up to N^3LO

	LO	NLO	NNLO	NNNLO
full	16,0	20,6	9,39	?
Laplace	16,0	11,3	2,67	-0,003
Laplace	16,0	14,1	5,20	0,629
Mellin	16,0	14,9	5,99	0,953

Ahrens *et al.* (SCET)

de Florian *et al.* (QCD)

(LHC@13 TeV, $\mu_r = \mu_f = m_H/2$)

2,0% of LO

- For different scale choices, matching corrections at LO and NLO can be larger (and similar) in both approaches.

2. Choice of the soft scale

$$\int_\tau^1 \frac{dz}{z}\, S(\sqrt{\hat{s}}(1-z),\mu)\, ff_{gg}(\tau/z,\mu)$$

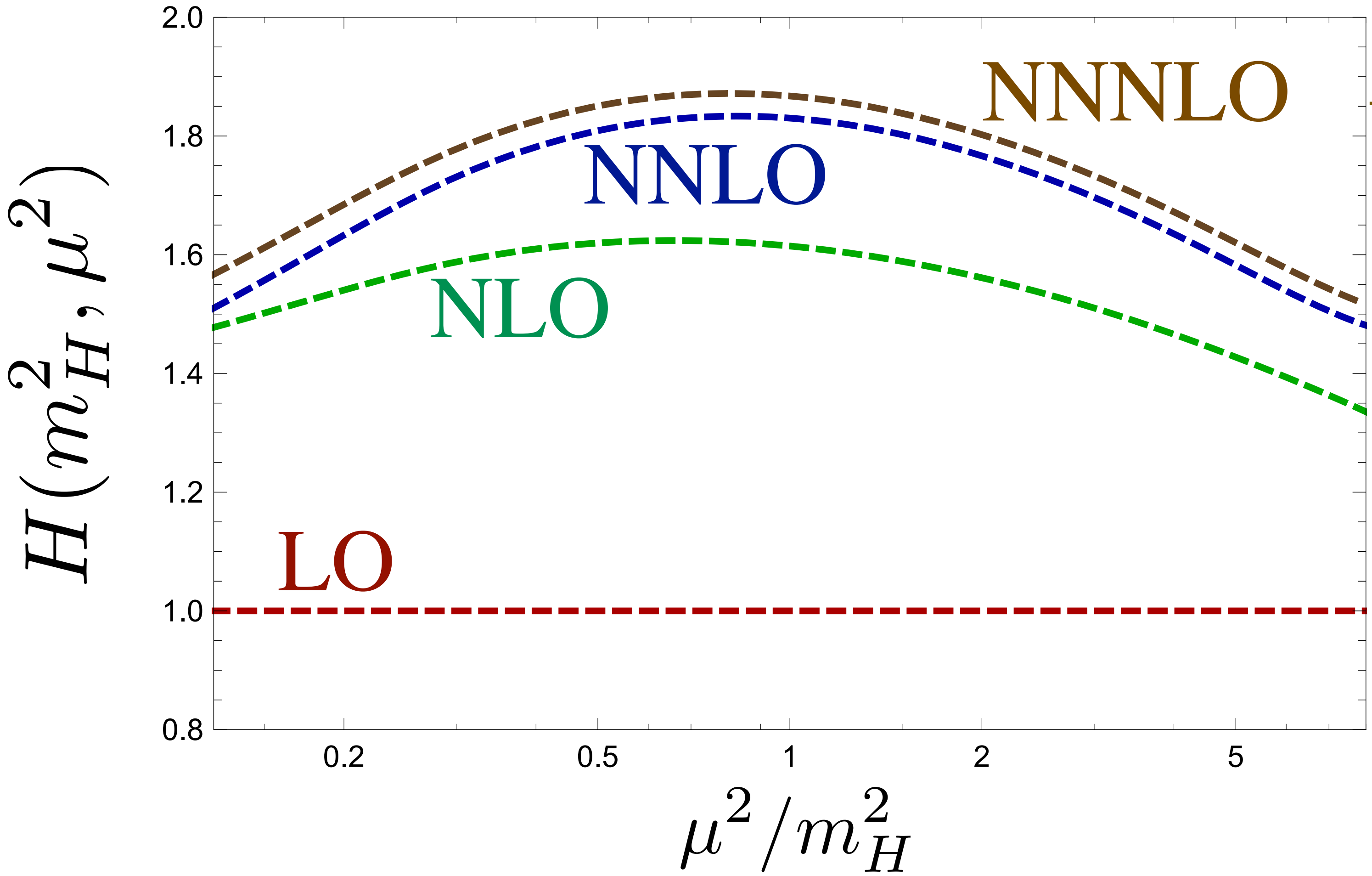
Appropriate scale μ in the soft radiation? Can set scale either at:

- **partonic level:** set $\mu = \sqrt{\hat{s}}(1-z)$ and give a prescription for Landau pole
- **hadronic level:** set μ equal to the average energy of soft radiation, determined numerically (result: $\mu \sim M_H/2$)

Numerically, the two prescriptions give very similar results

(see e.g.: Sterman, Zeng 1312.5397; Bonvini, Forte, Ridolfi, Rottoli 1409.0864)

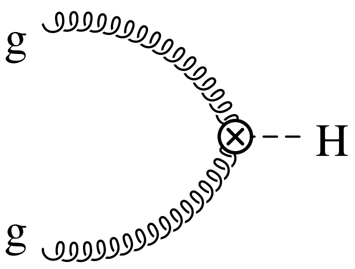
3. Choice of the hard scale



Heinrich *et al.* '09; Baikov *et al.* '09;
T. Gehrmann *et al.* '10

- Hard function is scale dependent.
- Naively, contains large corrections for any μ^2 !

Time-like gluon form factor



- Hard function $H(m_H^2, \mu^2) = \left| C_S(-m_H^2 - i\epsilon, \mu^2) \right|^2$

time-like gluon form factor

- Scalar form factor:

$$C_S(Q^2, \mu^2) = 1 + \sum_{n=1}^{\infty} c_n(L) \left(\frac{\alpha_s(\mu^2)}{4\pi} \right)^n , \qquad L = \ln(Q^2/\mu^2)$$

$$c_1(L) = C_A \left(-L^2 + \frac{\pi^2}{6} \right)$$

Sudakov double logarithm

- Perturbative expansions:

space-like: $C_S(Q^2, Q^2) = 1 + 0.393\, \alpha_s(Q^2) - 0.152\, \alpha_s^2(Q^2) + \dots$

time-like: $C_S(-q^2, q^2) = 1 + 2.75\, \alpha_s(q^2) + (4.84 + 2.07i)\, \alpha_s^2(q^2)$

Time-like gluon form factor

- Replacement $L \to \ln q^2/\mu^2 - i\pi$ in double logs gives rise to large π² terms, which can be resummed

Parisi '80; Ahrens, Becher, MN, Yang '08

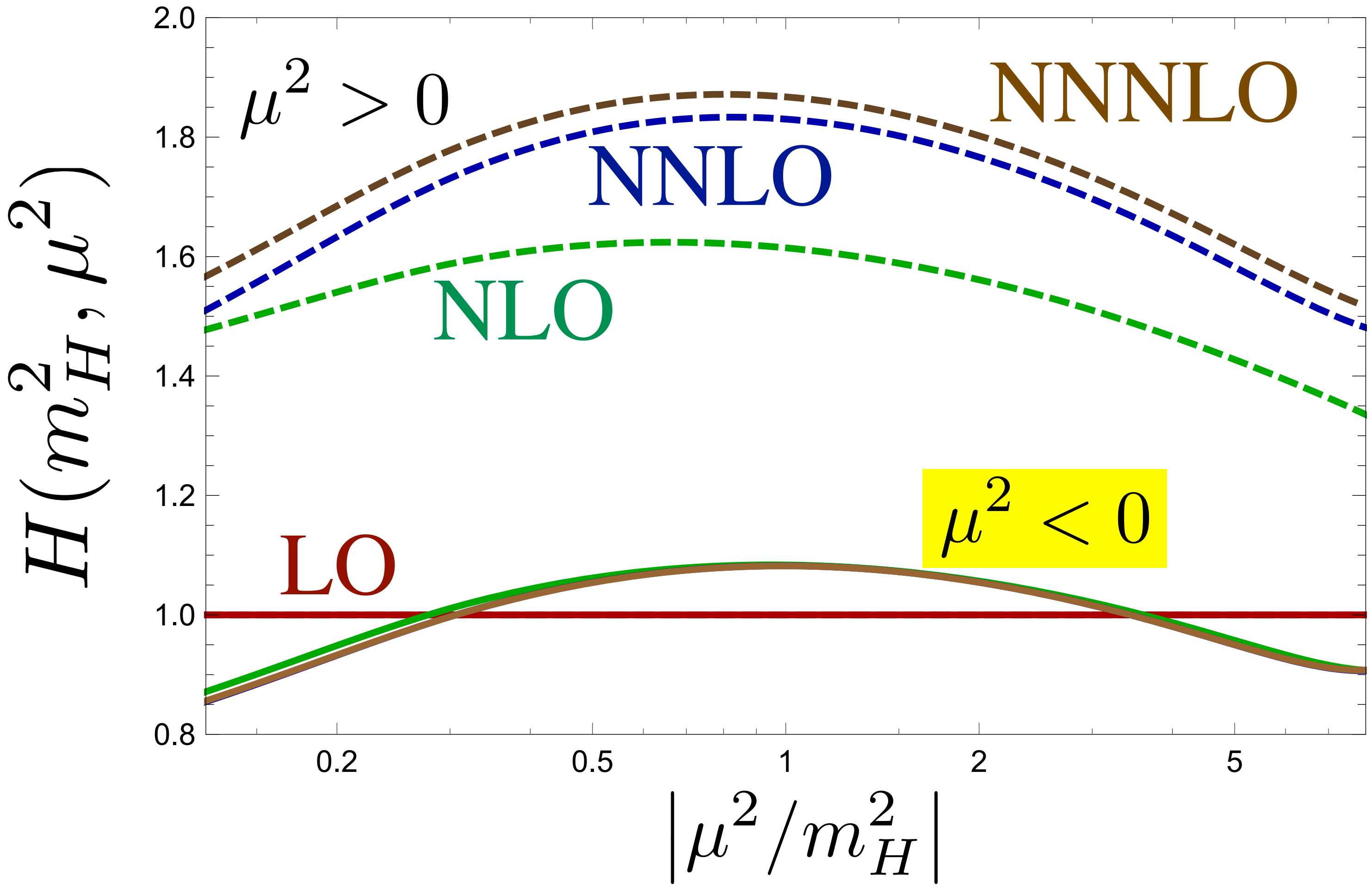
- We can avoid these terms in the matching by choosing a time-like value $\mu^2 = -q^2$

$$C_S(-q^2, -q^2) = 1 + 0.393\,\alpha_s(-q^2) - 0.152\,\alpha_s^2(-q^2) + \dots$$

→ same small expansion coefficients as for $C_S(Q^2, Q^2)$

- Large π² terms are resummed in the RG evolution of the hard function to the scale μ_f

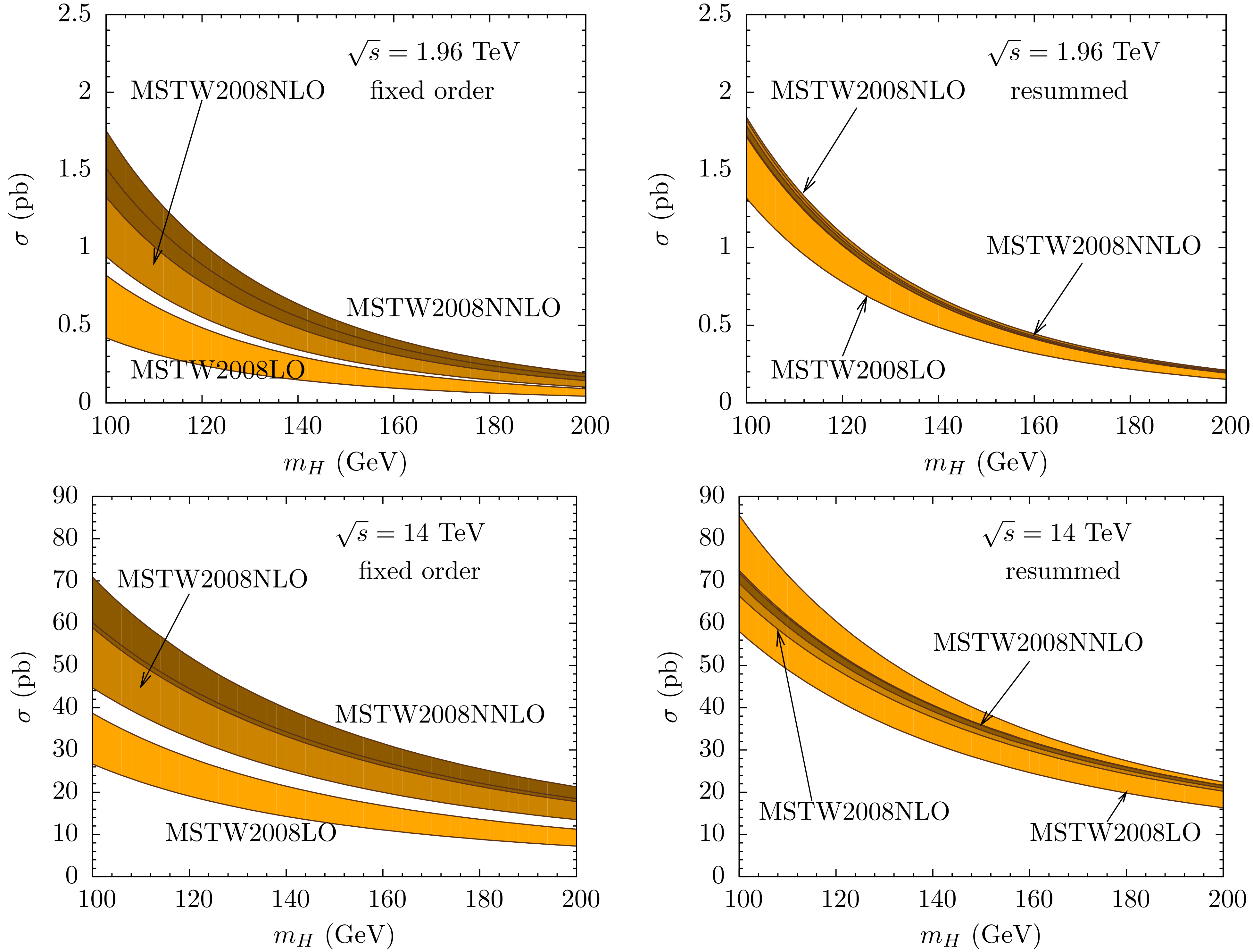
Time-like vs. space-like scale choice



Heinrich *et al.* ’09; Baikov *et al.* ‘09; T. Gehrmann *et al.* ’10

- Convergence is much better for $\mu^2 < 0$
- Evaluate *H* for $\mu^2 < 0$, where the convergence is good, and use RG to evolve it to an arbitrary scale

Fixed-order vs. resummed results



No large *K*-factors!

Conclusions

- Higgs production cross section is not strongly dominated by partonic threshold contribution; related expansion parameter is ~1/2 !
- As a result, a significant scheme dependence in soft-gluon resummation due to the truncation of "power corrections" remains at NNLO.
- Large corrections arising from time-like gluon form factor and contained in the hard function *H* can and should be resummed using RG methods!
- This might help reducing the ambiguities in the matching to recent partial N^3LO results (subleading terms in threshold expansion). Anastasiou *et al.* 1411.3584

Conclusions

Entire cross section: (LHC@13 TeV, $\mu_r = \mu_f = m_H$)

	LO	NLO	NNLO	NNNLO
full	13,7	17,6	10,6	?
Laplace	13,7	12,5	4,36	0,34
Laplace	13,7	14,6	7,90	2,64
Mellin	13,7	16,4	10,3	4,32

12% of LO

Cross section divided by the hard function:

	LO	NLO	NNLO	NNNLO
full	13,7	9,17	2,03	?
Laplace	13,7	4,05	-1,09	-0,478
Laplace	13,7	6,10	1,18	-0,011
Mellin	13,7	7,94	2,46	0,486

3,6% of LO

PRELIMINARY

Conclusions

- Today, theory uncertainties are dominated by PDF errors and the treatment of yet unknown subleading threshold terms (not by scale dependences)!
- These should be estimated by comparing different schemes, e.g. de Florain & Grazzini vs. Ahrens et al.
- Before presenting our final numbers in the SCET approach, we will:
  - include known N^3LO contributions to the hard and soft functions
  - match onto recently computed, partial N^3LO results for subleading threshold terms