A new tool for constrained vertex fitting in ATLAS

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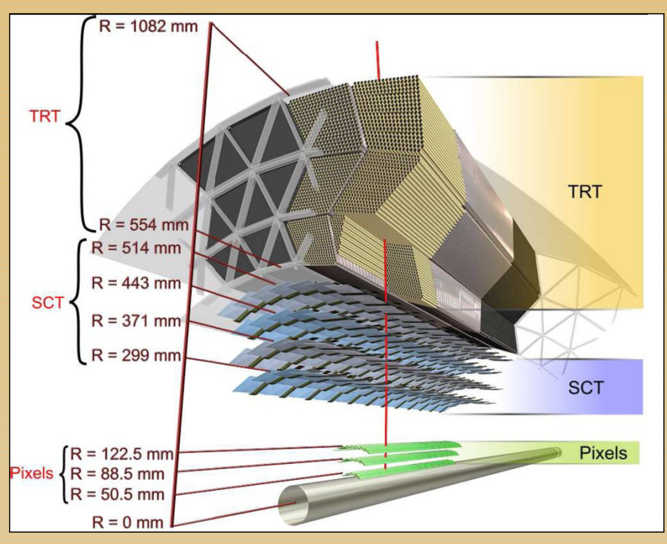
Introduction:

The precise reconstruction of trajectories of charged and neutral particles and their decay vertices is crucial for many physics analyses. Studying the tracking performance on well known benchmark channels helps to understand the properties of the ATLAS detector during the initial phase of the LHC. In order to exploit the correlations between parameters of final state tracks having the same mother particle, a new tool for vertex fitting with possibility of simultaneous application of kinematic constraints has been developed.

Track reconstruction in the ATLAS Inner Detector:

The inner detector is the closest detector to the interaction point in ATLAS. It is designed to reconstruct the trajectories of charged particles produced in the proton-proton collisions at the LHC. The inner detector is housed within a solenoid magnet producing a magnetic field of 2 T.



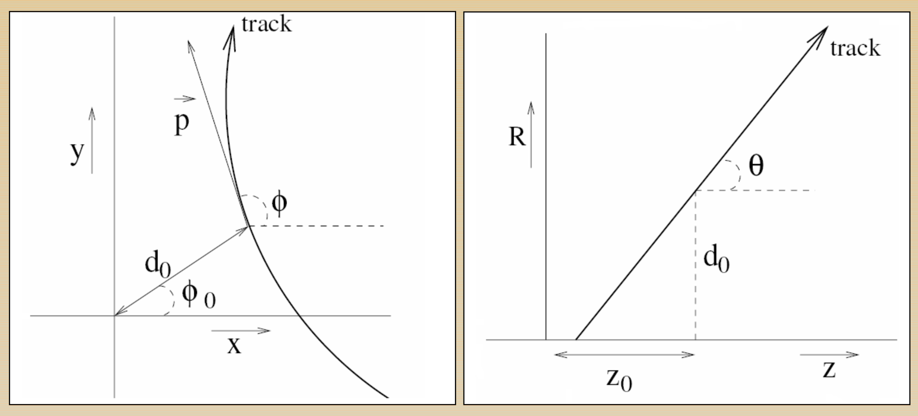
The inner detector consists of three different sub-detectors:

- **Pixel detector**: three hits per track with average resolutions of 14 µm (Rφ), 115 µm (z)
- **Silicon-strip detector** (SCT): 4 space-points (2 hits=1 space-point) per track with resolutions of 17 µm (Rφ), 580 µm (z)
- **Transition Radiation Tracker (TRT):** average of 36 hits per track with resolutions of 140 µm (Rφ)

The implementation of tracking and vertexing software follows a modular approach based on the use of object-oriented C++, a common set of abstract interfaces and a common Event Data Model.

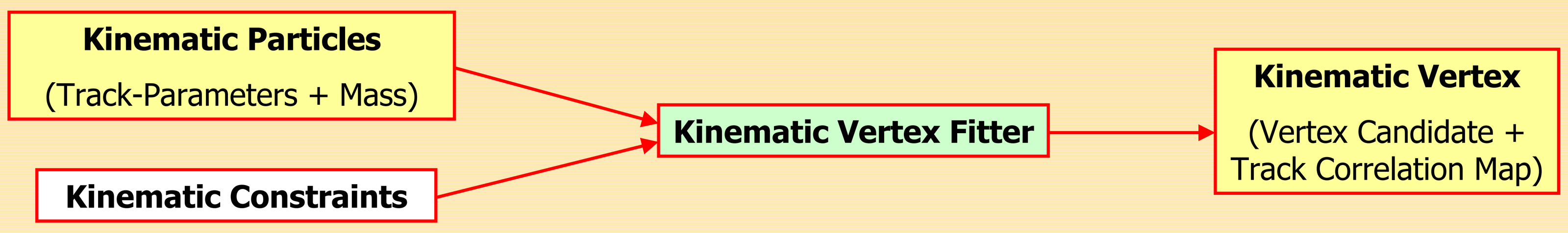
The reconstructed trajectory of a charged particle is described by the 5 **perigee parameters:**

d_0: the transverse impact parameter at the point of closest approach to the reference point (perigee point).
z_0: the z-coordinate of the perigee position
φ: the polar angle of the trajectory at the perigee
θ: the azimuthal angle of the trajectory at the perigee
q/p: the charge of the particle divided by the momentum



Modular tool for vertex fitting with kinematic constraints

The new tracking and vertexing framework of ATLAS is organised as set of tools and algorithms. This allows the implementation of the algorithm for constrained vertex fitting as an additional vertexing tool.
The input to the constrained vertex fitter is a set of tracks with associated particle masses. The output is the reconstructed vertex. The vertex object contains the vertex position and its covariance matrix as well as the updated track parameters defined at the vertex position. The tool is designed such that a list of kinematic constraints can be given as additional input to the vertex fit.



The calculations of the derivatives of the constraint-equations are contained in the implementations of the *IKinematicConstraint* class, while the main method only holds the constraint equations for the vertex constraint. When no additional constraints are present, the tool behaves as a standard vertex fitter.

Kinematic vertex fitting using Lagrange multipliers in the cartesian frame (http://www.phys.ufl.edu/~avery/fitting.html)

The tool is based on the χ^2 minimization with Lagrange multipliers. The calculations are performed in the cartesian frame.

The **track parameters** are expressed in terms of momentum, energy and position: $\vec{a} = (p_x, p_y, p_z, E, x, y, z)$
They are obtained by converting the perigee parameters and assigning a mass estimate to the final-state particle.
The use of cartesian representation simplifies the formulation of kinematic constraints.

Each constraint is represented by a **constraint equation** written as $\mathbf{H}(\vec{\alpha}) = 0$. (Where $\vec{\alpha}$ is a vector of 7 × N cartesian track paramers for N tracks that are used in the fit.)

The equations are linearized around a convenient expansion point A.

$$0 = [\frac{\partial \mathbf{H}(\vec{\alpha_A}, \vec{x_A})}{\partial \vec{\alpha}}](\vec{\alpha} - \vec{\alpha_A}) + [\frac{\partial \mathbf{H}(\vec{\alpha_A}, \vec{x_A})}{\partial \vec{x}}](\vec{x} - \vec{x_A}) + \mathbf{H}(\vec{\alpha_A}, \vec{x_A})$$
$$\equiv \mathbf{D}(\vec{\alpha} - \vec{\alpha_A}) + \mathbf{E}(\vec{x} - \vec{x_A}) + \mathbf{d}$$

The χ^2 to be minimized can be written as:

$$\chi^2 = (\vec{\alpha} - \vec{\alpha_0})^T V_{\alpha_0}^{-1} (\vec{\alpha} - \vec{\alpha_0}) + 2\lambda^T (\mathbf{D}(\vec{\alpha} - \vec{\alpha_A}) + \mathbf{E}(\vec{x} - \vec{x_A}) + \mathbf{d})$$

Contribution from the original track-fits
Contribution from kinemat-constraints with λ the vector of Lagrange multipliers

Minimizing this χ^2 with respect to λ, $\vec{\alpha}$ and $\vec{x}$ gives the solutions for finding the track-parameters and vertex-position that satisfy the given constraints.
Note that the constraint equations and their derivates can be seperated from the fit so that extra constraints can be easily added within the same framework.

Currently implemented constraints:

Vertex-constraint:
two constraint equations for each track in the fit:

$$0 = p_x \Delta y - p_y \Delta x - \frac{a}{2}(\Delta x^2 + \Delta y^2)$$
$$0 = \Delta z - \frac{p_z}{a} sin^{-1}\left(a(p_x \Delta x + p_y \Delta y)/p_T^2\right)$$

Where: $\Delta x = x_{trk} - x_{vtx}$, $\Delta y = y_{trk} - y_{vtx}$, $\Delta z = z_{trk} - z_{vtx}$. a represents the bending of the particle in the XY-plane and is given by $a = -0.2997BQ$

Mass-constraint:
one constraint equation which requires the invariant mass of the particles to be equal to a used defined value m_{constr}:

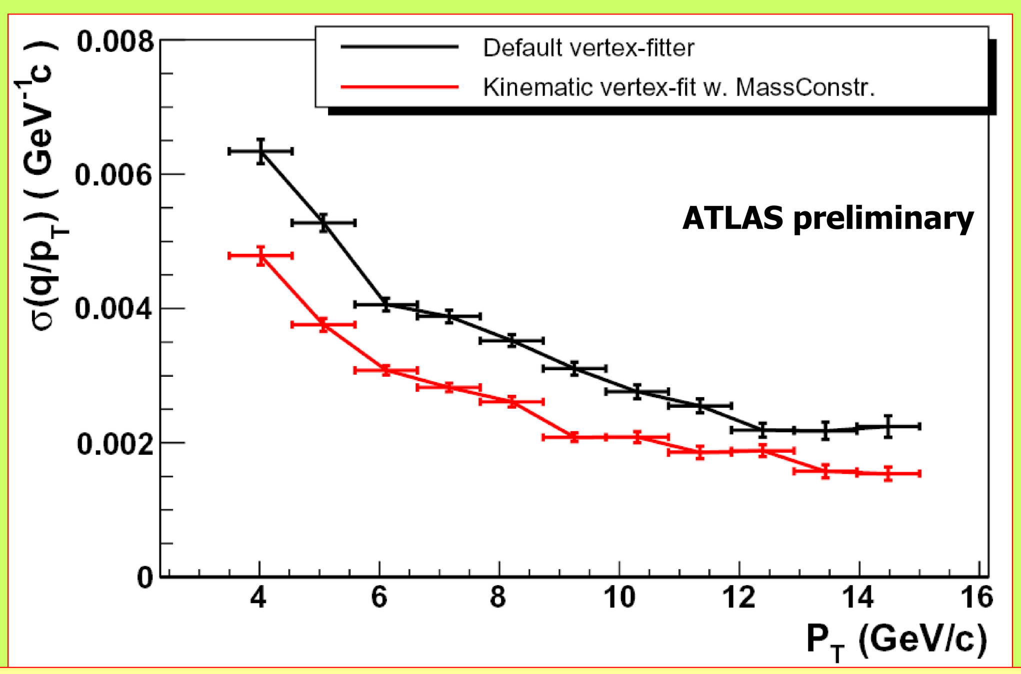
$$0 = (\sum_{i=0}^{ntrack} E_i)^2 - (\sum_{i=0}^{ntrack} p_{xi})^2 - (\sum_{i=0}^{ntrack} p_{yi})^2 - (\sum_{i=0}^{ntrack} p_{zi})^2 - (m_{constr})^2$$

Results

Vertex fit with mass constraint in $J/\psi \rightarrow \mu^+ \mu^-$ events

The kinematic vertex fit with mass constraint was tested using simulated events of J/ψ decaying to two muons. The result is compared with the conventional χ^2-based vertex fitting method in ATLAS.

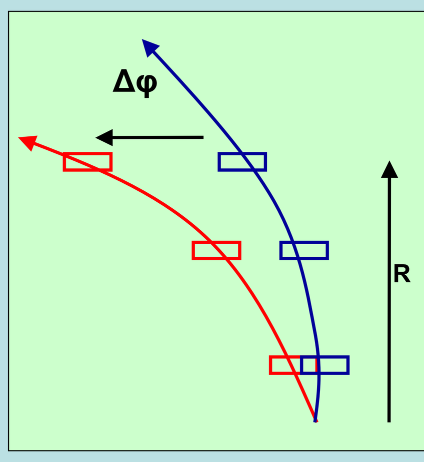
Using the mass constraint does not change the result for the resolution of the fitted vertex.



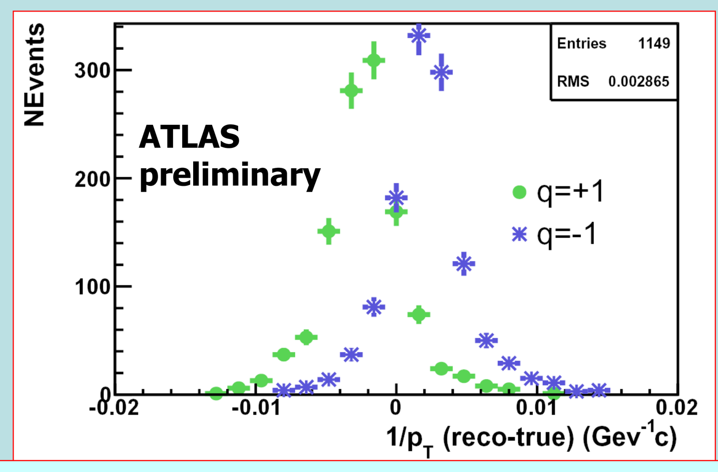
The inverse transverse momentum of the updated track parameters at the vertex is improved when using the mass constraint. (The resolution seen with the default vertex fitter is essentially that coming from the track fit alone.)

Weak-mode misalignments

Constrained vertex fitting helps to solve 'weak-mode' misalignments. These type of misalignments represent systematic deformations of the detector, that **affect the properties of reconstructed charged particles without increasing the hit residuals.** The standard alignment procedure is based on minimizing the hit residuals and is not sensitive to weak-mode misalignment. Adding information to the fitted track parameters on the vertex helps to find and resolve these type of misalignments.

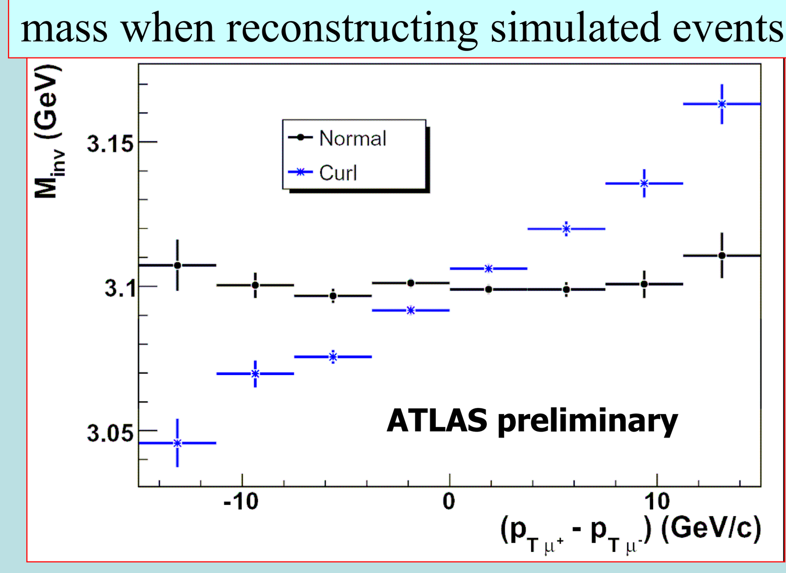


An example of a weak-mode in the ATLAS detector is the so-called 'curl' misalignment, which rotates the detector elements by a factor Δφ as a function of transverse radius (R). The effect of this misalignment can be studied by using simulated events reconstructed with a normal and 'curl' geometry. The 'curl'-geometry is based on reasonable shifts of the detector elements such that the position of the SCT modules are shifted by a maximum of 200 µm.

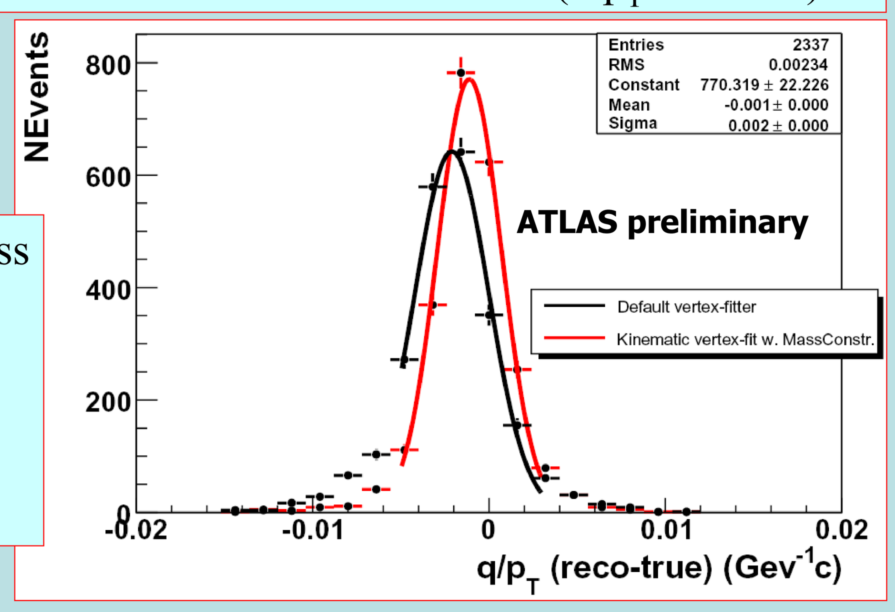


Effect of 'curl' geometry: Negatively charged tracks become more bent ($1/p_T$ increase) and **positively charged tracks become more 'stiff'** ($1/p_T$ decrease).

$J/\psi \rightarrow \mu^+ \mu^-$ events can be used as a benchmark channel to study weak-mode misalignments.
The invariant dimuon mass is no longer always equal to the J/ψ mass when reconstructing simulated events with a 'curl' geometry.



The kinematic vertex-fit with mass constraint can recover some the momentum shift of $J/\psi \rightarrow \mu^+ \mu^-$ events reconstructed with a 'curl' misalignment (result shown for muon $p_T > 9$ GeV).



Conclusions

A new tool for kinematic vertex fitting in ATLAS was developed. This tool allows the application of additional kinematic constraints during the vertex fit. The use of a mass constraint in a benchmark channel $J/\psi \rightarrow \mu^+ \mu^-$ significantly improves the experimental resolution on the momentum of the final-state muons. The use of this mass constraint also allows recovery of shifts in the particle momentum, which are present in the weak-mode misalignments of the ATLAS detector. The influence of the background to the benchmark channel on the correction of the weak-mode misalignment effects requires a further study.