Higgs b$\overline{\mathrm{b}}$ decay at NNLO: uncertainties of QCD predictions

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Abstract

The dominant channel of Higgs boson decay $H \to \bar{b}b$ for $M_H < 2M_W \simeq 160$ GeV is briefly reviewed. The perturbative QCD-corrections of higher orders up to (α_S^4) are considered. Various approaches for resummation of the QCD-corrections are discussed. An estimate for uncertainties of the theoretical approximations for width decay $\Gamma_{H\bar{b}b}$ is given.

Outline:

- Introduction
- Perturbative corrections to Higgs boson decay $H \to \bar{b}b$
- Some approaches of resummation of QCD corrections
- Summary

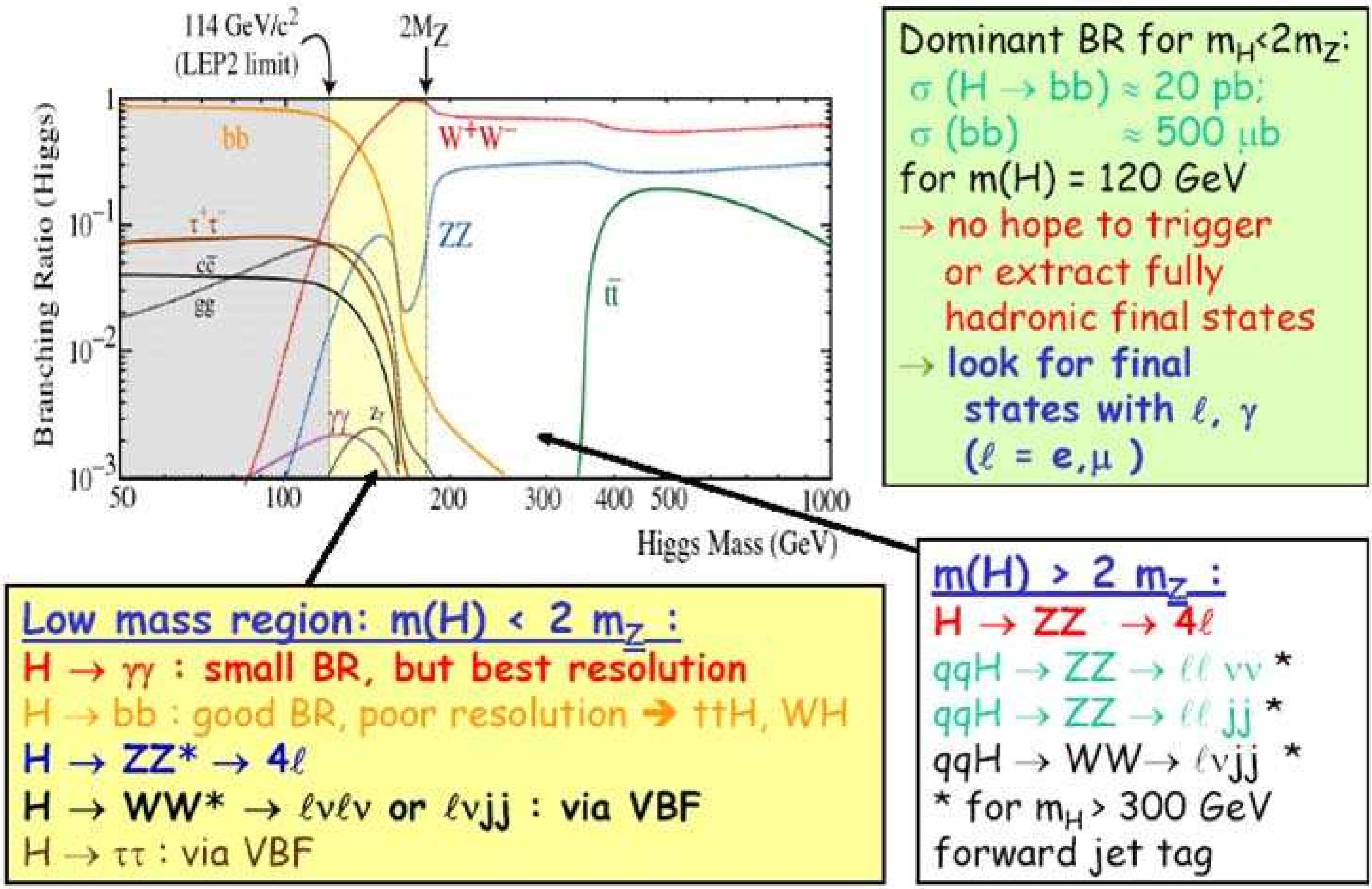
LEP and Tevatron fit: $M_H = 84\,^{+34}_{-26}$ GeV C.L. 68% (ICHEP-08)

$M_H \leq 144$ GeV C.L. 95% (without LEP-II) (pre-ICHEP08)

$M_H \leq 182$ GeV C.L. 95% (with LEP-II) (pre-ICHEP08)

LEP-II direct search: $M_H \geq 114.4$ GeV C.L. 95%

CMS in Standard Model $H^0 \to \gamma\gamma$ decay may detect $\mathrm{M_H} \leq 140$ GeV



LHC: ATLAS, CMS, Diffraction: CMS-TOTEM, US-British

Main quantity under study $\Gamma(H^0 \to b\bar{b})$=$\Gamma_{H\bar{b}b}$ with the mass 115 GeV $\leq M_H \leq 2M_W$, calculated in the $\overline{MS}$ up to the α_s^4 corrections. This decay mode is dominating in the sum of decay widths, and thus is dominating in the branching ratio of Higgs to $\gamma\gamma$, also interesting for **CMS- TOTEM**- main decay mode.

What is theoretical error of $\Gamma_{H\bar{b}b}$?

1. consideration in $\overline{MS}$-scheme and m_b-on shell [**Kataev** & **VK (93-94,07)**]

2. $\alpha_s(M_H)$ and $\overline{m}_b(M_H)$ $\overline{MS}$-scheme; calculated up to α_s^4-level; (α_s^4 massless term- [**Gorishny, Kataev, Larin, Surguladze (91)** – **Baikov,Chetyrkin** & **Kuhn (06)**]

3. invariant mass $\hat{m}_b$, the resummation of effects of analytic continuation within in β_0 approximation definition of special parameters in every order of PT (analog of [**Shirkov** & **Solovtsev (96)**] analytized perturbation theory) with fractional power of α_s, i.e. $\nu_0 = 2\gamma_0/\beta_0$, γ_0-first coefficient of anomalous dimension function [**Broadhurst, Kataev** & **Maxwell (01)**]

4. invariant mass $\hat{m}_b$, the resummation of effects of analytic continuation within analytized perturbation theory with fractional power [**Bakulev, Mikhailov** & **Stefanis (07)**]

Some definitions in terms of

$$\Gamma_{Hb\bar{b}} = \Gamma_0^b \left(\beta^3 [1 + \Delta\Gamma_{NLO} a_s + \Delta\Gamma_{NNLO} a_s^2 + \Delta\Gamma_{N^3LO} a_s^3 + \Delta\Gamma_{N^4LO} a_s^4 \right) \tag{1}$$

$$\Gamma_0^b = \frac{3\sqrt{2}}{8\pi} G_F M_H m_b^2 \ , \ a_s \equiv \alpha_s/\pi \ , \ \beta = \sqrt{1 - \frac{4m_b^2}{M_H^2}} \tag{2}$$

$$\Delta\Gamma_{NLO} = \frac{4}{3}\beta^2 A(\beta) + \frac{3 + 34\beta^2 - 13\beta^4}{16} \ln\frac{(1+\beta)}{(1-\beta)} + \beta\frac{3(-1+7\beta^2)}{8} \tag{3}$$

$$A(\beta) = (1+\beta^2)\left[4Li_2\left(\frac{1-\beta}{1+\beta}\right) + 2Li_2\left(-\frac{1-\beta}{1+\beta}\right) - 3\ln\frac{2}{1+\beta}\ln\frac{1+\beta}{1-\beta}\right. \tag{4}$$

$$\left. -2\ln\frac{1+\beta}{1-\beta}\ln\beta\right] - 3\beta\ln\frac{4}{1-\beta^2} - 4\beta\ln\beta \tag{5}$$

$Li_2(x) = \int_0^x (dt/t)\ln(1-t)$, only massive dependence of $\Delta\Gamma_{NNLO}$-term is known up to m_b^2/M_H^2 [**Kataev** & **VK (93-94)**]

The relation of the m_b-pole case with $\overline{m}_b(M_H)$-case and $a_s(M_H)$ is

$$\Gamma_{Hb\bar{b}} \quad = \quad \Gamma_0^{(b)} \frac{\overline{m}_b^{\,2}}{m_b^2} \left(1 + \Delta\Gamma_1 a_s + \Delta\Gamma_2 a_s^2 + \Delta\Gamma_3 a_s^3 + \Delta\Gamma_4 a_s^4 \right). \quad (6)$$

$$\overline{m}_b^2(M_H) = \overline{m}_b^2(m_b) \mathrm{exp} \left[-2 \int_{\alpha_\mathrm{s}(\mathrm{m_b})}^{\alpha_\mathrm{s}(\mathrm{M_H})} \frac{\gamma_\mathrm{m}(\mathrm{x})}{\beta(\mathrm{x})} \mathrm{dx} \right] \quad , \text{where} \quad (7)$$

$$\mu^2 \frac{da_s}{d\mu^2} = \beta(a) = -\beta_0 a_s^2 - \beta_1 a_s^3 - \beta_2 a_s^4 - \beta_3 a_s^5 - \beta_4^{Pade} a_s^6 + O(a_s^7) \quad (8)$$

$$\frac{d\ln\overline{m}_b}{d\ln\mu^2} = \gamma_m(a_s) = -\gamma_0 a_s - \gamma_1 a_s^2 - \gamma_2 a_s^3 - \gamma_3 a_s^4 - \gamma_4^{mPade} a_s^5 + O(a_s^6) \quad (9)$$

$$\overline{m}_b^2(M_H) = \overline{m}_b^2(m_b) \left(\frac{a_s(M_H)}{a_s(m_b)} \right)^{2\gamma_0/\beta_0} \left[\frac{AD(a_s(\mu))}{AD(a_s(\overline{m}_b))} \right]^2 \quad (10)$$

$$
\begin{aligned}
AD(a_s) &= \Bigg[1 + P_1 a_s + \left(P_1^2 + P_2\right)\frac{a_s^2}{2} + \left(\frac{1}{2}P_1^3 + \frac{3}{2}P_1P_2 + P_3\right)\frac{a_s^3}{3} \\
&+ \left(\frac{1}{6}P_1^4 + \frac{4}{3}P_1P_3 + P_1^2P_2 + P_4\right)\frac{a_s^4}{4}\Bigg]
\end{aligned}
$$

$$
\begin{aligned}
P_1 &= -\frac{\beta_1\gamma_0}{\beta_0^2} + \frac{\gamma_1}{\beta_0} \quad , P_2 = \frac{\gamma_0}{\beta_0^2}\left(\frac{\beta_1^2}{\beta_0} - \beta_2\right) - \frac{\beta_1\gamma_1}{\beta_0^2} + \frac{\gamma_2}{\beta_0} \qquad (11) \\
P_3 &= \left[\frac{\beta_1\beta_2}{\beta_0} - \frac{\beta_1}{\beta_0}\left(\frac{\beta_1^2}{\beta_0} - \beta_2\right) - \beta_3\right]\frac{\gamma_0}{\beta_0^2} + \frac{\gamma_1}{\beta_0^2}\left(\frac{\beta_1^2}{\beta_0} - \beta_2\right) - \frac{\beta_1\gamma_2}{\beta_0^2} + \frac{\gamma_3}{\beta_0} \\
P_4 &= \frac{\gamma_0}{\beta_0^4}\left[\frac{\beta_1^2}{\beta_0^2}\left(\frac{\beta_1^2}{\beta_0} - \beta_2) + \frac{\beta_2^2}{\beta_0} - \frac{2\beta_1}{\beta_0}\left(\frac{\beta_1\beta_2}{\beta_0} - \beta_3\right) - \beta_4^{Pade}\right]\right. \\
&+ \frac{\gamma_1}{\beta_0^2}\left[\frac{\beta_1\beta_2}{\beta_0} - \frac{\beta_1}{\beta_0}\left(\frac{\beta_1^2}{\beta_0} - \beta_2\right) - \beta_3\right] \\
&+ \frac{\gamma_2}{\beta_0^2}\left(\frac{\beta_1^2}{\beta_0} - \beta_2\right) - \frac{\gamma_3\beta_1}{\beta_0^2} + \frac{\gamma_4^{mPade}}{\beta_0} \qquad (12)
\end{aligned}
$$

The basic formula in terms of $\overline{m}_b(M_H)$, a_s for N_f=5:

$$\Gamma_{Hb\bar{b}} = \Gamma_0^{(b)}\frac{\overline{m}_b{}^2}{m_b^2}\Bigg[\Bigg(1+\Delta\Gamma_1 a_s+\Delta\Gamma_2 a_s^2+\Delta\Gamma_3 a_s^3+\Delta\Gamma_4 a_s^4\Bigg) \tag{13}$$

$$\Delta\Gamma_1 = \frac{17}{3} = 5.667 \quad , \Delta\Gamma_2 = d_2^E - \gamma_{\mathbf{0}}(\beta_{\mathbf{0}}+\mathbf{2}\gamma_{\mathbf{0}})\pi^{\mathbf{2}}/\mathbf{3} = \mathbf{29.147}$$

$$\Delta\Gamma_3 = d_3^E - \left[d_1(\beta_{\mathbf{0}}+\gamma_{\mathbf{0}})(\beta_{\mathbf{0}}+\mathbf{2}\gamma_{\mathbf{0}})+\beta_{\mathbf{1}}\gamma_{\mathbf{0}}+\mathbf{2}\gamma_{\mathbf{1}}(\beta_{\mathbf{0}}+\mathbf{2}\gamma_{\mathbf{0}})\right]\frac{\pi^{\mathbf{2}}}{\mathbf{3}} = \mathbf{41.178}$$

$$\begin{aligned}\Delta\Gamma_4 &= d_4^E - \Big[d_2(\beta_{\mathbf{0}}+\gamma_{\mathbf{0}})(\mathbf{3}\beta_{\mathbf{0}}+\mathbf{2}\gamma_{\mathbf{0}})+\mathbf{d_1}\beta_{\mathbf{1}}(\mathbf{5}\beta_{\mathbf{0}}+\mathbf{6}\gamma_{\mathbf{0}})/\mathbf{2}\\ &+ 4d_1\gamma_1(\beta_0+\gamma_0)+\beta_2\gamma_0+2\gamma_1(\beta_1+\gamma_1)+\gamma_2(3\beta_0+4\gamma_0)\Big]\pi^2/3\\ &\quad +\gamma_{\mathbf{0}}(\beta_{\mathbf{0}}+\gamma_{\mathbf{0}})(\beta_{\mathbf{0}}+\mathbf{2}\gamma_{\mathbf{0}})(\mathbf{3}\beta_{\mathbf{0}}+\mathbf{2}\gamma_{\mathbf{0}})\pi^{\mathbf{4}}/\mathbf{30} = \mathbf{-825.7}\end{aligned}$$

$\Gamma_0^b = \frac{3\sqrt{2}}{8\pi}G_F M_H m_b^2$, We transform from $m_b(M_H)$ to m_b-pole using

$$\overline{m}_b^2(m_b) = m_b^2\Bigg(1-2.67a_s(m_b)-18.57a_s(m_b)^2-175.79a_s^3(m_b)-1892a_s^4(m_b)\Bigg)$$

[Chetyrkin, Steinhauser (99), Melnikov, van Ritbergen (00), PMS/ECH estimate by Chetyrkin, Kniehl, Sirlin (97) motivated by Kataev, Starshenko(95)]

Approach N1:

Truncated series for $\Gamma_{Hb\bar{b}}$, the dependence from M_H in $\overline{m}_b(M_H)$ and $\alpha_s(M_H)$:

$$\begin{aligned}
\alpha_s(\mu)_{NLO} &= \frac{\pi}{\beta_0 \mathrm{Log}}\left[1-\frac{\beta_1 \ln(\mathrm{Log})}{\beta_0^2 \mathrm{Log}^2}\right] && (14)\\
\alpha_s(\mu)_{NNLO} &= \alpha_s(M_H)_{NLO}+\Delta\alpha_s(M_H)_{MNLO}\\
\alpha_s(\mu)_{N^3LO} &= \alpha_s(M_H)_{NNLO}+\Delta\alpha_s(M_H)_{N^3LO} && (15)\\
a_s(M_H)_{N^4LO} &= a_s(M_H)_{N^3LO}+\Delta a_s(M_H)_{N^4LO},\\
\Delta\alpha_s(M_H)_{NNLO} &= \frac{\pi}{\beta_0^5 \mathrm{Log}^3}\left(\beta_1^2\ln^2(\mathrm{Log})-\beta_1^2\ln(\mathrm{Log})+\beta_2\beta_0-\beta_1^2\right)\\
\Delta\alpha_s(\mu)_{N^3LO} &= \frac{\pi}{\beta_0^7 \mathrm{Log}^4}\left[\beta_1^3\left(-\ln^3(\mathrm{Log})+\frac{5}{2}\ln^2(\mathrm{Log})+2\ln(\mathrm{Log})-\frac{1}{2}\right)\right.\\
&\left. -3\beta_0\beta_1\beta_2\ln(\mathrm{Log})+\beta_0^2\frac{\beta_3}{2}\right], \text{ where } \mathrm{Log}=\ln(\mu^2/\Lambda_{\overline{\mathrm{MS}}}^{(\mathrm{f}=5)\ 2})
\end{aligned}$$

• m_b [**Penin & Steinhauser (02)**]

• $\Lambda^{(4)}_{\overline{\rm MS}}$ from the analysis of CCFR data by [**Kataev, Sidorov & Parente (01-03)**]- uncertainties and possible smaller values are not analysed

• $\Lambda^{(5)}_{\overline{\rm MS}}$ calculated here using matching conditions, (from $n_f = 4$ to $n_f = 5$

order	$\rm m_b$ GeV	$\Lambda^{(\rm n_f=4)}_{\overline{\rm MS}}$ MeV	$\Lambda^{(\rm n_f=5)}_{\overline{\rm MS}}$ MeV
LO	4.74	220	168
NLO	4.86	347	251
$\rm N^2LO$	5.02	331	238
$\rm N^3LO$	5.23	333	237
$\rm N^4LO$	5.45	333	241

Approach N2: Using truncated m_b parameterization

$$\Gamma_{Hb\bar{b}} = \Gamma_0^{(b)}\left(1 + \Delta\tilde{\Gamma}_1 a_s + \Delta\tilde{\Gamma}_2 a_s^2 + \Delta\tilde{\Gamma}_3 a_s^3 + \Delta\tilde{\Gamma}_4 a_s^4\right) \quad (16)$$

$\Delta\tilde{\Gamma}_1 = 3 - 2L$, where $L = ln(M_H^2/m_b^2)$
$\Delta\tilde{\Gamma}_2 = -4.52 - 18.138L + 0.084L^2$
$\Delta\tilde{\Gamma}_3 = -316.906 - 133.421L - 1.153L^2 + 0.05L^3$
$\Delta\tilde{\Gamma}_4^b = -4366.17 - 1094.62L - 55.867L^2 - 1.8065L^3 + 0.04774L^4$
Do not afraid of large $L - ogs$!!

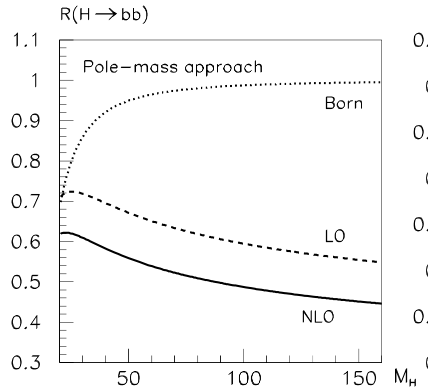


Fig.1

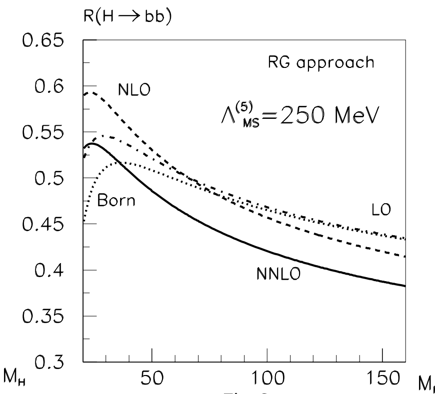


Fig.2

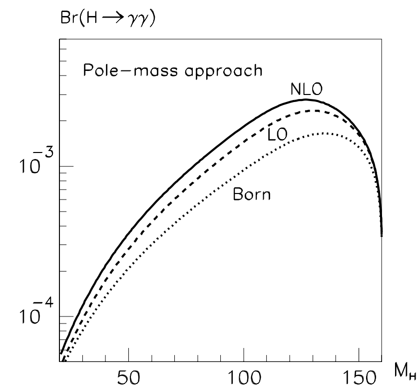


Fig.3

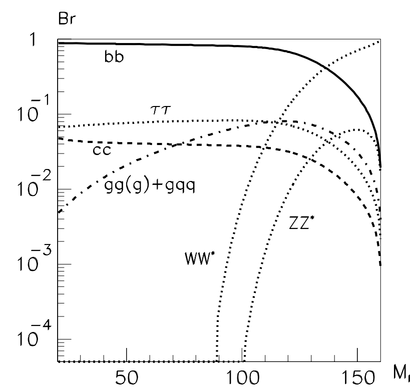


Fig.4

The results for $R_{H\bar{b}b} = \Gamma_{H\bar{b}b}/\Gamma_0^{(b)}$ $(\Gamma_0^{(b)} = \frac{3\sqrt{2}}{8\pi} G_F M_H m_b^2)$ at α_s^2-level. Approach 2 is compared with Aproach 1. At the next page the comparison is made at the α_s^4 level in the massless case

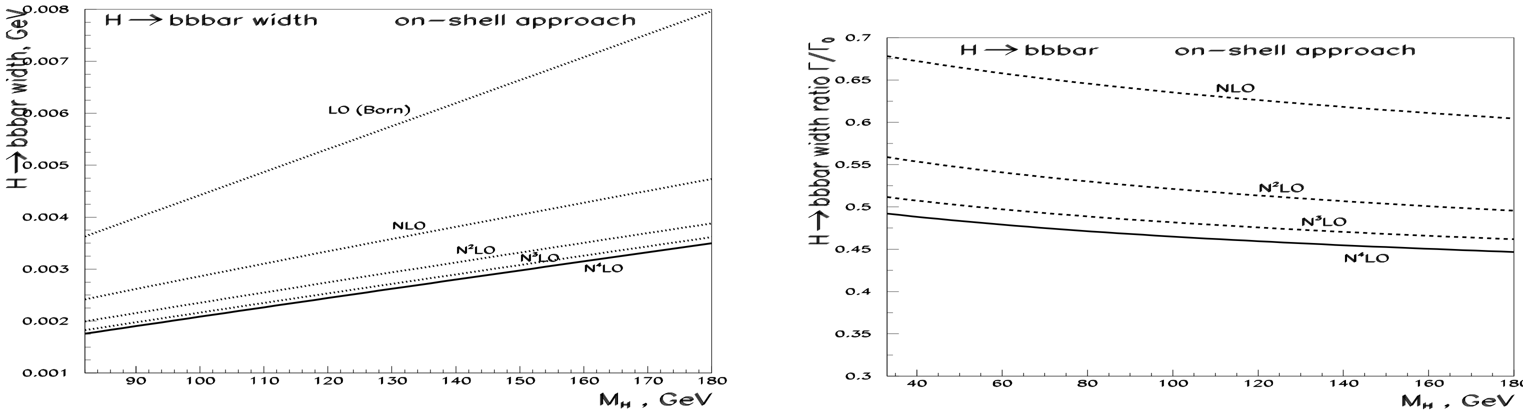


Figure 1: The analysed quantities in the OS-approach

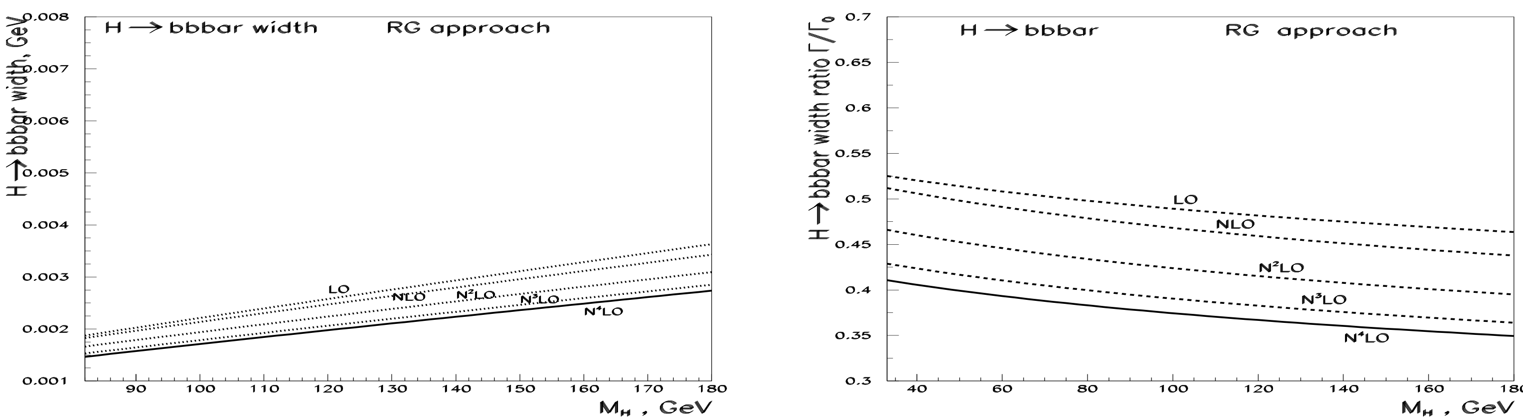


Figure 2: The analysed quantities in the RG-approach

1) Results for "running approach" are rather stable, effects of coefficient functions are not very large [**Kataev** & **VK (08)**]

2) In the on-shell scheme large logs are important, making them comparable with the "running case", in this case the corrections to RG function and coefficient functions are seen more clearly, in particular in the approaches with resummations of the π^2 terms ([**Krasnikov** & **A.Pivovarov (82), Radyushkin (82), Shirkov (00)**])-to be considered later.

3) For $M_H = 120\ GeV$ and $G_F = 1.166 \times 10^{-5}\ GeV^{-2}$ we get
$\Gamma^{OS}_{H\bar{b}b} \approx 2.50\ (\alpha_s^2 - level)\mathrm{MeV} - 1.75\ (\alpha_s^4 - level)\ \mathrm{MeV}$
$\Gamma^{RG\ \overline{MS}}_{H\bar{b}b} \approx 1.8\ (\alpha_s^2 - level)\mathrm{MeV} - 1.5\ (\alpha_s^4 - level))\mathrm{MeV}$
Difference between OS- and RG- approaches: $\Delta\Gamma_{H\bar{b}b} \approx 1$ MeV at α_s^2-level moved to 0.25 MeV at α_s^4

OS IS HIGHER!!

This is our current estimate of theoretical uncertainty.

The resummation of π^2-terms in $\Gamma_{Hb\bar{b}}$ in the case of running or invariant masses. In these cases the RG-evolution is starting from $\alpha_s(s)^{2\frac{\gamma_0}{\beta_0}}$. The summation of the π^2 leading terms with fractional power were used by **Gorishny, Kataev** & **Larin (84)**. It was considered more carefully in [**BKM (01)**] and in more detail by [**BMS (07)**]. Using notations of this paper let us define

$$\tilde{R}_S(M_H) = \frac{8\pi}{\sqrt{2}G_F M_H}\Gamma(H \to b\bar{b}) \tag{17}$$

In the $\overline{MS}$-scheme

$$\tilde{R}_S(M_H) = 3\overline{m}_b^2(M_H)\left[1 + \sum_{i=1}^{4} \Delta\Gamma_i a_s(M_H)^i\right] \tag{18}$$

The BKM expression with 1-loop coupling constant is

$$
\begin{aligned}
\widetilde{R}_{\rm S}^{\rm BKM} &= 3\,\hat{m}_b^2\,(a_s)^{\nu_0}\left[A_0^{\rm BKM}(a_s)+\sum_{n\geq 1} d_n\,A_n^{\rm BKM}(a_s)\right],\\
A_n^{\rm BKM}(a_s) &= \frac{4}{\beta_0\,\pi\,\delta_n}\left[1+\left(\frac{\beta_0\,\pi\,a_s}{4}\right)^2\right]^{-\delta_n/2}(a_s)^{n-1}\sin\left(\delta_n\arctan\left(\frac{b_0\,\pi\,a_s}{4}\right)\right),\\
\delta_n &= n+\nu_0-1\,,\quad \nu_0=2(\gamma_0/\beta_0).
\end{aligned}
$$

In the Fractional Analytic Perturbation Theory BMS obtained

$$
\widetilde{R}_{\rm S}^{(l){\rm BMS}} = 3\,\hat{m}_{(l)}^2\left[\mathfrak{a}_{\nu_0}^{(l)}+\sum_{n\geq 1}^{l} d_n\,\mathfrak{a}_{n+\nu_0}^{(l)}+\sum_{m\geq 1}^{l+4}\Delta_m^{(l)}\,\mathfrak{a}_{m+\nu_0}^{(l)}\right]
$$

The terms $\mathfrak{a}_{n+\nu_0}^{(l)}$ are summing β_0, γ_0 terms $(1\leq l\leq 4)$ and proportional to them π^2, higher orders in γ_i and β_i are accumulated in the coefficients $\Delta_m^{(l)}\ \mathfrak{a}_{n+\nu_0}^{(l)} = (a_s)^{\frac{2\gamma_0}{\beta_0}} A_n(a_s)$, the latter are rather closed to $A_n^{\rm BKM}(a_s)$. Next figure is from BMS paper.

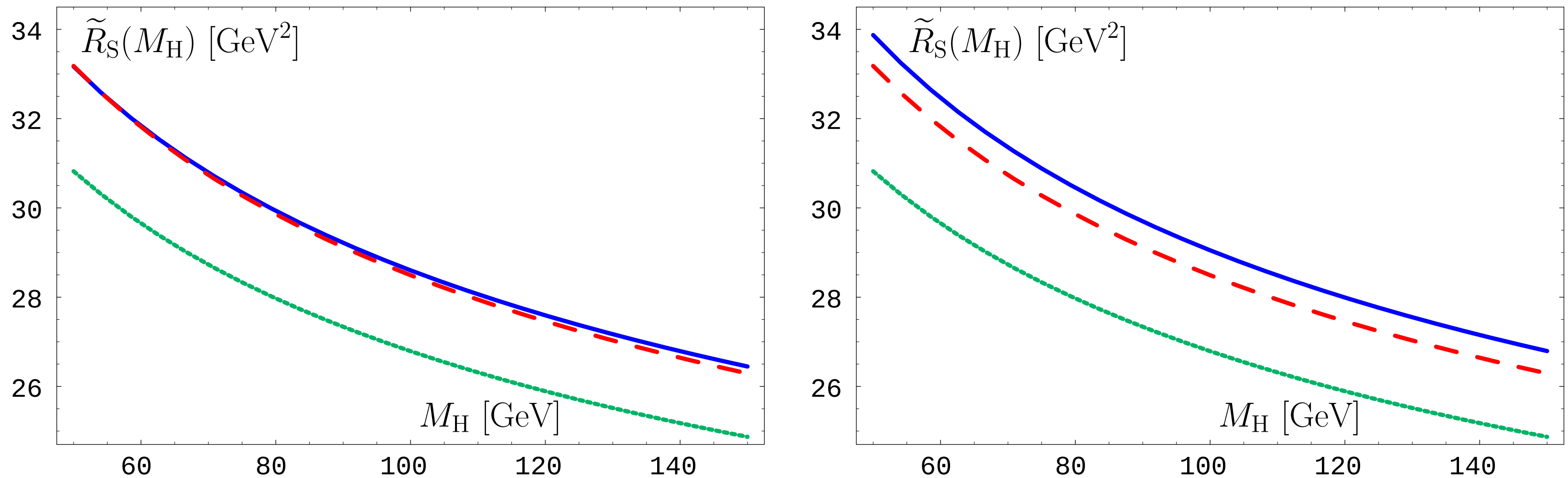
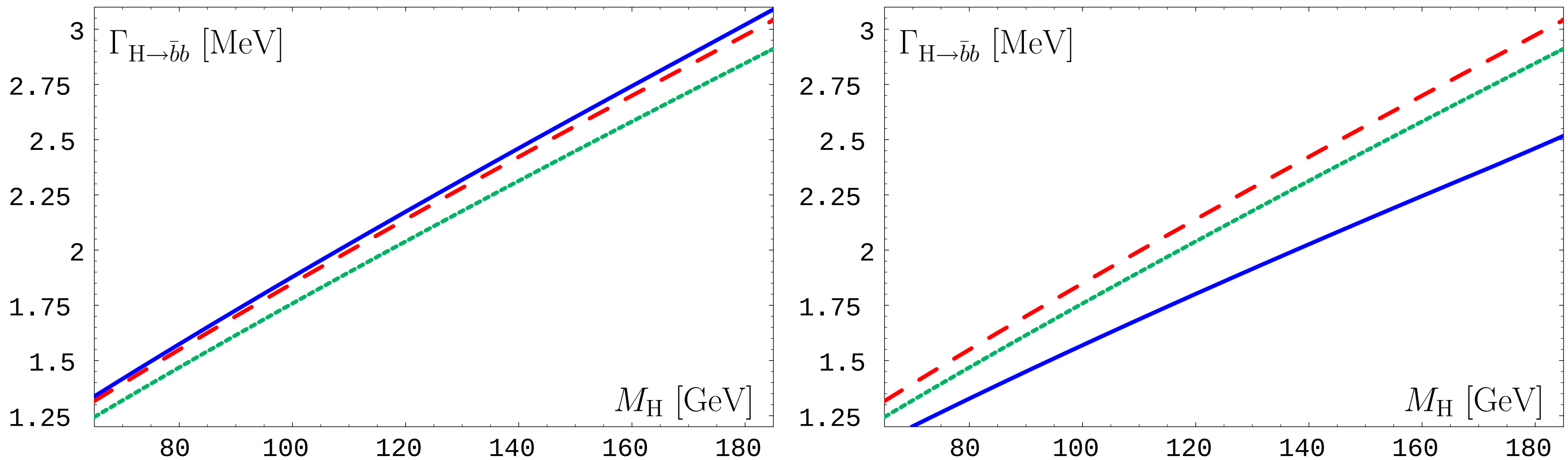


Illustration of the calculation of the perturbative series of the quantity $\widetilde{R}_{\rm S}(M_{\rm H}^2)$ in different approaches within the $\overline{MS}$ scheme: Standard perturbative QCD at the loop level $l = 4$ (dashed red line), BKM estimates, by taking into account the $O((a_s)^{\nu_0}\, A_4(a_s))$-terms, —(dotted green line), and finally MFAPT from for $N_f = 5$ (solid blue line), displayed for $l = 2$ (left panel) and $l = 3$ (right panel). These figures are from BMS (07) Erratum (08).

The transformation of these results into $\Gamma_{H\bar{b}b} = \frac{\sqrt{2}G_F}{8\pi} M_H \widetilde{R}_{\rm S}(M_H)$ was made by **Bakulev**- Contributed to Quarks-2008 Sergiev Posad, May 2008



Results of calculations of $\Gamma_{\rm H\to\bar{b}b}(M_{\rm H}^2)$ in different approaches. On both panels dashed (red) lines display the BCK results and dotted (green) lines — the BKM results. Solid line on the left panel corresponds to the results obtained in the variant of FAPT approach whereas on the right panel — to the results of the complete FAPT approach.

Theoretical uncertainties of $\Gamma_{H\bar{b}b}$ with resummed π^2 terms- using Bakulev studies Values of mass parameters and Λ a bit different! For $M_H = 120\ GeV$ α_s^4 approximtion $\Gamma_{H\bar{b}b}^{RG\ \bar{M}S} \approx 2 \times$ MeV (we got 1.75 MeV) $\Gamma_{H\bar{b}b}^{BKM} \approx (1.75\ -\ 1.85) \times$ MeV (depending on the values of parameters used). $\Gamma_{H\bar{b}b}^{FAPT} \approx (2\ -\ 1.65) \times$ MeV

Maximal difference $\Delta\Gamma_{H\bar{b}b} \approx 0.35 \times$ MeV (resummed is lower!)
Our estimate is 0.25 MeV (on-shell is higher).

$\overline{MS}$ prediction is in the corridor $\Delta\Gamma_{Hb\bar{b}} \approx \pm 0.3$ MeV. **Conclusions**

- The results of different analysis and resummation of $O(\alpha_s)$-effects are considerd (including resummed π^2-terms in higher orders of α_s).
- The estimate of theoretical precision of $\Gamma_{H\bar{b}b}$ is proposed. More detailed comparison may be useful in future.
- Matching with Higgs production should be done with care (number of calculated terms should be taken into account)