LHC phenomenology at next-to-leading order QCD: theoretical progress and new results

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Content:

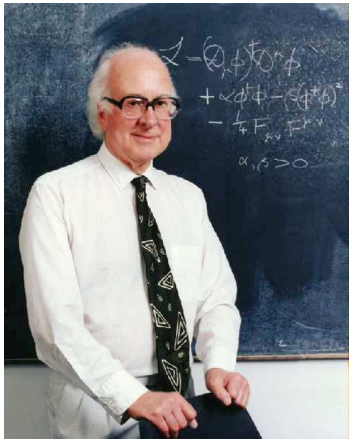
- Why LHC phenomenology at next-to-leading order?
- New methods for one-loop amplitudes
  - Unitarity based approaches
  - Feynman diagrammatic approaches
  - comment on performance
- Summary and Outlook

"...religious battle between Feynmanians and Unitarians..."

Joey Huston

The advent of the LHC era

LHC: Large Hadron Collider at CERN, $\sqrt{s} = 14$ TeV, switched on September 10 !

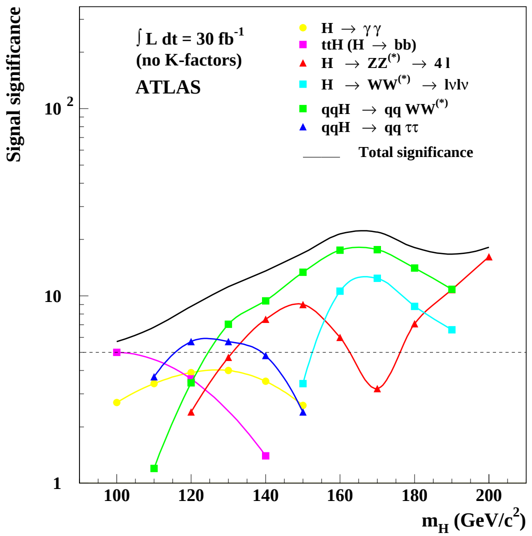


What do we expect?

- test Higgs mechanism
  - $\longrightarrow$ SM Higgs boson: 114.4 GeV $< m_H <$ 200 GeV (!)
  - $\longrightarrow$ $V(H) = \frac{1}{2} M_H^2 H^2 + \lambda_3 \, H^3 + \lambda_4 \, H^4$
    SM: $\lambda_4 = \lambda_3 / v = 3 \, M_H^2 / v^2$
- explore physics beyond the Standard Model
  - $\longrightarrow$ SM $\subset$ "Extra Dimensions", "Little Higgs", "Strong interaction" Model
    SM $\subset$ MSSM $\subset$ SUSY GUT $\subset$ Supergravity $\subset$ Superstring $\subset$ $\mathcal{M}$-Theory
  - $\longrightarrow$ BSM something around 1 TeV (?)
- nothing ?!
  - $\longrightarrow$ hint of a hidden sector (?)
  - $\longrightarrow$ hint of strong interactions in the e.w. sector (?)

Discovery potential of the Higgs boson at the LHC

- most studies based on LO Monte Carlo tools
  - $\longrightarrow$ large uncertainties
  - $\longrightarrow$ some loop induced LO processes not included [e.g. $gg \to WW$]



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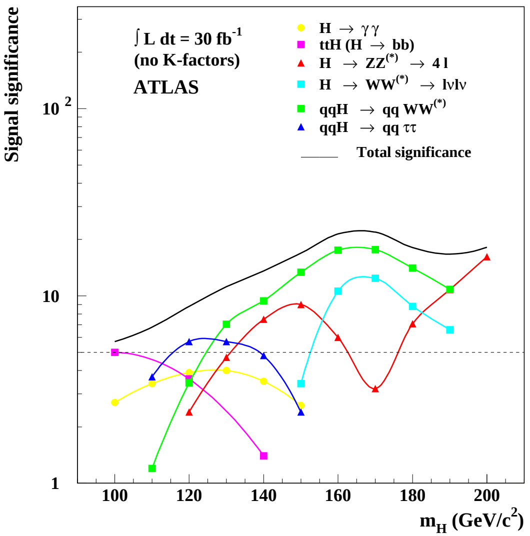
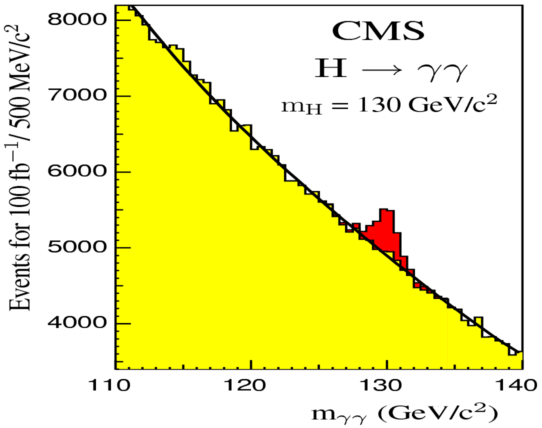
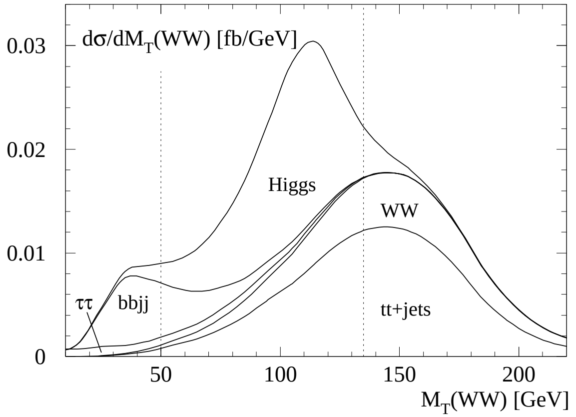


- Not all backgrounds can be measured
- Quantitative analysis of SM/BSM physics needs background control
- Nothing @ LHC = Bkgnd(experiment) - Bkgnd(theory) !

$PP \to H + X \to \gamma\gamma + X$

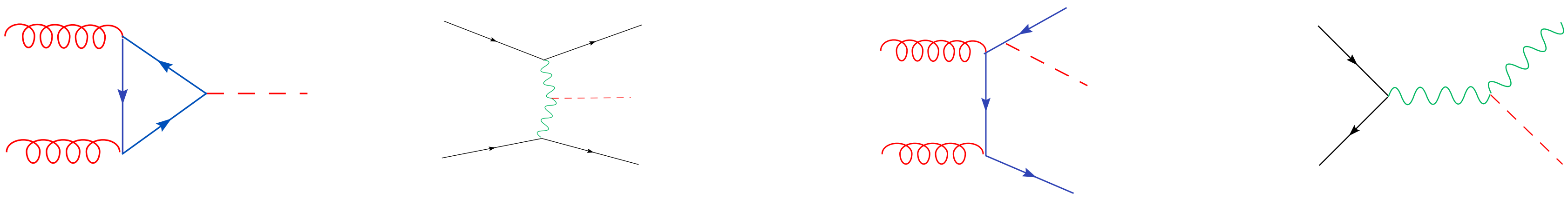


WBF: $H \to WW \to l^+ l^- + \not{p}_T$



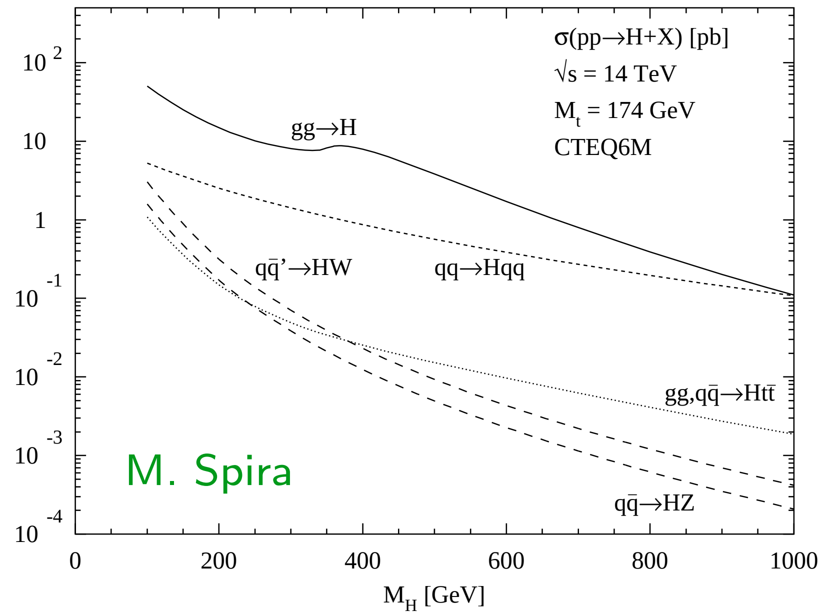
Kauer, Plehn, Rainwater, Zeppenfeld (2001)

S+B for the Higgs boson



Signal:

- Decays: $H \to \gamma\gamma$, $H \to WW^{(*)}$, $H \to ZZ^{(*)}$, $H \to \tau^+\tau^-$
- $PP \to H + 0, 1, 2$ jets Gluon Fusion
- $PP \to Hjj$ Weak Boson Fusion
- $PP \to H + t\bar{t}$
- $PP \to H + W, Z$



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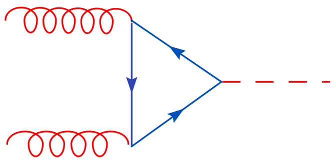
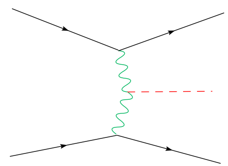
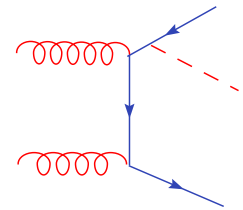
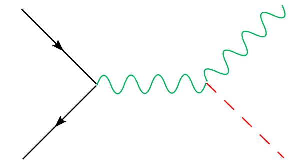

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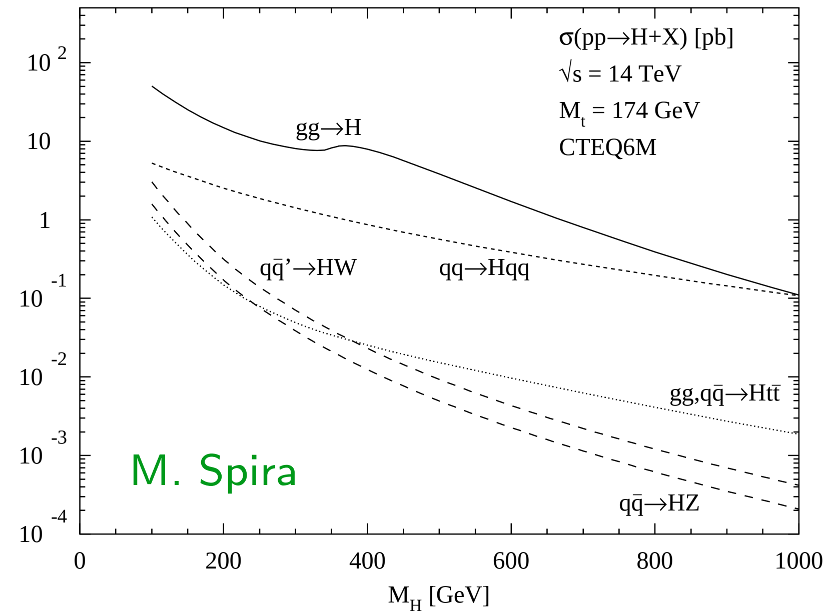


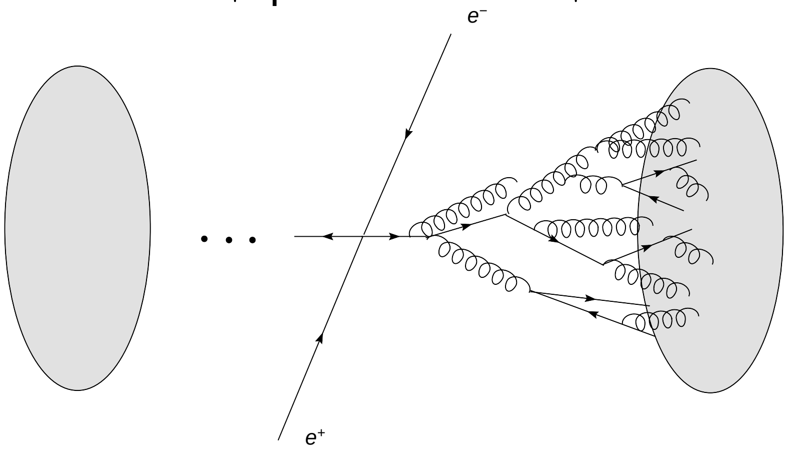
Backgrounds:

- $PP \to \gamma\gamma + 0, 1, 2$ jets
- $PP \to WW^*, ZZ^* + 0, 1, 2$ jets
- $PP \to t\bar{t} + 0, 1, 2$ jets
- $PP \to V +$ up to 3 jets $\quad (V = \gamma, W, Z)$
- $PP \to VVV + 0, 1$ jet

Multi purpose tools for experimental analysis

Pythia Sjöstrand, Lonnblad, Mrenna, Skands
Herwig Corcella, Knowles, Marchesini, Moretti, Odagiri, Richardson, Seymour, Webber
Sherpa Gleisberg, Hoche, Krauss, Schalicke, Schumann, Winter

- LO Matrixelements + parton shower + hadronization model



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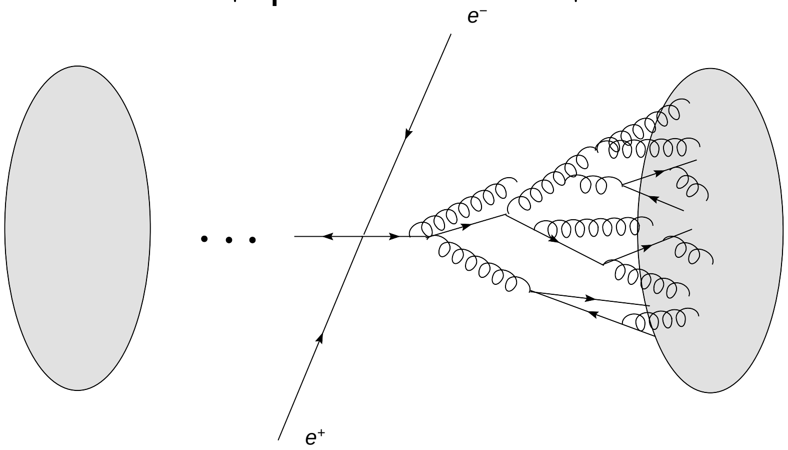
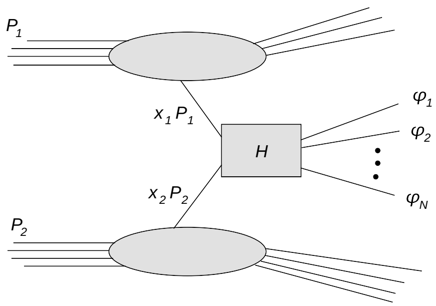


- $2 \to N$ Matrixelements: shapes, jet structure, well described after tuning
- many general purpose LO ME generators available: Alpgen,CompHEP talks Arbuzov, Kolesnikov, Comix, FeynArts/Formcalc → Hahn, GRACE → Kurihara, Helac,Madgraph, Whizard...
- merging procedure for matrix elements with a parton shower
- LO rates intrinsically unreliable!

Parton model and scale uncertainties



$$d\sigma(H_1 H_2 \to \phi_1 + \ldots \phi_N + X) = \sum_{j,l} \int dx_1 dx_2 f_{j/H_1}(x_1, \mu_F) f_{l/H_2}(x_2, \mu_F)$$

$$\times \, d\hat{\sigma}(\mathrm{parton}_j(x_1 P_1) + \mathrm{parton}_l(x_2 P_2) \to \varphi_1 + \ldots \varphi_N, \alpha_s(\mu), \mu_F)$$

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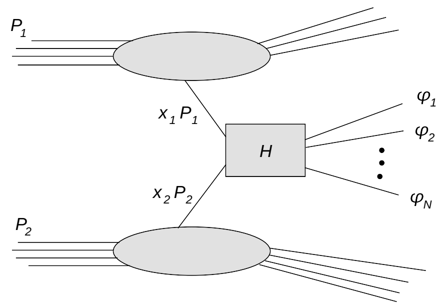

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Scale dependence remnant of UV/IR divergencies: $\frac{Q^\epsilon}{\epsilon} - \frac{\mu^\epsilon}{\epsilon} = \log(Q/\mu)$

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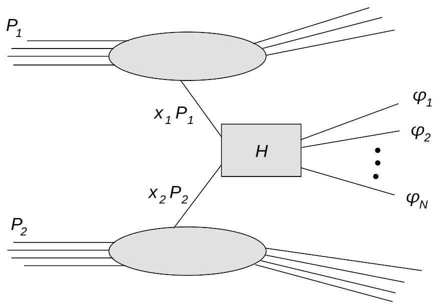

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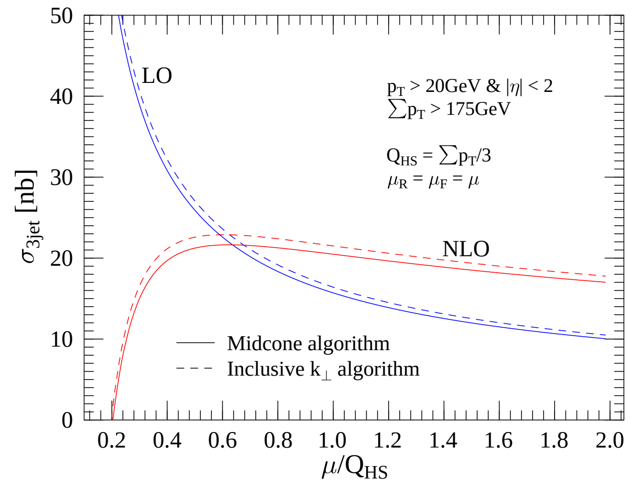
Scale dependence remnant of UV/IR divergencies: $\frac{Q^\epsilon}{\epsilon} - \frac{\mu^\epsilon}{\epsilon} = \log(Q/\mu)$

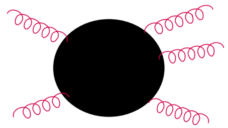
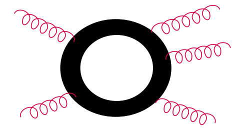
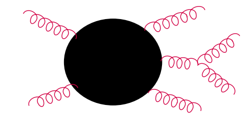
Example: 3 jet cross section at NLO

[Z. Nagy, Phys.Rev. D68 (2003)]



Higher order QCD calculations are mandatory to soften scale dependence !!!

Framework for NLO calculations

$\mathcal{A}_{\mathrm{LO}}$:  $\mathcal{A}_{\mathrm{NLO,Virtual}}$:  $\mathcal{A}_{\mathrm{NLO,Real}}$: 

$$
\begin{aligned}
\sigma &= \sigma_{LO} + \sigma_{NLO} \\
\sigma_{LO} &= \int dPS_N \, \frac{1}{2s} \; |\mathcal{A}_{\mathrm{LO}}|^2 \\
\sigma_{NLO} &= \int dPS_N \, \frac{1}{2s} \, \alpha_s \, \Big(\Big[\mathcal{A}_{\mathrm{LO}} \mathcal{A}^*_{\mathrm{NLO,V}} + \mathcal{A}^*_{\mathrm{LO}} \mathcal{A}_{\mathrm{NLO,V}} \Big] \\
&\qquad + \int dPS_1 \; |\mathcal{M}_{\mathrm{NLO,R}}|^2 \Big)
\end{aligned}
$$

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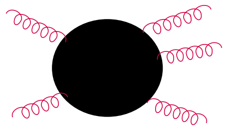
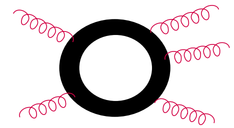
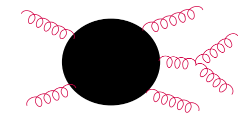
$\mathcal{A}_{\rm LO}$:  $\mathcal{A}_{\rm NLO,Virtual}$:  $\mathcal{A}_{\rm NLO,Real}$: 

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$$

- Use matrix element generators for tree amplitudes
- IR subtraction:Frixione, Kunszt, Signer, Soper,...; dipole method à la Catani, Seymour (massless); Dittmaier, Trocsanyi, Weinzierl, Phaf (massive).
- automated dipole subtraction: Gleisberg, Krauss (2007); Seymour, Tevlin (2008); Hasegawa, Moch, Uwer (2008); Frederix, Gehrmann, Greiner (2008).
- Bottleneck: virtual corrections

Status QCD@NLO for LHC:

$2 \to 2:$ everything you want (see e.g. MCFM by Campbell/Ellis)

- combination with parton shower MC@NLO Frixione et al.

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$2 \to 3:$ before 2005:

- $pp \to jjj$, $pp \to \gamma\gamma\ j$, $pp \to Vjj$
- $pp \to Hjj$ [WBF], $pp \to Hjj$ [GF], $pp \to Ht\bar{t}$

after 2005:

- $pp \to HHH$ (2005)
- $pp \to VVjj$ [WBF] (2006)
- $pp \to ZZZ$, $pp \to t\bar{t}j$, $pp \to WWj$ (2007)
- $pp \to VVV$, $pp \to t\bar{t}Z, b\bar{b}Z$ (2008)

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$2 \to 4:$ No complete LHC cross section prediction done yet!

Many people involved: Berger, Bern, Binoth, Bredenstein, Campbell,Dawson, Denner, Dittmaier, Dixon, Ellis, Febres Cordero, Forde, Giele, Guillet, Forde, Ita, Jager, Kallweit, Karg, Kosower, Kunszt, Lazopoulos, Maitre, Mahmoudi, Melnikov, Nagy, Oleari, Orr, Ossola, Papadopoulos, Pittau, Pozzorini, Reina, Sanguinetti, Soper, Uwer, Wackeroth, Weinzierl, Zanderighi, Zeppenfeld,...and many others

Methods for one-loop computations

Based on Feynman diagrams:

- Passarino-Veltman reduction (momentum space)
  Passarino, Veltman (1979)
- Feynman parameter space reduction
  Davydychev (1991); Tarasov (1996) Bern, Dixon, Kosower (1993)
- Reduction of N-point intetgrals for $N > 5$
  Melrose (1965); Van Neerven, Vermaseren (1984); Oldenburgh, Vermaseren (1990); Binoth, Guillet, Heinrich (1999); Fleischer, Jegerlehner, Tarasov (1999); Denner, Dittmaier (2002) Duplancic, Nizic (2003); Binoth, Guillet, Heinrich, Pilon, Schubert (2005). (see also talk Tord Riemann)

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Based on unitarity:

- old idea Cutkosky (1960)
  Eden, Landshoff, Olive, Polkinghorne, "The analytic S-matrix" (1966)
- cutting & sewing (two-particle cuts)
  Bern, Dixon, Dunbar, Kosower (1994)
- new inspiration from twistor space Witten (2003) (see talk W. Perkins)
  Britto, Cachazo, Feng, Witten (2005); Brandhuber, McNamara, Spence, Travaglini (2005); Britto, Cachazo, Feng, Witten (2005);
  Britto, Cachazo, Mastrolia (2006)

Helicity management

Helicity amplitudes $\rightarrow$ work only with physical degrees of freedom
Spinor-helicity formalism: work with Weyl spinors for massless fermions

$$|k^{\pm}\rangle = \Pi^{\pm}u(k) \quad , \quad \langle k^{\pm}| = \bar{v}(k)\Pi^{\mp} \quad , \quad \not{k}|k^{\pm}\rangle = 0$$

spinor products: $\langle kq\rangle = \langle k^{-}|q^{+}\rangle \quad , \quad [kq] = \langle k^{+}|q^{-}\rangle$

massless gluon/photon (two helicity states, axial gauge: $\varepsilon^{\pm} \cdot k = \varepsilon^{\pm} \cdot q = 0$)

$$\varepsilon_{\mu}^{+}(k,q) = \frac{1}{\sqrt{2}} \frac{\langle q^{-}|\gamma_{\mu}|k^{-}\rangle}{\langle q|k\rangle} \quad , \quad \varepsilon_{\mu}^{-}(k,q) = \frac{1}{\sqrt{2}} \frac{\langle q^{+}\gamma_{\mu}|k^{+}\rangle}{[kq]}$$

$\Rightarrow$ Compact representations for tree and loop helicity amplitudes !

Colour management:

wildly used: colour flow representation for $SU(N_C)$
'tHooft (1974); Maltoni, Paul, Stelzer, Willenbrock (2001)

$$
\begin{aligned}
i f^{abc} T^c_{ik} &= T^a_{ij} T^b_{jk} - T^b_{ij} T^a_{jk} \\
T^a_{ik} T^a_{jl} &= \frac{1}{2}\left(\delta^i_l \delta^j_k - \frac{1}{N_C} \delta^i_k \delta^j_l\right)
\end{aligned}
$$

maps to colour basis (N $=$ # gluons $\leftrightarrow$ #quark lines)

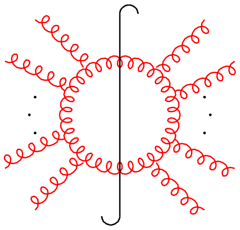
$$
\mathcal{A} = \sum_{\sigma \in S_N} \mathcal{A}_\sigma |c_\sigma\rangle \quad , \quad |c_\sigma\rangle = \delta^{j_{\sigma(1)}}_{i_1} \delta^{j_{\sigma(2)}}_{i_2} \ldots \delta^{j_{\sigma(N)}}_{i_N} \tag{1}
$$

for N-gluon amplitude $(N-2)!$ independent colour states (factorial growth)

One-loop amplitudes from unitarity in a nut-shell

Unitarity of $S = 1 + iT$ matrix: $S^\dagger S = 1 \Rightarrow 2\mathrm{Im}(T) = T^\dagger T$
implies for scattering amplitudes evaluted in perturbation theory

$$\mathrm{Im}\ \mathcal{A}_{1-\mathrm{loop}} \sim \sum_C \int d\mathrm{PS}_C$$



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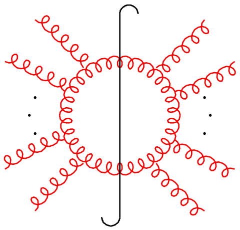

$$\mathcal{A}_{1-\mathrm{loop}} = \sum_{k=2,3,4} C_k I_k + \mathcal{R} \quad \Rightarrow \quad \mathrm{Im}\mathcal{A}_{1-\mathrm{loop}} = \sum_{k=2,3,4} C_k \mathrm{Im}(I_k)$$

- tree amplitudes as input, gauge invariance built in
- $D = 4$ cuts determine amplitude only up to rational part $\mathcal{R}$
- choice: evaluate $D = 4 - 2\epsilon$ cuts *or* use more sophisticated methods
- analytical structure encoded in known scalar integrals

$I_{k=2,3,4} \sim$

The Blackhat implementation of the unitarity method

Blackhat collaboration: Berger, Bern, Dixon, Febres Cordero, Forde, Ita, Kosower

- numerical implementation of unitarity methods
- uses complex external momenta (complex on-shell condition)
  $\Rightarrow$ efficient use of spinor helicity formalism
- rational terms by on-shell recursion relations
- uses "generalized" unitarity, i.e. 2-,3-,4-particle cuts

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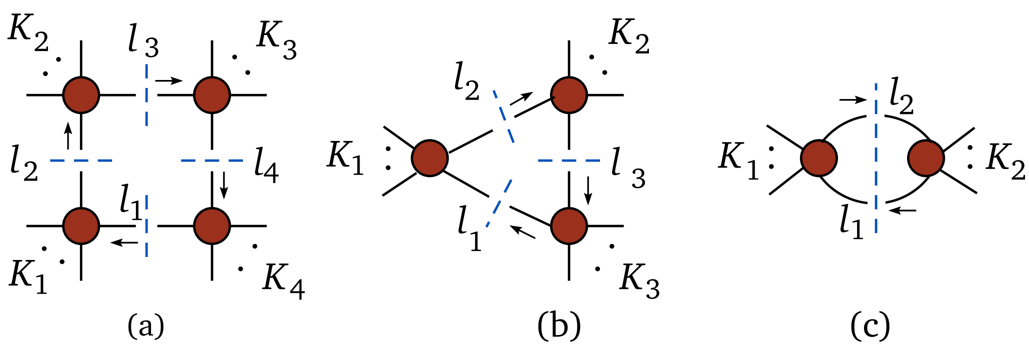

E.g. box coefficients from quadruple cuts:

$$\mathcal{A}_{1-\text{loop}} = \sum_{k=2,3,4} C_k I_k + \mathcal{R} \quad \Rightarrow \quad C_4 = \frac{1}{2} \sum_{\sigma=\pm} A^{(1)}_{\text{tree}} A^{(2)}_{\text{tree}} A^{(3)}_{\text{tree}} A^{(4)}_{\text{tree}}$$

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- N=6,7,8-gluon "MHV" one-loop amplitude: $g^- g^- g^+ \ldots g^+ \to 0$
- leading colour of $q\bar{q} \to Vggg$

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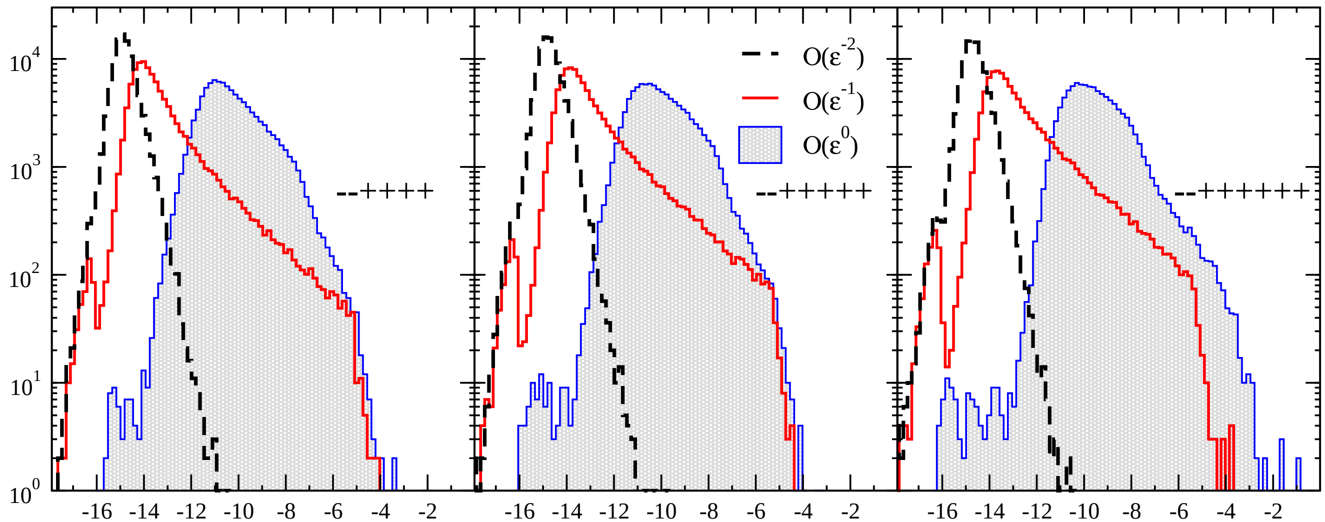
arXiv:0807.3705 [hep-ph] $\log_{10}\left(\frac{|A_n^{num} - A_n^{target}|}{|A_n^{target}|}\right)$ $\mathcal{A} \sim a/\epsilon^2 + b/\epsilon + c$



- evaluation of 100,000 phase space point to check numerical performance
- event-by-event check of numerical instabilities $\longrightarrow$
  use of multi-precision library for a few percent of phase space points

The Ossola, Papadopoulos, Pittau (OPP) method

- ”reconstructing 1-loop amplitudes from the integrand” (2006)

leg ordered amplitudes: $\mathcal{A}_{1,\ldots,N} = \sum \mathcal{G}_{1,\ldots,N} \sim \int d^D k \frac{\mathcal{N}(k)}{D_1 \ldots D_N}$

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$$\mathcal{N}(k) \sim \sum_{boxes} [C_4 + \tilde{C}_4(k)] \prod_{j \notin box}^{N} D_j + \sum_{triangles} [C_3 + \tilde{C}_3(k)] \prod_{j \notin triangles}^{N} D_j$$

$$+ \sum_{bubbles} [C_2 + \tilde{C}_2(k)] \prod_{j \notin bubbles}^{N} D_j + \ldots$$

- $\mathcal{N}(k)$ from Feynman diagrams, $\tilde{C}_l(k)$ vanish upon integration

How are $\tilde{C}_{4,3,2,\ldots}(k)$ defined without integration?

- putting propagators on-shell for box, (triangle, bubble...), i.e. multiple cuts!, $\Rightarrow$ polynomial in loop momentum with 2, (7,9,...) coefficients defined
- coefficients can be extracted by numerical interpolation !!!
- rational term from $\mathcal{O}(\epsilon)$ part in C_2

The Ossola, Papadopoulos, Pittau (OPP) method

- public fortran code: `CutTools` 0711.3596 [hep-ph]
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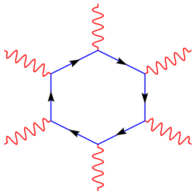
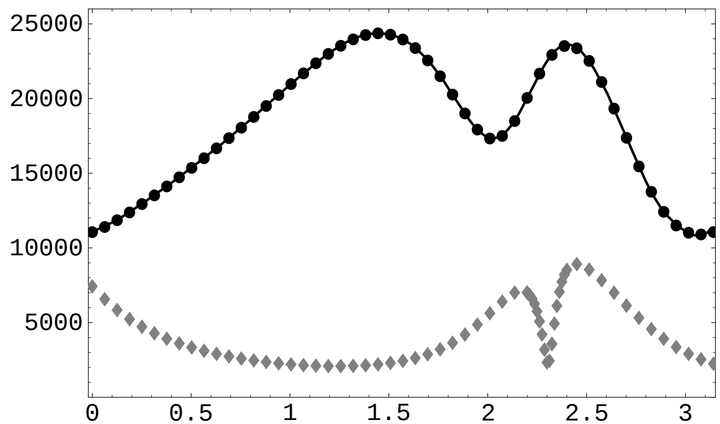
Applications:



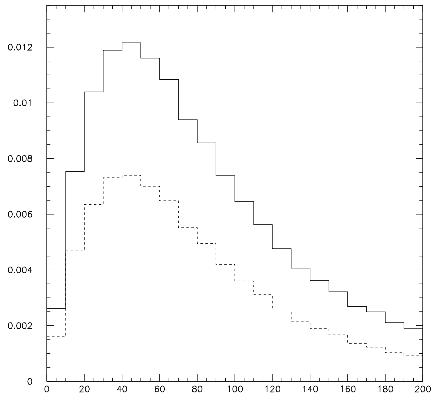
- six-photon amplitudes (massless and massive leptons)
- triple vector boson production at LHC: $pp \to VVV, V = W^{\pm}, Z^0$

$\mathcal{A}^{+++\pm--}_{\gamma\gamma\to\gamma\gamma\gamma\gamma}$ along slice in phase space



OPP 0704.1271 [hep/ph]

$p_T(W)$-distribution at LO/NLO



TB, OPP 0804.0350 [hep/ph]

Full one-loop amplitudes from tree amplitudes

D-dimensional OPP method: R.K. Ellis, Giele, Kunszt, Melnikov, Zanderighi

$$\begin{aligned}\mathcal{N}^D &= \mathcal{N}_0 + (D-4)\mathcal{N}_1 \\ &= \sum_{pentagons} [C_5 + \tilde{C}_5(k)] \prod_{j \notin pentagon}^{N} D_j + \ldots\end{aligned}$$

- uses 5-particle cuts, 5-point functions needed
- rational part by evaluating amplitude in D=4 and D=6 dimensions.
- numerator function directly from tree amplitudes

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Asymptotic (large N) behaviour for colour ordered amplitudes:

- recursion relation for ordered tree amplitudes: $\sim N^4$
- summing over all cuts: $\sim N^5$
- $\Rightarrow$ colour ordered 1-loop amplitudes $\sim N^9$
  polynomial complexity algorithm [$\mathcal{P}$-algorithm]
- But: colour dressing will spoil polynomial behaviour $\rightarrow$ $\mathcal{E}$-algorithm

$$\mathcal{A}(Ng) \sim \sum_{\sigma \in S_{N-1}} \delta_{i_1}^{j_{\sigma(1)}} \delta_{i_2}^{j_{\sigma(2)}} \ldots \delta_{i_N}^{j_{\sigma(N)}} \mathcal{A}(1\sigma(2)\ldots\sigma(N))$$

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- implementation of multi-gluon amplitudes in computer code Rocket
- evaluation of single phase space points up to $N \leq 20$!
- evaluation of $t\bar{t}ggg \to 0$ amplitude
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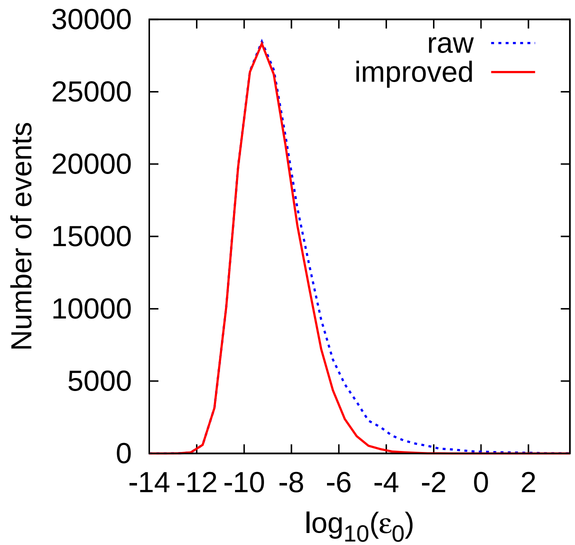
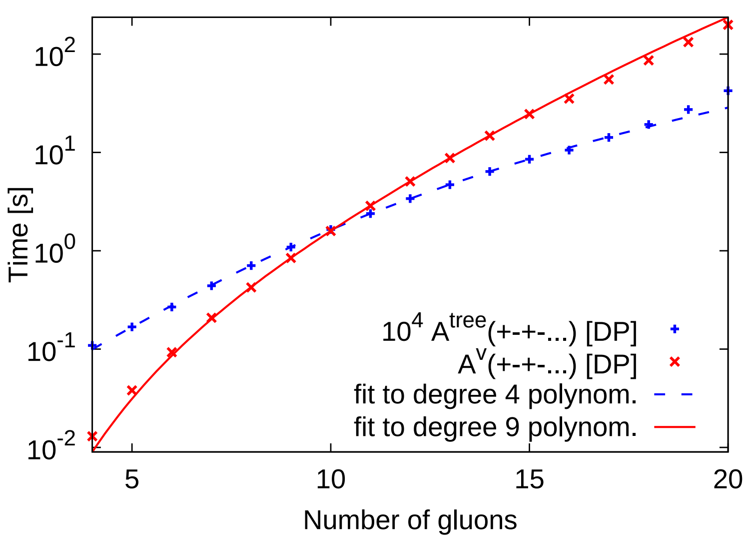
Numerical performance:



Ellis, et. al. 0810.2762 [hep/ph]

Large-N behaviour of N-gluon tree/loop amplitudes:



Giele, Zanderighi 0805.2152 [hep/ph]

Feynman diagrammatic approach:

$$\mathcal{A}^{c,\lambda}(p_j, m_j) = \sum_{\{c_i\},\alpha} f^{\{c_i\}} \mathcal{G}_\alpha^{\{\lambda\}}$$

$$\mathcal{G}_\alpha^{\{\lambda\}} = \int \frac{d^n k}{i\pi^{n/2}} \frac{\mathcal{N}^{\{\lambda\}}}{D_1 \ldots D_N} = \sum_R \mathcal{N}^{\{\lambda\}}_{\mu_1,\ldots,\mu_R} \, I_N^{\mu_1\cdots\mu_R}(p_j, m_j)$$

$$I_N^{\mu_1\cdots\mu_R}(p_j, m_j) = \int \frac{d^n k}{i\pi^{n/2}} \frac{k^{\mu_1} \ldots k^{\mu_R}}{D_1 \ldots D_N} \,, \; D_j = (k - r_j)^2 - m_j^2, \; r_j = p_1 \cdots + p_j$$

- Passarino-Veltman: $\rightarrow 1/\det(G)^R$, $G_{ij} = 2\, r_i \cdot r_j$ induce numerical problems
- Lorentz Tensor Integrals $\longrightarrow$ form factor representation à la Davydychev
- Reduction in Feynman parameter space

$$I_N^{\mu_1\cdots\mu_R} = \sum \tau^{\mu_1\cdots\mu_R}(r_{j_1}, \ldots, r_{j_r}, g^m) \, I_N^{n+2m}(j_1, \ldots, j_r)$$

$$I_N^D(j_1, \ldots, j_r) = (-1)^N \Gamma(N - \frac{D}{2}) \int_0^\infty d^N z \, \delta(1 - \sum_{l=1}^N z_l) \, \frac{z_{j_1} \ldots z_{j_r}}{(-\frac{1}{2} z \cdot \mathcal{S} \cdot z)^{N-D/2}}$$

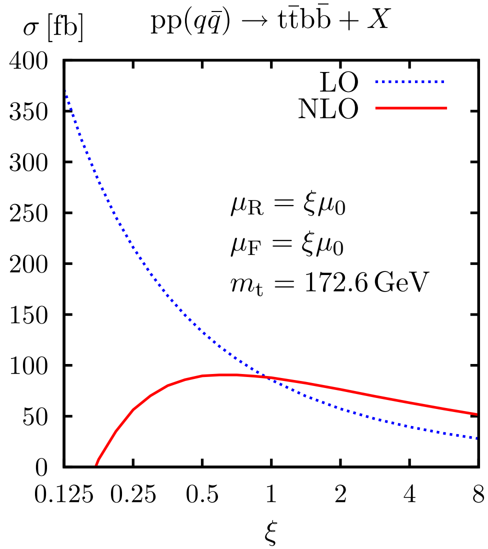
$$\mathcal{S}_{ij} = (r_i - r_j)^2 - m_i^2 - m_j^2$$

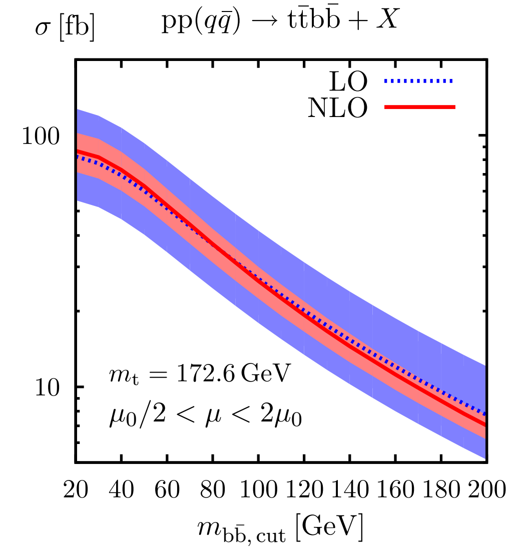
Progress for $pp \to b\bar{b}t\bar{t}$

- $pp \to b\bar{b}t\bar{t}$ important background process at the LHC
- efficient Feynman diagrammatic computation in progress, $q\bar{q} \to b\bar{b}t\bar{t}$ completed
- Form factor approach, numerical tensor reduction (rank 3 6-point problem)

$$\mathcal{A}(q\bar{q} \to b\bar{b}t\bar{t}) \sim \sum C_{j_1 j_2 j_3} I^{j_1 j_2 j_3}_{N \le 6}$$

Bredenstein, Denner, Dittmaier, Pozzorini 2008





The GOLEM project

General One Loop Evaluator for Matrix elements

- Evaluation of 1-loop amplitudes bottleneck for LHC@NLO
- Combinatorial complexity $\leftrightarrow$ numerical instabilities
  $\Rightarrow$ switching between algebraic/numerical representations
- Aim: Automated evaluation of numerically stable one-loop amplitudes for multi-leg processes

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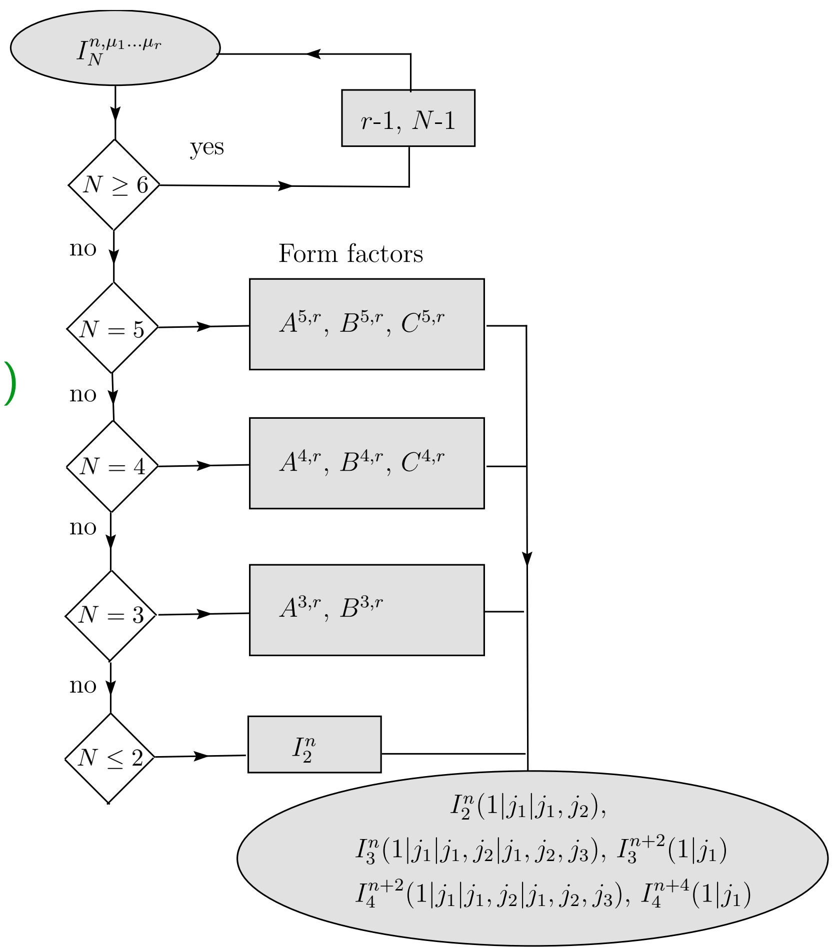
- The GOLEM team: T.B., A. Guffanti, J.Ph. Guillet, G. Heinrich, S. Karg, N. Kauer, E. Pilon, T. Reiter, J. Reuter

Schematic overview of N-point tensor integral reduction

T.B., J.P. Guillet, G. Heinrich (2000); T.B., Guillet, Heinrich, Pilon, Schubert (2005).

- works for general N
- no inverse Gram determinants
- isolation of IR divergences simple
- tractable expressions
- form factors for $N \leq 6$ implemented in Fortran95 code "golem95"
  T.B., Guillet, Heinrich, Pilon, Reiter (2008)
- optional reduction to scalar integrals
- evaluation of rational terms



$$I_{N=3,4}^{n,n+2}(j_1, \ldots, j_r) \quad \sim \quad \int_0^1 \prod_{i=1}^{4} dz_i \, \delta(1 - \sum_{l=1}^{4} z_l) \, \frac{z_{j_1} \ldots z_{j_r}}{(-\frac{1}{2}\, z \cdot \mathcal{S} \cdot z - i\delta)^{3-n/2}}$$

Implementation of the algorithm in a nutshell

Preparation:

- Diagram generation: QGRAF P. Nogueira, FeynArts 3.2 T. Hahn
- Perform colour algebra
- Projection on helicity amplitudes

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From here two independent set-ups:

a) Convert to form factor representation, link to `Fortran95` library "golem95"
   - $\mathcal{M}^{\{\lambda\}} \to C^{ijk}_{box} I_4^{n+2,n+4}(z_i z_j z_k) + C^{ijk}_{tri} I_3^{n,n+2}(z_i z_j z_k) + \ldots$
   - In numerically critical phase space regions:
     – compile/run code in quadruple precision
     – use one-dimensional integral representations for $I_{N=3,4}^{n+2,n+4}(z_i z_j z_k)$

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b) Symbolic reduction to scalar integrals based on `FORM` and `MAPLE`

- $\mathcal{M}^{\{\lambda\}} \to C_{box} I_4^{d=6} + C_{tri} I_3^{d=4-2\epsilon} + C_{bub} I_2^{d=4-2\epsilon} + C_{tad} I_1^{d=4-2\epsilon} + \mathcal{R}$
- automated method to evaluate $\mathcal{R}$ T.B., Guillet, Heinrich (2006)
- introduces $1/\det G$ but allows to apply symbolic simplifications

Computations with GOLEM:

Algorithm coded in FORM and FORTRAN 90

some recent applications ...

- $gg \to W^*W^* \to l\nu l'\nu'$, GG2WW code
  T.B., M. Ciccolini, M. Kramer, N. Kauer (2006)
- $gg \to HH, HHH$
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- $pp \to Hjj$ GF/WBF NLO interference $\mathcal{O}(\alpha^2\alpha_s^3)$
  J.R. Andersen, T.B., G. Heinrich, J. Smillie (2007)
- $\gamma\gamma \to \gamma\gamma\gamma\gamma$
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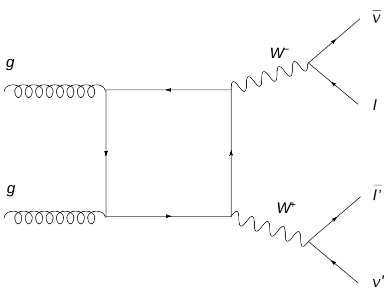

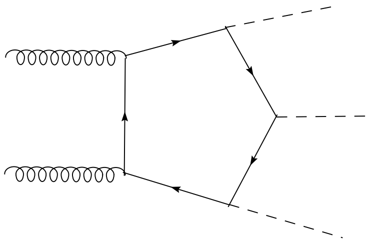

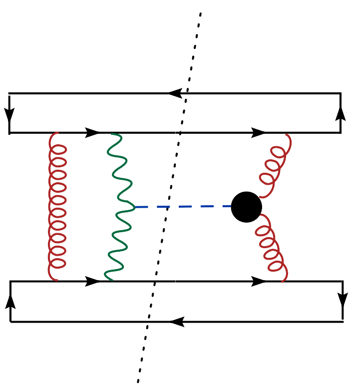

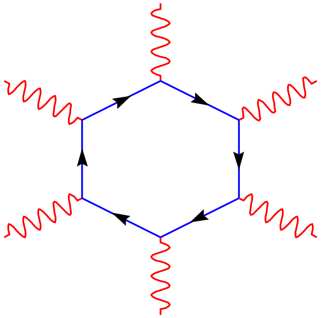

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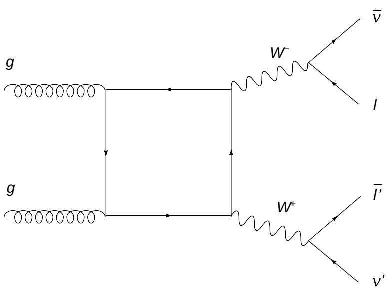

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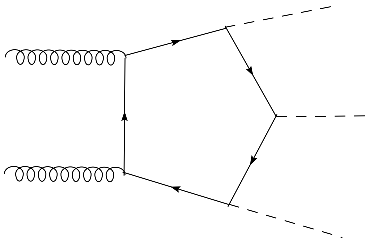

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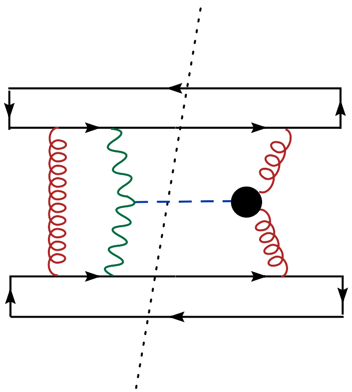

- $\gamma\gamma \to \gamma\gamma\gamma\gamma$
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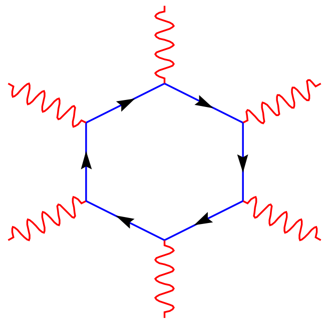
... and ongoing work:

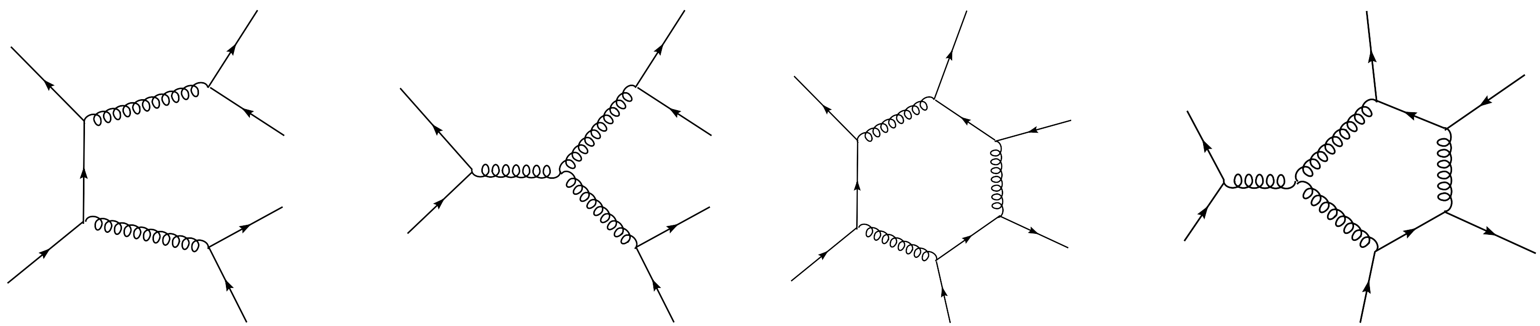


- $gg \to Z^*Z^*, \gamma^*Z^*, \gamma^*\gamma^* \to l\bar{l}l'\bar{l}'$, GG2ZZ code
- $pp \to WWj, ZZj$, $gg \to WWg, ZZg$
- $pp \to bbbb$

The process $pp \to b\bar{b}b\bar{b}$ at NLO QCD

Motivation: Higgs search in two Higgs doublet models/MSSM included in experimentalist "Les Houches wish list" (Heinrich, Huston 2007)

- two subprocesses: $q\bar{q} \to b\bar{b}b\bar{b}$, $gg \to b\bar{b}b\bar{b}$
- $\sim$ 250 Feynman diagrams, 25 pentagon and 8 hexagon diagrams



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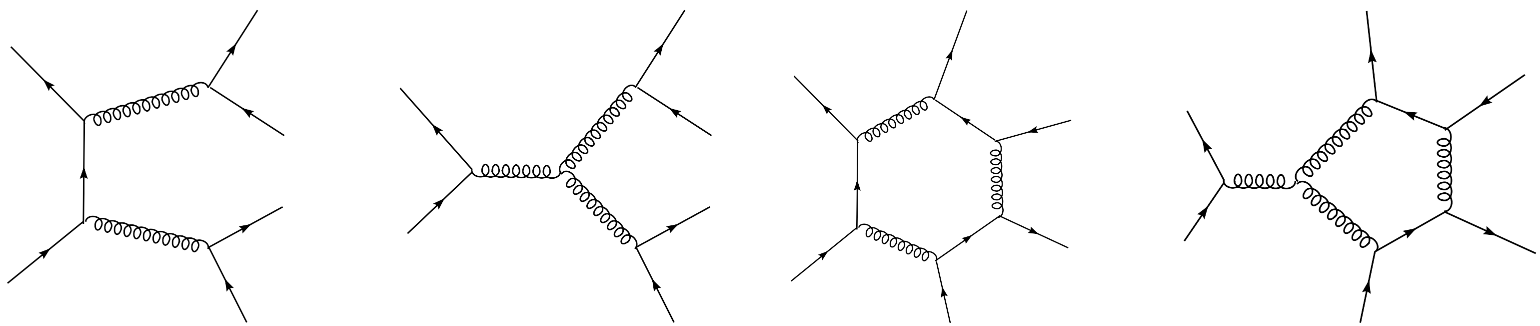


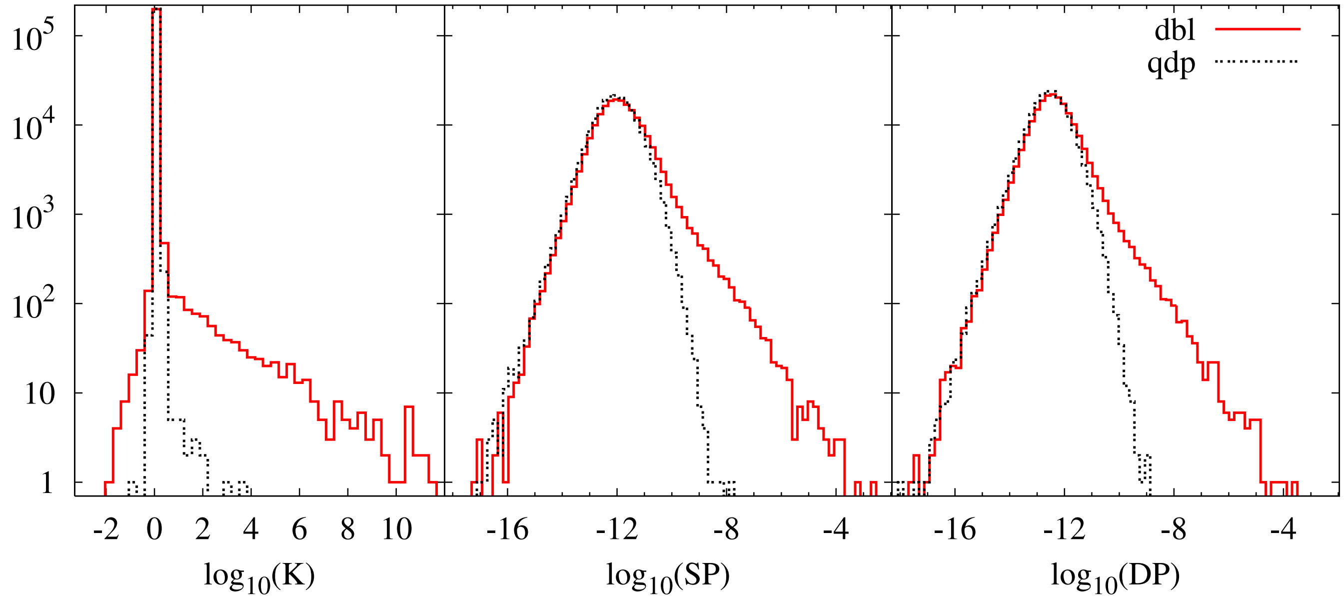
- virtual part of $q\bar{q} \longrightarrow b\bar{b}b\bar{b}$ completed, $gg \longrightarrow b\bar{b}b\bar{b}$ under construction
- GOLEM provides finite combination

$$|\mathcal{A}_{LO+V}|^2 - \text{ UV counterterms } - \text{ IR subtraction terms}$$

Numerical precision of virtual correction evaluation

- evaluation of 200.000 random phase space points
- cuts: $\eta < |2.5|$, $\Delta R > 0.4$, $p_T > 25$ GeV

$K = |\mathcal{A}_{LO} + \mathcal{A}_{NLO,V}|^2 / |\mathcal{A}_{LO}|^2$



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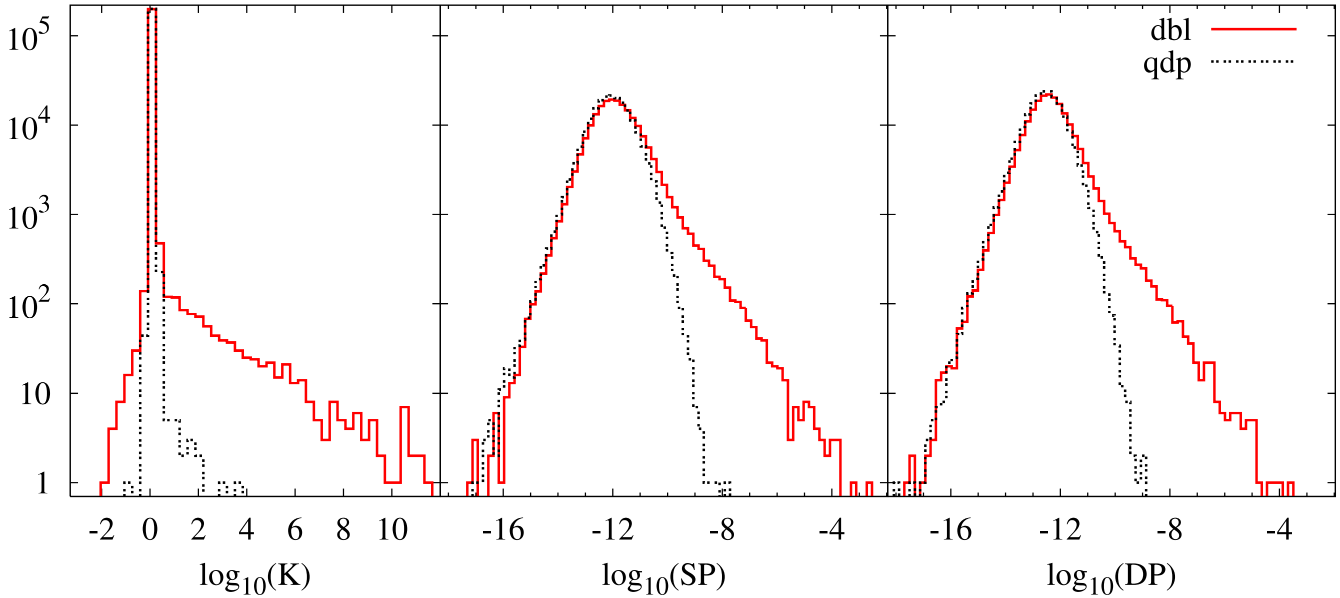
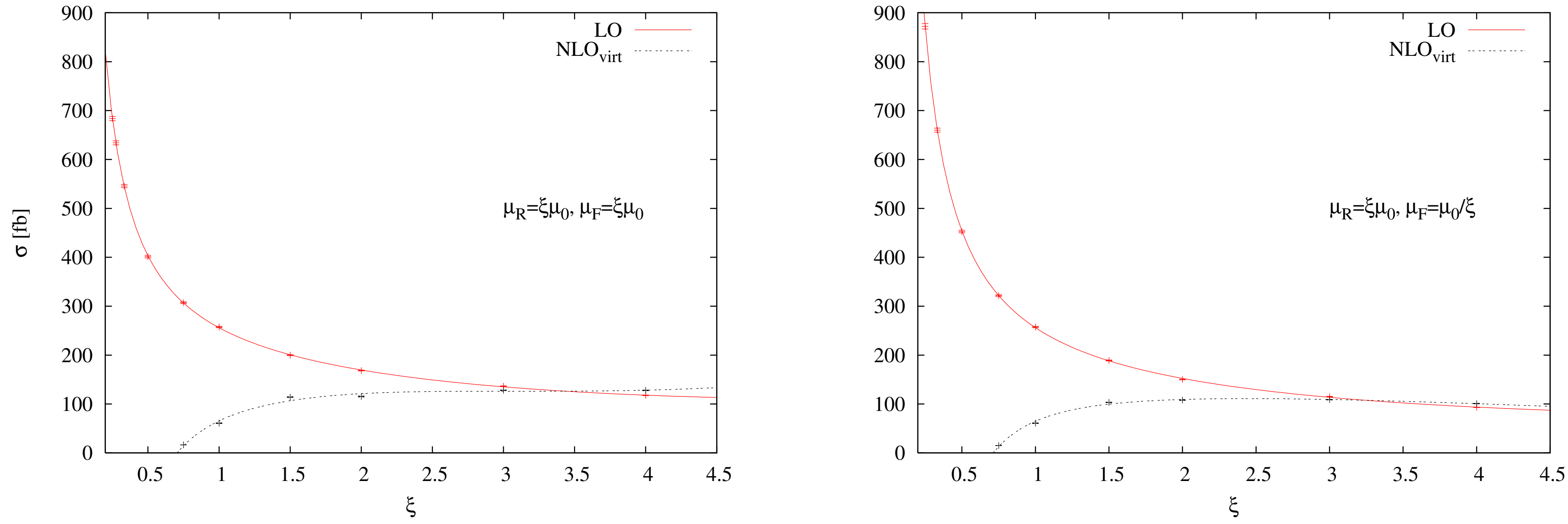


- imperfect numerical cancellations lead to inaccuracies
  $\longrightarrow$ dangerous for adaptive integration methods
- size of local K-factor good indicator for numerical problem
- GOLEM integration strategy:
  - generate unweighted LO events, reweight by local K-factor

  $\longrightarrow$ numerical inaccuracies do not interfere with integration !

Scale variations

Standard scale choice: $\mu_R = \mu_F = \sum_{j=1}^{4} p_{T\,j}/4$



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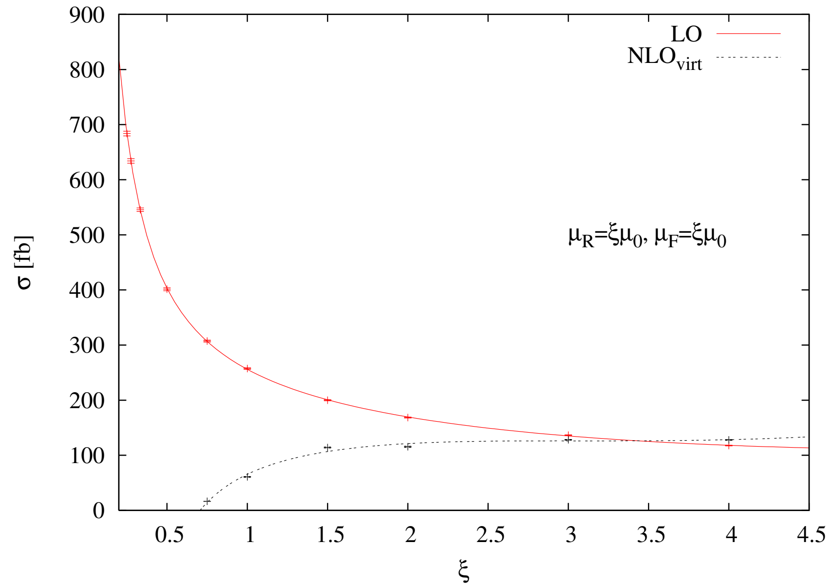
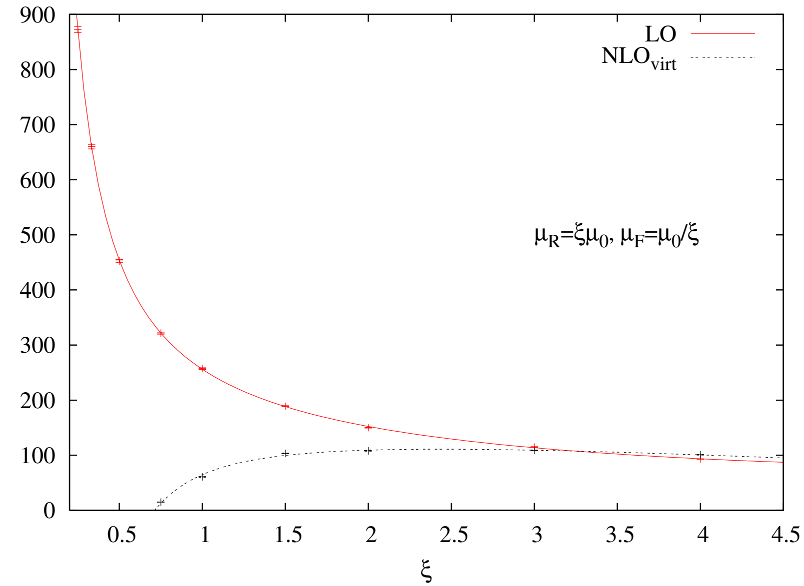
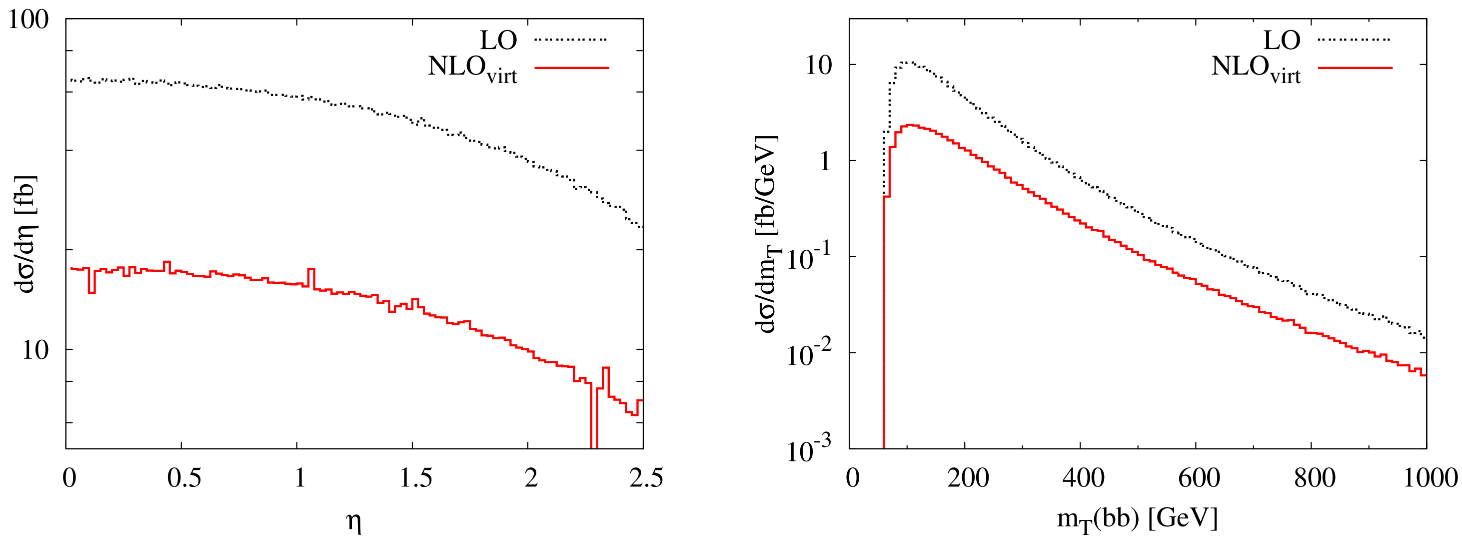




- real emission contribution not yet added, further compensation of $\alpha_s \log(\mu_F)$
- under construction using Whizard, MadDipole

Experimental distributions η- and $m_{T\,bb}$



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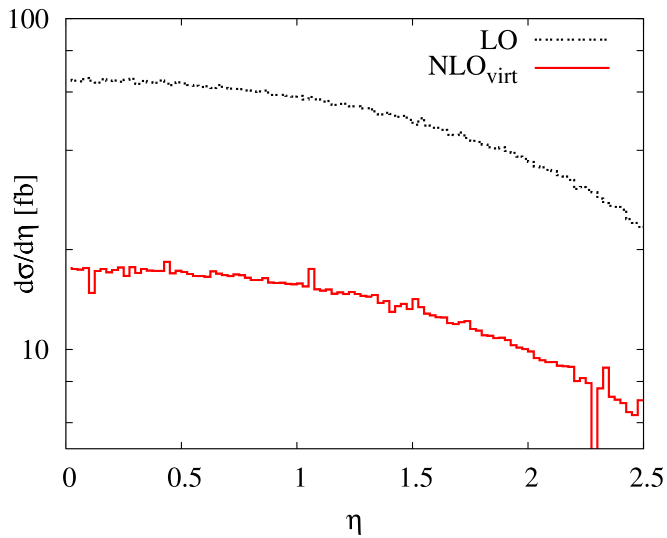
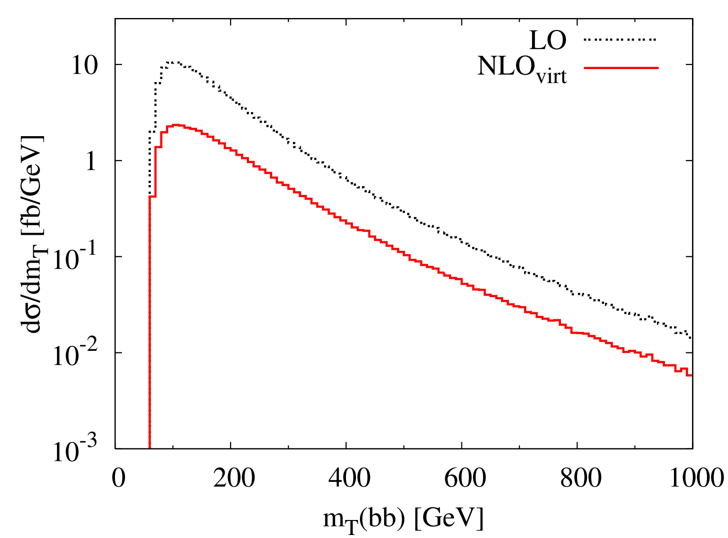
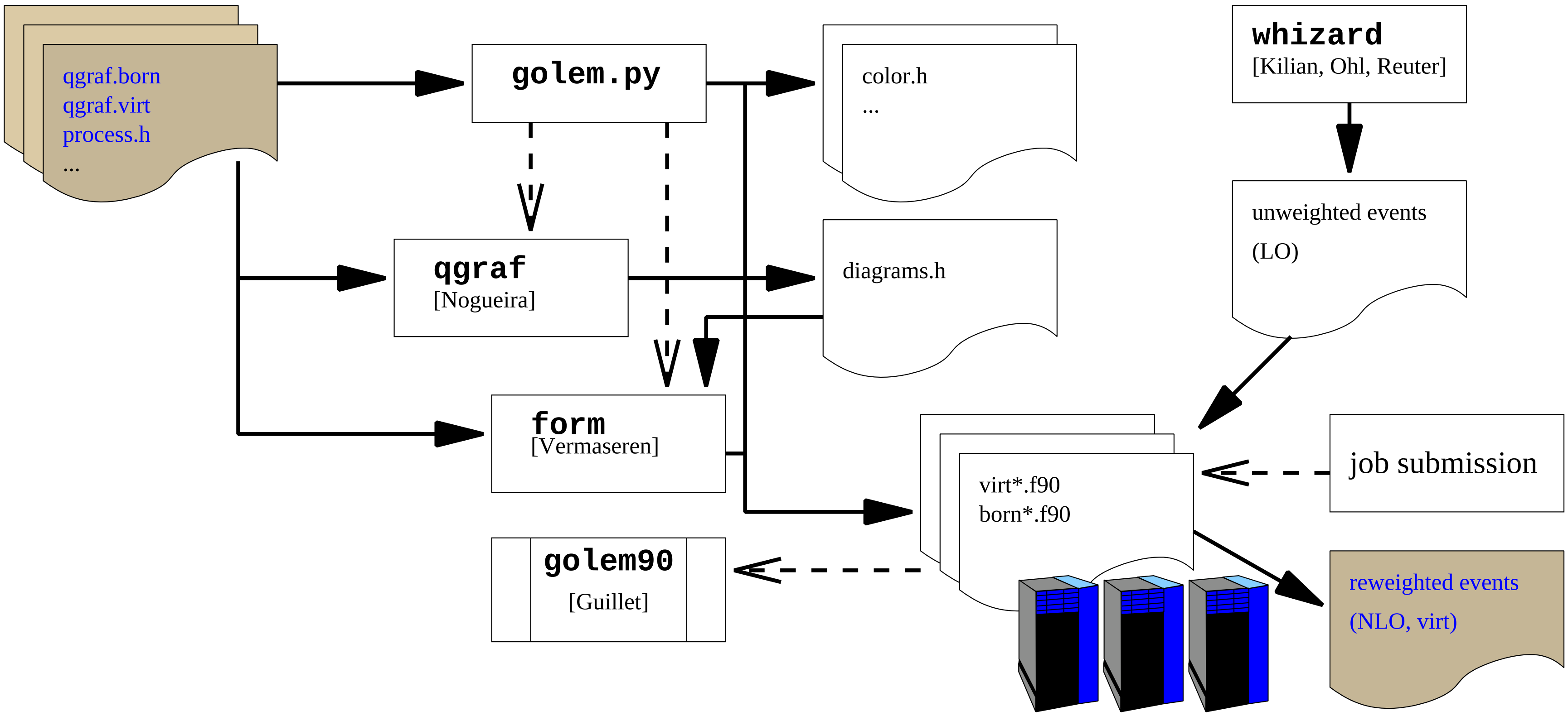




Philosophy and vision: no standalone NLO computations but rather...

- Monte Carlo LO event generator including dipole subtraction
- Tree matrix elements merged with parton shower
- data base with one-loop matrix-elements in one format
  $\Rightarrow$ K-factor reweighting done by experimentalists

Flow chart of computation



(from Thomas Reiter's PhD thesis)

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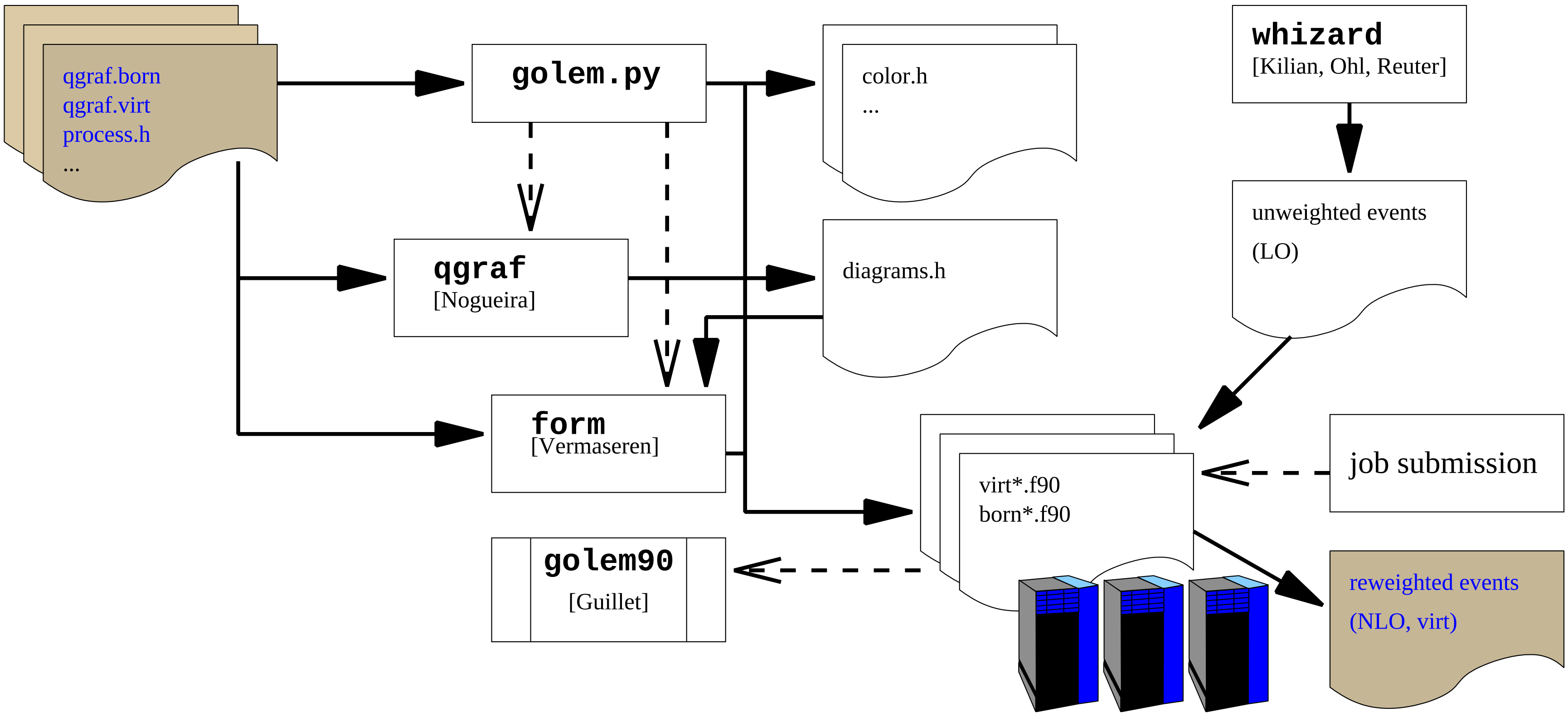

(from Thomas Reiter's PhD thesis)

- reweighting of events done in parallel on Edinburgh (ECDF) cluster
- General set-up for NLO computations, to be used for other processes

Performance Feynman diagrams vs. unitarity based methods

"...religious battle between Feynmanians and Unitarians..."

Joey Huston

Look at colour ordered multi-gluon amplitudes:

- unitarity based method $\sim \tau_{\text{Tree}} \times \tau_{\#\text{ cuts}} \sim N^9$
- Feynman diagrams $\sim \tau_{\#\text{Formfactors}} \times \#\text{Diagrams} \sim \Gamma(N)\, 2^N$

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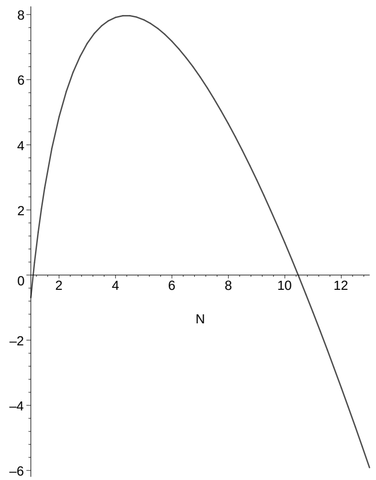

$\log[N^9\,/\,(\Gamma(N)\,2^N)]$

- asymptotic behaviour not relevant for LHC region $N \leq 8$
- for LHC both methods can/will do the job!

Summary

LHC phenomenology needs and deserves predictions at the NLO level

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NLO multileg processes still challenging, lot of progress

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Summary

LHC phenomenology needs and deserves predictions at the NLO level

NLO multileg processes still challenging, lot of progress

- ... unitarity based methods
- ... improvements in Feynman diagram calculations
- ... for LHC both methods can/will do the job!

The issue is automation, public & experimentalist friendly code !!!

- `Blackhat, Rocket, CutTools, SANC, GRACE, GOLEM, FormCalc, ...`
- NLO virtual corrections are just reweighting exercise of event NTUPLES
- problem is "embarrassingly" parallel
- combine with parton showers $\Rightarrow$ MC with NLO precision!
- topic of the Les Houches workshop in June 2009 !

Summary

LHC phenomenology needs and deserves predictions at the NLO level

NLO multileg processes still challenging, lot of progress

- ... unitarity based methods
- ... improvements in Feynman diagram calculations
- ... for LHC both methods can/will do the job!

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LHC = Long and Hard Calculations ...

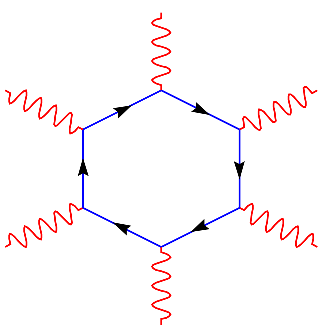
- ...but the future is bright !

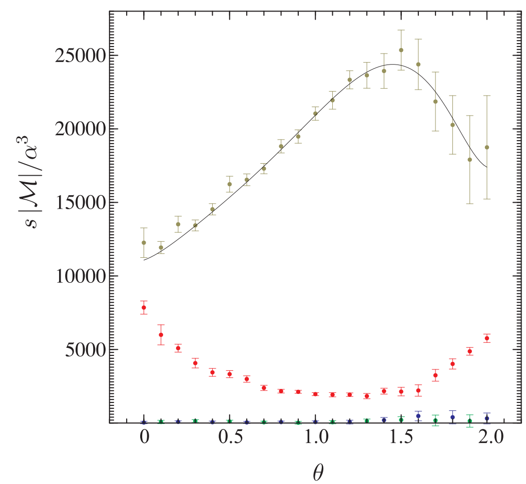
Fully numerical evaluation of multileg amplitudes

- Singularity structure of one-loop integrals well understood
- dedicated numerical integration method e.g. talk F. Yuasa

$$I_N^D(j_1,\ldots,j_r) \quad \sim \quad \int_0^\infty d^N z\, \delta(1-\sum_{l=1}^{N} z_l)\, \frac{z_{j_1}\ldots z_{j_r}}{(-\frac{1}{2} z\cdot\mathcal{S}\cdot z)^{N-D/2}}$$

6-photon amplitude using "multi-dimensional contour deformation" Soper, Nagy 2006





Many ideas: Soper (2000); Ferroglia, Passera, Passarino, Uccirati (2002); T.B., Heinrich, Kauer (2002), T.B., Guillet, Heinrich, Pilon, Schubert (2005) Y. Kurihara, T. Kaneko, (2005); Anastasiou, Daleo (2005); Czakon (2005); Lazopoulos, Melnikov, Petriello (2007).