Jets: seeing quarks, gluons and more at the LHC

Grégory Soyez

IPhT, CEA Saclay

Natal, October 21-31, 2014

- Lecture 1: Jets \sim QCD parton basic concepts
- Lecture 2: How close is a jet to a parton? Analytic estimates of perturbative and non-perturbative effects between a parton and a jet
- **•** Lecture 3: A jet can be something else too! Boosted jets and jet substructure

What is a jet?

- Concept of a jet
- \bullet Jet algorithm/jet definition
- **•** Fundamental requirements
- A little bit of history from LEP...
- ... to the LHC
- Practical implementation [if time permits]

How are the hadrons distributed in a collision event?

- (a) Uniformly across the event
- (b) Along a few directions
- (c) I do not understand the question

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How are the hadrons distributed in a collision event?

(b) Along a few directions

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Final-state events are pencil-like already observed in e^+e^- collisions:

 QQ

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"Jets" \equiv bunch of collimated particles

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Is that expected in QCD?

(a) Yes

(b) No

(c) What is QCD?

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Is that expected in QCD?

(a) Yes

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Origin in QCD

This is expected from QCD:

- **collinear divergence: enhancement of small-angle branchings**
- Same as in DGLAP/PDF evolution

$$
|\mathcal{M}_{n+1}|^2 = |\mathcal{M}_n|^2 \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} P(z) dz
$$

Physical origin: as $\theta \rightarrow 0$, $\rho^2_a \rightarrow 0$ (assuming b and c are massless)

"Jets" ≡ bunch of collimated particles ≅ hard partons

How many jets?

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"Jets" ≡ bunch of collimated particles ≅ hard partons

obviously 2 jets

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"Jets" ≡ bunch of collimated particles ≅ hard partons

How many jets

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"Jets" ≡ bunch of collimated particles [≅] hard partons

3 jets

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"Jets" ≡ bunch of collimated particles [≅] hard partons

3 jets... or 4?

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"Jets" ≡ bunch of collimated particles ≅ hard partons

 \rightarrow

3 jets... or 4?

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- "collinear" is arbitrary
- "parton" concept strictly valid only at LO

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Jet definition

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What is a "jet"?

jet definition(s)

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Jet definition

- A jet definiton is supposed to
	- give finite jet cross sections (th)
	- \circ be fast enough (exp)
	- be (as) consistent (as possible) across different view of an event (both)

Divergences in QCD

UV divergences

- Re-absorbed into the parameters of the Lagrangian $(\Psi_f, A_a^{\mu}, g, m_f)$
- QCD is a renormalisable theory
- Renormalisation-group equation e.g. $\alpha_{\bm{s}}(\mu^2)$ \bullet
- **•** Asymptotic freedom in QCD

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IR divergences

• In the initial state: re-absorbed in the PDFs DGLAP evolution equation

Divergences in QCD

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- **•** Asymptotic freedom in QCD

IR divergences

- In the initial state: re-absorbed in the PDFs DGLAP evolution equation
- what about the final-state?

Parton branching at small angle

Can be calculated explicitly (3 combinations: $q \rightarrow qg$, $g \rightarrow gg$, $g \rightarrow q\bar{q}$):

Parton branching at small angle

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Or if you're looking for something more "concrete": $e^+e^-\to$ hadrons

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Or if you're looking for something more "concrete": $e^+e^-\to$ hadrons

(only gives
$$
q \rightarrow qg
$$
)

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In the end:

$$
|\mathcal{M}_{n+1}|^2 = |\mathcal{M}_n|^2 \frac{\alpha_s}{2\pi} \frac{d\theta^2}{\theta^2} P(z) dz, \qquad P(z) \stackrel{z \ll 1}{\propto} \frac{2C_R}{z}
$$

At leading log: P's are the same splitting fcts a[s f](#page-24-0)[or](#page-26-0) [D](#page-22-0)[G](#page-25-0)[L](#page-26-0)[A](#page-17-0)[P](#page-18-0)

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Two IR divergences: collinear $(\theta \rightarrow 0)$; soft $(z \rightarrow 0)$

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Kinoshita-Lee-Nauenberg/Block-Nordsieck theorem

Soft and collinear divergences cancel between real and virtual diagrams at all orders of the perturbation theory

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Infrared-and-collinear safe observables

This cancellation must be preserved!

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Soft and collinear divergences cancel between real and virtual diagrams at all orders of the perturbation theory

Infrared-and-collinear safe observables

- This cancellation must be preserved!
- Observables (e.g. jets) must be insensitive to collinear branchings and soft emissions

Question 3

How often are jets used at the LHC?

- (a) Never (QCD is dirty, I live with leptons and photons)
- (b) in about 20% of the analysis
- (c) in about 40% of the analysis
- (d) in about 60% of the analysis
- (e) in about 80% of the analysis
- (f) always (and I buy all QCD lecturers a beer)

How often are jets used at the LHC?

(d) in about 60% of the analysis

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Jets are omnipresent

Jets are used in $\sim 60\%$ of the LHC analyses

Jets are important

A robust jet definition is needed: it guarantees a precise access to the quarks and gluons in "hard" collisions

[What is a jet?](#page-34-0) [jet definition\(s\)](#page-34-0)

A brief/rough flight over the history of jets

[What is a jet?](#page-35-0) [jet definition\(s\)](#page-35-0)

A brief/rough flight over the history of jets

- • Cone algorithms and IRC safety
- Clustering at the LHC: the anti- k_t algorithm
- FastJet: speed and implementation
- Now is a good time to stand up if you want to hear about sth else \bullet

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What is a "jet"?

Cone algorithms and IRC safety

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Basic concepts

Central idea: stable cones

- A jet is a direction of stable energy flow
- Stable cone: the sum of all momenta in a cone of (fixed) radius R points in the direction of the centre of the cone

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Basic concepts

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Cone algorithms with split–merge

- find stable cones (Usually iteratively starting from a set of seeds)
- run a split–merge procedure to get rid of overlaps CDFJetClu, CDFMidPoint, D0MidPoint, ATLASCone, SISCone

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Cone algorithms with progressive removal

- **•** find the hardest stable cone (Usually iterating from the hardest particle in the event)
-

call it a jet and iterate CMSIte[rat](#page-45-0)[iv](#page-47-0)[e](#page-43-0)[C](#page-44-0)[o](#page-46-0)[n](#page-47-0)[e,](#page-42-0) [S](#page-53-0)[I](#page-54-0)[S](#page-2-0)[C](#page-3-0)[o](#page-79-0)[n](#page-80-0)[e-](#page-0-0)[PR](#page-88-0)

[What is a jet?](#page-47-0) [Cone algorithms and IRC safety](#page-47-0)

IR safety: JetClu v. SISCone

JetClu

Finds stable cones stating from all the particles in the event

SISCone

Finds ALL stable cones

Consequences

- **•** JetClu is IR-unsafe
- SISCone is IR-safe

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[What is a jet?](#page-48-0) [Cone algorithms and IRC safety](#page-48-0)

IR safety: JetClu v. SISCone

cancellation between real and virtual spoiled

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[What is a jet?](#page-49-0) [Cone algorithms and IRC safety](#page-49-0)

IR safety: JetClu v. SISCone

Same stable cones found everywhere: all OK

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Question 4

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Question 4

Does it matter?

(a) Yes

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Seeds are bad!

Consequences of IRC unsafety

- JetClu, ATLASCone IR-unsafe for 2 (nearby) particles $+1$ soft Trust jets only at lowest order α_s^0
- CDF/D0MidPoint IR-unsafe for 3 (nearby) particles $+1$ soft Trust jets only at lowest order α_s^1
- CMSIterativeCone collinear-unsafe for 3 (nearby) particles $+1$ soft Trust jets only at lowest order α_s^1
- SISCone(-PR) find ALL stable cones: safe at all orders

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• IR-unsafety usually(!) beyond Tevatron precision • Not sufficient for the LHC

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What is a "jet"?

Anti- k_t and jets at the LHC

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The anti- $\mathit{k_{t}}$ jets

• All experiments use the anti- k_t algorithm:

[M. Cacciari, G. Salam, GS, 2008]

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The anti- k_t algorithm

• From all the objects, define the distances

$$
d_{ij} = \min(p_{t,i}^{-2}, p_{t,j}^{-2})(\Delta y_{ij}^2 + \Delta \phi_{ij}^2), \qquad d_{iB} = p_{t,i}^{-2}R^2
$$

• repeatedly find the minimal distance if d_{ii} : recombine *i* and *j* into $k = i + j$ if d_{iB} : call i a jet

 \bullet R is a size parameter (e.g. CMS: 0.5,0.7,0.4(soon) ATLAS: 0.4,0.6) It determines the "angular size" of jets.

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- \bullet R is a size parameter (e.g. CMS: 0.5,0.7,0.4(soon) ATLAS: 0.4,0.6) It determines the "angular size" of jets.
- $p_t^{-2} \rightarrow p_t^{2p} \colon \, p = -1, 0, 1$ is anti- k_t , Cambridge/Aachen and k_t

Question 5

Are anti- $k t$, C/A and k_t IRC-safe?

- (a) Yes
- (b) No: IR-unsafe
- (c) No: collinear-unsafe
- (d) No: IR and collinear unsafe

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Are anti- $k t$, C/A and k_t IRC-safe?

(a) Yes

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Clustering in action: anti- k_t ($R = 0.7$)

Start with your favourite picture

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Clustering in action: anti- k_t ($R = 0.7$)

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Clustering in action: anti- k_t ($R = 0.7$)

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Clustering in action: anti- k_t ($R = 0.7$)

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Why anti- k_t ?

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The anti- $\mathit{k_{t}}$ jets

Main property: hard jets are circular

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Examples

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Implementation and speed

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Question 7

What complexity to cluster N particles?

- (a) $\mathcal{O}(N)$
- (b) $\mathcal{O}(N^2)$
- (c) $\mathcal{O}(N^3)$
- (d) $\mathcal{O}(\exp(N))$

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Question 7

What complexity to cluster N particles?

(c)
$$
\mathcal{O}(N^3)
$$

Naively:

- Compute all pairwise distances to find the minimum: $\mathcal{O}(N^2)$
- Do that $\sim N$ times

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FastJet $(1/2)$

Before 2005: k_t deemed too slow by the Tevatron

 10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} $10⁰$ 10^{1} $10¹$ 10^2 10^3 $10⁴$ 10⁵ 10⁶ time (s) N LHC LHC LHC low lumi now KtJet Intel $@$ i5 760 FastJet 3.0

[M.Cacciari, G.Salam, 05]

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Tevatron era: k_t too slow: $\mathcal{O}(N^3)$ for N particles Cone preferred (easier calibration too)

 \Box

FastJet $(1/2)$

2005: FastJet: a fast implementation of k_t

[M.Cacciari, G.Salam, 05]

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Tevatron era: k_t too slow: $\mathcal{O}(N^3)$ for N particles Cone preferred (easier calibration too)

Realistic timing for the LHC! $(\mathcal{O}(N^2))$ or even $\mathcal{O}(N \log(N)))$

FastJet $(1/2)$

[M.Cacciari, G.Salam, 05]

2014: FastJet timings for various algorithms

- Tevatron era: k_t too slow: $\mathcal{O}(N^3)$ for N particles Cone preferred (easier calibration too)
- Realistic timing for the LHC! $(\mathcal{O}(N^2))$ or even $\mathcal{O}(N \log(N)))$
- •The situation to[d](#page-0-0)ay: 10-50ms for $R = 0.4$ ([inc](#page-84-0)[lu](#page-86-0)d[i](#page-83-0)[n](#page-85-0)[g](#page-86-0) [p](#page-79-0)[il](#page-80-0)[eu](#page-88-0)[p](#page-79-0) [an](#page-88-0)d [area](#page-88-0)s)

FastJet (2/2)

[M.Cacciari, G.Salam, GS, 2007-2013]

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Grown way beyond just fast recombinations:

- plugins for used jet definitions
- jet areas and background subtraction
- tools for manipulating jets
- more to come...
- FastJet 3.1.0 released in September 2014 see <www.fastjet.fr>
- Standard interface for jet physics for both theorists and experimentalists

FastJet contrib (since Feb 2013)

· fastjet.fr

· fastiet-contrib

· contrib syn

FastJet Contrib

The fastjet-contrib space is intended to provide a common location for access to 3rd party extensions of Fastlet.

Download the current version: ficontrib-1.011 (released 6 April 2014), which contains these contributions. Changes relative to earlier versions are briefly described in the NEWS file.

- a quick and uniform access to 3rd-party code
- **e** contributors are welcome

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(please contact us)

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The FastJet lemma

It (i,j) is the pair that minimize the k_t distance and $k_{t,i} < k_{y,j}$, then j is i's nearest neighbour

Proof: assume it is not, then $\exists k$ s.t. $\Delta R_{ik} < \Delta R_{ii}$ and

$$
\begin{array}{rcl}\n\min(k_{t,i}^2, k_{t,i}^2) \Delta R_{il}^2 & \leq & k_{t,i}^2 \Delta R_{il}^2 \\
& \leq & \min(k_{t,i}^2, k_{t,j}^2) \Delta R_{il}^2 \\
& < & \min(k_{t,i}^2, k_{t,j}^2) \Delta R_{il}^2\n\end{array}
$$

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