$\ln^2(-1) = -\pi^2 \neq -6\mathrm{Li}_2(1) = -\pi^2$

(for lack of a better title and sleep)

Frank Tackmann

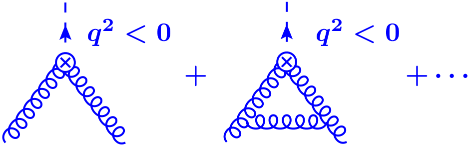
Deutsches Elektronen-Synchrotron

ggH meeting
November 18, 2014

Gluon Form Factor.

Consider the on shell gluon form factor at spacelike momentum transfer $q^2 < 0$

$$C_g(q^2) = \langle g | G^a_{\mu\nu} G^{a\mu\nu} | g \rangle =$$



$C_g(q^2)$ is IR-divergent

- Can consider it in $d = 4 - 2\epsilon$ dimensions: $C_g(q^2, \epsilon)$
- Equivalently, can "renormalize" it at some scale μ: $C_g(q^2, \mu)$
  - This is what happens in SCET, where $C_g(\mu)$ becomes (part of) the matching coefficient of the $O_{gg}(\mu)$ operator which is renormalized, usually in $\overline{\mathrm{MS}}$
- It depends on q^2 only through Sudakov logarithms

$$C_g(q^2, \mu) = \sum_{n, m \leq 2n} c_{mn}\, \alpha_s^n(\mu)\, \ln^m(q^2/\mu^2)$$

Gluon Form Factor.

$$C_g(q^2, \mu) = \sum_{n,m \leq 2n} c_{mn}\, \alpha_s^n(\mu)\, \ln^m(q^2/\mu^2)$$

Ratio $|C(-q^2 - \mathrm{i}0)/C(q^2)|^2$ of timelike to spacelike form factors

- is IR finite (ϵ or μ-independent) but contains leftover Sudakov logarithms

  $\alpha_s^n \ln^m(-1 - \mathrm{i}0) = \alpha_s^n(-\mathrm{i}\pi)^m$

  - Responsible for large corrections in Drell-Yan/DIS (quark form factor)
  - Can resum these by RG evolving from $-\mathrm{i}\mu \to \mu$
  - Very well known since many years

    [..., Parisi '80; Sterman '87; Magnea, Sterman '90]

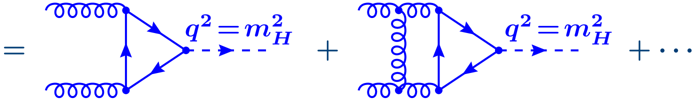
$\Rightarrow$ Same happens in $gg \to H$ where $C_g(-m_H^2 - \mathrm{i}0)$ at timelike q^2 appears which contributes to large pert. corrections

[Ahrens, Becher, Neubert, Yan '08]

ggH Form Factor.

Now consider full $t\bar{t}H$ induced ggH form factor

$$C_{ggH}(m_H, m_t) = \langle H | t\bar{t}H | gg \rangle$$



In $m_t \to \infty$ limit: $C_{ggH}(m_H, m_t; \mu) = C_t(m_t; \mu)\, C_g(-m_H^2 - \mathrm{i}0; \mu)$

- $C_t(m_t; \mu)$ contains single logarithms $\ln(m_t/\mu)$
- $C_g(-m_H^2 - \mathrm{i}0; \mu)$ contains Sudakov logarithms $\ln(-m_H^2 - \mathrm{i}0/\mu^2)$

General finite m_t: Write $C_{ggH}(m_H, m_t; \mu)$ in terms of

- Single logs $\ln(-\mathrm{i}m_H/m_t)$ (very small effect, starting at NNLO)
- Sudakov logs $\ln(-m_H^2 - \mathrm{i}0/\mu^2)$ (dominant)

C_{ggH} Perturbative Series.

$H_{gg}(m_H, m_t; \mu) = |C_{ggH}(m_H, m_t; \mu)|^2$ at different scales and with/without finite m_t (up to 1-loop)

$$\mu = m_H, m_t \to \infty : \qquad H_{gg} = H_{gg}^{(0)}(1 + 0.810 + 0.356 + \cdots)$$

$$\mu = m_H, \text{finite } m_t : \qquad H_{gg} = H_{gg}^{(0)}(1 + 0.815 + 0.360 + \cdots)$$

$$\mu = m_H/2, m_t \to \infty : \qquad H_{gg} = H_{gg}^{(0)}(1 + 0.786 + 0.258 + \cdots)$$

$$\mu = m_H/2, \text{finite } m_t : \qquad H_{gg} = H_{gg}^{(0)}(1 + 0.791 + 0.261 + \cdots)$$

$$\mu = -\mathrm{i}m_H, m_t \to \infty : \qquad H_{gg} = H_{gg}^{(0)}(1 + 0.270 + 0.0417 + \cdots)$$

$$\mu = -\mathrm{i}m_H, \text{finite } m_t : \qquad H_{gg} = H_{gg}^{(0)}(1 + 0.274 + 0.0425 + \cdots)$$

(where the numbers are total NLO and NNLO contributions)

Clear improvement in the pert. series of the form factor, so the relevant question is:

Can we use this to improve the inclusive cross section, and if so how?

Total Cross Section.

Total σ in d dimensions to regulate IR

$$\sigma = \underbrace{H_{gg}(\epsilon)}_{\text{virtual}} \times \underbrace{[\mathcal{I}_{gi}(\epsilon, \mu_f) \otimes_z f_i(\mu_f)]^2}_{\text{real collinear}} \times \underbrace{S_{gg}(\epsilon)}_{\text{real soft}} + \text{IR finite}$$

- $1/\epsilon$ IR divergences cancel between virtual and real collinear+soft

Renormalized EFT version

$$\sigma = H_{gg}(\mu) \times [\mathcal{I}_{gi}(\mu, \mu_f) \otimes_z f_i(\mu_f)]^2 \times S_{gg}(\mu) + \text{IR finite}$$

- Now μ dependence cancels between first three terms
- Collinear contributions contain $1 - z$ dependence
  (in collinear limit but for arbitrary z, includes qg, qq channels at (N)NLO)
- Threshold limit $z \to 1$ takes the further limit that collinear are soft as well, combining them into overall virtual+soft
  - $\Rightarrow$ Above virtual+soft+collinear limit is weaker and sufficient to discuss how H_{gg} enters

Combine collinear+soft: $\sigma = \underbrace{H_{gg}(\mu)\, sc(\mu)}_{\text{“singular”}} + \text{IR-finite}$

Consider the singular and ignore the IR-finite pieces (only for a moment)

$$\sigma = \big[1 + h^{(1)}(\mu) + sc^{(1)}(\mu) + h^{(2)}(\mu) + sc^{(2)}(\mu) + h^{(1)}(\mu) sc^{(1)}(\mu) + \mathcal{O}(\alpha_s^3)\big]$$

- $h^{(i)}(\mu)$ contains large corrections from $\ln^2(-1) = -\pi^2$ *at each order*.
- To avoid them evaluate $H_{gg}(-\mathrm{i}\mu)$ and evolve it back to μ (simply evaluating $sc(-\mathrm{i}\mu)$ would just move the large terms from $h^{(i)}$ to $sc^{(i)}$)

$$H_{gg}(\mu) = H_{gg}(-\mathrm{i}\mu)\, U_H(-\mathrm{i}\mu, \mu)$$

$\Rightarrow$ Gives “π^2-resummed” singular

$$\sigma = U_H(-\mathrm{i}\mu, \mu)\big[1 + h^{(1)}(-\mathrm{i}\mu) + sc^{(1)}(\mu) + h^{(2)}(-\mathrm{i}\mu) + sc^{(2)}(\mu) + h^{(1)}(-\mathrm{i}\mu) sc^{(1)}(\mu) + \mathcal{O}(\alpha_s^3)\big]$$

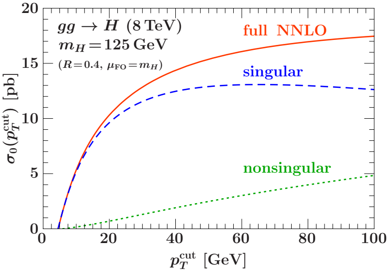
Singular vs. Nonsingular.

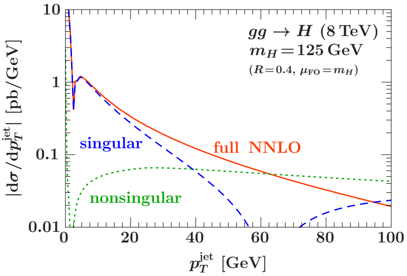
To explicitly define $sc(\mu)$ we can cut on any IR-sensitive kinematic variable $p \equiv \{p_T^H, p_T^{\rm jet}, E_T, \mathcal{T}^{\rm jet}, \mathcal{T}_B, \mathcal{T}_C, \ldots\} < p_{\rm cut}$

$$\sigma = \underbrace{\underbrace{H_{gg}(\mu)\, sc(\mu, p_{\rm cut})}_{\text{singular}} + \underbrace{\sigma^{\rm ns}(p_{\rm cut})}_{\text{nonsingular}}}_{\sigma(p_{\rm cut})} + \int_{p_{\rm cut}}^{\infty} \mathrm{d}p\, \frac{\mathrm{d}\sigma}{\mathrm{d}p}$$

$\sigma(p_{\rm cut})$ is a *physical* cross section

- For $p_{\rm cut} \ll m_H$ it is always dominated by the singular, since $\sigma^{\rm ns}(p_{\rm cut})$ is power-suppressed (by $\sim p_{\rm cut}/m_H$)





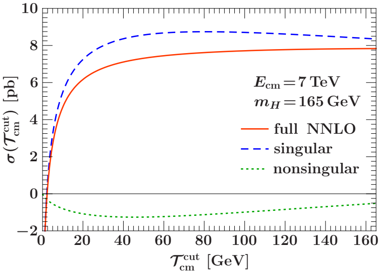
Singular vs. Nonsingular.

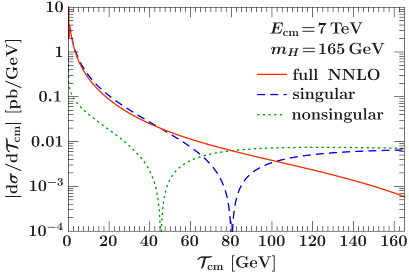
To explicitly define $sc(\mu)$ we can cut on any IR-sensitive kinematic variable $p \equiv \{p_T^H, p_T^{\text{jet}}, E_T, \mathcal{T}^{\text{jet}}, \mathcal{T}_B, \mathcal{T}_C, \ldots\} < p_{\text{cut}}$

$$\sigma = \underbrace{\underbrace{H_{gg}(\mu)\, sc(\mu, p_{\text{cut}})}_{\text{singular}} + \underbrace{\sigma^{\text{ns}}(p_{\text{cut}})}_{\text{nonsingular}}}_{\sigma(p_{\text{cut}})} + \int_{p_{\text{cut}}}^{\infty} \mathrm{d}p\, \frac{\mathrm{d}\sigma}{\mathrm{d}p}$$

$\sigma(p_{\text{cut}})$ is a *physical* cross section

- For $p_{\text{cut}} \ll m_H$ it is always dominated by the singular, since $\sigma^{\text{ns}}(p_{\text{cut}})$ is power-suppressed (by $\sim p_{\text{cut}}/m_H$)





Spectrum and $p_{\rm cut}$ Independence.

$$\sigma = H_{gg}(\mu)\, sc(\mu, p_{\rm cut}) + \sigma^{\rm ns}(p_{\rm cut}) + \int_{p_{\rm cut}}^{\infty} \mathrm{d}p\, \frac{\mathrm{d}\sigma}{\mathrm{d}p}$$

must be $p_{\rm cut}$ independent, which means

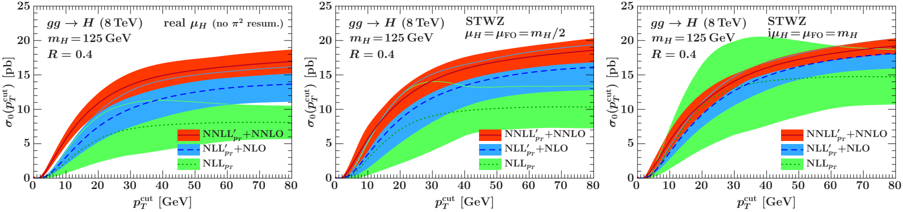
$$\frac{\mathrm{d}\sigma}{\mathrm{d}p} = H_{gg}(\mu)\frac{\mathrm{d}sc(\mu)}{\mathrm{d}p} + \frac{\mathrm{d}\sigma^{\rm ns}}{\mathrm{d}p}$$
$$= U_H(-\mathrm{i}\mu, \mu)\Big[\frac{\mathrm{d}sc^{(1+2)}(\mu)}{\mathrm{d}p} + h^{(1)}(-\mathrm{i}\mu)\frac{\mathrm{d}sc^{(1)}(\mu)}{\mathrm{d}p}\Big] + \frac{\mathrm{d}\sigma^{\rm ns(1+2)}}{\mathrm{d}p}$$

- At large $p \gtrsim m_H/2$: singular and nonsingular spectra are separately meaningless and must recombine into correct fixed-order spectrum $\mathrm{d}\sigma^{\rm FO}$

$$\mathrm{d}\sigma^{\rm FO(1)} = \mathrm{d}sc^{(1)} + \mathrm{d}\sigma^{\rm ns(1)}$$
$$\mathrm{d}\sigma^{\rm FO(2)} = \mathrm{d}sc^{(2)} + h^{(1)}(\mu)\mathrm{d}sc^{(1)} + \mathrm{d}\sigma^{\rm ns(2)}$$

- Either make U_H $p_{\rm cut}$-dependent and turn it off at some large $p_{\rm cut}$
- Or adjust $\sigma^{\rm ns}$ to $U_H[\sigma^{\rm ns(1)}(1 - U_H^{(1)}) + \sigma^{\rm ns(2)}]$
  - Extends π^2-resummation to $\mathrm{d}\sigma^{\rm ns}$ and thereby to full $\mathrm{d}\sigma/\mathrm{d}p$ and σ

- π^2-resummation improves singular which directly translates into a much improved convergence for $\sigma(p_{\rm cut})$ at small $p_{\rm cut}$
- Once we reach $p_{\rm cut} \gtrsim m_H/2$ (and turn off the $p_{\rm cut}$ resummation), we have accumulated most of the cross section



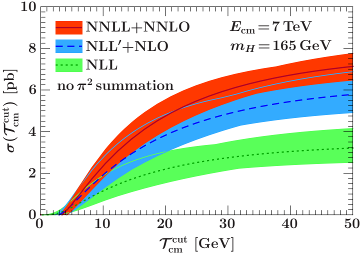
[Stewart, FT, Walsh, Zuberi '13]

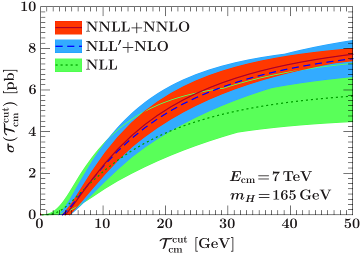
$\Rightarrow$ Better choice is to extend π^2-resummation to $\sigma^{\rm ns}(p_{\rm cut})$ and full σ

- Numerically, nonsingular also shows improved convergence
- Safer than forcing π^2-resummation to turn off, which could then easily lead to a decreasing cumulant (unphysical negative spectrum)

Singular vs. Nonsingular.

- π^2-resummation improves singular which directly translates into a much improved convergence for $\sigma(p_{\rm cut})$ at small $p_{\rm cut}$
- Once we reach $p_{\rm cut} \gtrsim m_H/2$ (and turn off the $p_{\rm cut}$ resummation), we have accumulated most of the cross section





[Berger, Marcantonini, Stewart, FT, Waalewijn '10]

$\Rightarrow$ Better choice is to extend π^2-resummation to $\sigma^{\rm ns}(p_{\rm cut})$ and full σ

- Numerically, nonsingular also shows improved convergence
- Safer than forcing π^2-resummation to turn off, which could then easily lead to a decreasing cumulant (unphysical negative spectrum)

Benchmark Results.

(Disclaimer: Numbers are preliminary and have not been cross-checked)

Using MSTW2008NNLO with $\alpha_s(m_Z) = 0.1171$ and 3-loop running, $m_t = 172.5\,\mathrm{GeV}$ in EFT limit

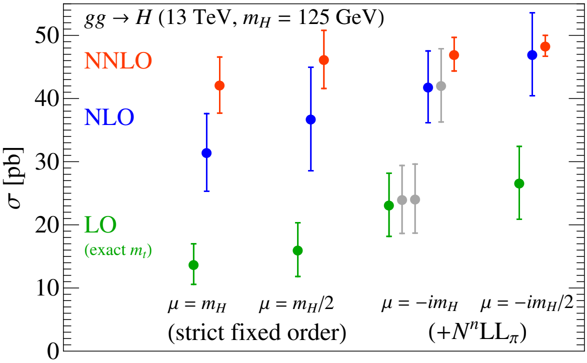
	$\mu = m_H/2$	$\mu = m_H$	$\mu = 2m_H$
$\mu_F = m_H/2$	48.35	46.70	
$\mu_F = m_H$	48.48	47.02	44.36
$\mu_F = 2m_H$		47.38	44.83

	$\mu = m_H/4$	$\mu = m_H/2$	$\mu = m_H$
$\mu_F = m_H/4$	47.88	48.36	
$\mu_F = m_H/2$	47.68	48.35	46.70
$\mu_F = m_H$		48.48	47.02

Benchmark Results.

(Disclaimer: Numbers are preliminary and have not been cross-checked)

Using MSTW2008NNLO with $\alpha_s(m_Z) = 0.1171$ and 3-loop running at each order



NNLO+NNLL$_\pi$
$47.02 \pm 2.66\ (5.7\%)$

NLO+NLL$_\pi$ [+NNLL$_\pi$]
$41.85 \pm 5.68\ (13.6\%)$

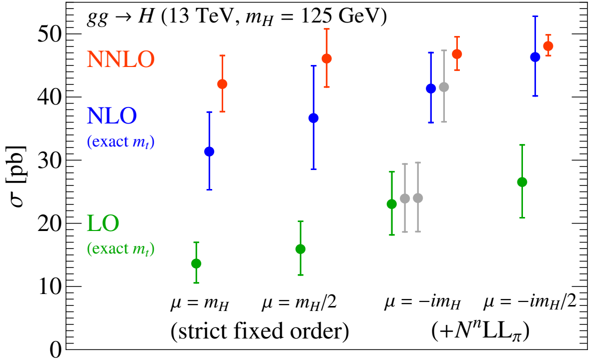
LO+LL$_\pi$ [+(N)NLL$_\pi$]
$23.17 \pm 5.00\ (21.6\%)$

- Perturbative corrections get more moderate
- Scale variations are reduced and provide better coverage
  (uncertainties use maximal absolute difference of 6-point variation from central)

Benchmark Results.

(Disclaimer: Numbers are preliminary and have not been cross-checked)

Using MSTW2008NNLO with $\alpha_s(m_Z) = 0.1171$ and 3-loop running at each order



$\mathrm{NNLO+NNLL}_\pi$

$46.90 \pm 2.63\,(5.6\%)$

$\mathrm{NLO+NLL}_\pi$ $[+\mathrm{NNLL}_\pi]$

$41.49 \pm 5.54\,(13.3\%)$

$\mathrm{LO+LL}_\pi$ $[+(\mathrm{N})\mathrm{NLL}_\pi]$

$23.17 \pm 5.00\,(21.6\%)$

- Perturbative corrections get more moderate
- Scale variations are reduced and provide better coverage
  (uncertainties use maximal absolute difference of 6-point variation from central)

Summary.

"π^2-resummation" follows from the Sudakov resummation of the gluon form factor at timelike momentum transfer

- Does not rely on threshold limit
  - Can be used standalone or in conjunction with other exclusive resummations
- Significantly improves perturbative behavior in exclusive region of physical observables
- Can be extended to inclusive cross section
  - Improves perturbative series
  - Smaller perturbative uncertainties with better coverage
  - By itself does not justify any partial $\mathrm{N^3LO}$ result, but can be added to your favourite one if you like, since any approximate result includes the $\mathrm{N^3LO}$ form factor