

# *Higgs-boson production cross section at approximate $N^3LO$*

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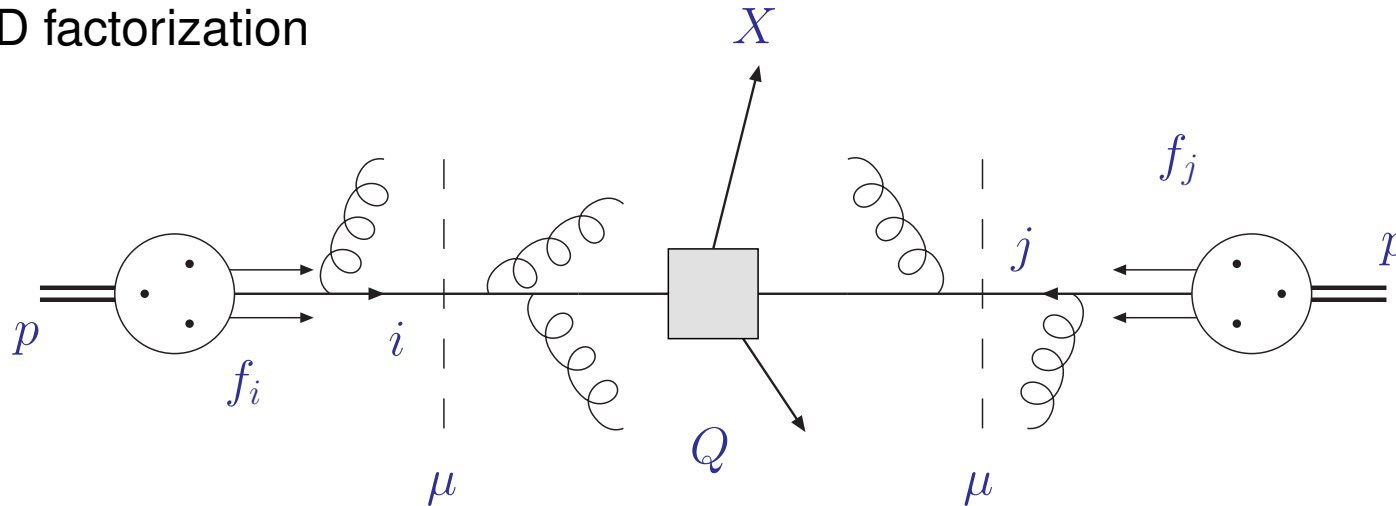
HiggsXS Working Group *ggH inclusive cross section meeting*, CERN, Nov 18, 2014

## Work done in collaboration with:

- *Approximate  $N^3$  LO Higgs-boson production cross section using physical-kernel constraints*  
D. de Florian, J. Mazzitelli, S. M. and A. Vogt [arXiv:1408.6277](https://arxiv.org/abs/1408.6277)

# QCD factorization

- QCD factorization

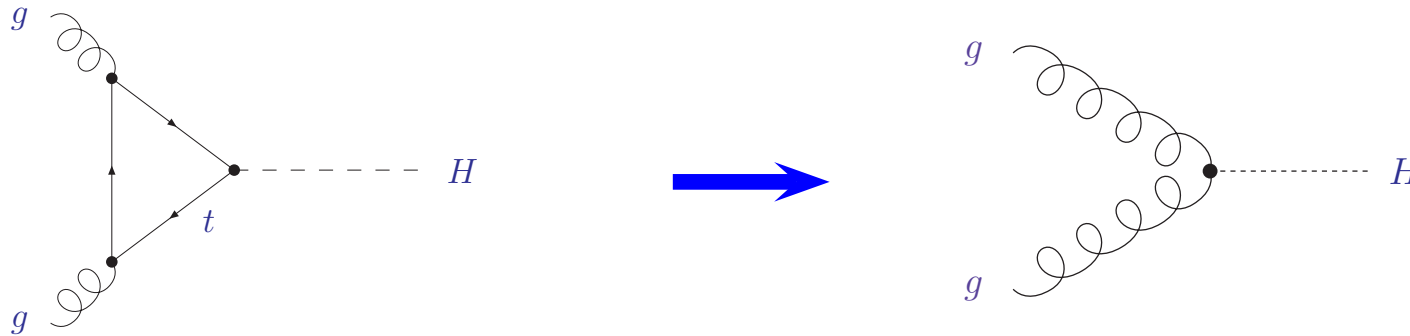


$$\sigma_{pp \rightarrow X} = \sum_{ij} f_i(\mu^2) \otimes f_j(\mu^2) \otimes \hat{\sigma}_{ij \rightarrow X}(\alpha_s(\mu^2), Q^2, \mu^2, m_X^2)$$

- Hard parton cross section  $\hat{\sigma}_{ij \rightarrow X}$  calculable in perturbation theory
  - known to NLO, NNLO, ... ( $\mathcal{O}(\text{few}\%)$  theory uncertainty)
- Non-perturbative parameters: parton distribution functions  $f_i$ , strong coupling  $\alpha_s$ , particle masses  $m_X$ 
  - known from global fits to exp. data, lattice computations, ...

# Higgs production in $gg$ -fusion

## Effective theory



- Hard scattering cross section  $\hat{\sigma}_{ij \rightarrow H+X}$  dominated by gluon-gluon fusion

- typically treated in effective theory in limit  $m_t \rightarrow \infty$ ; Lagrangian

$$\mathcal{L} = -\frac{1}{4} \frac{H}{v} C_H G^{\mu\nu a} G_{\mu\nu}^a$$

- QCD corrections significant

- NNLO corrections still large

Harlander, Kilgore '02; Anastasiou, Melnikov '02; Ravindran, Smith, van Neerven '03

- improvement with soft N<sup>3</sup>LO corrections S.M., Vogt '05; Laenen, Magnea '05:  
in  $x$ -space '+'-distributions  $\alpha_s^k \ln^{2k-1}(1-x)/(1-x)$

- NNLL resummation Catani, de Florian, Grazzini, Nason '03; Ahrens et al. '10; [...];  
Ahmed, Mahakhud, Rana, Ravindran '14

- soft-virtual corrections completed:  $\delta(1-x)$ -term at N<sup>3</sup>LO

Anastasiou et al. '14

# Physical kernel

- Cross section for Higgs boson production function of  $x = m_H^2/s$

$$\sigma(H) = \sigma_0 \sum_{a,b} f_a \otimes f_b \otimes c_{ab}$$

- coefficient functions for hard scattering  $c_{ab}(\alpha_s) = \alpha_s^k c_{ab}^{(k)}$ :  
**LO**  $c_{gg}^{(0)} = \delta(1-x)$ , **NLO**  $c_{ab}^{(1)}$ , **NNLO**  $c_{ab}^{(2)}$ , **N<sup>3</sup>LO**  $c_{ab}^{(3)}$ , ...
- $c_{ab}(\alpha_s)$  in  $x$ -space show  $x \rightarrow 1$  double logarithmic enhancement:  
'+'-distributions  $\alpha_s^k \ln^l(1-x)/(1-x)$  with  $0 \leq l \leq 2k-1$

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 '+'-distributions  $\alpha_s^k \ln^l(1-x)/(1-x)$  with  $0 \leq l \leq 2k-1$
- Alternative factorization: partonic structure functions  $\sigma(H) = \sum_{a,b} \sigma_0 \mathcal{F}_{ab}$
- gg-channel dominant for  $x \rightarrow 1$
- evolution equation for  $\mathcal{F}_{gg}$  defines physical kernel  $K_{gg}$

$$\frac{d}{d \ln m_H^2} \mathcal{F}_{gg} = \underbrace{\left\{ 2P_{gg}(\alpha_s) + \beta(\alpha_s) \frac{dc_{gg}(\alpha_s)}{d\alpha_s} \otimes (c_{gg}(\alpha_s))^{-1} \right\}}_{\text{scheme invariant}} \otimes \mathcal{F}_{gg}$$

**scheme invariant**

$$\equiv K_{gg}(\alpha_s) \otimes \mathcal{F}_{gg}$$

- $K_{gg}(\alpha_s)$  for  $x \rightarrow 1$  exhibits single logarithmic enhancement  
 $\alpha_s^k \ln^l(1-x)/(1-x)$  with  $0 \leq l \leq k \longrightarrow$  constraints on  $c_{ab}(\alpha_s)$

# Coefficient functions

- Constraints from physical kernel  $K_{gg}(\alpha_s)$  exact to all powers in  $(1-x)^n$ 
  - access to power suppressed terms with logarithmic enhancement  $\alpha_s^k \ln^l(1-x)$  with  $1 \leq l \leq 2k$

- Coefficient function at **N<sup>3</sup>LO**  $c_{gg}^{(3)}$

$$c_{gg}^{(3)}(x) = c_{gg}^{(3)}(x) \Big|_{\mathcal{D}_k, \delta(1-x)} - 512C_A^3 \ln^5(1-x) + \left\{ 1728C_A^3 + \frac{640}{3}C_A^2\beta_0 \right\} \ln^4(1-x) \\ + \left\{ \left( -\frac{1168}{3} + 3584\zeta_2 \right) C_A^3 - \left( \frac{2512}{3} + \frac{1}{3}\xi_H \right) C_A^2\beta_0 - \frac{64}{3}C_A\beta_0^2 \right\} \ln^3(1-x) + \dots$$

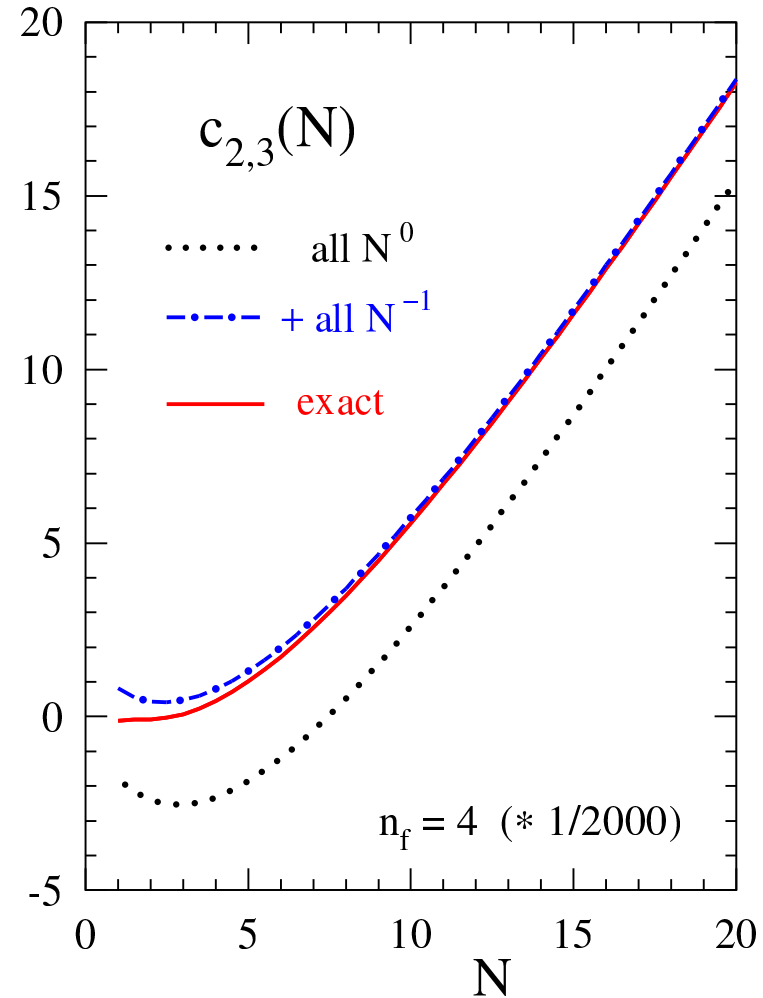
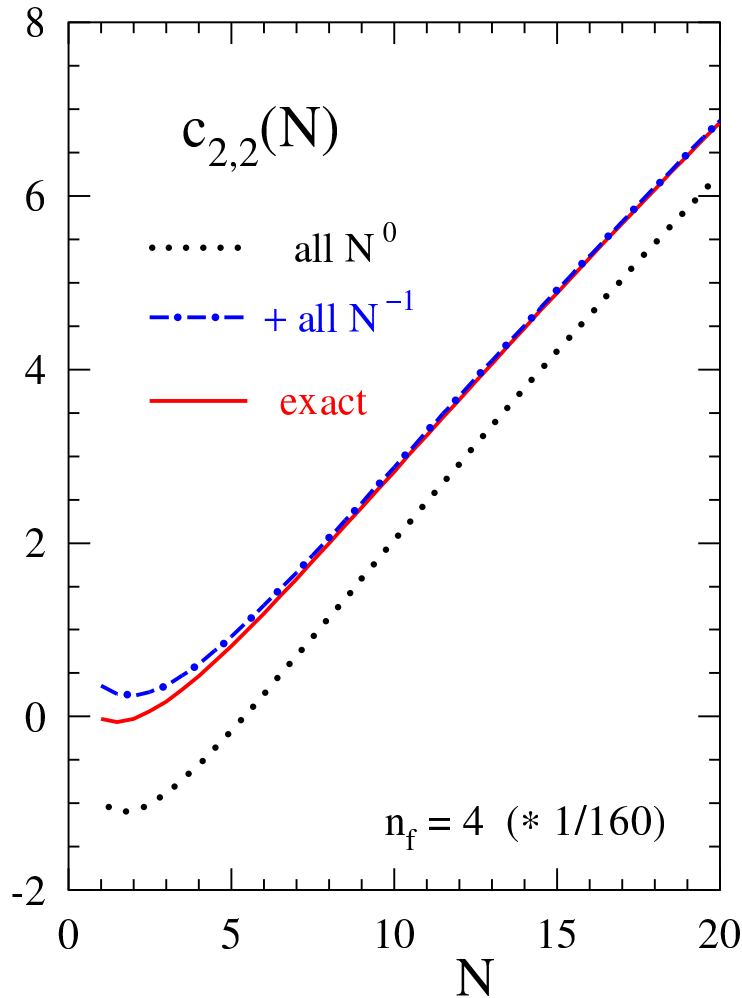
- unknown coefficient for  $\ln^3(1-x)$  estimated  $\xi_H = 300$
- exact computation  $\xi_H = 896/3$

Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger arXiv:1411.3584

- Space of Mellin moments  $N$ :

- leading contributions (SV):  $\alpha_s^k \ln^l N$  with  $1 \leq l \leq 2k$
- sub-leading contributions (SV+1/N):  $\alpha_s^k (\ln^l N)/N$  with  $1 \leq l \leq 2k$

# Quality check

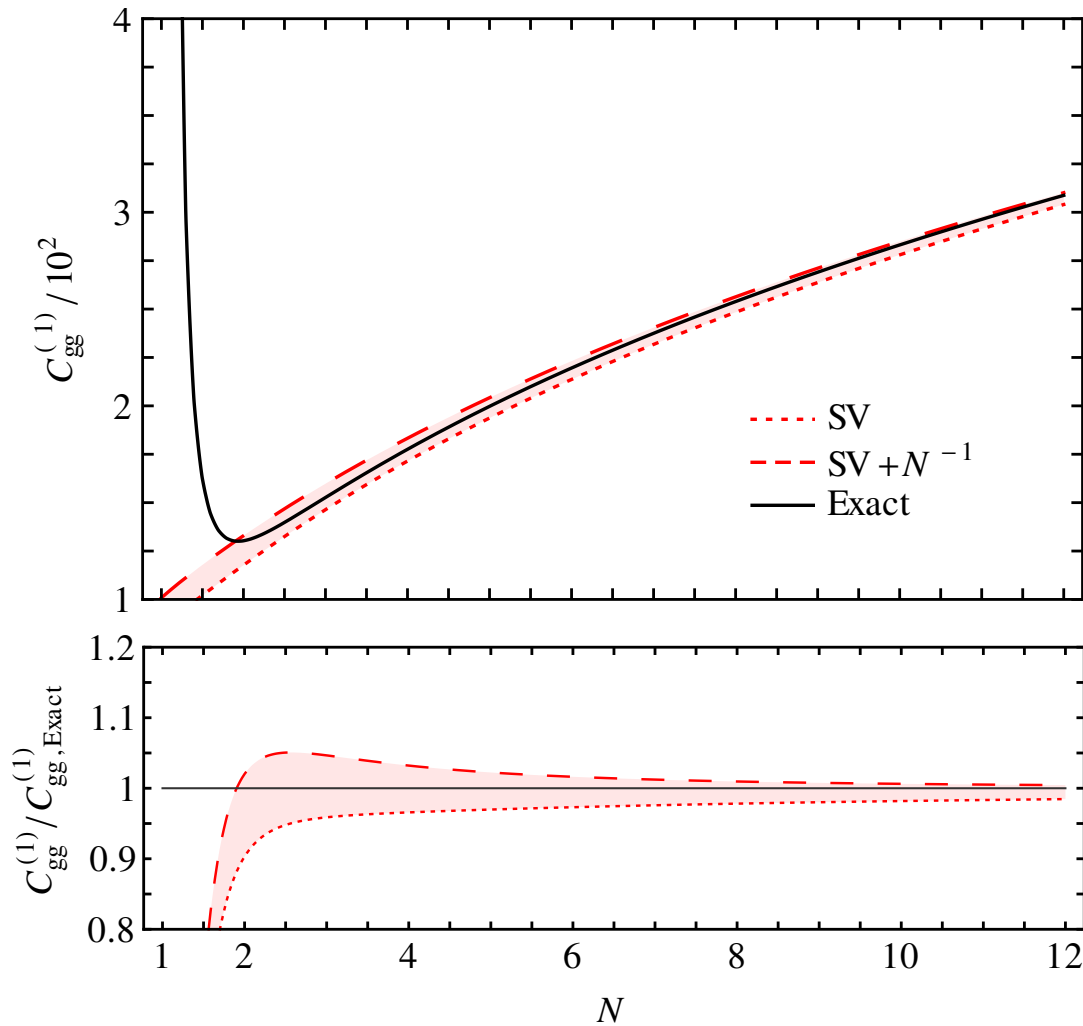


- SV and SV+1/N approximations provide uncertainty band [S.M., Vogt '09](#)
  - tested for large number of observables
  - $\gamma$ -DIS and  $\phi$ -DIS known exactly to  $N^3$ LO  
[S.M., Vermaseren, Vogt '05](#), [Soar, S.M., Vermaseren, Vogt '09](#)



# Higgs boson production: coefficient function

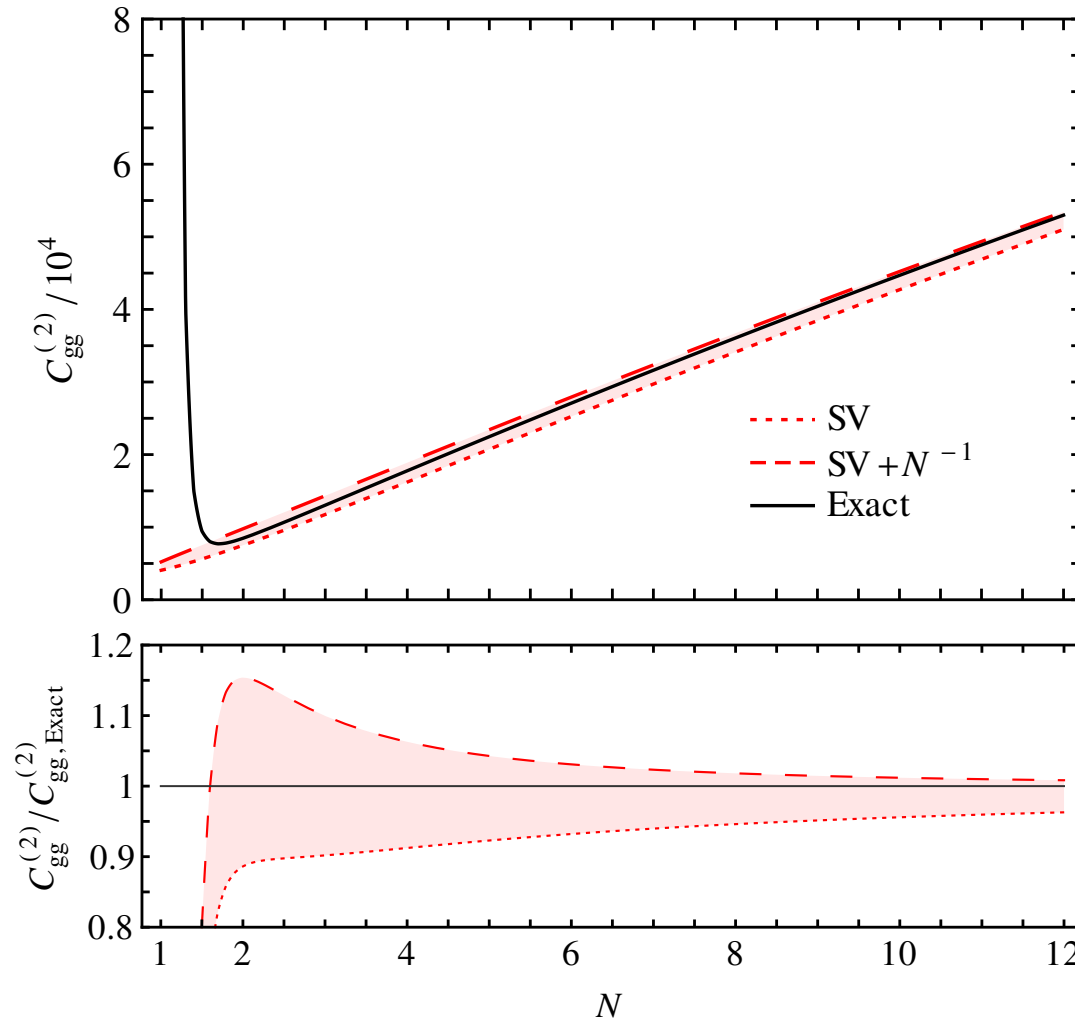
## Coefficient function at NLO



- SV and SV+1/N approximations work even for  $N$  not so large

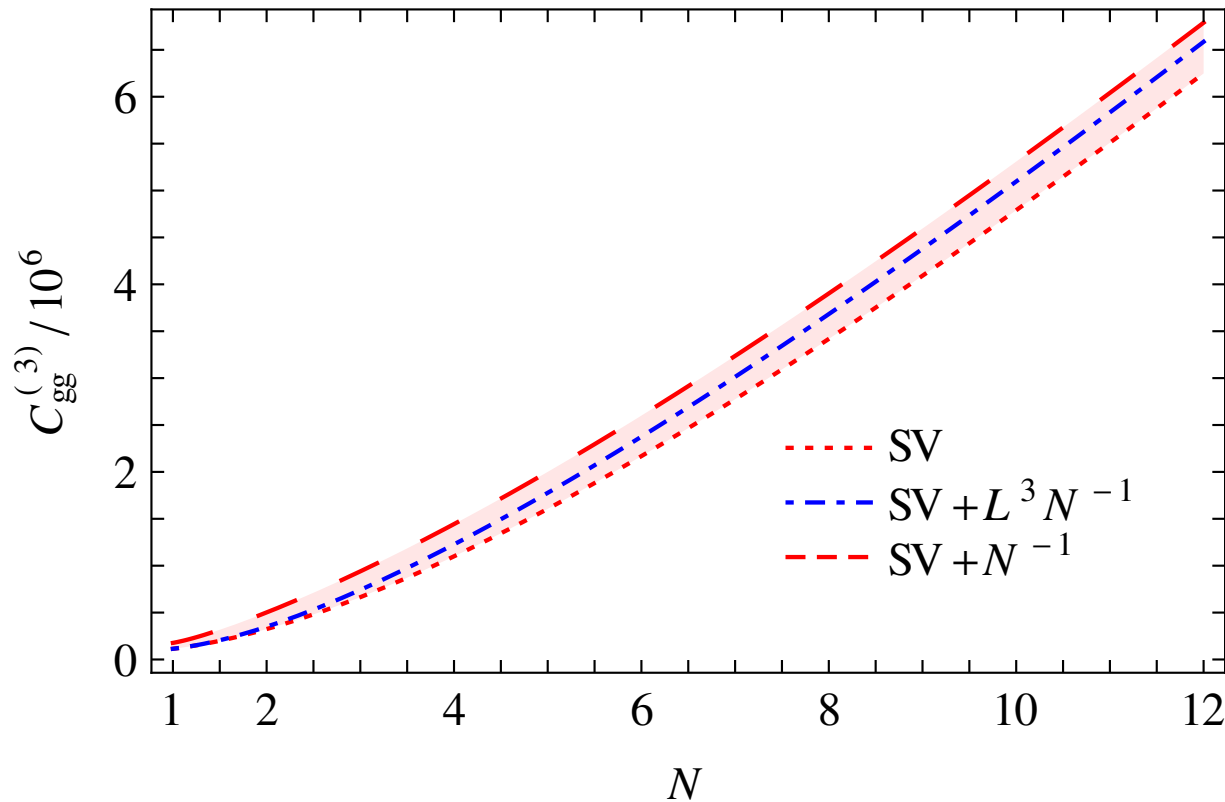
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# Approximate $N^3$ LO: coefficient function



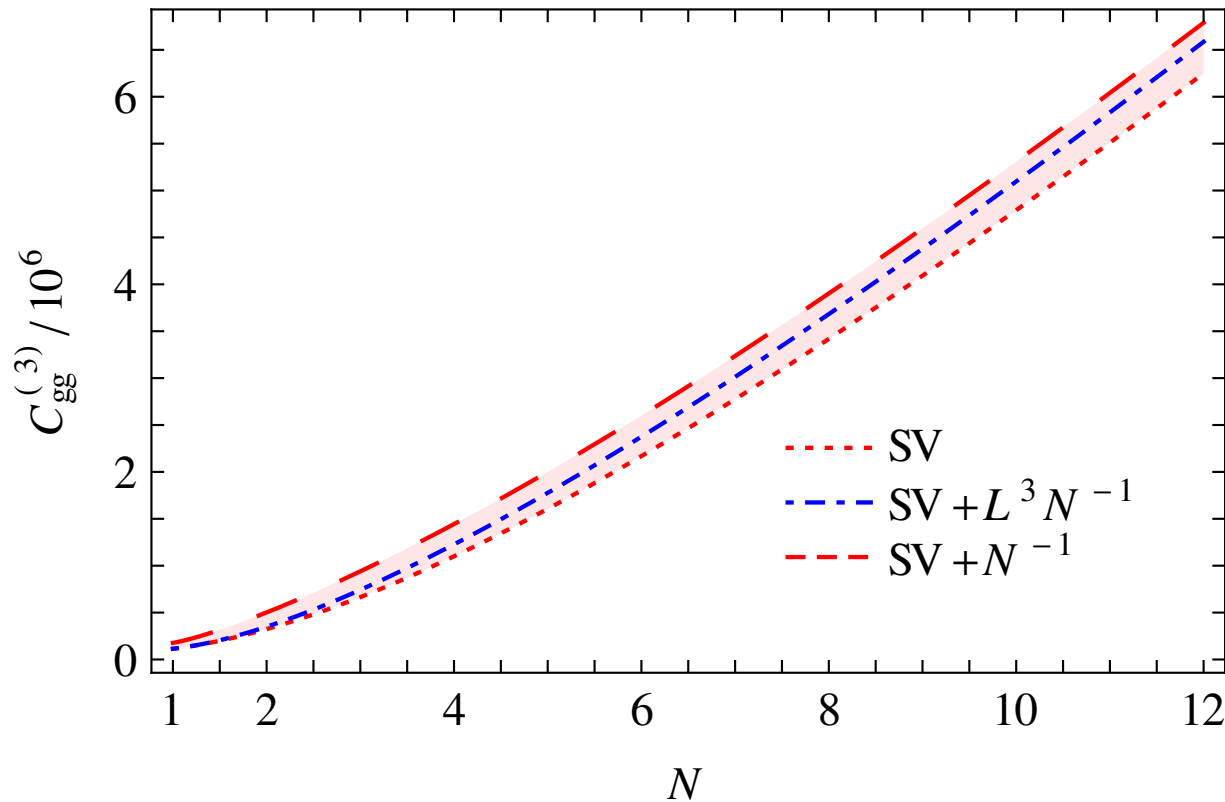
- Coefficient function at  $N^3$ LO (only subleading  $1/N$  terms displayed)

$$c_{gg}^{(3)}(N) \sim (108 \ln^5 N + 615.7 \ln^4 N + 2041.2 \ln^3 N + \underbrace{4968 \ln^2 N + 1944 \ln N + 972}_{\text{estimate from } \phi\text{-DIS}}) / N$$

- Leading logarithm agrees with estimate by factorization of splitting function  $P_{gg}^{(0)} \rightarrow c_{gg}^{(3)}(N) \sim 108 \ln^5 N / N$

Krämer, Laenen, Spira '96; Ball, Bonvini, Forte, Marzani, Ridolfi '13

# Approximate $N^3$ LO: coefficient function



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- Exact computation

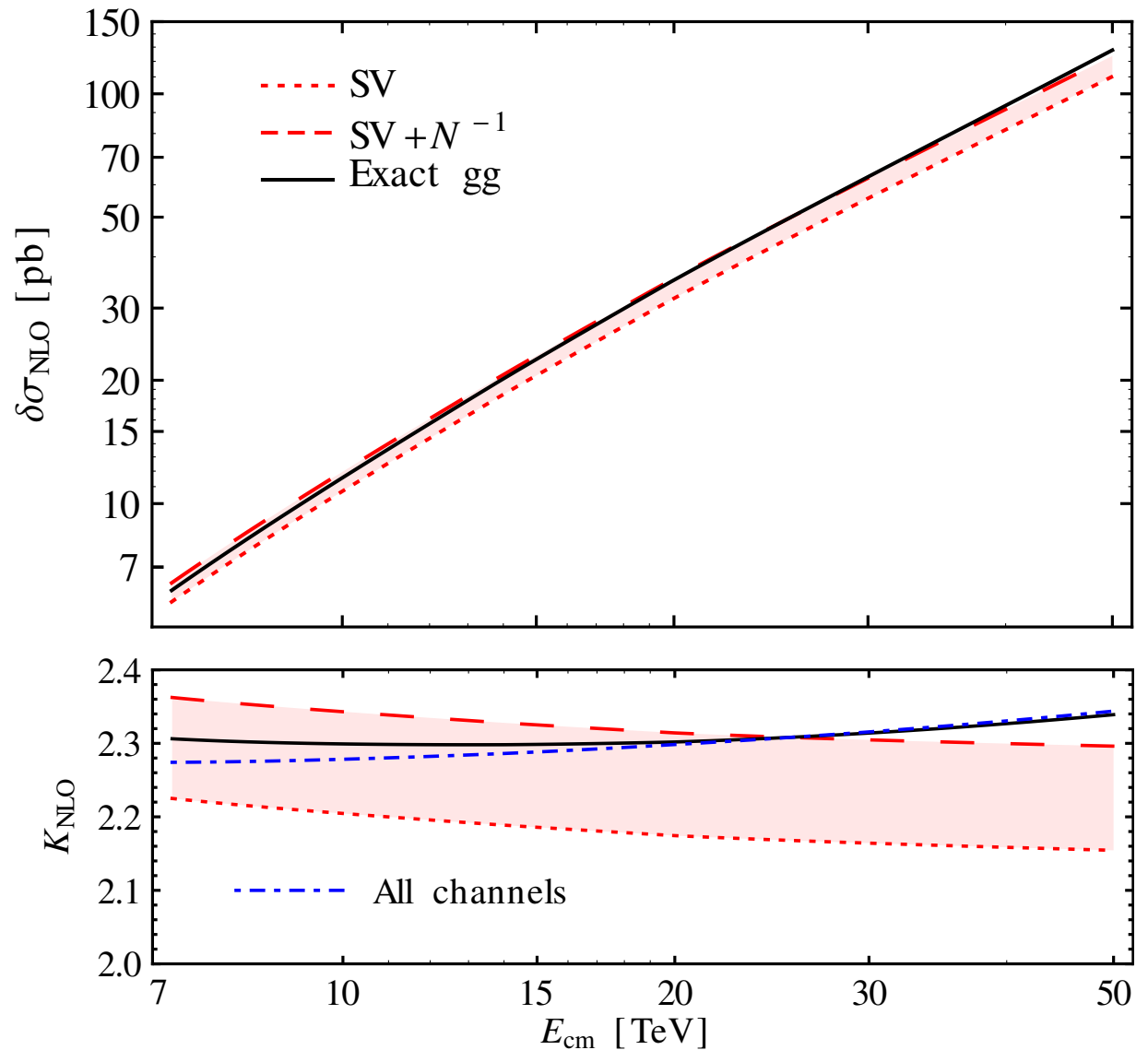
Anastasiou, Duhr, Dulat, Furlan, Gehrmann, Herzog, Mistlberger arXiv:1411.3584

$$c_{gg}^{(3)}(N) \sim (108 \ln^5 N + 615.7 \ln^4 N + 2036.4 \ln^3 N + 3305 \ln^2 N + 3459 \ln N + 703)/N$$

# Higgs boson production: cross section

## Cross section at NLO

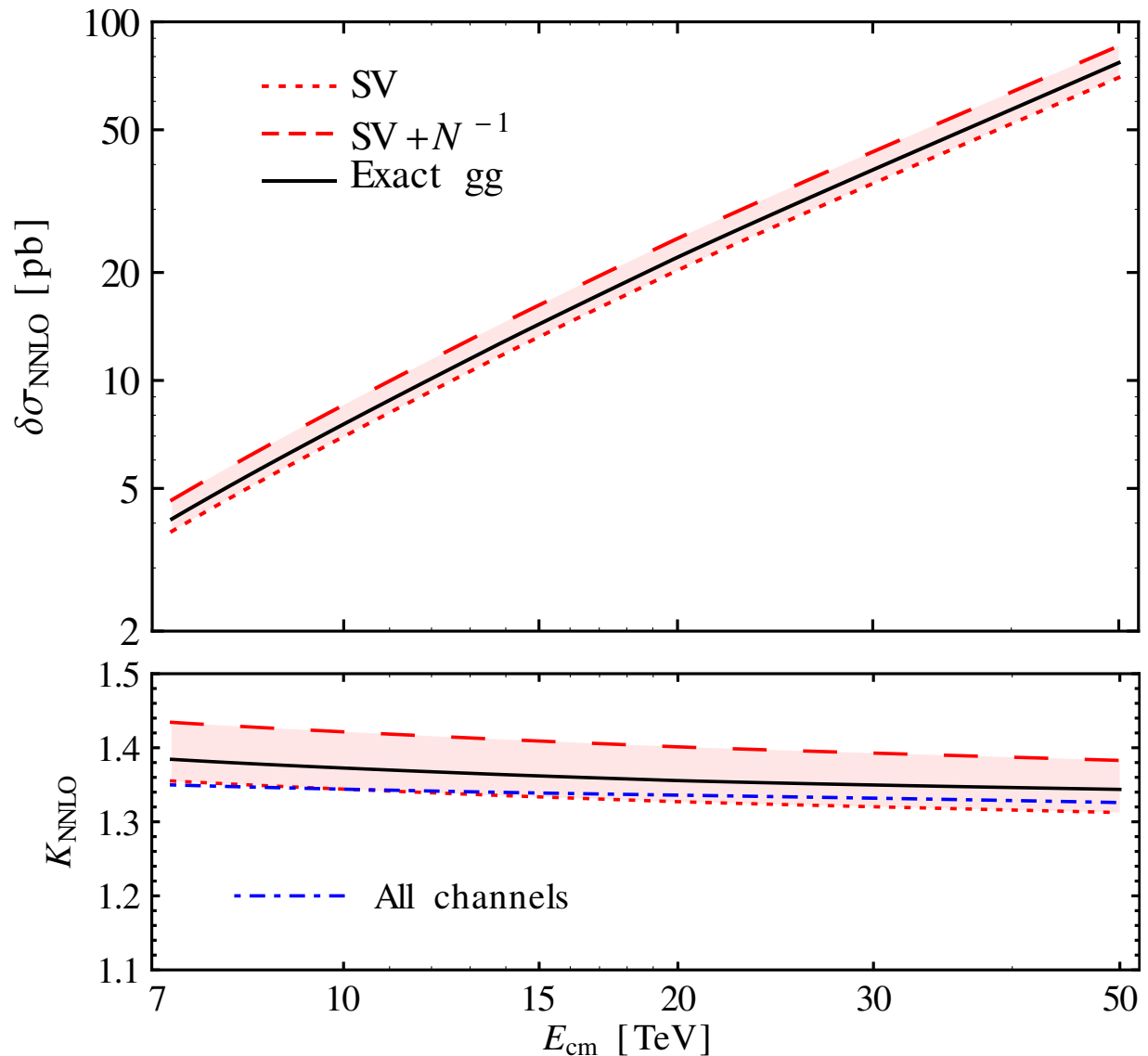
- Approximation based on SV and SV+1/N works for current LHC energies  $\sqrt{s} = 13$  TeV very well



# Higgs boson production: cross section

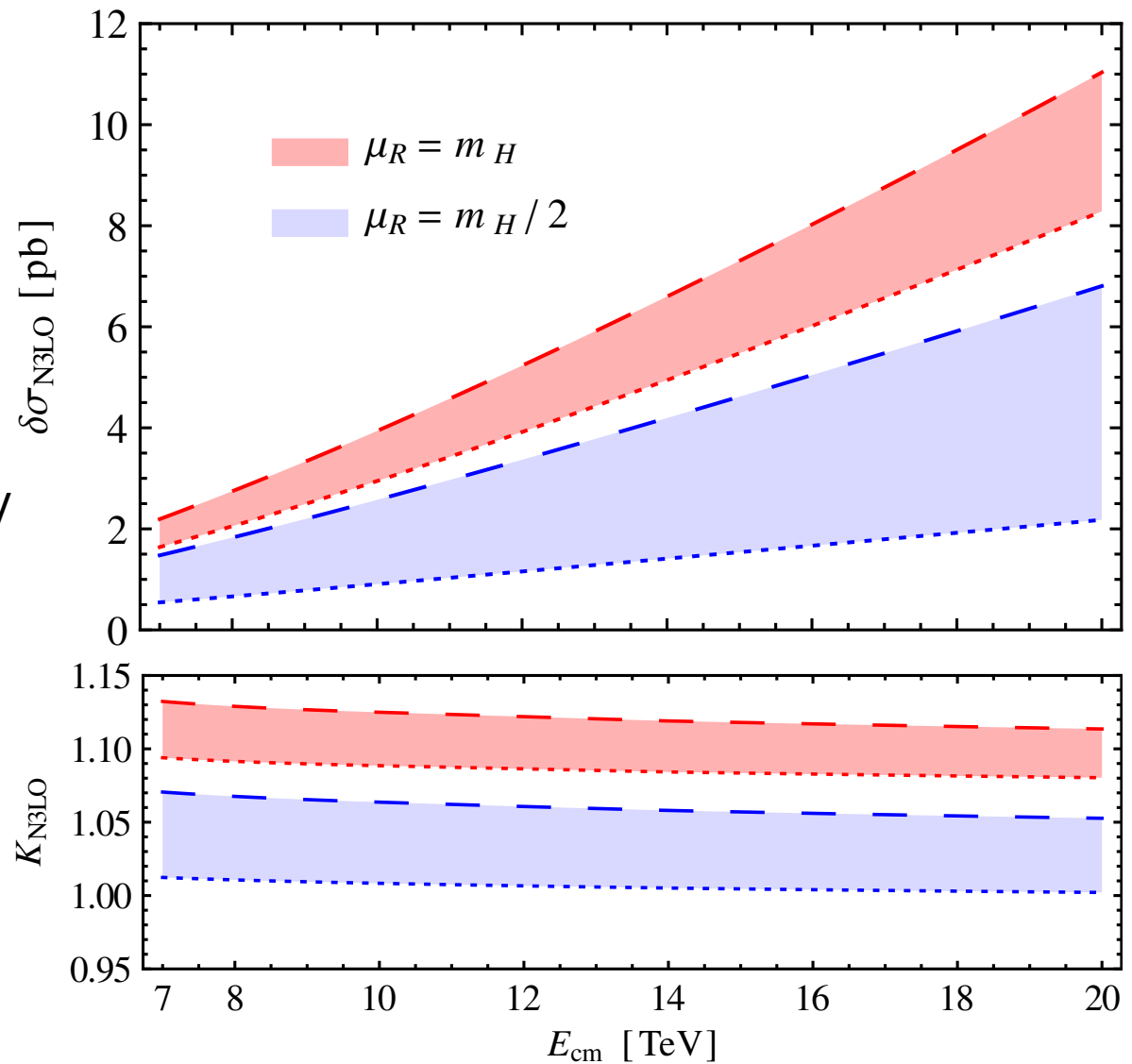
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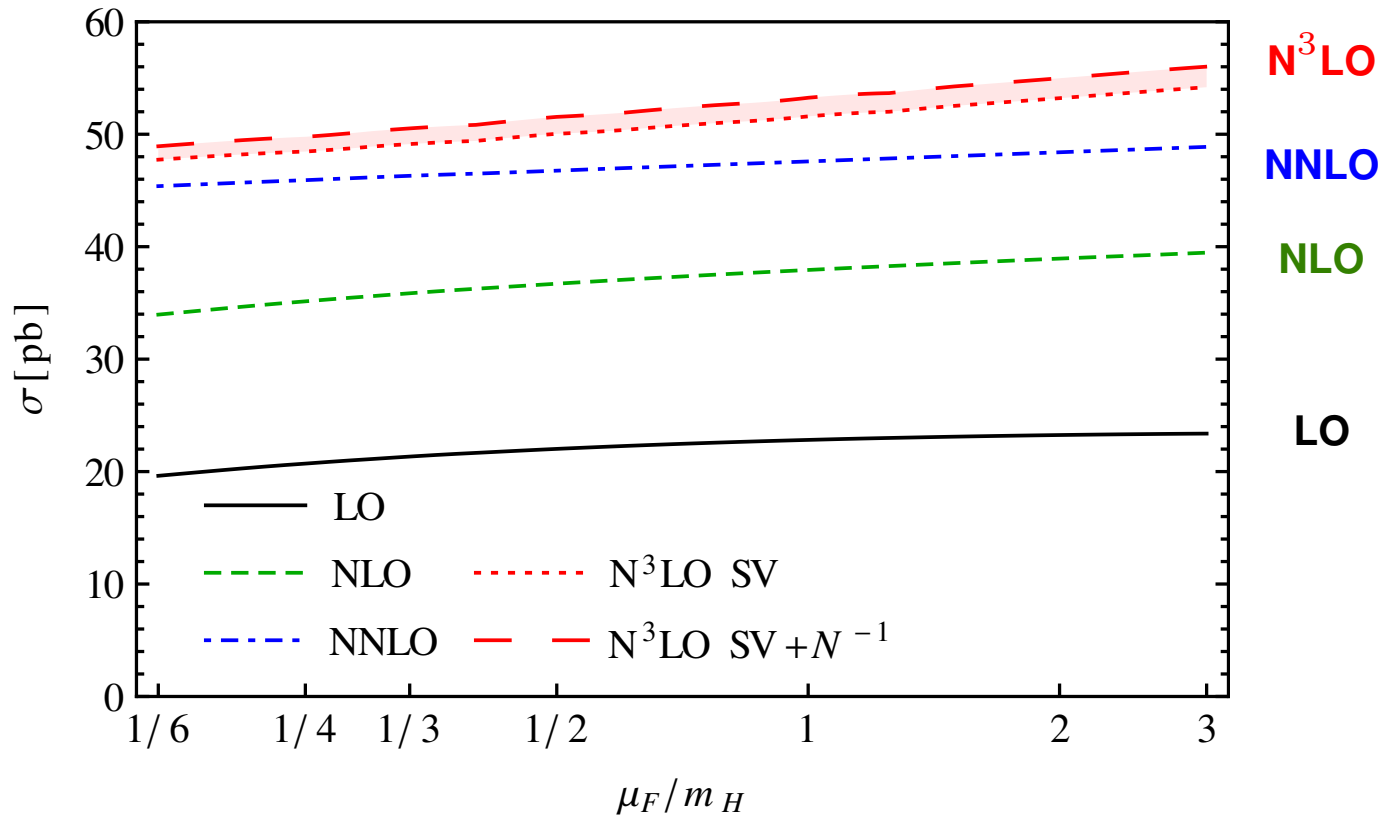


# Approximate $N^3LO$ : cross section

- $N^3LO$  corrections at two scales  
 $\mu = m_H$  and  $\mu = m_H/2$
- Point of minimal sensitivity around  $\mu = m_H/2$  with  $K$ -factor 1...5% at  $\sqrt{s} = 13$  TeV



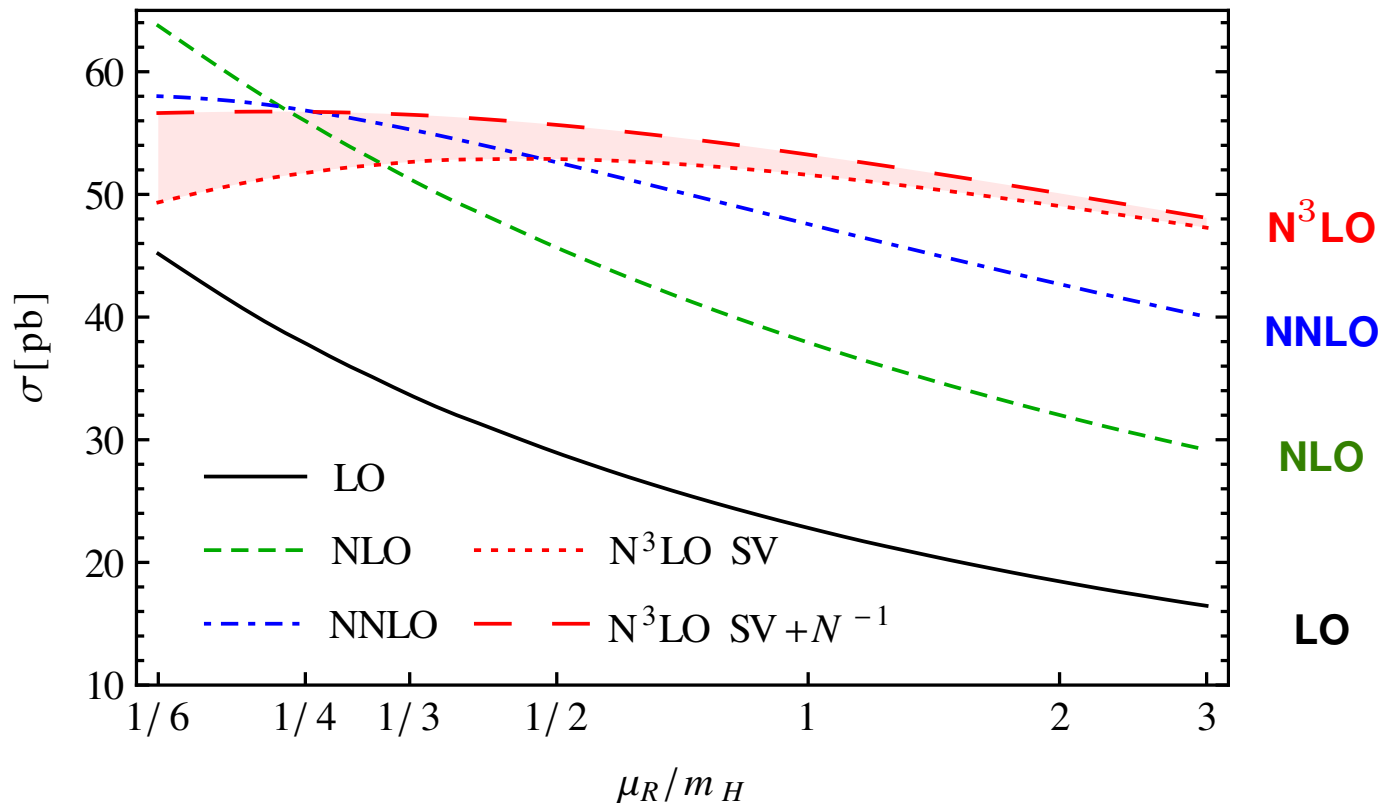
# Factorization scale variation



- Variation of factorization scale  $\mu_F$  for fixed  $\mu_R = m_H$  at  $\sqrt{s} = 14$  TeV
- Dependence on  $\mu_F$  at **N<sup>3</sup>LO** larger than at **NNLO**
  - missing four-loop splitting functions  $P_{ij}^{(3)}$
  - gluon PDF only known up to **NNLO**
  - omission of the quark-gluon and quark-(anti)quark channels

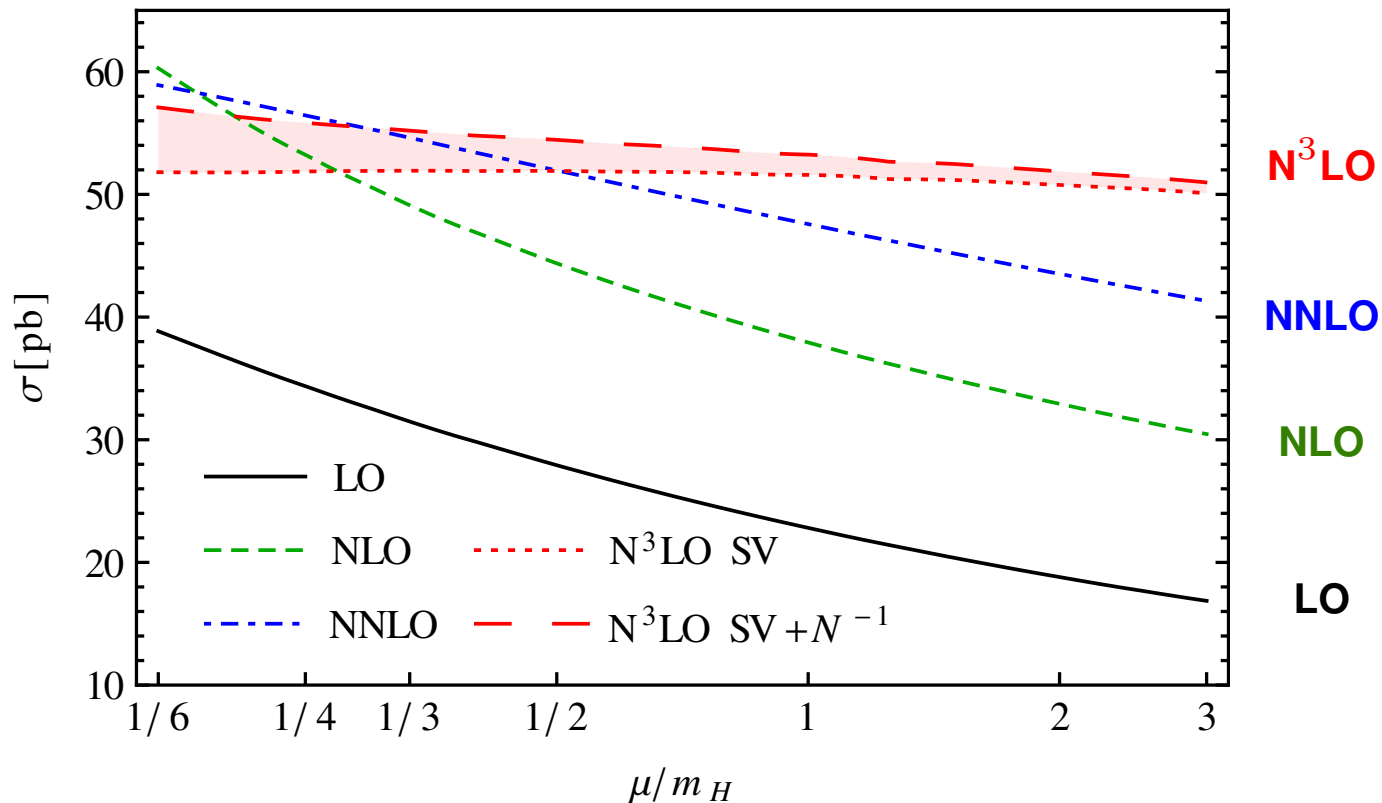


# Renormalization scale variation



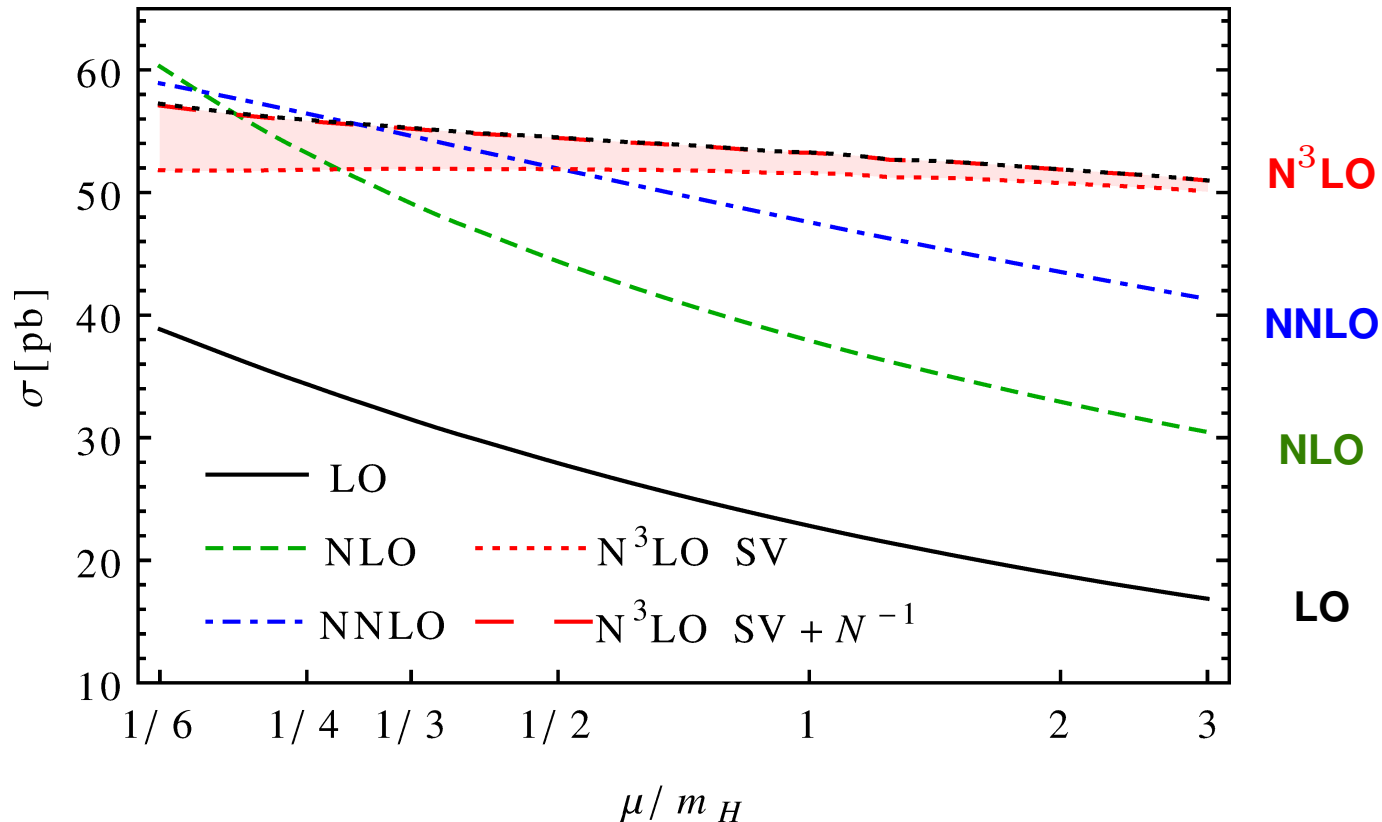
- Variation of renormalization scale  $\mu_R$  for fixed  $\mu_F = m_H$  at  $\sqrt{s} = 14$  TeV
- Perturbative stability under renormalization scale variation
  - dependence on  $\mu_R$  flattens off at **N<sup>3</sup>LO**
  - four-loop beta function  $\beta_3$  known
- Point of minimal sensitivity around  $\mu_R = m_H/2$

# Combined scale variation



- Simultaneous variation of scales  $\mu_R = \mu_F$  around  $m_H$  at  $\sqrt{s} = 14$  TeV
- Small improvement with respect  $\mu_R$  variation for fixed  $\mu_F$ 
  - should not be trusted quantitatively
- Conservative error estimate: variation of  $\mu_R$  for fixed  $\mu_F$ 
  - overall theoretical uncertainty  $\Delta\sigma(H)$  less than 5%

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# Benchmark cross section

- Cross section  $\sigma(H)$  at LHC  $\sqrt{s} = 13$  TeV
- N<sup>3</sup>LO from average of SV and SV+1/N approximation
  - parameters:  $m_H = 125$  GeV,  $m_t = 172.5$  GeV pole mass,  $\alpha_s(M_Z) = 0.1171$  with MSTW2008 PDFs (nnlo68cl)
  - scale choice  $\mu_R = \mu_F = m_H$

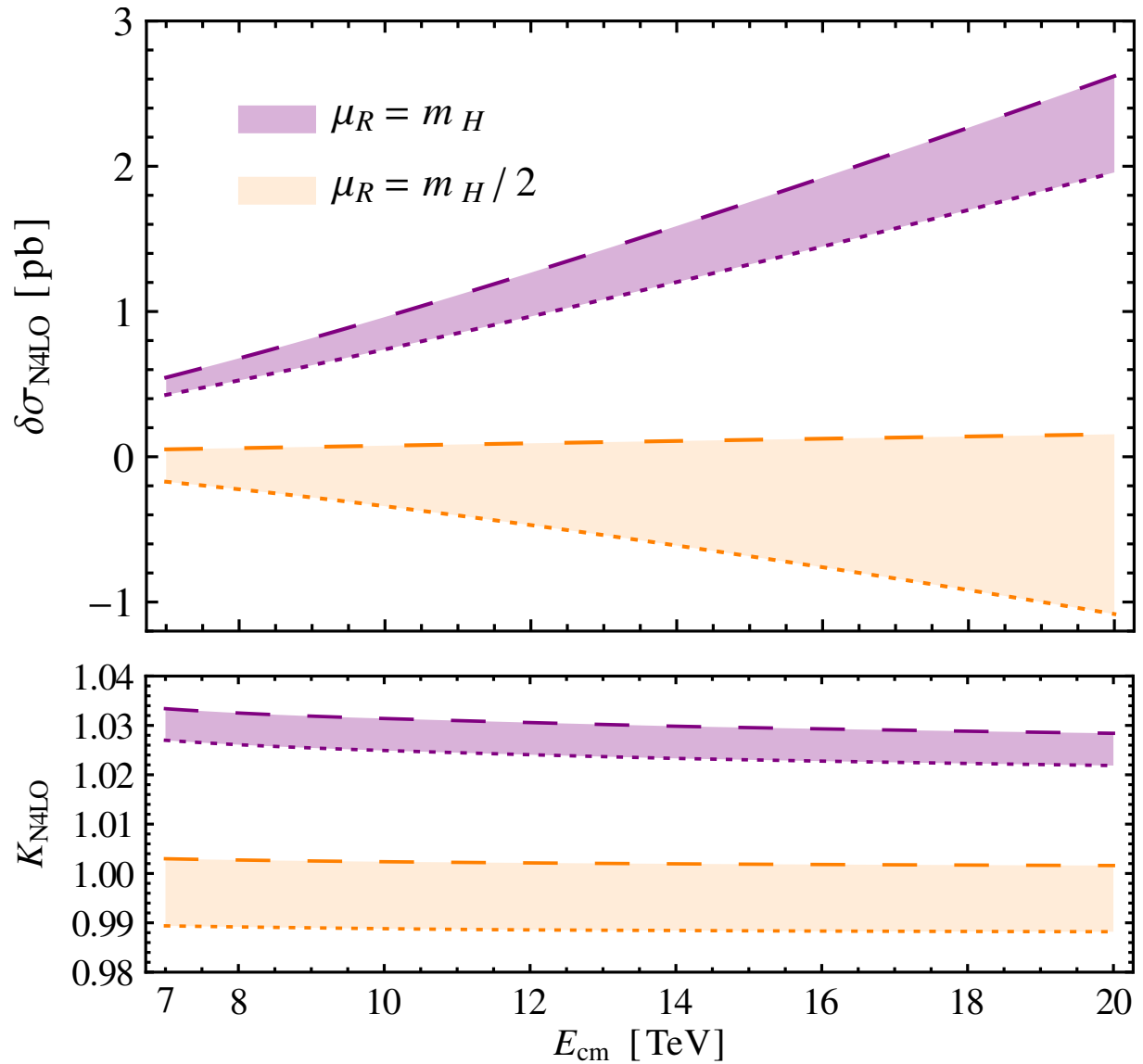
$\mu_0 = m_H$	$\mu_R/\mu_0 = 0.5$	$\mu_R/\mu_0 = 1$	$\mu_R/\mu_0 = 2$
$\mu_F/\mu_0 = 0.5$	$48.05 \pm 1.17$	$45.98 \pm 0.70$	—
$\mu_F/\mu_0 = 1$	$48.91 \pm 1.29$	$47.22 \pm 0.76$	$44.63 \pm 0.47$
$\mu_F/\mu_0 = 2$	—	$48.50 \pm 0.82$	$46.07 \pm 0.51$

- scale choice  $\mu_R = \mu_F = m_H/2$

$\mu_0 = m_H/2$	$\mu_R/\mu_0 = 0.5$	$\mu_R/\mu_0 = 1$	$\mu_R/\mu_0 = 2$
$\mu_F/\mu_0 = 0.5$	$48.66 \pm 1.86$	$47.16 \pm 1.04$	—
$\mu_F/\mu_0 = 1$	$48.73 \pm 2.11$	$48.05 \pm 1.17$	$45.98 \pm 0.70$
$\mu_F/\mu_0 = 2$	—	$48.91 \pm 1.29$	$47.22 \pm 0.76$

# Approximate $N^4LO$ : cross section

- Consistency check with approximate  $N^4LO$  corrections at two scales  $\mu = m_H$  and  $\mu = m_H/2$
- $K$ -factor  $\simeq 1\%$  for  $\mu = m_H/2$  with at  $\sqrt{s} = 13$  TeV



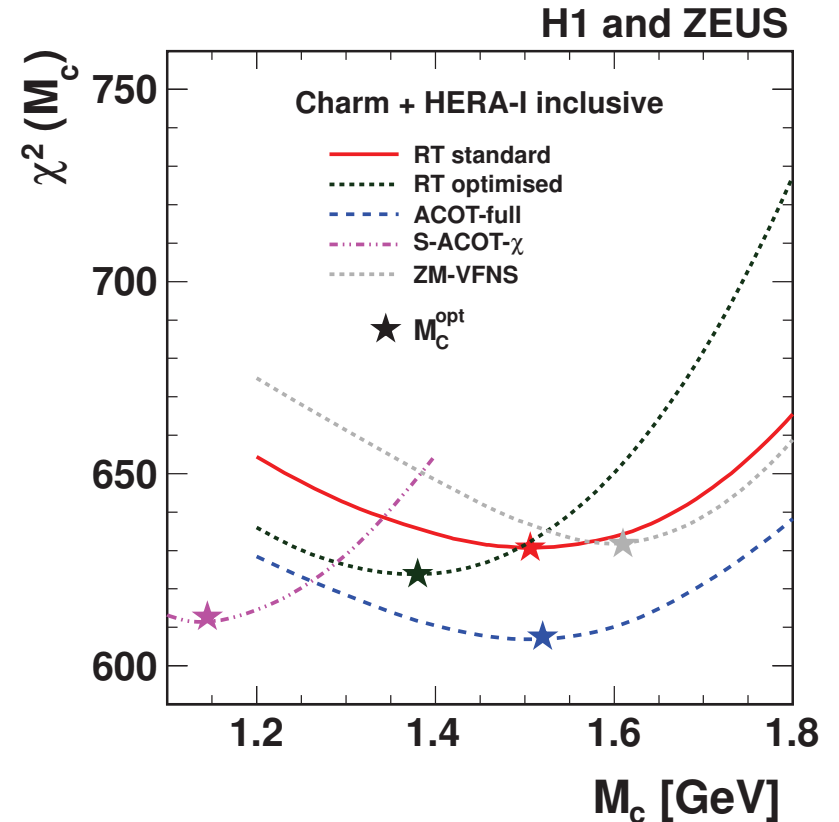
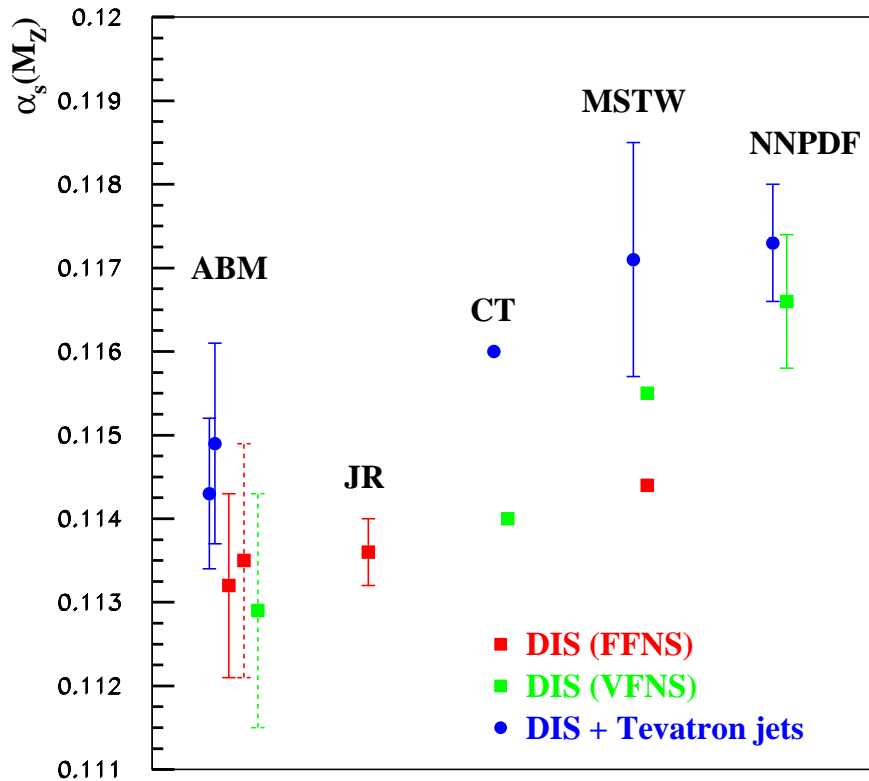
# Dependence on parton distributions

- Cross section  $\sigma(H)$  at NNLO with uncertainties:  $\sigma(H) + \Delta\sigma(\text{PDF} + \alpha_s)$

ABM11	ABM12	CT10	MSTW	NN23
$39.58 \pm 0.77$	$39.70 \pm 0.84$	$41.84^{+1.30}_{-1.69}$	$42.12^{+0.44}_{-0.63}$	$43.75 \pm 0.41$

- Comparison for PDF sets at NNLO
  - ABM11, ABM12 Alekhin, Blümlein, S.M. '13, CT10 Gao et al. '13, MSTW Martin, Stirling, Thorne, Watt '09, NNPDF (NN23) Ball et al. '12
- Large spread for predictions from different PDFs  $\sigma(H) = 39.6 \dots 43.8$
- PDF and  $\alpha_s$  differences between sets amount to up to 10%
  - significantly larger than residual theory uncertainty due to incomplete N<sup>3</sup>LO QCD corrections
- Observed spread due to differences in theory considerations and analysis procedures  $\longrightarrow$  correlations between  $\alpha_s$ ,  $g(x)$  and  $m_q$ 
  - target mass corrections and higher twist in DIS
  - treatment of heavy quarks
  - error correlations among data sets
  - fits to compatible data sets

# Non-perturbative parameters



- Differences in  $\alpha_s$  values due to different physics models for heavy quarks
- Charm mass  $m_c$  “tuning” parameter for variable flavor number schemes  
H1 coll. arxiv:1211.1182
- Effect of Higgs cross section
  - linear rise in  $\sigma(H) = 40.6 \dots 43.8$  for  $m_c = 1.05 \dots 1.75$  with MSTW PDFs Martin, Stirling, Thorne, Watt ‘10

# Summary

## *Inclusive Higgs cross section at the LHC*

- Radiative corrections at higher orders in QCD well under control
  - approximate N<sup>3</sup>LO in QCD with overall theoretical uncertainty  $\Delta\sigma(H)$  less than 5%
  - good stability around point of minimal sensitivity  $\mu = m_H/2$
  - conservative error estimate: variation of  $\mu_R$  for fixed  $\mu_F$

## *Parton distributions, $\alpha_s$ and all that*

- Precision determinations of non-perturbative parameters is dominating source of uncertainty
  - parton content of proton (PDFs)
  - coupling constants  $\alpha_s(M_Z)$
  - masses  $m_c, m_b, m_t, m_H, \dots$
- Spread in predictions amounts currently up to 10%