The Higgs cross-section at N3LO beyond threshold

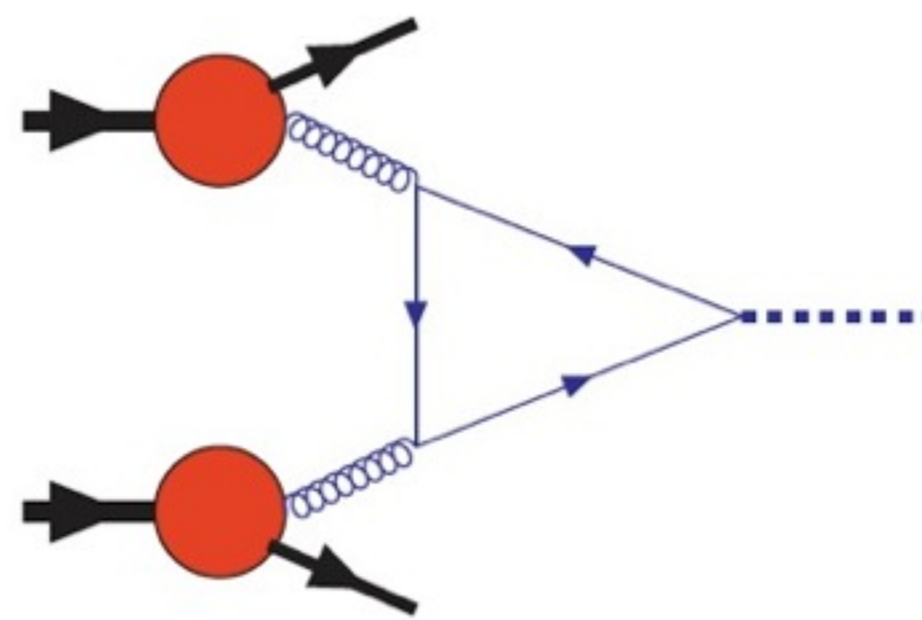
Babis Anastasiou
ETH Zurich

Based on common and co-ordinated work with:

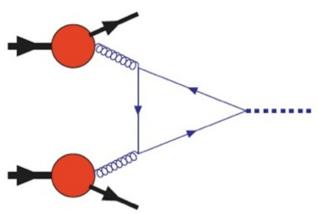
Stephan Buehler, ETHZ
Claude Duhr, Louvain & CERN
Falko Dulat, ETHZ
Elisabetta Furlan, Fermilab
Thomas Gehrmann, UZH
Franz Herzog, CERN & NIKHEF
Achilleas Lazopoulos, ETHZ
Bernhard Mistlberger. ETHZ

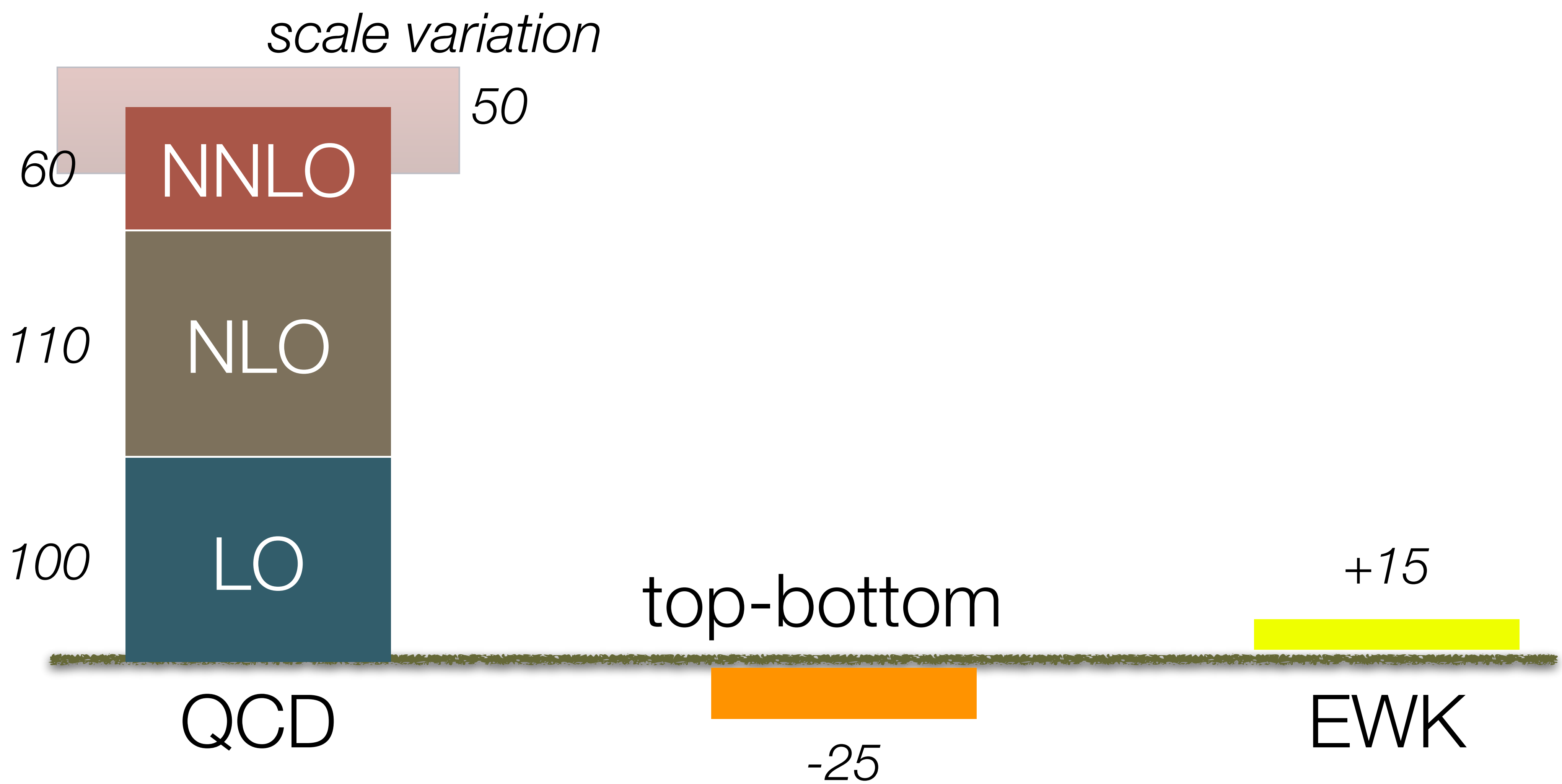
Gluon fusion cross-section

- How well do we know now the gluon fusion cross-section?
- Are current recommendations to experiments the best knowledge?
- What is our opinions for a new recommendation?



Gluon fusion cross-section in fixed order perturbation theory





LHC 14TeV

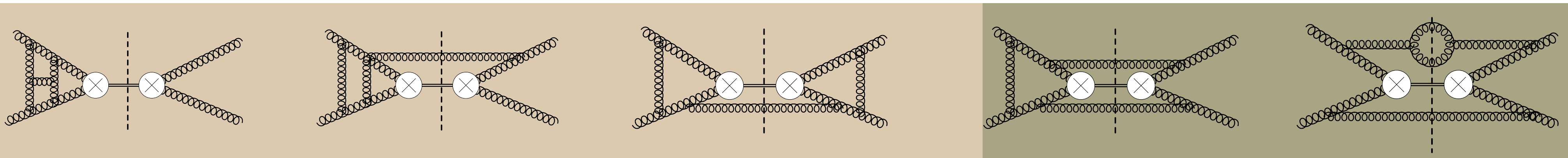
$\mu = M_H = 125\,\mathrm{GeV}$

MSTWNNLO2008

Progress in N3LO corrections

	1st term	2nd term	Full
VVV	-	-	yes
RVV	yes	yes	yes
(RV)(RV)	yes	yes	yes
RRV	yes	yes	no
RRR	yes	yes	no
IR+UV	yes	yes	yes

04/2014 11/2014



VVV RVV (RV)(RV) RRV RRR

$$\frac{P_{gg}^{(1)}}{\epsilon} \otimes \sigma^{\text{NNLO}}(\epsilon)$$

IR+UV

Progress in N3LO corrections

$$z = \frac{M_h^2}{s}$$

	$\frac{1}{(1-z)_+}$	$(1-z)^0$	ALL
$\delta(1-z)$	$-$	$-$	yes
$\log^5(1-z)$	yes	yes	yes
$\log^4(1-z)$	yes	yes	yes
$\log^3(1-z)$	yes	yes	yes
$\log^2(1-z)$	yes	yes	no
$\log^1(1-z)$	yes	yes	no
$\log^0(1-z)$	yes	yes	no
	04/2014	11/2014	

These results constitute the state-of-the-art beyond NNLO

Progress in N3LO corrections

	$\frac{1}{(1-z)_+}$	$(1-z)^0$	ALL
$\delta(1-z)$	$-$	$-$	5.1%
$\log^5(1-z)$	93.72%	115.33%	205.63%
$\log^4(1-z)$	20.01%	101.07%	113.88%
$\log^3(1-z)$	−39.30%	−32.15%	−78.50%
$\log^2(1-z)$	−52.45%	−89.41%	?
$\log^1(1-z)$	−22.88%	−55.50%	?
$\log^0(1-z)$	−5.85%	−14.31%	?

LHC 14TeV

$\mu = M_H = 125\,\text{GeV}$

MSTWNNLO2008

Mellin-space

$$
\begin{aligned}
M\left[\eta_{gg}^{(3)}\right](N) \simeq\, & 36 \log^6 N && (\to 0.0013\%) \\
& + 170.679\ldots \log^5 N && (\to 0.0226\%) \\
& + 744.849\ldots \log^4 N && (\to 0.2570\%) \\
& + 1405.185\ldots \log^3 N && (\to 1.0707\%) \\
& + 2676.129\ldots \log^2 N && (\to 4.0200\%) \\
& + 1897.141\ldots \log N && (\to 5.1293\%) \\
& + 1783.692\ldots && (\to 8.0336\%) \\
& + 108 \frac{\log^5 N}{N} && (\to 0.0105\%) \\
& + 615.696\ldots \frac{\log^4 N}{N} && (\to 0.1418\%) \\
& + 2036.407\ldots \frac{\log^3 N}{N} && (\to 0.9718\%) \\
& + 3305.246\ldots \frac{\log^2 N}{N} && (\to 2.9487\%) \\
& + 3459.105\ldots \frac{\log N}{N} && (\to 5.2933\%) \\
& + 703.037\ldots \frac{1}{N} && (\to 1.7137\%).
\end{aligned}
$$

LHC 14TeV

$\mu = M_H = 125\,\mathrm{GeV}$

MSTWNNLO2008

	NLO	NNLO	N^3LO
$z-$space	63.42%	376.5%	−1106.5%
Mellin − space	14.02%	32.71%	59.78%

- The formal hierarchy of logs is not reflected in the hadronic cross-section (neither in z-space nor in Mellin-space).
- Next-to-soft corrections are large
- Next-to-soft corrections become increasingly important at higher orders.
- No theoretical justification to stop at next-to-soft.
- Empirical arguments from NLO and NNLO experience?

Hadronic integral and an ambiguity

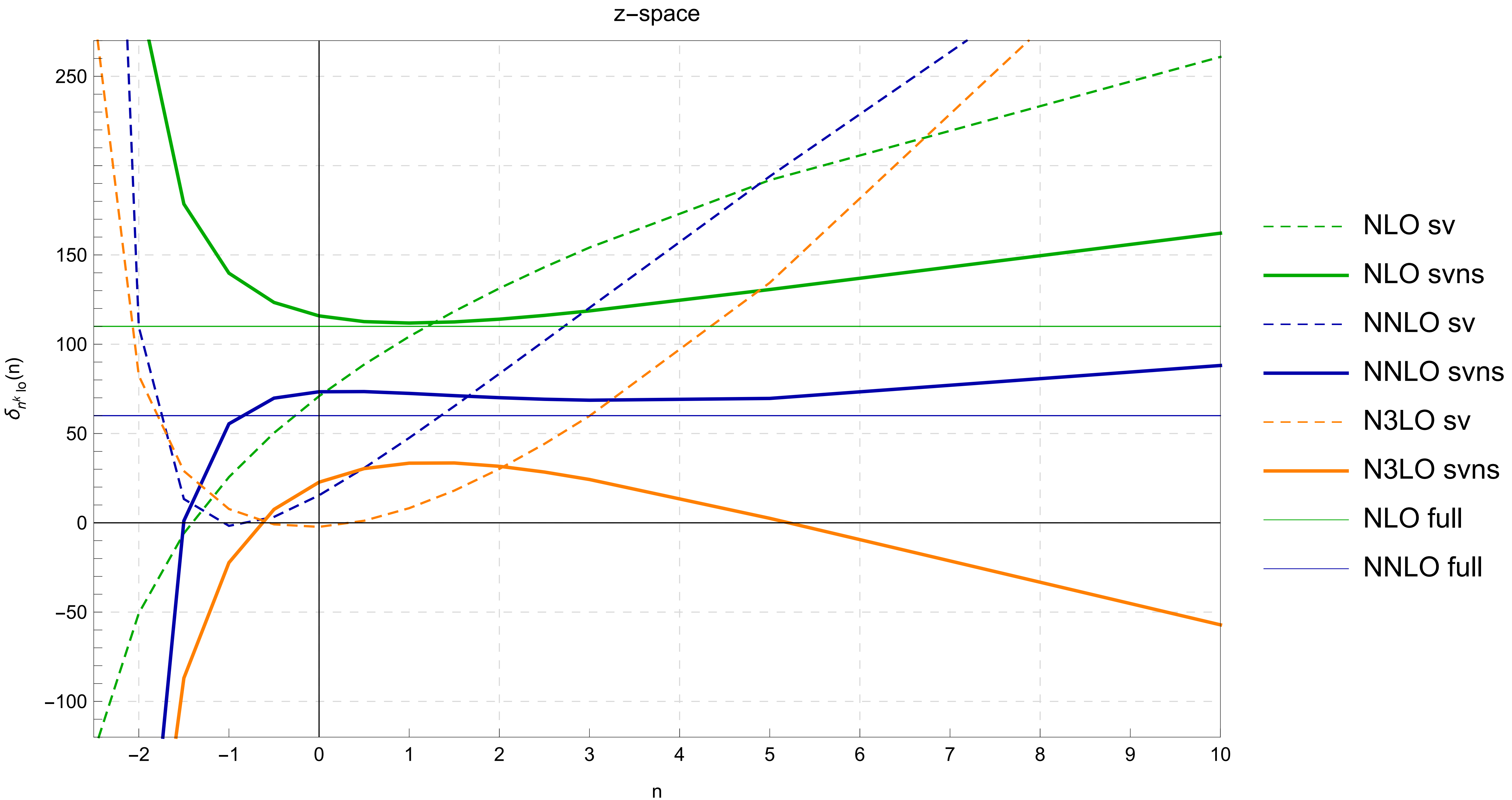
$$\sigma = \tau^{1+n} \sum_{ij} \left(f_i^{(n)} \otimes f_j^{(n)} \otimes \frac{\hat{\sigma}_{ij}(z)}{z^{1+n}} \right)(\tau)$$

$$f_i^{(n)}(z) \equiv \frac{f_i(z)}{z^n}.$$

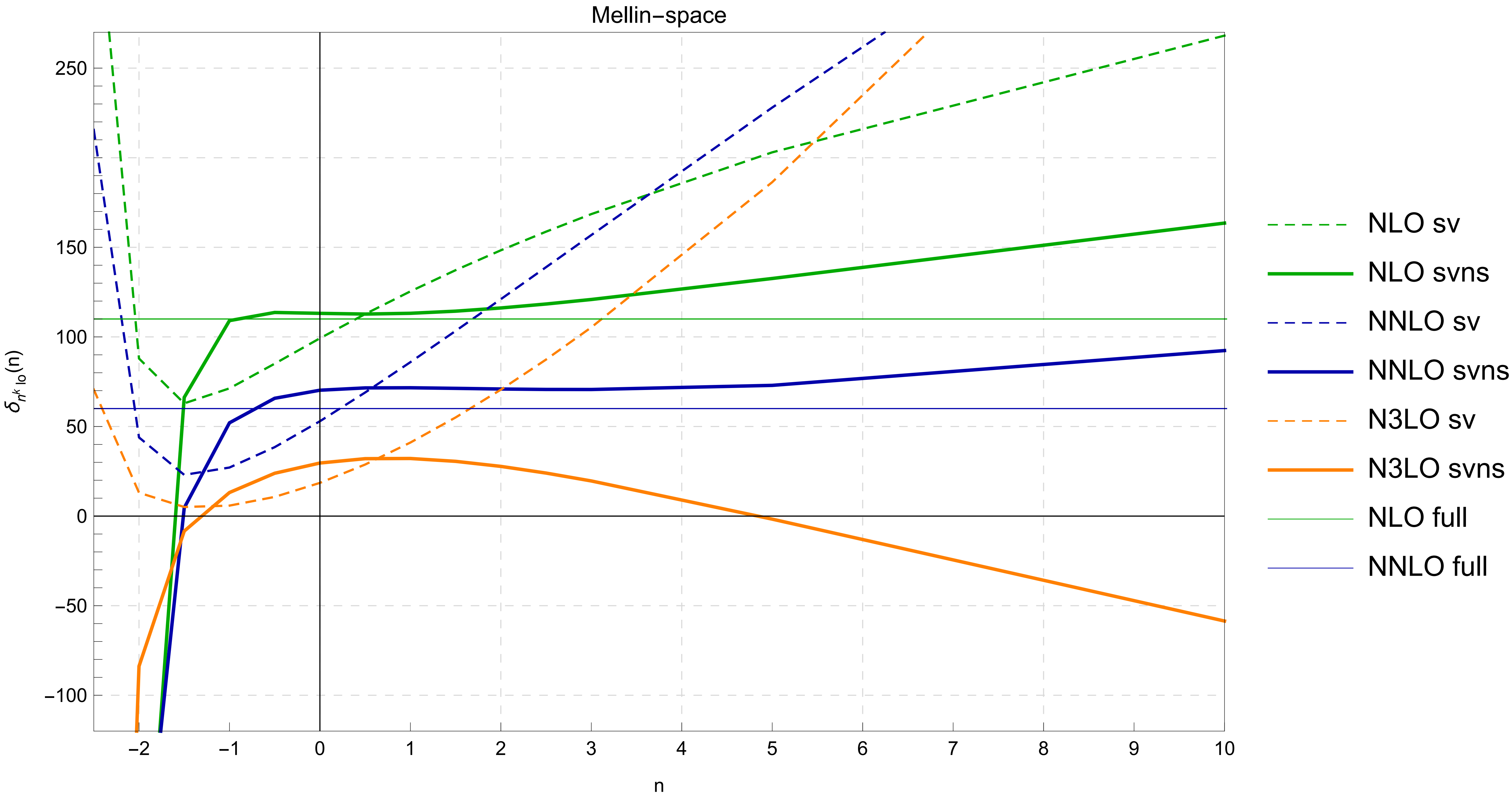
$$M\left[\frac{\sigma}{\tau^{1+n}}\right](N) = \sum_{ij} M\left[f_i^{(n)}\right](N)\, M\left[f_j^{(n)}\right](N)\, M\left[\frac{\hat{\sigma}(z)}{z^{1+n}}\right](N)$$

$$\frac{\hat{\sigma}_{ij}(z)}{z^{1+n}} \simeq \hat{\sigma}_{ij}(z)|_{(1-z)^{-1}} + \hat{\sigma}_{ij}(z)|_{(1-z)^0} + n(1-z)\, \hat{\sigma}_{ij}(z)|_{(1-z)^{-1}} + \mathcal{O}(1-z)^1$$

n-variaton in z-space

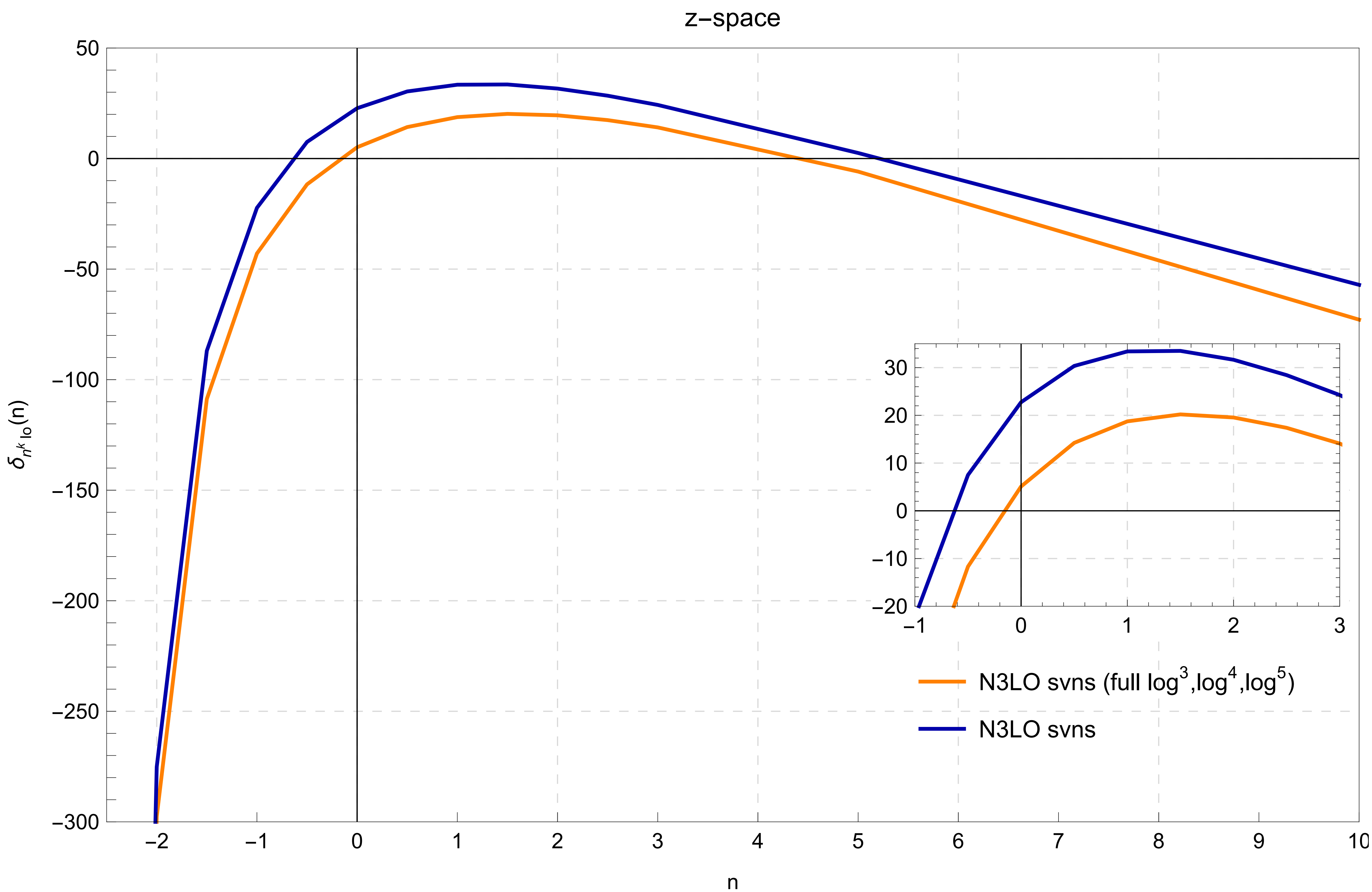


n-variation Mellin-space

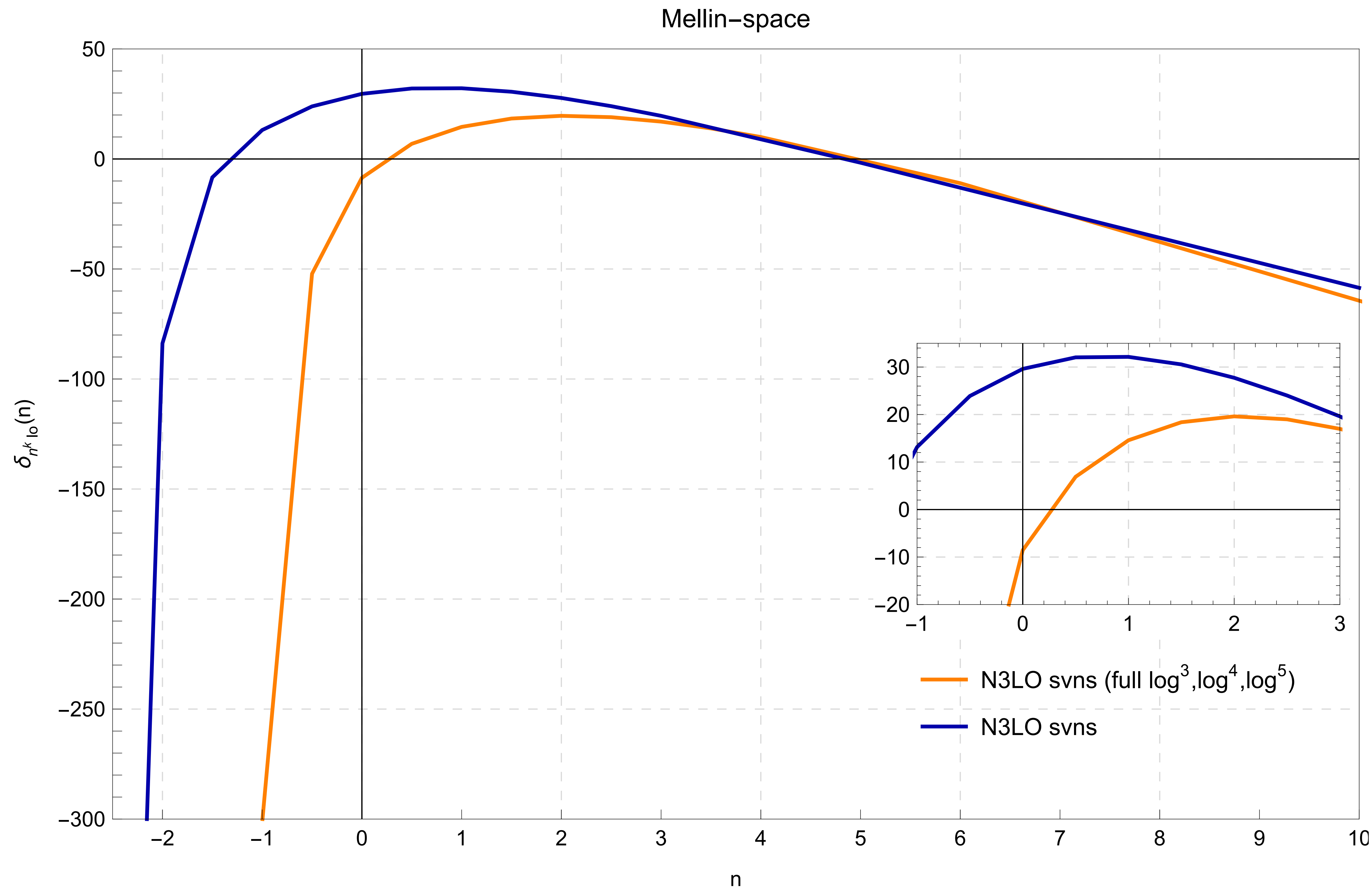


- n-variation is very large for the soft-virtual terms alone.
- sensitivity decreases by including the next-to-soft terms.
- sensitivity increases at N3LO, rendering empirical arguments even more dangerous.
- Entertaining the idea that NLO and NNLO may show us the way, we can restrict n in [-1,3].
- This gives a range of predictions for N3LO from -22% to 33% of the Born.
- Larger uncertainty than the -12% to 12% of the Born scale variation at N3LO.

n-variation z-space/ Full leading logs



n-variation Mellin-space/ Full leading logs



- including the exact log^k(1-z), k=5,4,3 produces a downward shift to the cross-section.
- The shift is more dramatic in N-space.
- It is bigger that the N3LO scale variation.

Summary

- I have presented a calculation of the N3LO cross-section through the first two terms in the threshold expansion.
- We also computed the exact coefficients of the three leading logarithms.
- Unfortunately, these results are not yet the full N3LO cross-section. What is missing is important!

My personal recommendations (not meant to represent my collaborators)

- We cannot trust the soft approximation and soft-gluon resummation to “capture the bulk of N3LO and beyond corrections”. It may even be misleading, since large cancelations are at play.
- Resort to full corrections in fixed order perturbative QCD.
- Wait for the full N3LO result.
- In the mean time, use the NNLO corrections (without resummation) and assign a generous uncertainty.