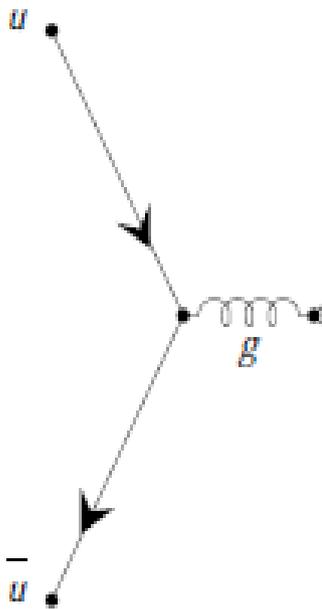
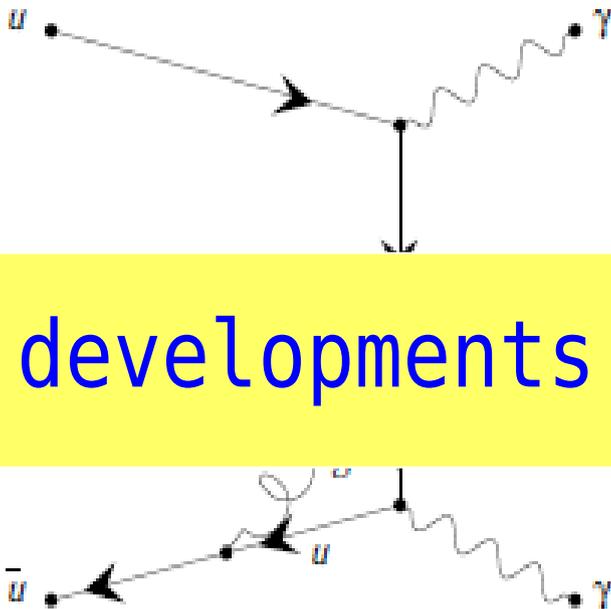


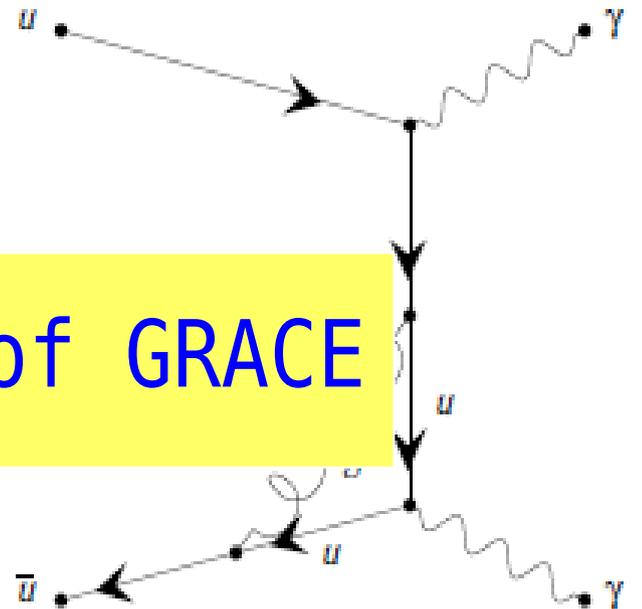
Graph 12



Graph 13

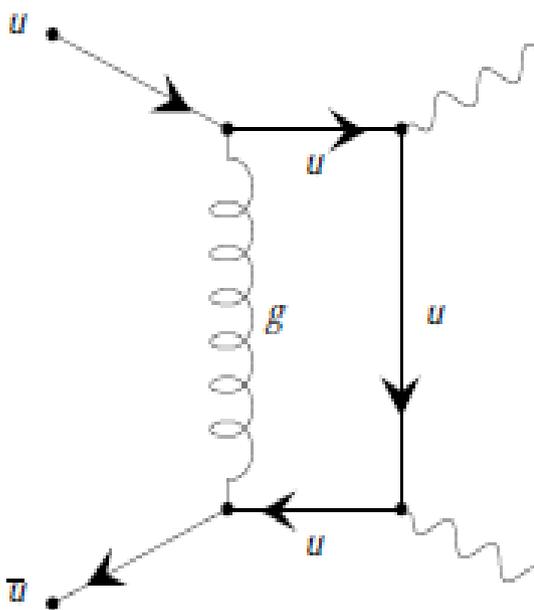


Graph 14

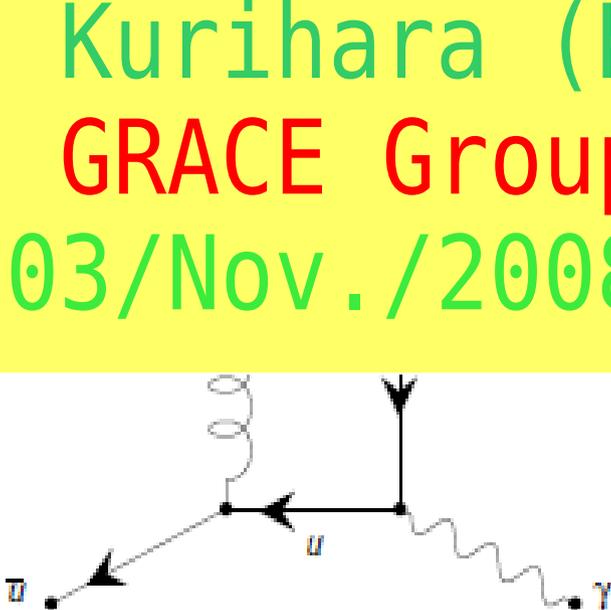


Recent developments of GRACE

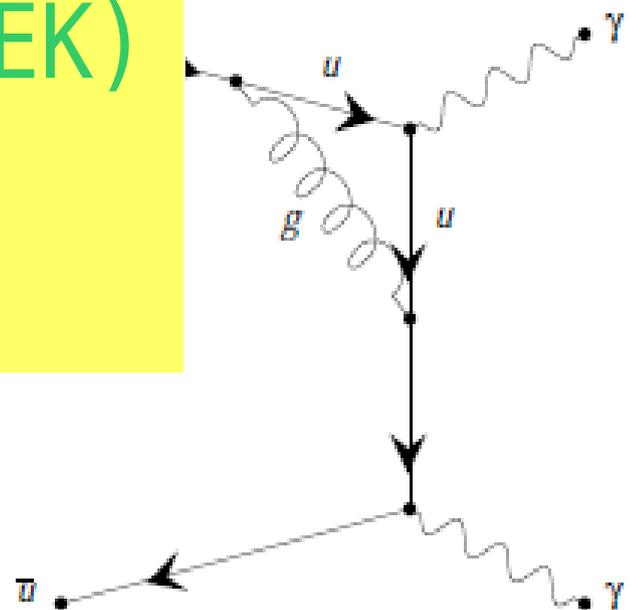
Graph 15



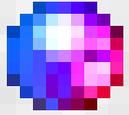
Graph 16



Graph 17

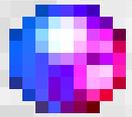


Y. Kurihara (KEK)  
GRACE Group  
03/Nov./2008



# Outline

- Introduction
- Overview of GRACE
- GRACE/SUSY
- GRACE/NLO-QCD
- GR@PPA-NLO
- Summary



# Introduction

- Automatic System is very important
  - ★ May processes
  - ★ Many particles
  - ★ Many loop
- Automatic Calculation System in the world

GRACE

Prog. Theor. Phys. Suppl. 138 (2000) 18

Comput. Phys. Commun. 153 (2003) 106

CompHEP

Nucl. Instrum. Meth. A534 (2004) 250

CalcHEP

hep-ph/0412191

FeynArts/FormCalc

Comput. Phys. Commun. 140 (2001) 418;

143 (2002) 54

MadGraph

JHEP 0302 (2003) 027

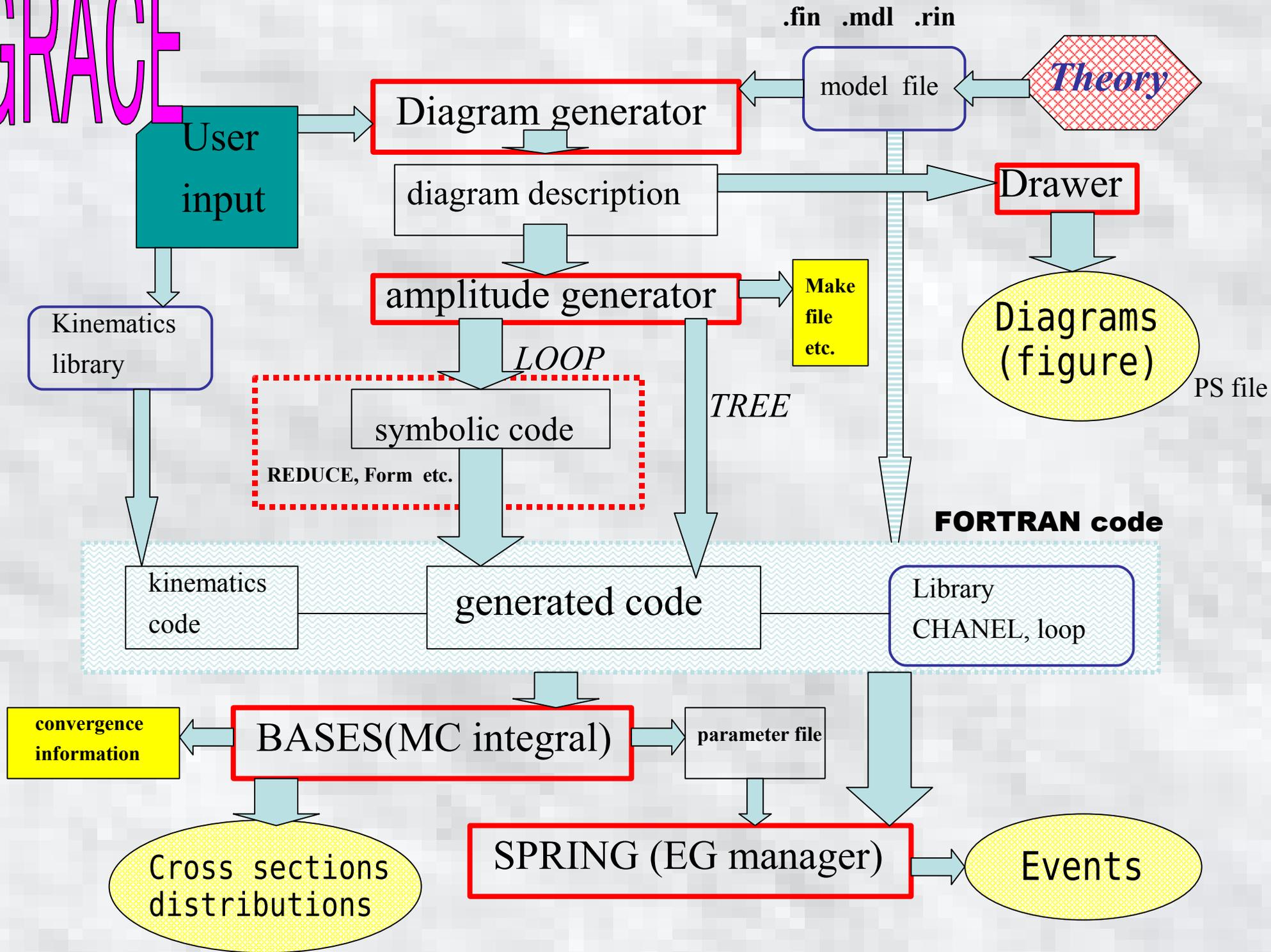


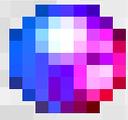
# What can be done by GRACE

- ✓ Generate Feynman diagrams automatically
- ✓ Generate FORTRAN code of helicity amplitudes automatically (@ tree level)
- ✓ Generate symbolic (REDUCE, Form) source code for generating FORTRAN code of amplitudes automatically (@ one-loop level)
- ✓ Calculate phase-space integration of amplitudes numerically

- Event generation in framework of GR@PPA

# GRACE





# GRACE for SUSY Processes

- Tree Level System: GRACE/SUSY

M. Kuroda, Complete Lagrangian of MSSM, hep-ph/9902340

J. Fujimoto et al., Comput. Phys. Commun. 153 (2003) 106

- 1-Loop Level System: GRACE/SUSY/loop

Renormalization scheme;

Physical results of chargino production and decay;

J. Fujimoto et al., Phys. Rev. D75 (2007) 113002

# Non-linear Gauge Fixing

$$F_{W^\pm} = (\partial_\mu \pm ie\tilde{\alpha}A_\mu \pm igc_W\tilde{\beta}Z_\mu)W^{\pm\mu} \\ \pm i\xi_W^{\pm} \frac{g}{2} (v + \tilde{\delta}_h h^0 + \tilde{\delta}_H H^0 \pm i\tilde{\kappa}G^0)G^\pm$$

$$F_Z = \partial_W Z^\mu + \xi_Z \frac{g_Z}{2} (v + \tilde{\epsilon}_h h^0 + \tilde{\epsilon}_H H^0)G^0$$

$$F_\gamma = \partial_\mu A^\mu$$

$(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}_h, \tilde{\delta}_H, \tilde{\kappa}, \tilde{\epsilon}_h, \tilde{\epsilon}_H)$ : NLG parameters

# Numerical Check of the Amplitude

$$e^- e^+ \rightarrow \tilde{\chi}_1^0 \tilde{\chi}_2^0$$

For  $\sigma[(\text{Full loop-diagrams}) \times (\text{A tree-diagram})]$

Gauge invariance check (NLG)

31 digits

- $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}_n, \tilde{\delta}_H, \tilde{\kappa}, \tilde{\epsilon}_n, \tilde{\epsilon}_H) = (0, 0, 0, 0, 0, 0, 0)$

$$\sigma(\text{Virtual}) = -1.9729879385817752800946250998469395\text{E-}02$$

- $(\tilde{\alpha}, \tilde{\beta}, \tilde{\delta}_n, \tilde{\delta}_H, \tilde{\kappa}, \tilde{\epsilon}_n, \tilde{\epsilon}_H) = (1, 2, 3, 4, 5, 6, 7)$

$$\sigma(\text{Virtual}) = -1.9729879385817752800946250998462461\text{E-}02$$

Ultraviolet check (CUV)

22 digits

- $\text{CUV} = 0$

$$\sigma(\text{Virtual}) = -1.9729879385817752800946250998469395\text{E-}02$$

- $\text{CUV} = 1000$

$$\sigma(\text{Virtual}) = -1.9729879385817752800947890626998724\text{E-}02$$

# Infra-red finiteness check

## Fictitious photon mass ( $\lambda$ )

- $\lambda = 1.0\text{E-}24$

$$\sigma(\text{Virtual}+\text{Soft}) = -3.95636186320988\text{E-}2$$

- $\lambda = 1.0\text{E-}27$

$$\sigma(\text{Virtual}+\text{Soft}) = -3.95636186322991\text{E-}2$$

## Cut off photon energy ( $\kappa_C$ )

- $\kappa_C = 1.0\text{E-}3$

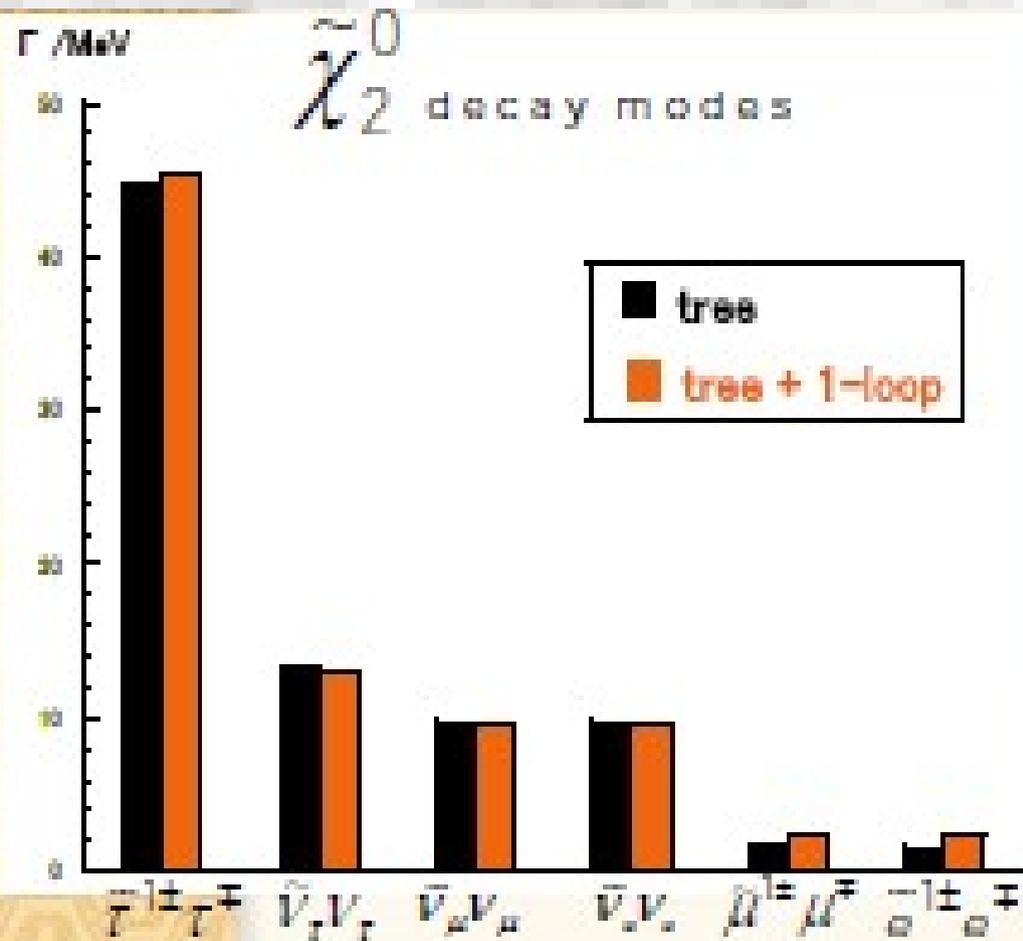
[Hard photon: MC]

$$(\text{Virtual}+\text{Soft}+\text{Hard}) = 2.97876\text{E-}2$$

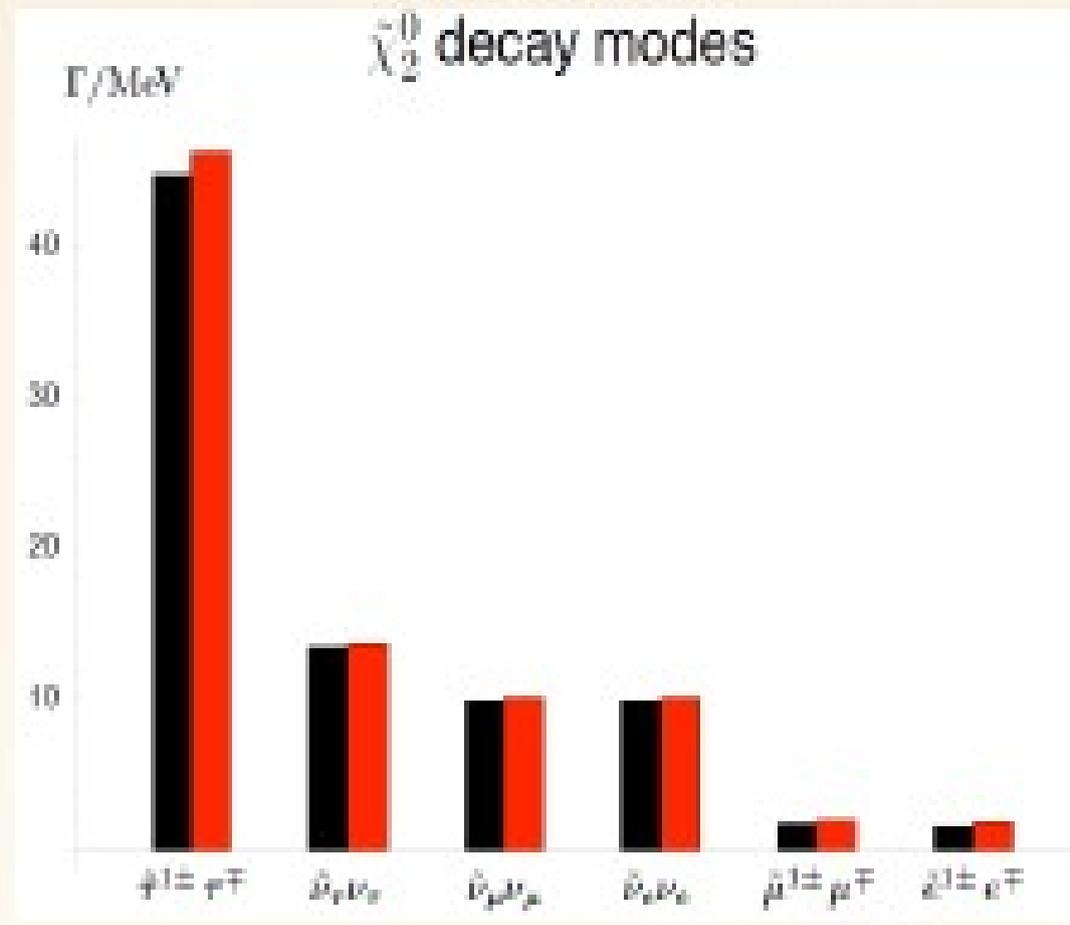
- $\kappa_C = 1.0\text{E-}1$

$$(\text{Virtual}+\text{Soft}+\text{Hard}) = 2.97886\text{E-}2$$

# Comparison with FeynArt/FeynCalc



GRACE/SUSY

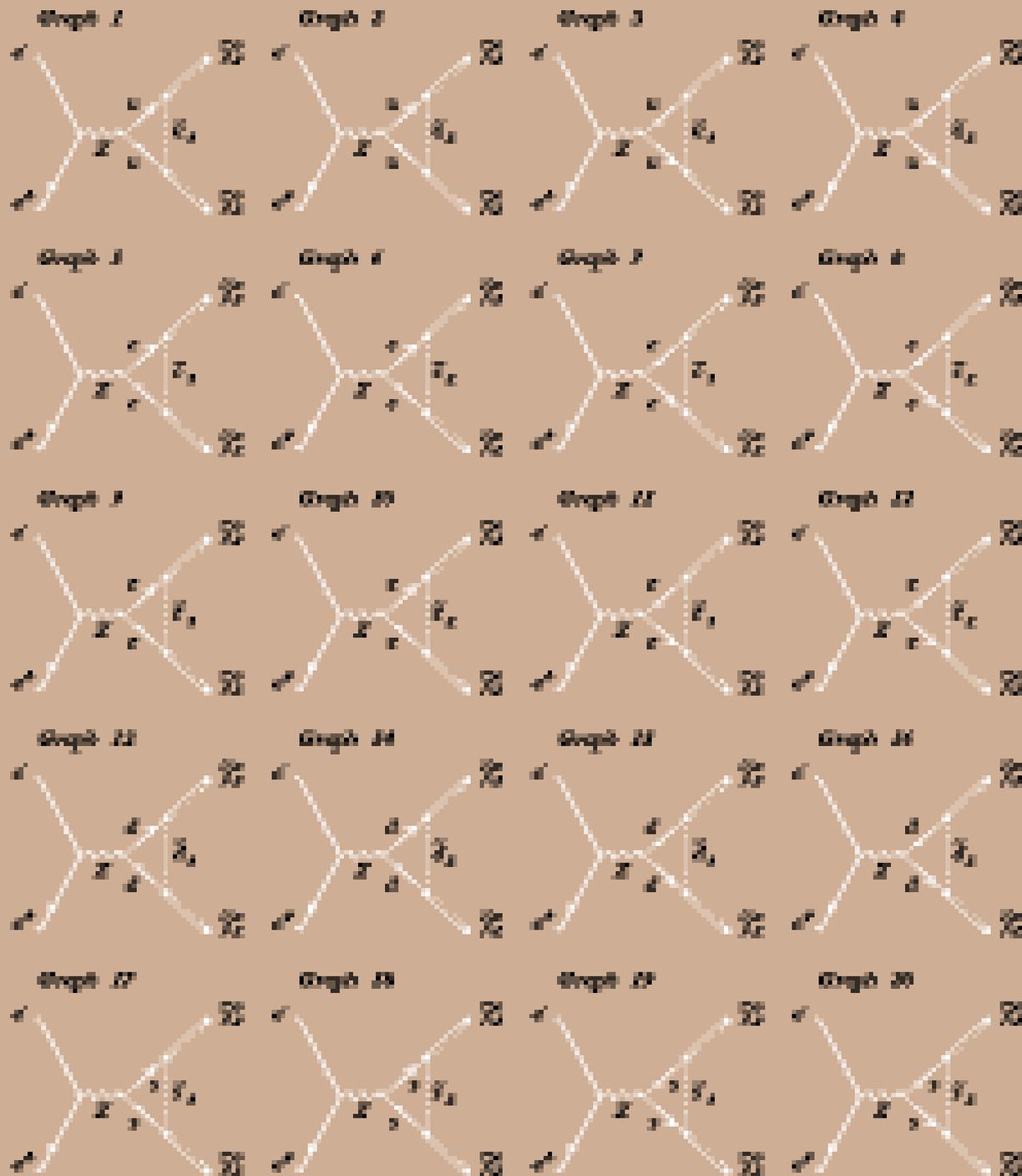


FeynArt/FeynCalc



$$e^+e^- \rightarrow \chi^0_1 \chi^0_2$$

3554 loop diagrams  
9 tree diagrams

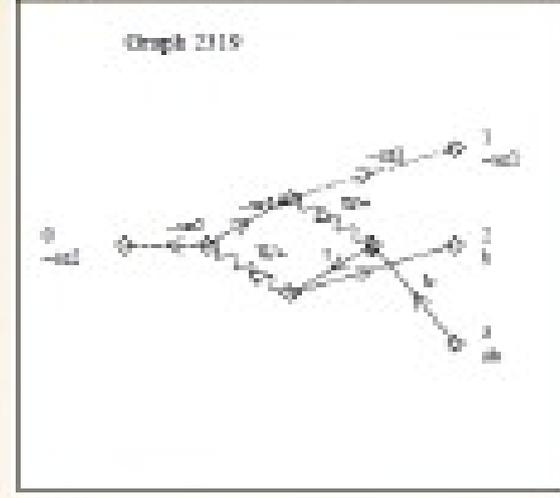
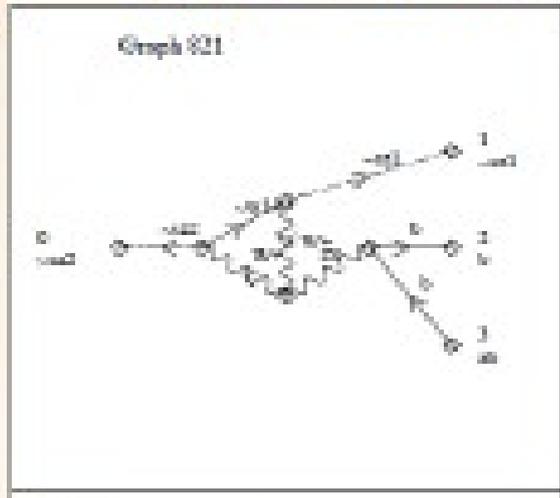
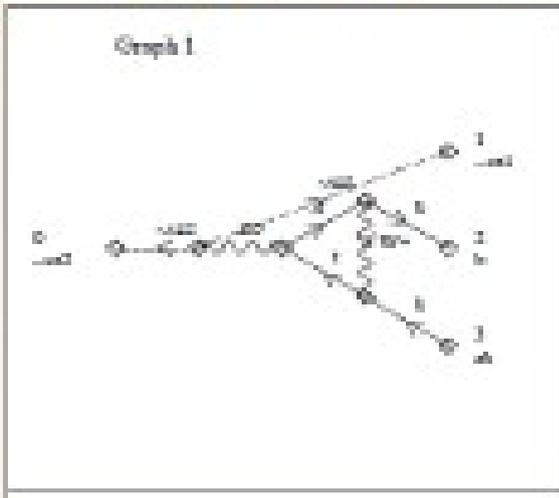




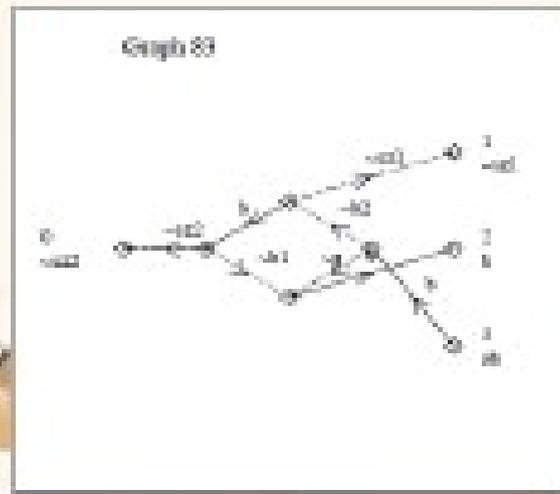
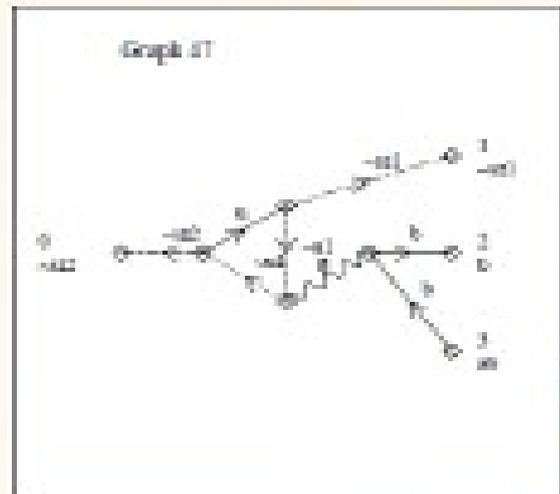
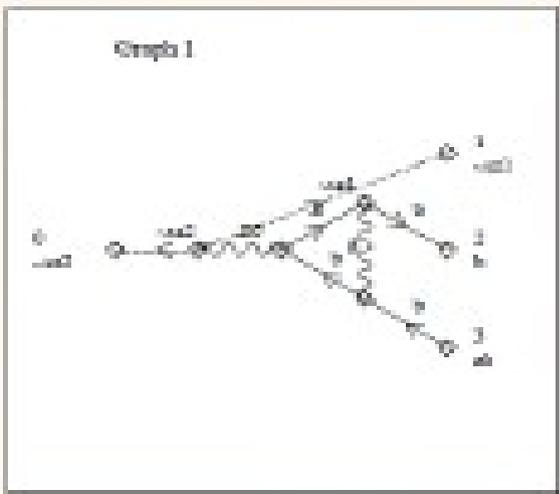
# Three body decay diagrams

$$\tilde{\chi}_2^0 \rightarrow \tilde{\chi}_1^0 b \bar{b}$$

## ELWK One-loop (3716 diagrams)

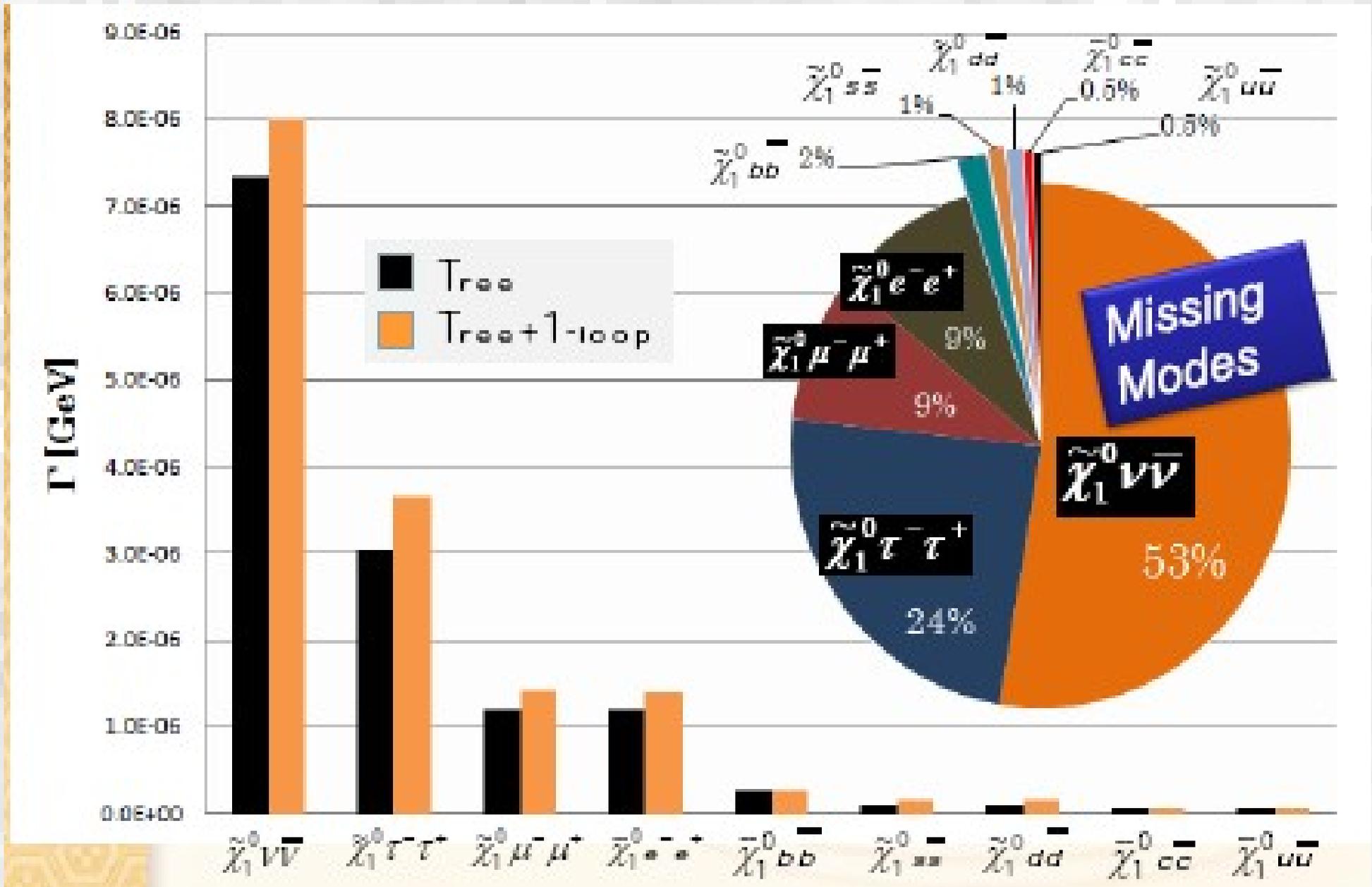


## QCD One-loop (126 diagrams)



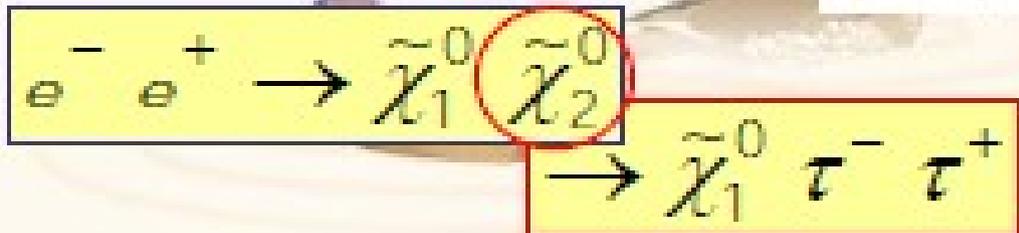
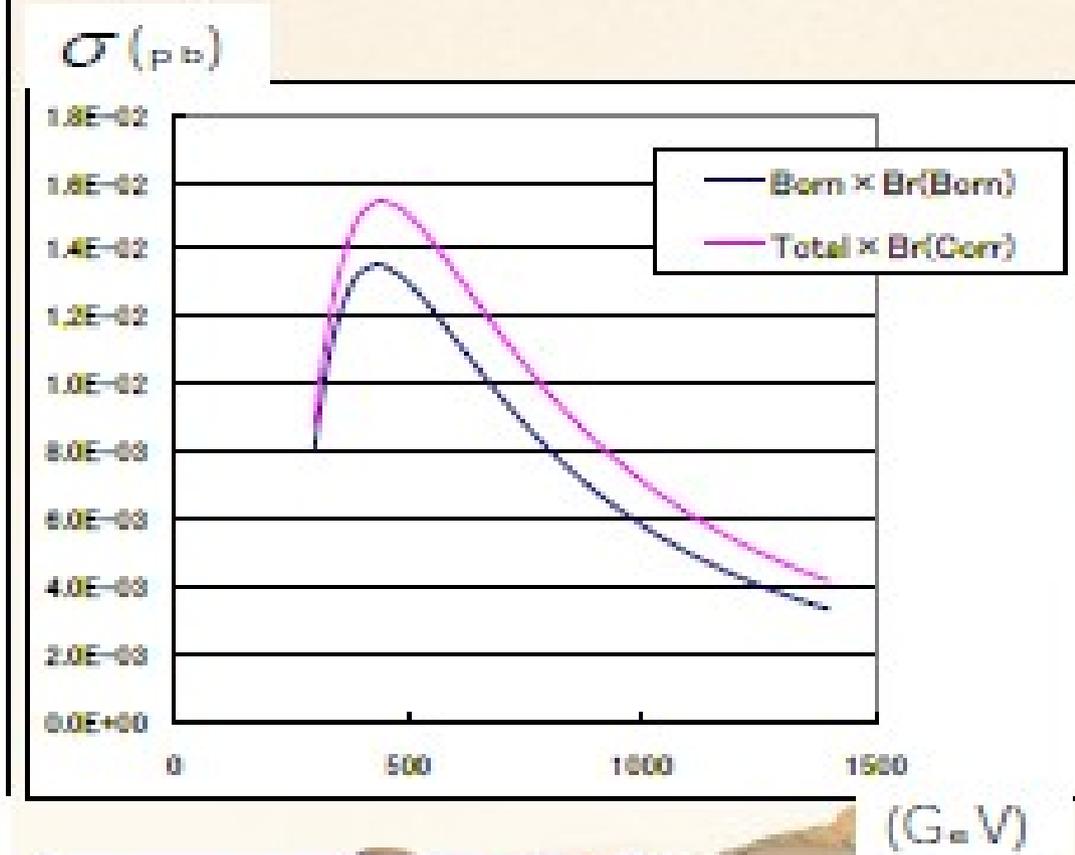
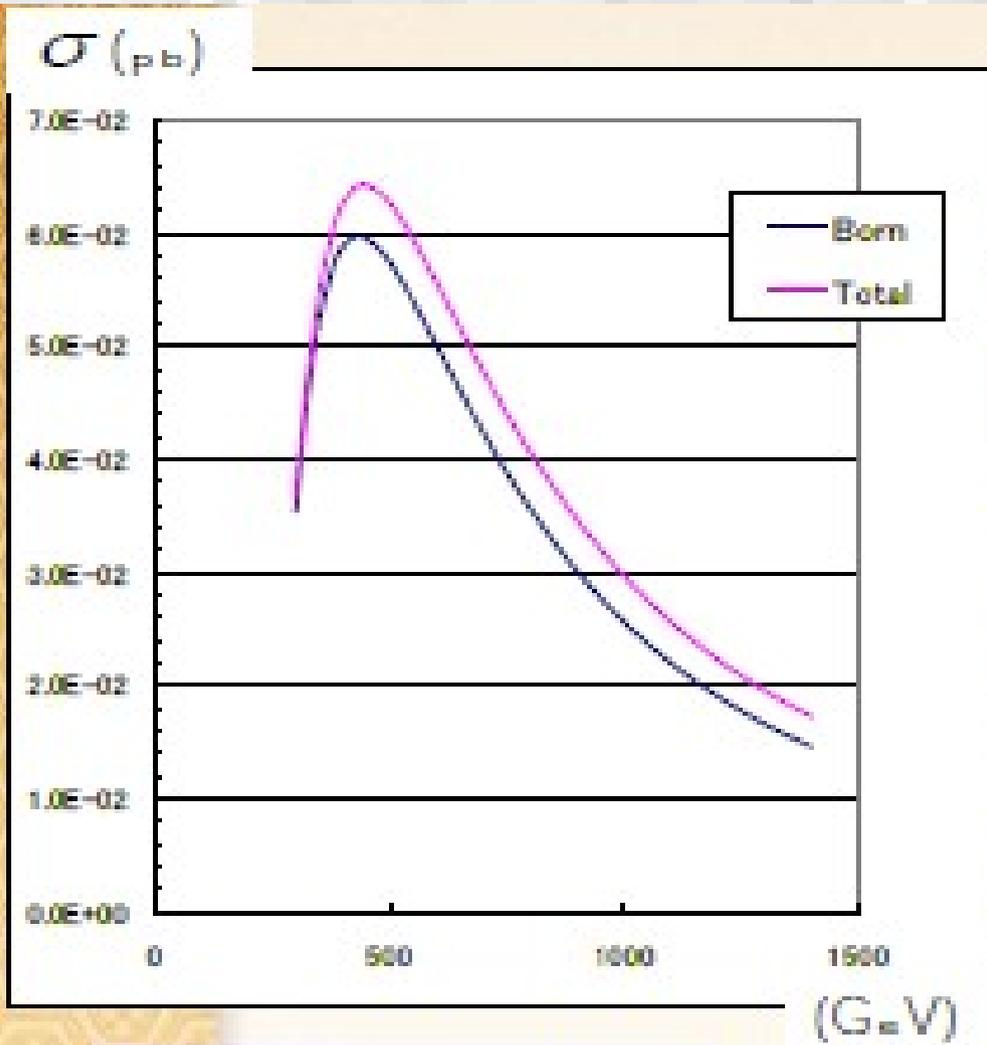


# Three body decay of $\chi^0_2$



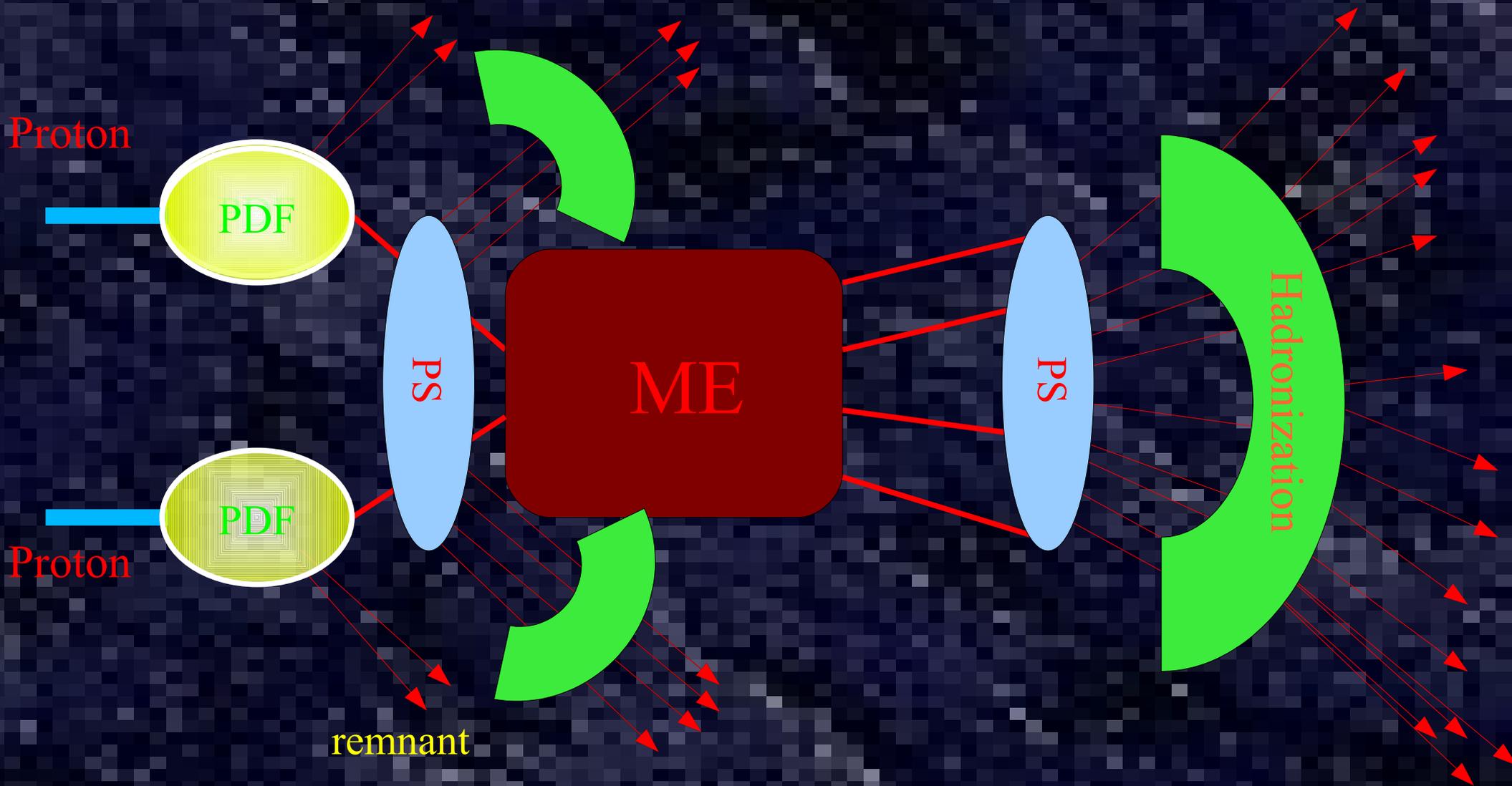


# Production and decay of $\chi^0_2$





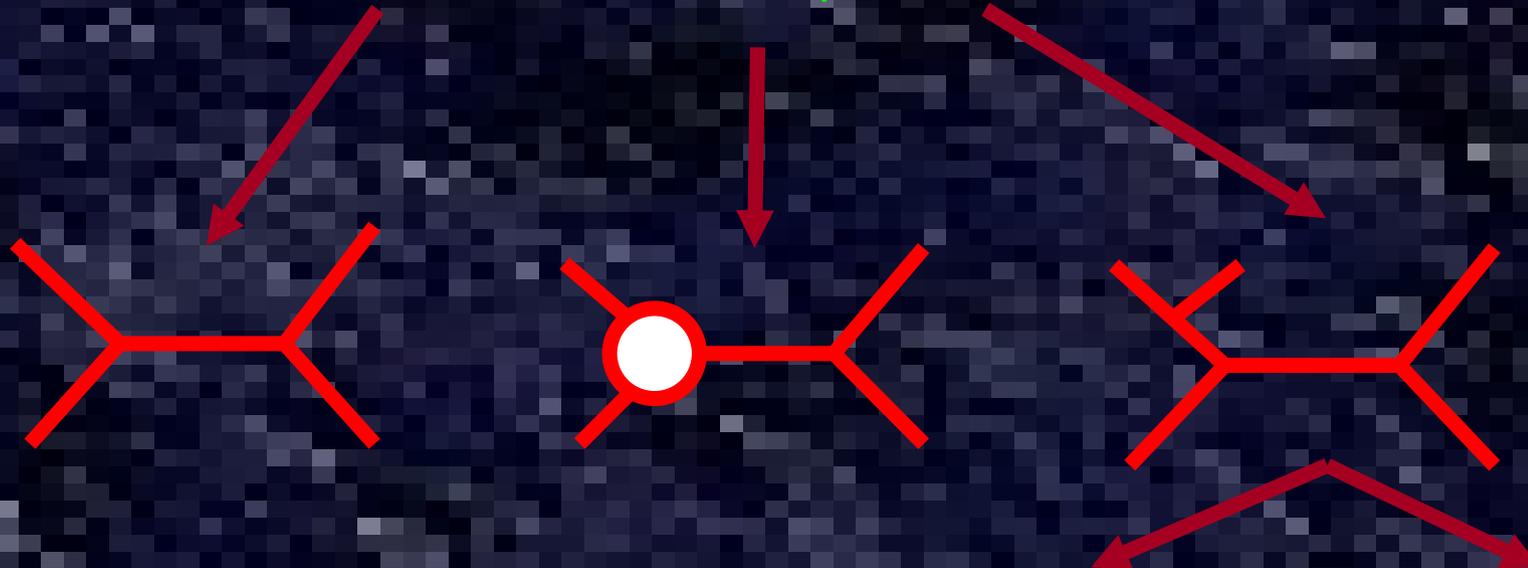
# Structure of Proton-Proton Scattering





# NLO Cross sections

$$\sigma_{\text{NLO}} = \sigma_{\text{tree}} + \sigma_{\text{loop}} + \sigma_{\text{R}}$$



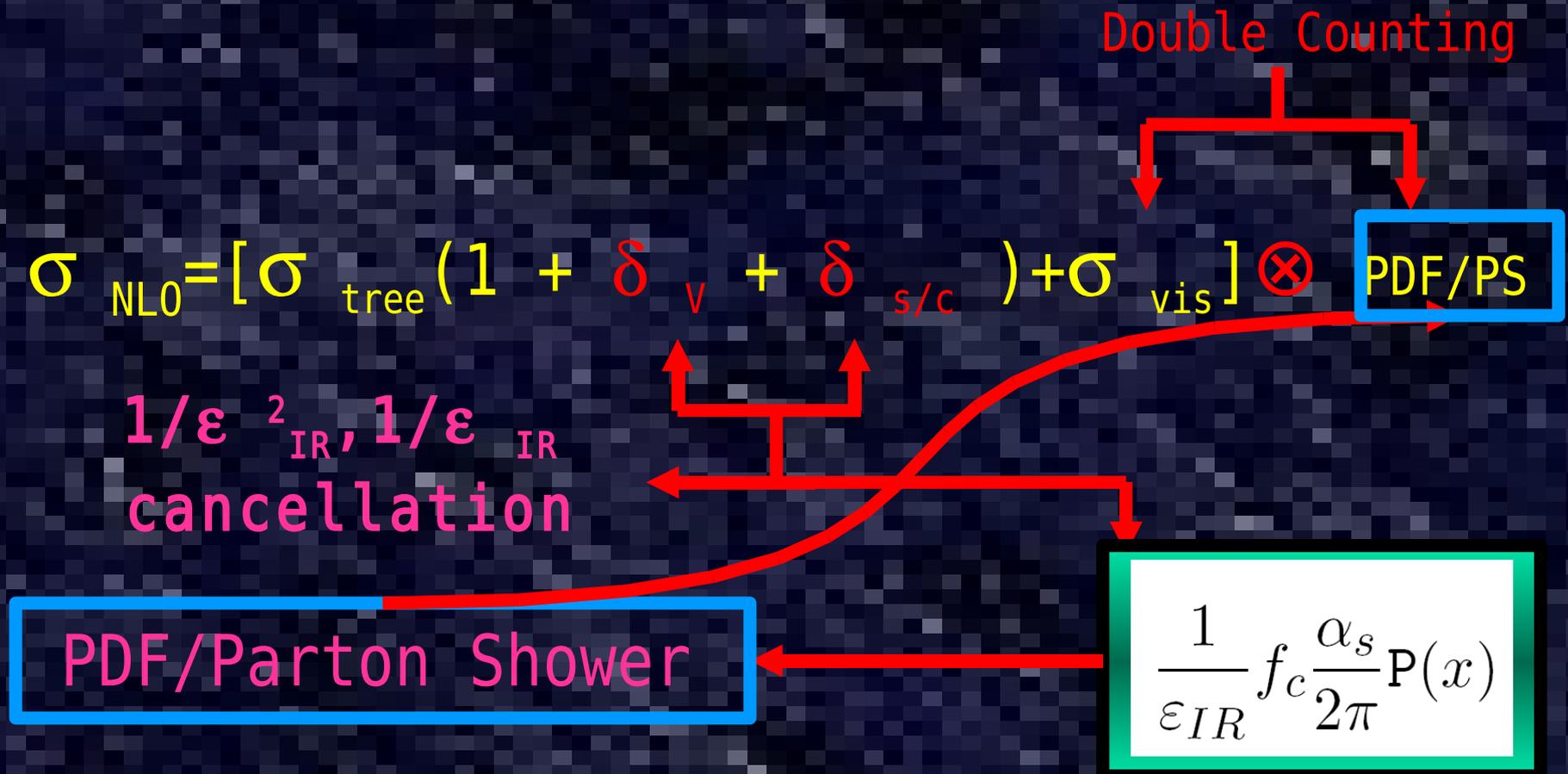
$$= \sigma_{\text{tree}} (1 + \delta_{\text{V}} + \delta_{\text{s/c}}) + \sigma_{\text{vis}}$$

$\delta_{\text{V}}$  : Virtual (loop) correction

$\delta_{\text{s/c}}$  : Soft/Collinear correction

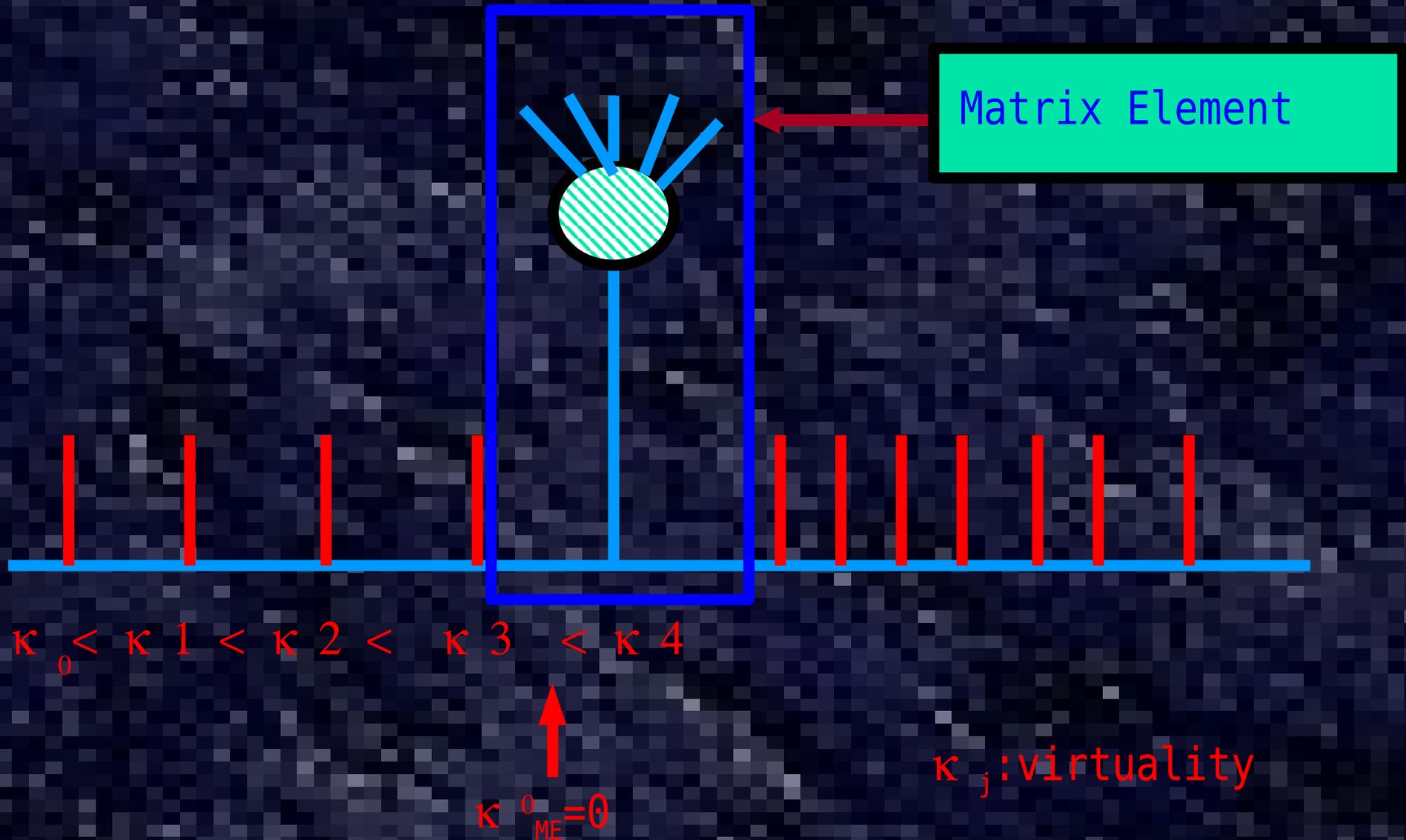
# Double Counting 1<sup>st</sup> category

No colored parton in final state at tree level.

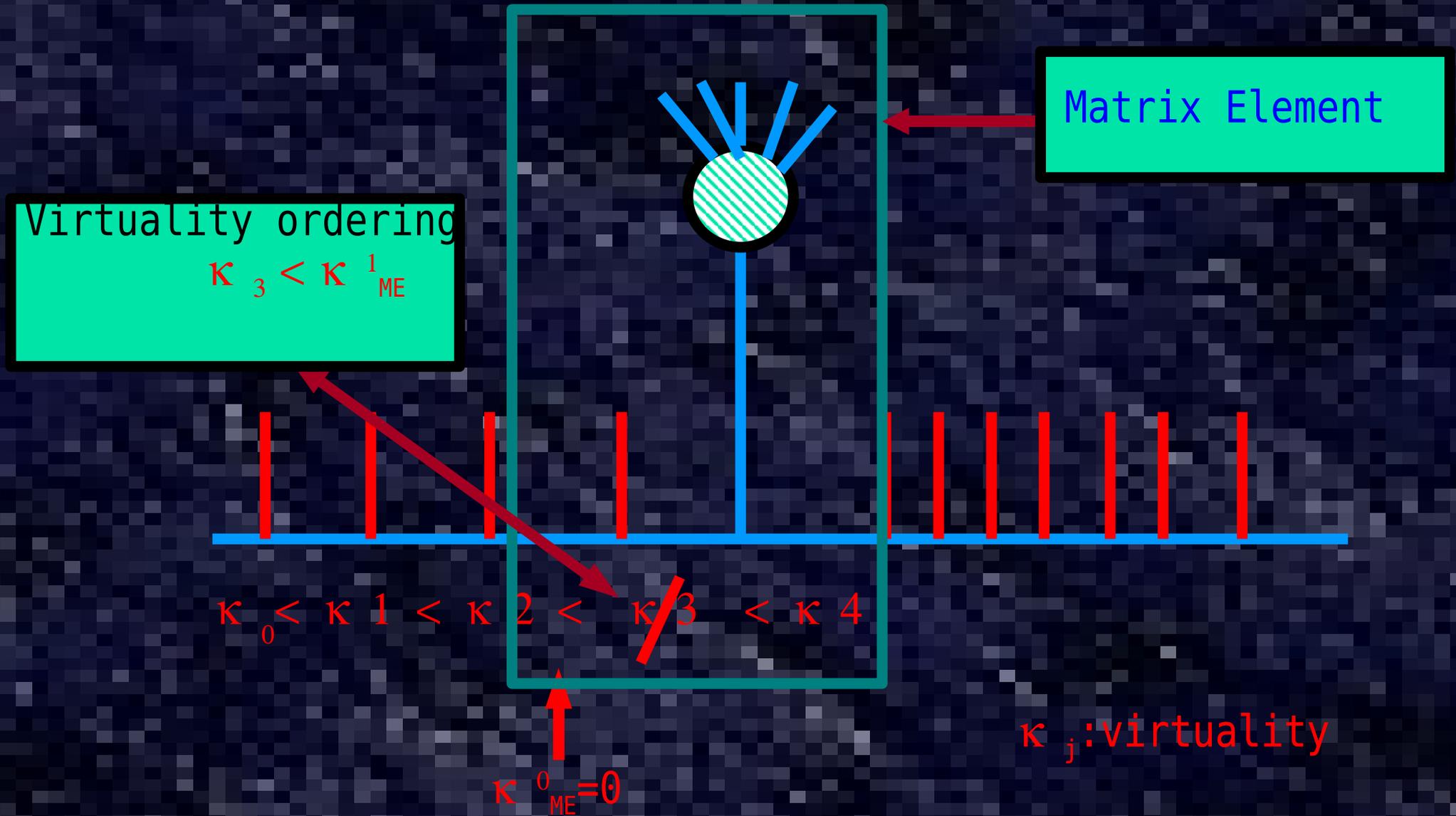


$P(x)$  : Splitting function      Space/time dimension :  $d=4+2\epsilon_{\text{IR}}$

# Phase space limitation

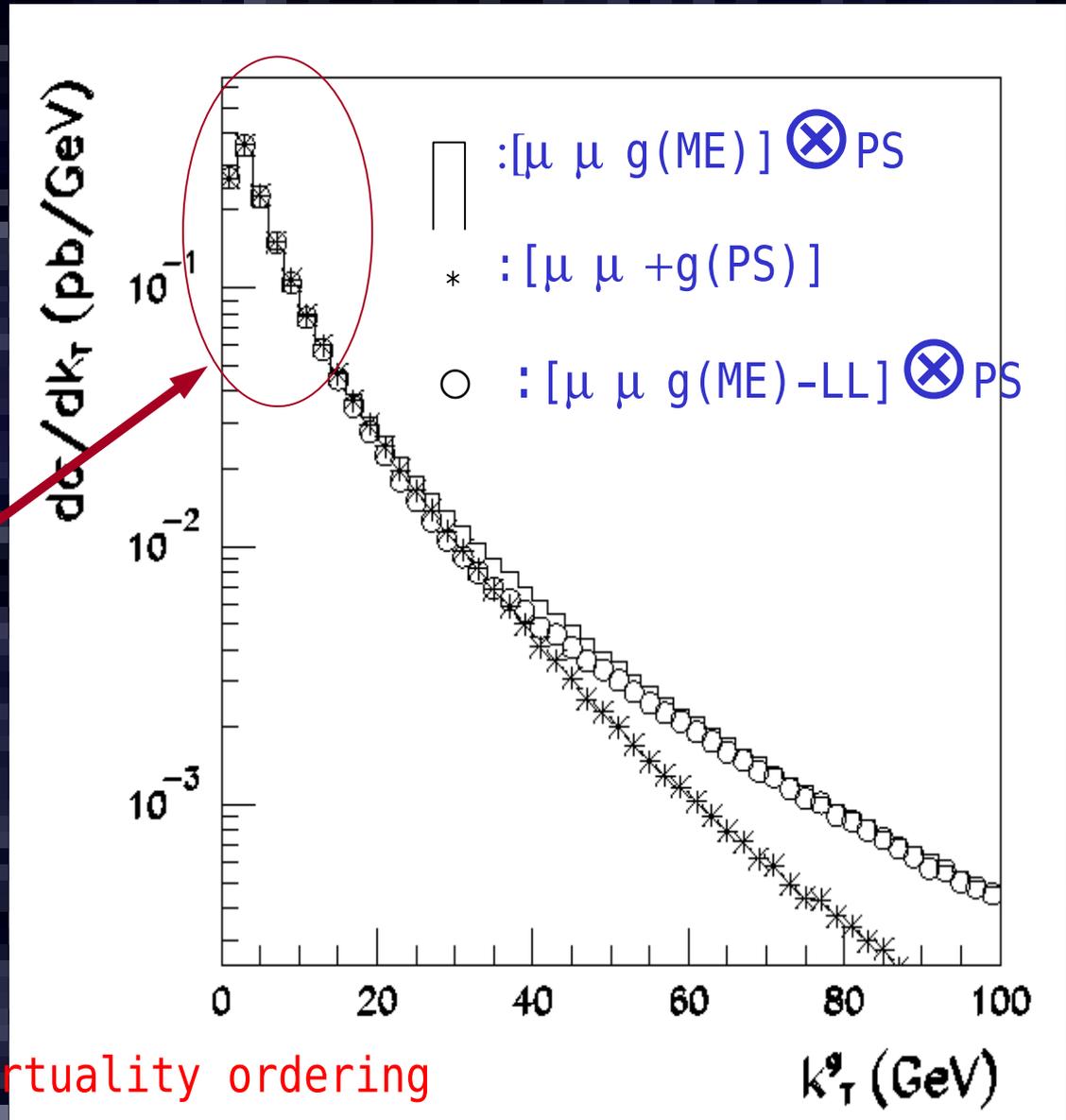
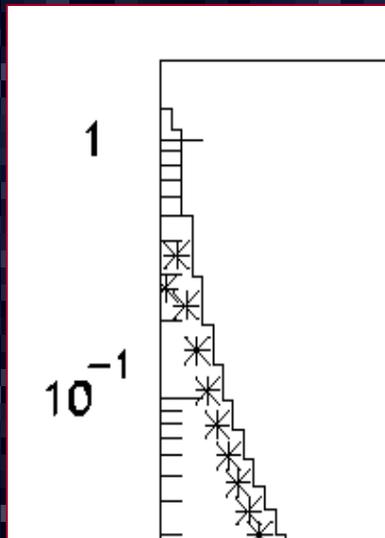


# Phase space limitation



# Test for phase space limitation

- Process :  
 $uu \rightarrow \mu^+ \mu^- (+\text{gluon})$   
 in pp collision
- Cuts:  
 $\sqrt{s_{\mu\mu}} > 40 \text{ GeV}$   
 $k_T^g > 1 \text{ GeV}$



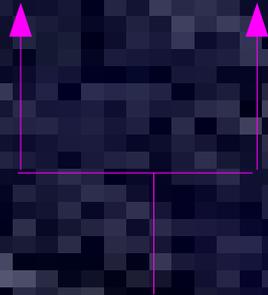
# Double Counting 2<sup>nd</sup> category

$$\sigma_{\text{NLO}} = [\sigma_{\text{tree}} (1 + \delta_{\text{V}} + \delta_{\text{s/c}}) + \sigma_{\text{vis}}] \otimes \text{PDF/PS}$$

Tree level cross section

$$\rightarrow [\sigma_{\text{tree}} + \sigma_{\text{vis}}] \otimes \text{PDF/PS}$$

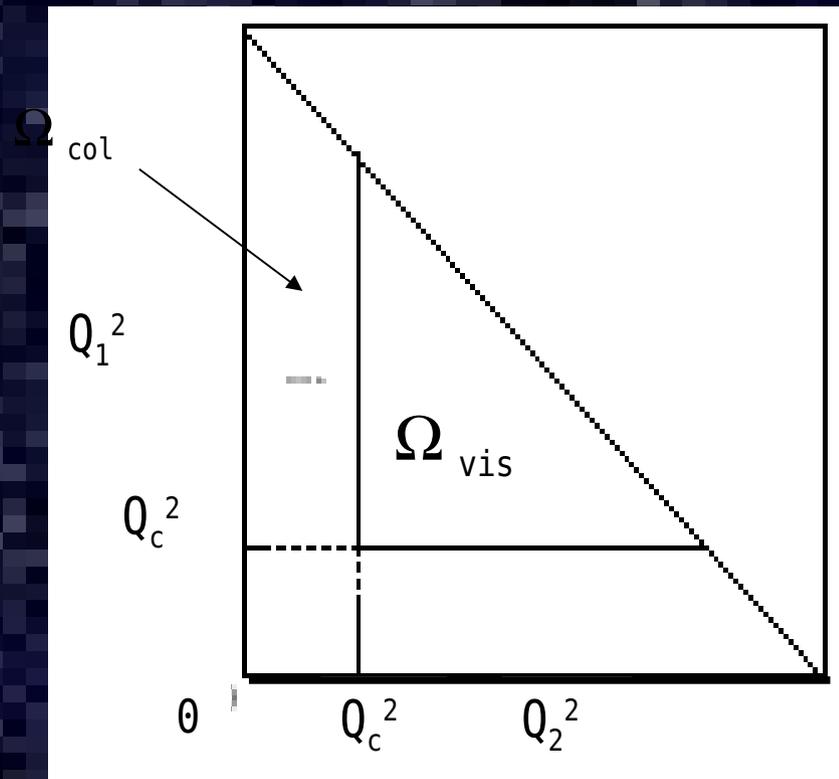
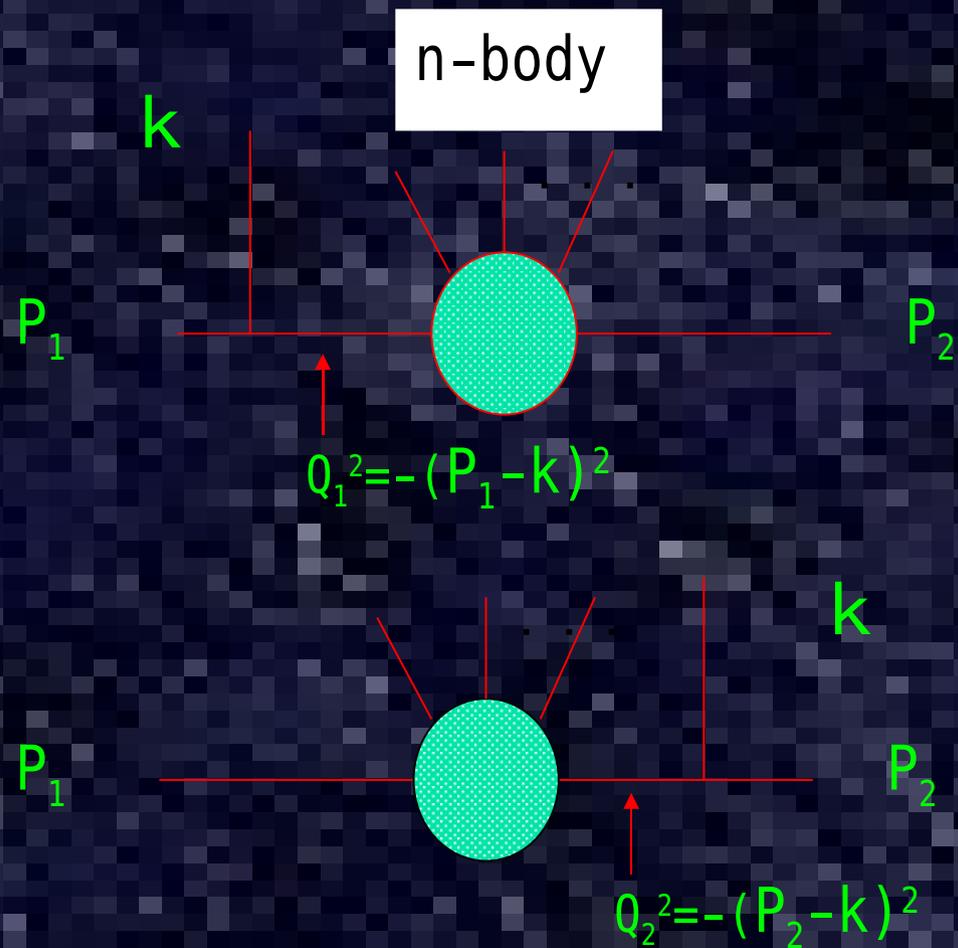
$$= \sigma_{\text{tree}} \otimes \text{PDF/PS} + \sigma_{\text{vis}} \otimes \text{PDF/PS}$$



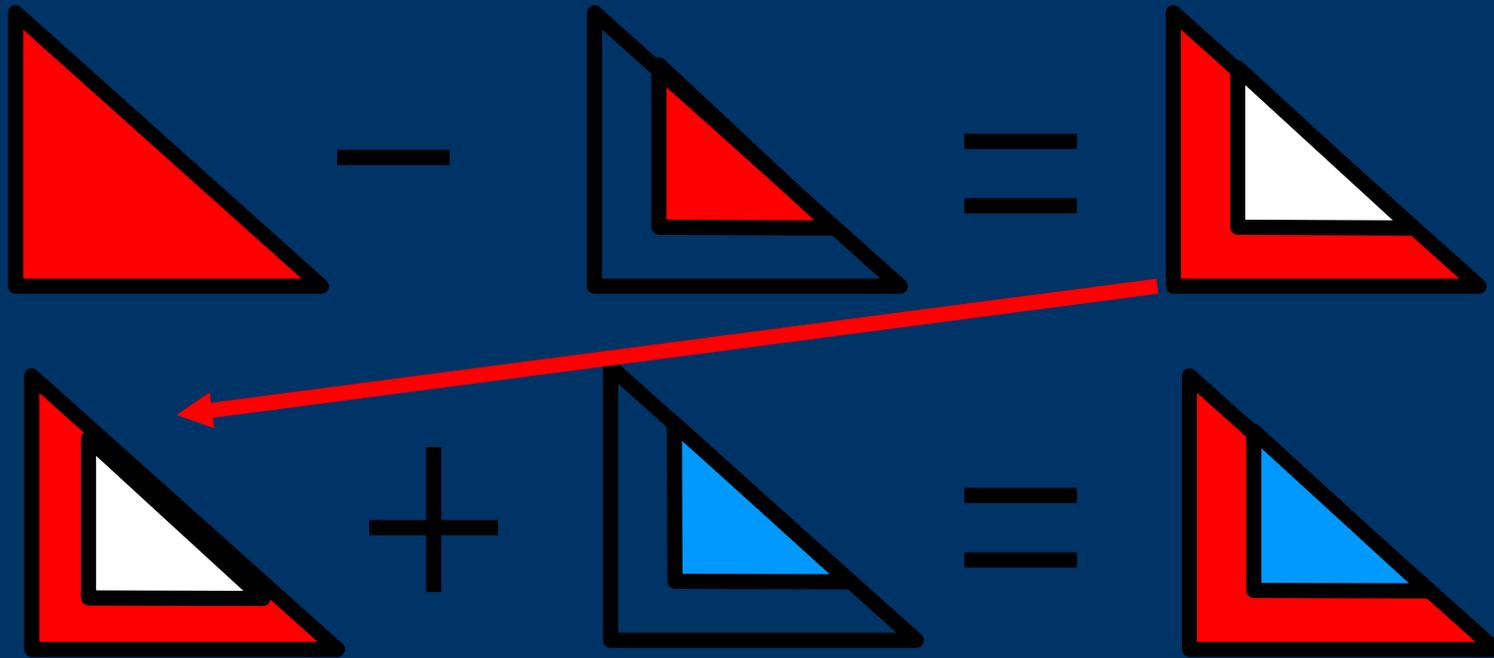
Double Counting



# Subtraction method (MC@NLO, GRACE)

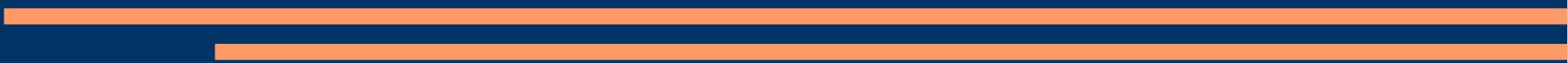


# Phase space slicing

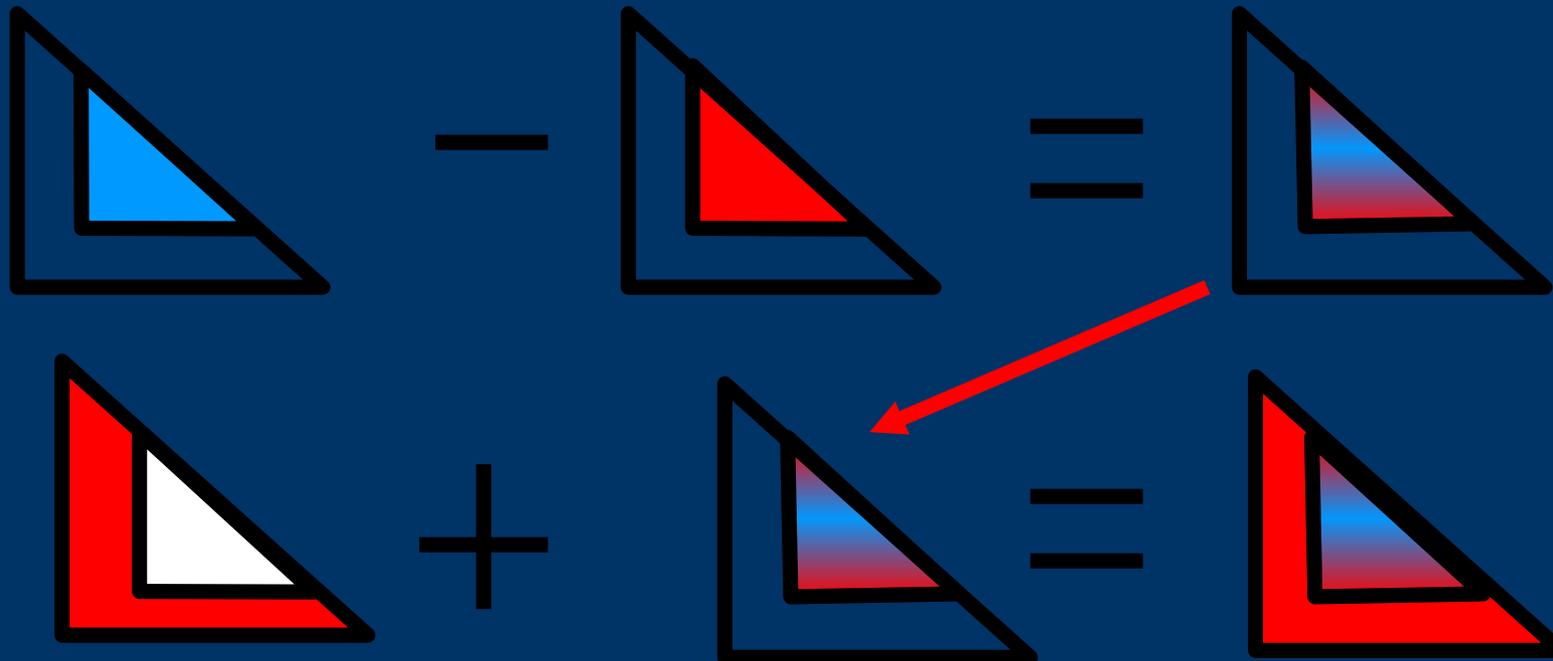


● : Collinear (Leading Log) Approx.

● : Exact Matrix Elements

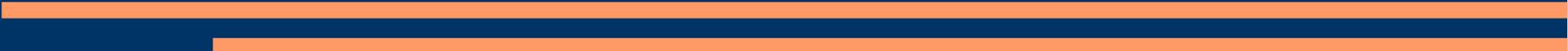


# Limited Leading Log Subtraction



● : Collinear (Leading Log) Approx.

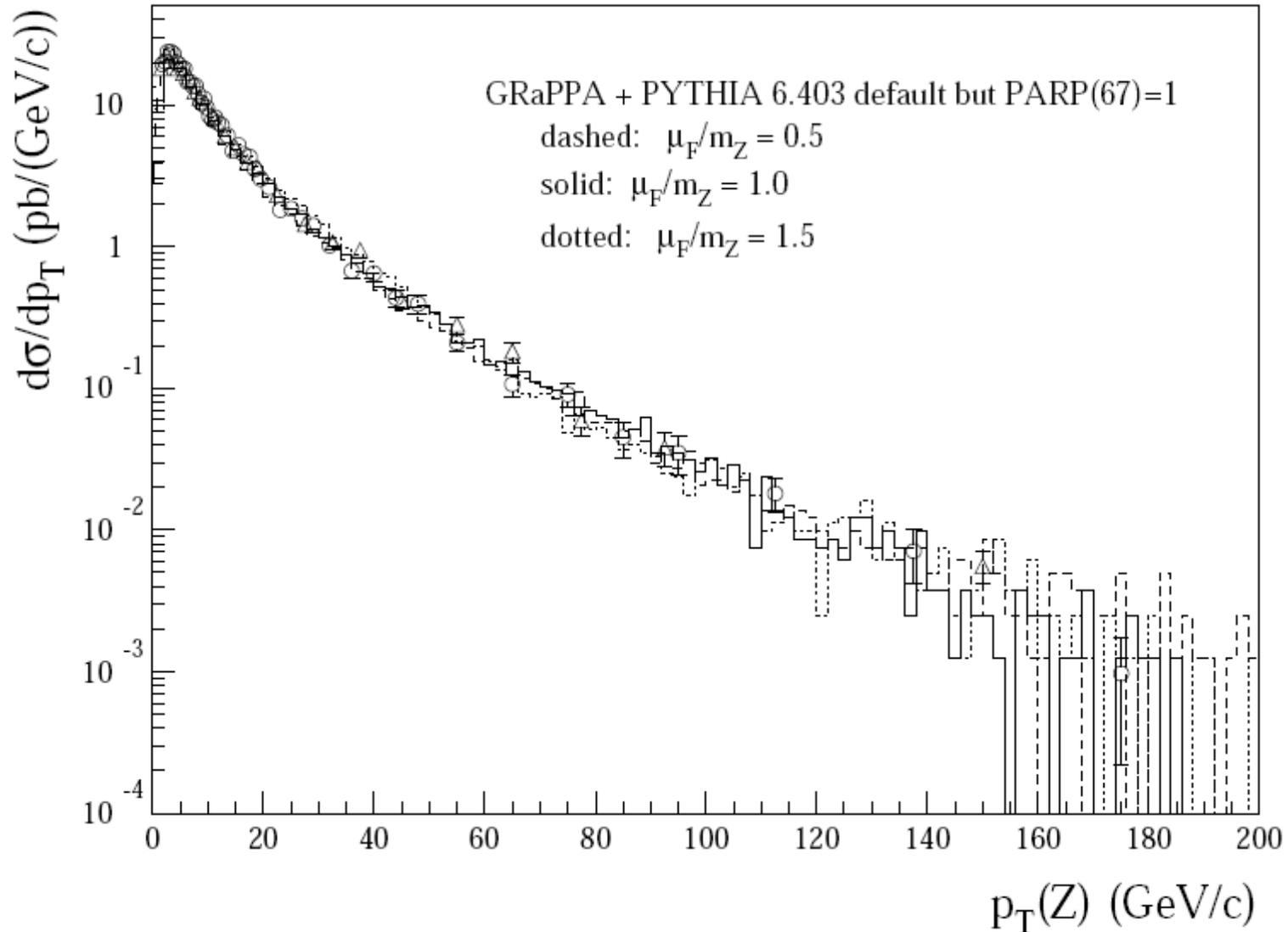
● : Exact Matrix Elements



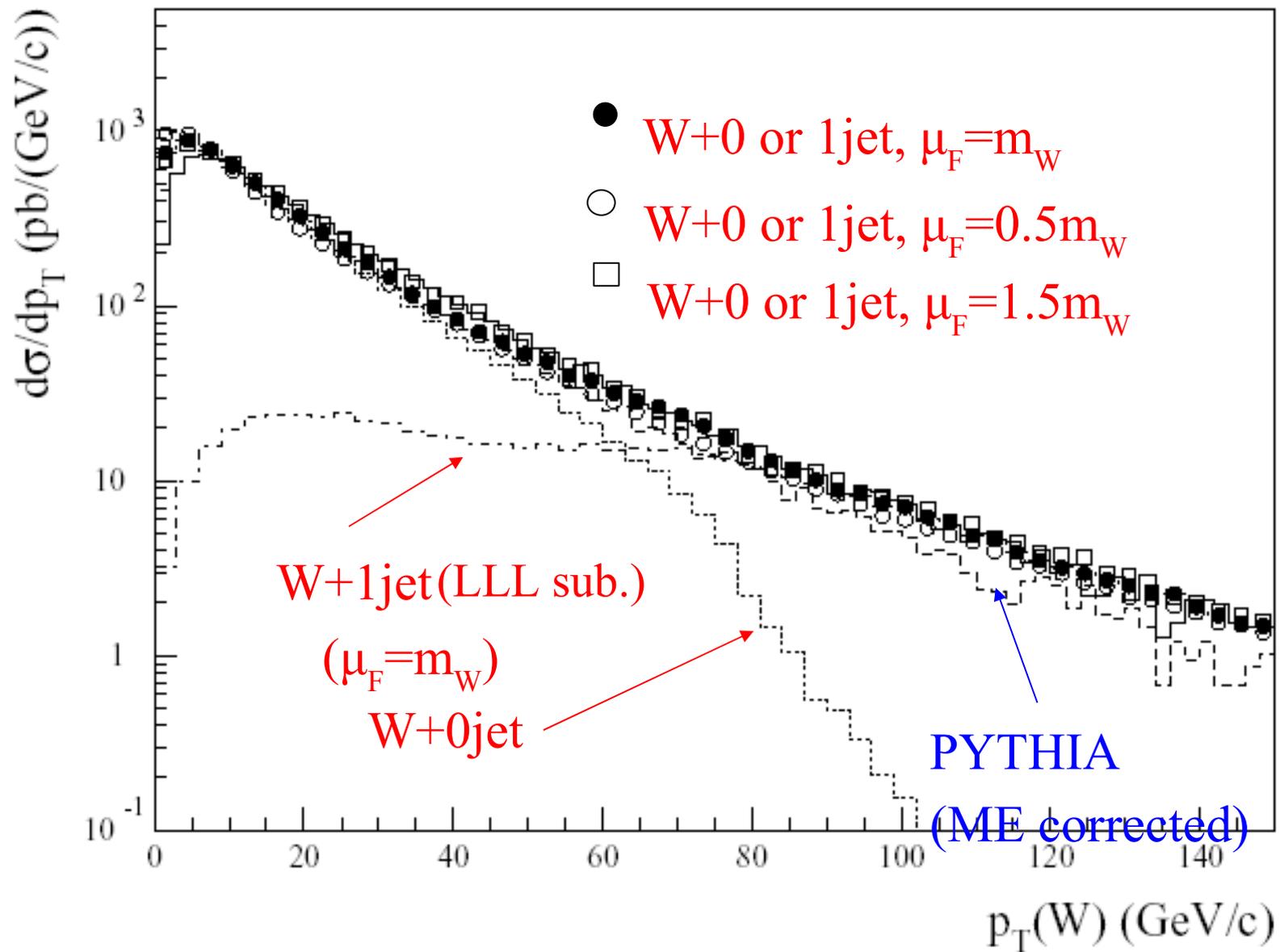


# Result of Subtraction Method

$Z(\rightarrow e^+e^-)$  production at Tevatron Run 1







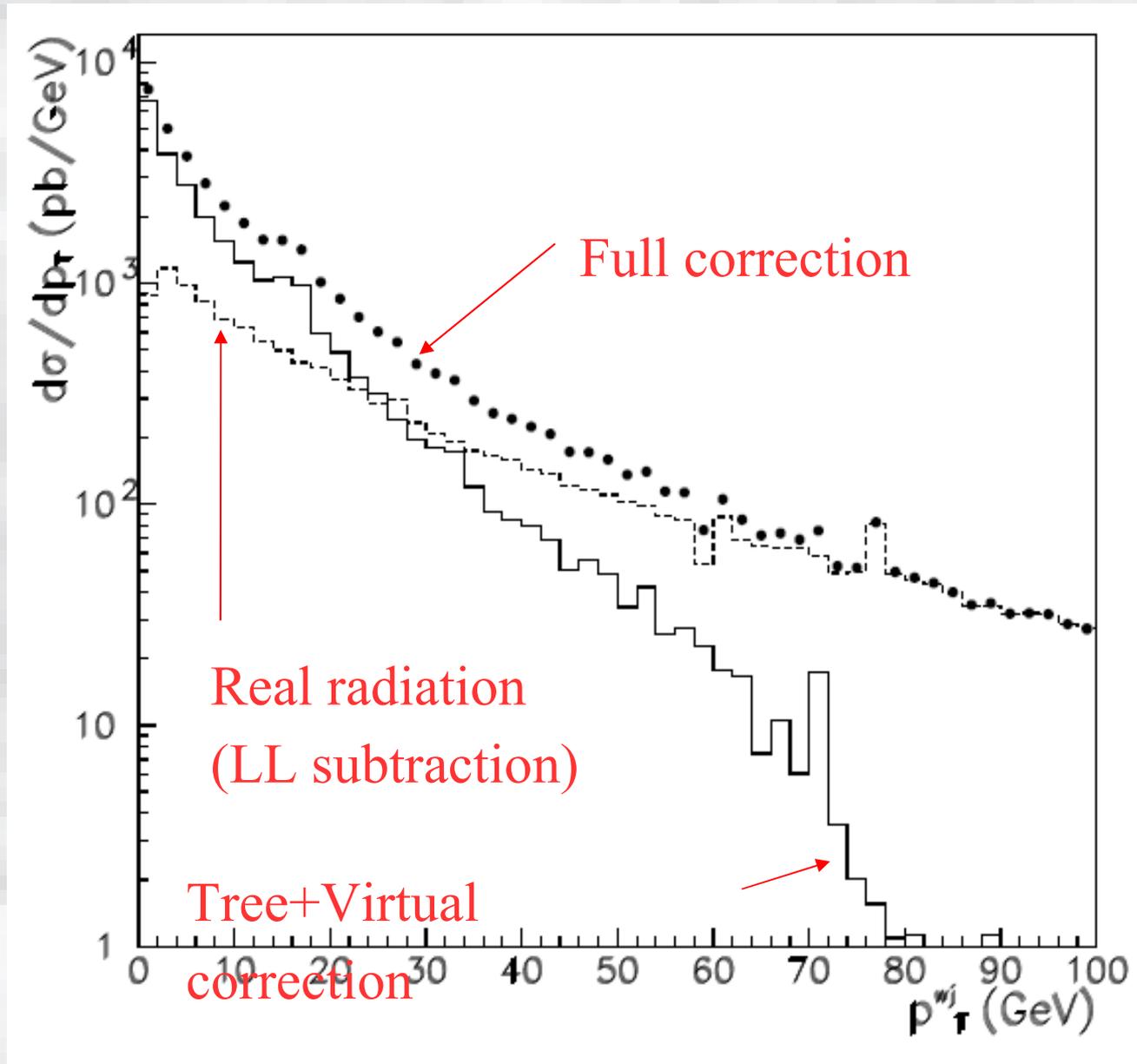
## Test process: $PP \rightarrow W + \text{jet}$

- $\sqrt{s} = 14 \text{ TeV}$
- PDF: CTEQ5L
- Cuts:  $p_T^W > 20 \text{ GeV}$ ,  $p_T^g > 20 \text{ GeV}$
- $\mu_F = \mu_R = m_W = 80.2 \text{ GeV}$
- $\sigma_{\text{tree}} = 3.56 \cdot 10^4 \text{ pb}$  (W/O double counting rejection)
- $\sigma_{\text{tree}} = 4.26 \cdot 10^4 \text{ pb}$  (W/ double counting rejection)
- double counting effect is 20%!

## IR cancellation

$$1/\epsilon_{\text{IR}}^2, 1/\epsilon_{\text{IR}} \quad O(10^{-10})$$

# Transverse momentum distribution of W-jet



$$P_T^W > 20 \text{ GeV}$$

$$\bullet \sigma_{\text{NLO}} = 7.06 \cdot 10^4 \text{ pb}$$



## Test process: $PP \rightarrow \gamma\gamma$

- $\sqrt{s} = 14 \text{ TeV}$
- PDF: CTEQ5L
- Cuts:  $E_\gamma > 10 \text{ GeV}, 10^\circ < \theta_\gamma < 170^\circ, \theta_{\gamma\gamma} > 10^\circ$
- $\mu_F = \mu_R = s_0$ 
  - $\sigma_{\text{tree}} = 7.35 \cdot 10^2 \text{ pb}$
  - $\sigma_{\text{tree+virtual}} = 1.32 \cdot 10^3 \text{ pb}$
  - $\sigma_{\text{real radiation}} = 9.32 \cdot 10^2 \text{ pb}$

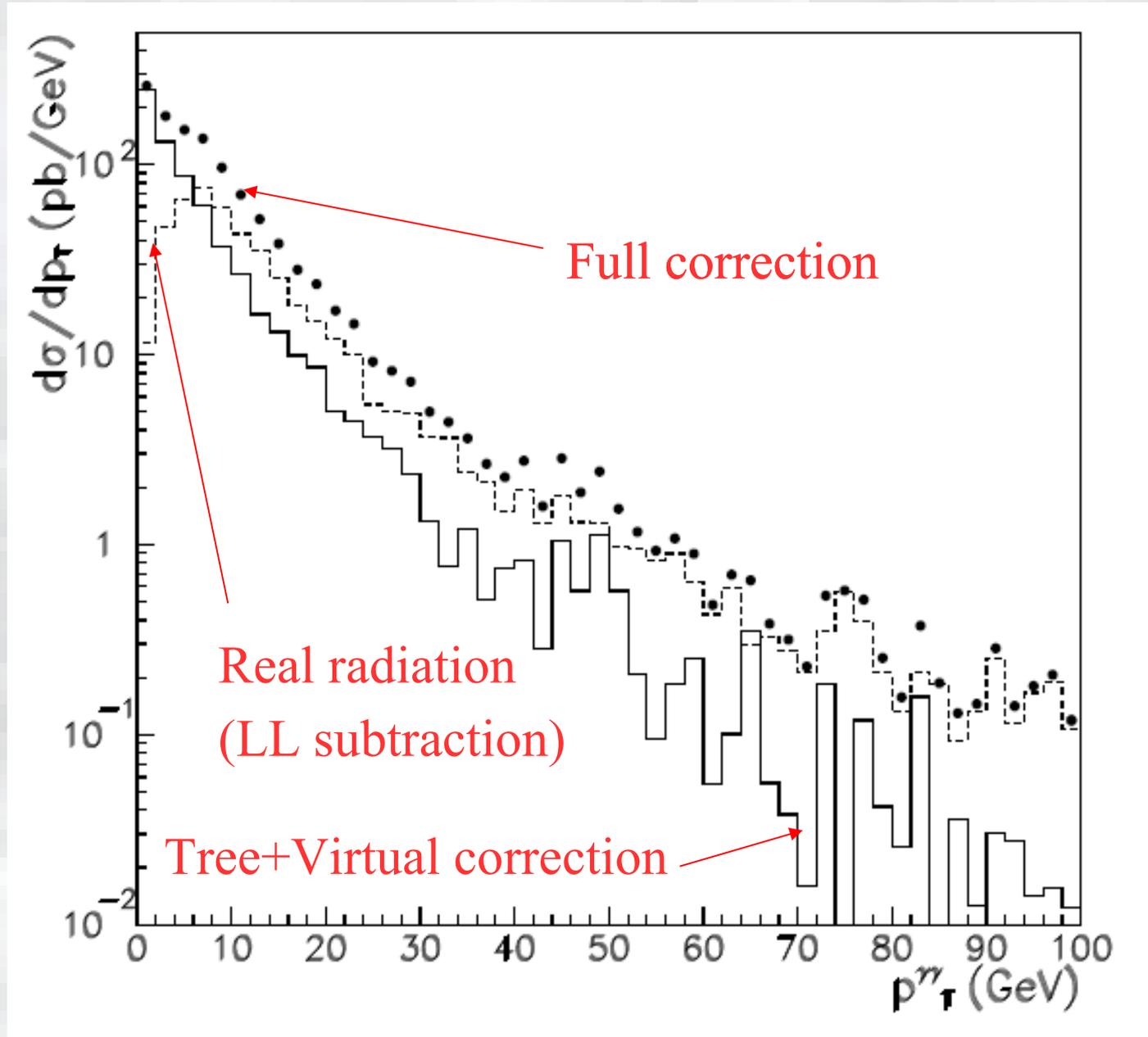


## IR cancellation

$$1/\epsilon_{\text{IR}}^2, 1/\epsilon_{\text{IR}} \quad O(10^{-10})$$

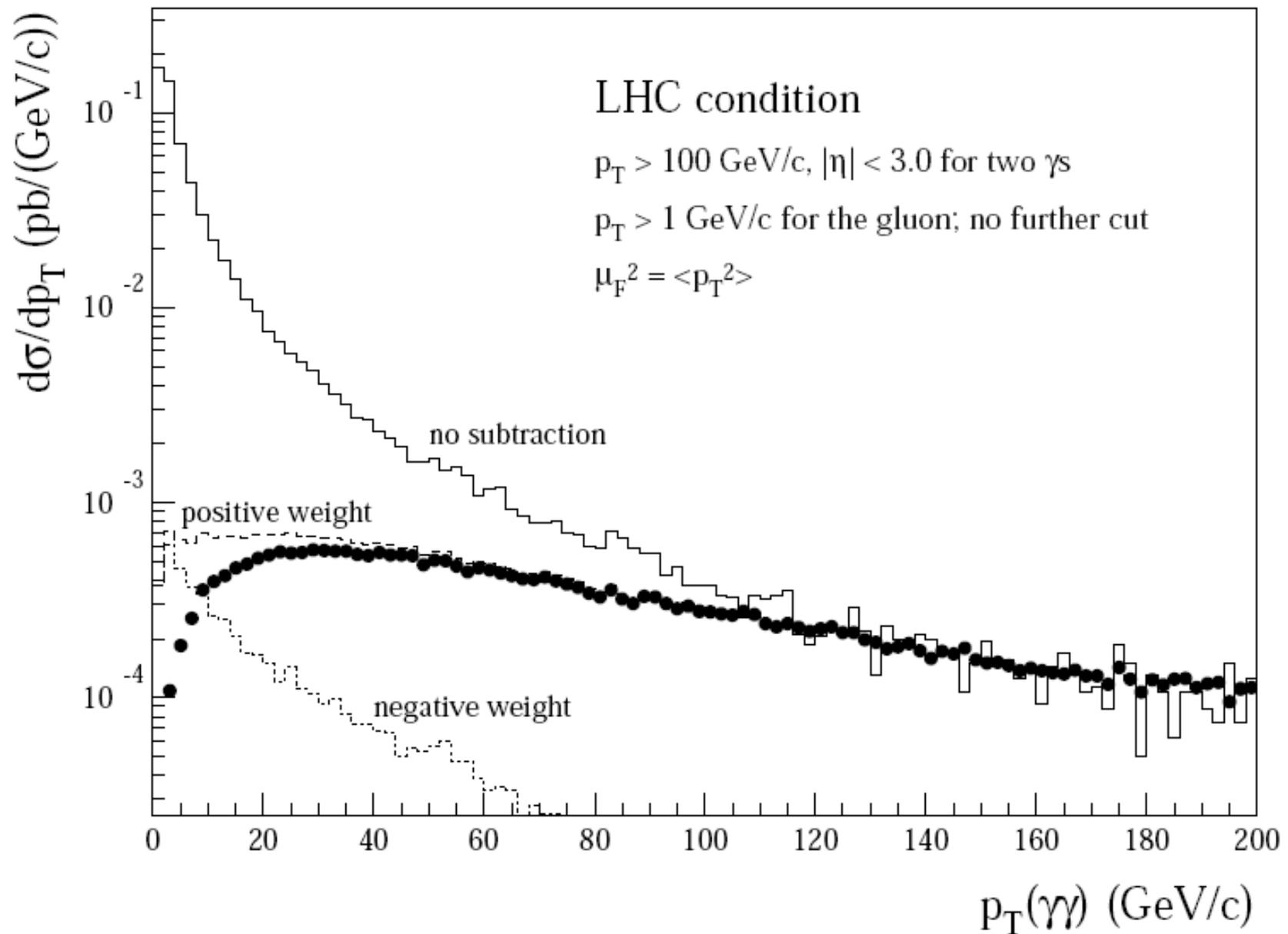


# Transverse momentum distribution of $\gamma\gamma$



- $\sigma_{\text{NLO}} = 2.25 \cdot 10^3 \text{ pb}$

# LLL subtraction for $q\bar{q} \rightarrow \gamma\gamma g$



# GR@PPA event generator

Integrated system of hadronic event generators for PP and PP-bar.

- QCD jets (2~4 jets)
- V+ 0~4 Jets
- VV+ 0~2 Jets
- tt+jet

NLO event generators will be implemented into GR@PPA

# Process list in GR@PPA

IGSUB	Coupling order	Command parameter	Process description
100	$\alpha_{\text{em}}^2$	w0j	$pp(\bar{p}) \rightarrow W(2f) + 0 \text{ jet } (+X)$
101	$\alpha_{\text{em}}^2 \alpha_s$	w1j	$pp(\bar{p}) \rightarrow W(2f) + 1 \text{ jet } (+X)$
102	$\alpha_{\text{em}}^2 \alpha_s^2$	w2j	$pp(\bar{p}) \rightarrow W(2f) + 2 \text{ jets } (+X)$
103	$\alpha_{\text{em}}^2 \alpha_s^3$	w3j	$pp(\bar{p}) \rightarrow W(2f) + 3 \text{ jets } (+X)$
104	$\alpha_{\text{em}}^2 \alpha_s^4$	w4j	$pp(\bar{p}) \rightarrow W(2f) + 4 \text{ jets } (+X)$
110	$\alpha_{\text{em}}^2$	z0j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f) + 0 \text{ jet } (+X)$
111	$\alpha_{\text{em}}^2 \alpha_s$	z1j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f) + 1 \text{ jet } (+X)$
112	$\alpha_{\text{em}}^2 \alpha_s^2$	z2j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f) + 2 \text{ jets } (+X)$
113	$\alpha_{\text{em}}^2 \alpha_s^3$	z3j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f) + 3 \text{ jets } (+X)$
114	$\alpha_{\text{em}}^2 \alpha_s^4$	z4j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f) + 4 \text{ jets } (+X)$
120	$\alpha_{\text{em}}^4$	ww0j	$pp(\bar{p}) \rightarrow W^+(2f)W^-(2f') + 0 \text{ jet } (+X)$
121	$\alpha_{\text{em}}^4 \alpha_s$	ww1j	$pp(\bar{p}) \rightarrow W^+(2f)W^-(2f') + 1 \text{ jet } (+X)$
122	$\alpha_{\text{em}}^4 \alpha_s^2$	ww2j	$pp(\bar{p}) \rightarrow W^+(2f)W^-(2f') + 2 \text{ jets } (+X)$
130	$\alpha_{\text{em}}^4$	zw0j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f)W(2f') + 0 \text{ jet } (+X)$
131	$\alpha_{\text{em}}^4 \alpha_s$	zw1j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f)W(2f') + 1 \text{ jet } (+X)$
132	$\alpha_{\text{em}}^4 \alpha_s^2$	zw2j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f)W(2f') + 2 \text{ jets } (+X)$
140	$\alpha_{\text{em}}^4$	zz0j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f)Z/\gamma^*(2f') + 0 \text{ jet } (+X)$
141	$\alpha_{\text{em}}^4 \alpha_s$	zz1j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f)Z/\gamma^*(2f') + 1 \text{ jet } (+X)$
142	$\alpha_{\text{em}}^4 \alpha_s^2$	zz2j	$pp(\bar{p}) \rightarrow Z/\gamma^*(2f)Z/\gamma^*(2f') + 2 \text{ jets } (+X)$
160	$Y_b^2 \alpha_s^2$	b4hbb	$pp(\bar{p}) \rightarrow h_0(bb) + bb (+X)$
161	$\alpha_{\text{em}}^2 \alpha_s^2$	b4zbb	$pp(\bar{p}) \rightarrow Z/\gamma^*(b\bar{b}) + b\bar{b} (+X)$
162	$\alpha_s^4$	b4qcd	$pp(\bar{p}) \rightarrow bbbb (+X)$
163	$Y_b^2 \alpha_{\text{em}}^2$	b4hz	$pp(\bar{p}) \rightarrow h_0(bb) + Z/\gamma^*(b\bar{b}) (+X)$
164	$\alpha_{\text{em}}^4$	b4zz	$pp(\bar{p}) \rightarrow Z/\gamma^*(b\bar{b}) + Z/\gamma^*(b\bar{b}) (+X)$
170	$\alpha_{\text{em}}^4 \alpha_s^2$	tt6bdy	$pp(\bar{p}) \rightarrow t\bar{t} \rightarrow 6f (+X)$
171	$\alpha_{\text{em}}^4 \alpha_s^3$	ttj7bdy	$pp(\bar{p}) \rightarrow t\bar{t} + 1 \text{ jet} \rightarrow 6f + 1 \text{ jet } (+X)$
182	$\alpha_s^2$	qcd2j	$pp(\bar{p}) \rightarrow 2 \text{ jets } (+X)$
183	$\alpha_s^3$	qcd3j	$pp(\bar{p}) \rightarrow 3 \text{ jets } (+X)$
184	$\alpha_s^4$	qcd4j	$pp(\bar{p}) \rightarrow 4 \text{ jets } (+X)$

V+N jets

N jets	Tevatron Run-II		LHC	
	W( $e\nu_e$ )	Z( $e^+e^-$ )	W( $e\nu_e$ )	Z( $e^+e^-$ )
0	$1.576(2) \times 10^3$	$1.598(3) \times 10^2$	$1.116(2) \times 10^4$	$9.57(3) \times 10^2$
1	$1.852(3) \times 10^2$	$1.829(4) \times 10^1$	$2.854(5) \times 10^3$	$2.614(7) \times 10^2$
2	$3.461(7) \times 10^1$	3.485(6)	$1.143(3) \times 10^3$	$1.082(2) \times 10^2$
3	6.29(2)	$6.35(2) \times 10^{-1}$	$4.82(1) \times 10^2$	$4.53(1) \times 10^1$
4	1.201(5)	$1.173(3) \times 10^{-1}$	$2.19(1) \times 10^2$	$2.045(5) \times 10^1$
2 ( $\geq 1b$ )	$3.260(6) \times 10^{-1}$	$9.24(2) \times 10^{-2}$	2.720(7)	8.68(2)
3 ( $\geq 1b$ )	$1.019(2) \times 10^{-1}$	$2.266(5) \times 10^{-2}$	4.305(9)	3.740(7)
4 ( $\geq 1b$ )	$2.947(8) \times 10^{-2}$	$3.817(5) \times 10^{-3}$	3.90(3)	$9.47(2) \times 10^{-1}$
$t\bar{t} + 0$	$5.269(9) \times 10^{-4}$	$2.248(3) \times 10^{-4}$	$3.774(7) \times 10^{-2}$	$2.682(6) \times 10^{-2}$
$t\bar{t} + 1$	$1.357(2) \times 10^{-4}$	$6.302(9) \times 10^{-5}$	$4.98(2) \times 10^{-2}$	$3.59(1) \times 10^{-2}$
$t\bar{t} + 2$	$6.86(1) \times 10^{-5}$	$5.707(5) \times 10^{-7}$	$6.52(2) \times 10^{-2}$	$1.156(2) \times 10^{-2}$

WV+N jets

N jets	Tevatron Run-II		
	WW	WZ	ZZ
0	$7.91(2) \times 10^{-2}$	$6.06(1) \times 10^{-3}$	$1.541(3) \times 10^{-3}$
1	$1.986(5) \times 10^{-2}$	$2.013(4) \times 10^{-3}$	$3.721(6) \times 10^{-4}$
2	$4.77(1) \times 10^{-3}$	$5.16(2) \times 10^{-4}$	$7.698(9) \times 10^{-5}$
2 ( $\geq 1b$ )	$1.582(2) \times 10^{-4}$	$6.00(2) \times 10^{-6}$	$1.629(3) \times 10^{-6}$
$t\bar{t} + 0$	$5.504(5) \times 10^{-7}$	$5.065(7) \times 10^{-8}$	$8.871(9) \times 10^{-9}$

N jets	LHC		
	WW	WZ	ZZ
0	1.339(5)	$4.64(1) \times 10^{-2}$	$1.080(2) \times 10^{-2}$
1	1.071(3)	$1.873(4) \times 10^{-1}$	$6.06(1) \times 10^{-3}$
2	$9.41(2) \times 10^{-1}$	$1.536(4) \times 10^{-1}$	$3.419(5) \times 10^{-3}$
2 ( $\geq 1b$ )	$7.36(1) \times 10^{-2}$	$1.164(3) \times 10^{-4}$	$1.887(8) \times 10^{-4}$
$t\bar{t} + 0$	$7.432(9) \times 10^{-4}$	$1.213(2) \times 10^{-5}$	$3.040(5) \times 10^{-6}$



# Summary

1. GRACE/SUSY and GRACE/SUSY/loop
2. NLO-QCD event generators in GRACE
3. NLO event generators in GR@PPA framework
5. Tool development (talks in Wednesday)

# Renormalization scheme

On-shell renormalization condition:

Gauge bosons, Fermions, Scalar fermions,

Higgs bosons ( $A^0, H^0$ ), Neutralino ( $\tilde{\chi}_1^0$ ),

Charginos ( $\tilde{\chi}_1^+, \tilde{\chi}_2^+$ )

Renormalization of  $\tan\beta$ :

$$\delta \tan \beta = -\frac{1}{2} \tan \beta \left( \delta Z_{H_1} - \delta Z_{H_2} - 2 \frac{\delta v_1}{v_1} + 2 \frac{\delta v_2}{v_2} \right)$$

Gauge invariance check with Non-linear gauge (NLG)!!

$$\frac{\delta v_1}{v_1} = \frac{\delta v_2}{v_2} \quad \longrightarrow \quad \text{Gauge}$$


# Renormalization of scalar fermions

## 1) Scheme Variation 1 (with SU(2) relation)

$$\delta m_f^2 = -\text{Re} \Sigma_{\bar{f}f} (m_f^2)$$

$$\delta m_{\nu}^2 = 2 \cos \theta_e \sin \theta_e \delta \theta_e (m_{\nu_2}^2 - m_{\nu_1}^2) + \cos^2 \theta_e \delta m_{\nu_1}^2 + \sin^2 \theta_e \delta m_{\nu_2}^2 + \delta (M_W^2 \cos 2\beta - m_{\nu}^2) \longrightarrow \delta \theta_e$$

$$\delta \theta_e = \frac{1}{2} \frac{\Sigma_{\bar{\nu}_1 \nu_2} (m_{\nu_2}^2) + \Sigma_{\bar{\nu}_2 \nu_1} (m_{\nu_1}^2)}{m_{\nu_2}^2 - m_{\nu_1}^2}$$

$$\begin{aligned} \sin^2 \theta_e \delta m_{\nu_2}^2 &= 2 \cos \theta_e \sin \theta_e \delta \theta_e (m_{\nu_2}^2 - m_{\nu_1}^2) + \cos^2 \theta_e \delta m_{\nu_1}^2 \\ &+ \sin^2 \theta_e \delta m_{\nu_2}^2 - 2 \cos \theta_e \sin \theta_e \delta \theta_e (m_{\nu_2}^2 - m_{\nu_1}^2) - \cos^2 \theta_e \delta m_{\nu_1}^2 \\ &+ \delta (M_W^2 \cos 2\beta + m_{\nu_2}^2 - m_{\nu_1}^2) \longrightarrow \delta \theta_e \end{aligned}$$

## 2) Scheme Variation 2

$$\delta m_r^2 = -\text{Re} \sum_{\overline{r}} (m_{\overline{r}}^2)$$

$$\delta \theta_s = \frac{1}{2} \frac{\sum_{\overline{s1} \overline{s2}} (m_{\overline{s2}}^2) + \sum_{\overline{s2} \overline{s1}} (m_{\overline{s1}}^2)}{m_{\overline{s2}}^2 - m_{\overline{s1}}^2}$$

$$\delta \theta_q = \frac{1}{2} \frac{\sum_{\overline{q1} \overline{q2}} (m_{\overline{q2}}^2) + \sum_{\overline{q2} \overline{q1}} (m_{\overline{q1}}^2)}{m_{\overline{q2}}^2 - m_{\overline{q1}}^2} \quad q = u, d$$

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