From Colour Glass Condensate to Quark–Gluon Plasma

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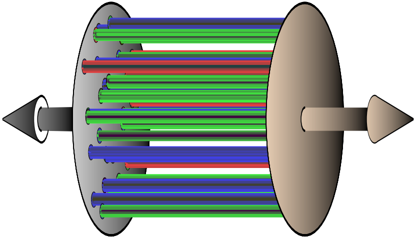
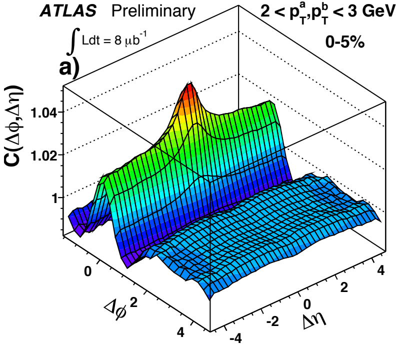
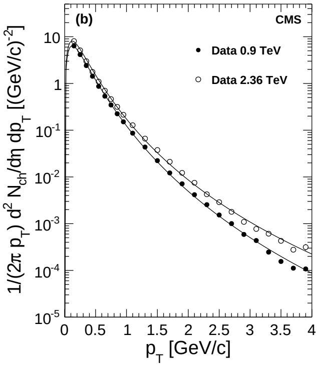
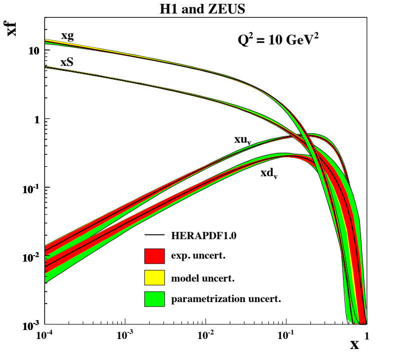


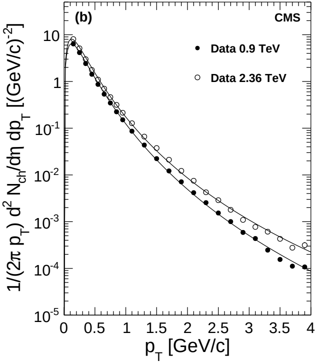


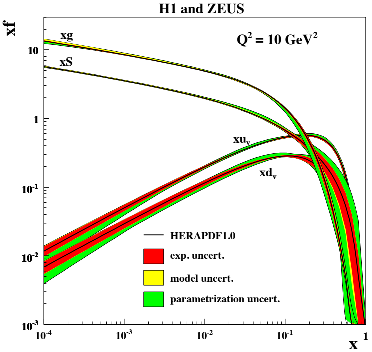
Recall: The prominence of the small–x gluons





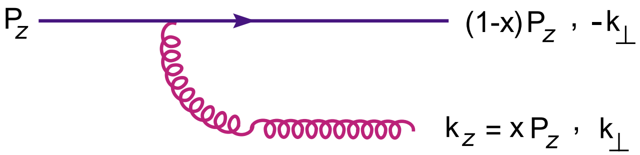
- 99% of the total multiplicity in pp and AA lies below $p_\perp = 2$ GeV
- The bulk of particle production is controlled by partons at small $x \ll 1$
- DIS demonstrates these partons are predominantly gluons
- Need to better understand gluon evolution at small x





- 99% of the total multiplicity in pp and AA lies below $p_\perp = 2$ GeV
- The bulk of particle production is controlled by partons at small $x \ll 1$
- DIS demonstrates these partons are predominantly gluons
- With due respect to Emmanuel: we do have a good theory at small x !

Bremsstrahlung



$$\mathrm{d}\mathcal{P}_{\rm Brem} \equiv \sum_{a,\lambda} \left|\mathcal{M}^a_\lambda(k_z, \boldsymbol{k}_\perp)\right|^2 \simeq \frac{\alpha_s(k_\perp^2)\, C_R}{\pi^2} \frac{\mathrm{d}^2 k_\perp}{k_\perp^2} \frac{\mathrm{d}x}{x}$$

- Phase–space enhancement for the emission of
  - collinear $(k_\perp \to 0)$
  - and/or soft (low–energy) $(x \to 0)$ gluons
- The parent parton can be either a quark or a gluon

$$C_F = t^a t^a = \frac{N_c^2 - 1}{2N_c} = \frac{4}{3}\,, \quad C_A = T^a T^a = N_c = 3$$

The gluon distribution of a single quark

- To leading order in α_s : single gluon emission by the quark $\Longrightarrow$

$$\frac{\mathrm{d}N_{\mathrm{gluon}}}{\mathrm{d}x\mathrm{d}^2k_\perp} = \frac{\mathrm{d}\mathcal{P}_{\mathrm{Brem}}}{\mathrm{d}x\mathrm{d}^2k_\perp}$$

▷ "unintegrated gluon distribution"

- The gluon distribution $xG(x, Q^2)$: # of gluons with a given energy fraction x and any transverse momentum $k_\perp \lesssim Q$

$$xG(x, Q^2) = \int^Q \mathrm{d}^2\boldsymbol{k}\, x\frac{\mathrm{d}N_{\mathrm{gluon}}}{\mathrm{d}x\mathrm{d}^2k_\perp} = \frac{\alpha_s C_F}{\pi}\int_{\Lambda^2}^{Q^2}\frac{\mathrm{d}k_\perp^2}{k_\perp^2} = \frac{\alpha_s C_F}{\pi}\ln\frac{Q^2}{\Lambda^2}$$

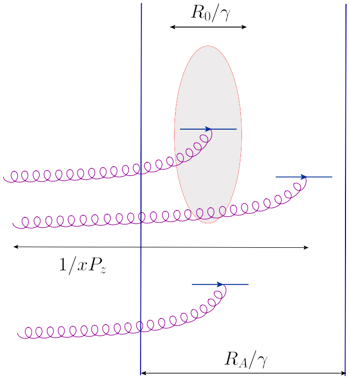
▷ logarithmic sensitivity to the confinement scale Λ

▷ the first 'transverse' logarithm of the DGLAP resummation

▷ no dependence upon energy (x) since gluon spin $j = 1$: s^{j-1}

The gluon distribution of a large nucleus

- 'Large nucleus' : incoherent superposition of A nucleons, each one made with N_c valence quarks *(McLerran–Venugopalan model, 1994)*



$$xG_A(x,Q^2) \;=\; AN_c\, xG_q(x,Q^2)$$

$$xG_q(x,Q^2) \;=\; \frac{\alpha_s C_F}{\pi}\ln\frac{Q^2}{\Lambda^2}$$

$$R_A \;=\; R_0\, A^{1/3} \quad \text{(nuclear radius)}$$

$$\gamma = 100\,(RHIC) \div 1000\,(LHC)$$

- The small–x gluons are delocalized over a large longitudinal distance:

$$\Delta z \sim \frac{1}{xP_z} \gg \frac{R_A}{\gamma}$$

Gluon saturation in a large nucleus

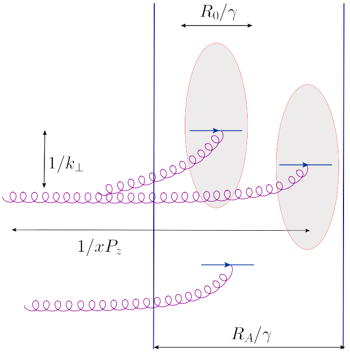
- $AN_c \sim 600$ for Au or Pb : can we simply superpose the different emissions as if they were independent from each other ?

  ▷ can one ignore gluon recombination ?

- In order to interact, gluons must overlap with each other

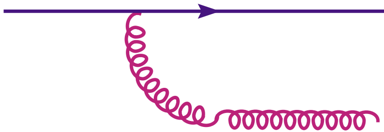
  ▷ they naturally overlap in longitudinal direction ...

  ▷ but what about their overlap in the transverse plane ?



- numerous enough : large density per unit $\perp$ area $\propto A^{1/3} \simeq 6$
- large enough : relatively small $k_\perp$
- large occupation numbers $\sim 1/\alpha_s$

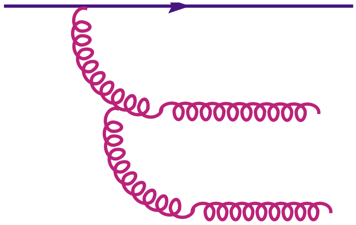
Bremsstrahlung strikes back



$$\mathrm{d}\mathcal{P}_{\rm Brem} \simeq \frac{\alpha_s C_R}{\pi^2}\,\frac{\mathrm{d}^2k_\perp}{k_\perp^2}\,\frac{\mathrm{d}x}{x} \propto \alpha_s \frac{\mathrm{d}x}{x} = \alpha_s\,\mathrm{d}Y$$

- $Y \equiv \ln(1/x) = \eta_{\rm quark} - \eta_{\rm gluon}$:
  rapidity difference between the parent quark and the emitted gluon
- A probability of $\mathcal{O}(\alpha_s)$ to emit one gluon per unit rapidity
- If $\alpha_s Y \sim 1$, the emitted gluon can in turn emit an even softer one
- The origin of the 'BFKL cascades' (high energy evolution in QCD)

Two gluons



- The 'cost' of the additional gluon :

$$\alpha_s \int_x^1 \frac{dx_1}{x_1} = \alpha_s \ln\frac{1}{x} = \alpha_s Y$$

- Formally, a process of higher order in α_s, but which is enhanced by the large available rapidity interval
- When $\alpha_s Y \gtrsim 1 \Longrightarrow$ need for resummation !

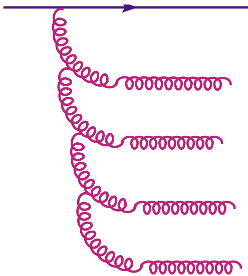
Gluon cascades

- n gluons strictly ordered in x

  $x \ll x_n \ll x_{n-1} \cdots \ll x_1 \ll 1$

- The n–gluon cascade contributes

$$\frac{1}{n!}\,(\alpha_s Y)^n$$



- Gluons are strongly ordered also in their lifetimes :

$$\Delta t \;\sim\; \frac{\hbar}{\Delta E} \;=\; \frac{2k_z}{k_\perp^2} \;=\; \frac{2xP}{k_\perp^2}$$

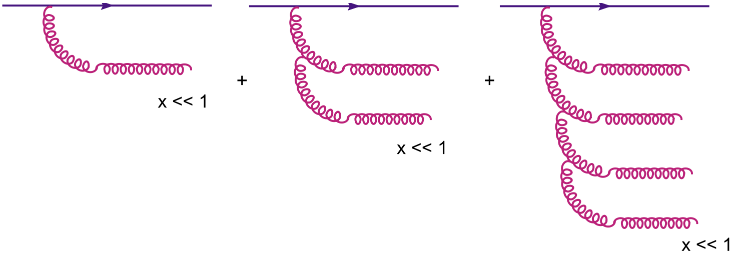
▷ the smaller x, the shorter the lifetime ! (Lorentz time dilation)

- During its short lifetime, the gluon at x overlaps with all its parent gluons at $x' \gg x$, which appear to it as frozen in some random configuration

- The sum of all the cascades exponentiates :

$$\sum_n \frac{1}{n!}\,(\alpha_s Y)^n \;\propto\; \mathrm{e}^{\omega\alpha_s Y} \;\sim\; \frac{1}{x^{\omega\alpha_s}}$$

- BFKL really applies to the unintegrated gluon distribution



$$\frac{\mathrm{d}N_{\rm gluon}}{\mathrm{d}Y\,\mathrm{d}k_\perp^2} \;\simeq\; \frac{\alpha_s C_F}{\pi}\,\frac{1}{k_\perp^2}\,\mathrm{e}^{\omega\alpha_s Y}$$

BFKL evolution *(Balitsky, Fadin, Kuraev, Lipatov, 75–78)*

- The sum of all the cascades exponentiates :

$$\sum_n \frac{1}{n!}(\alpha_s Y)^n \propto e^{\omega \alpha_s Y} \sim \frac{1}{x^{\omega \alpha_s}}$$

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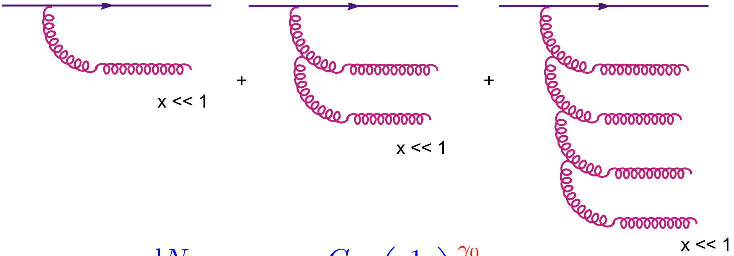
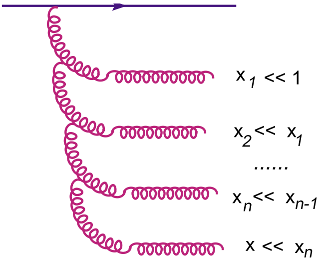
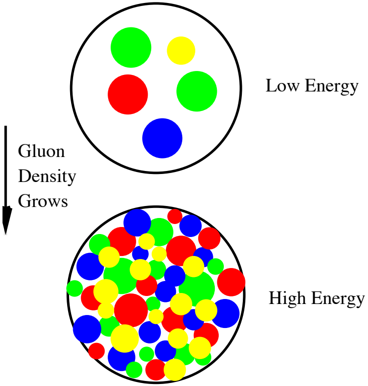


$$\frac{dN_{\rm gluon}}{dY\, dk_\perp^2} \simeq \frac{\alpha_s C_F}{\pi}\left(\frac{1}{k_\perp^2}\right)^{\gamma_0} e^{\omega_0 \alpha_s Y}$$

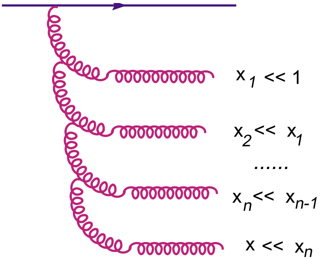
- $\gamma_0 = 1/2$: 'BFKL anomalous dimension'
- $\omega_0 = 4 \ln 2 (N_c/\pi)$: 'BFKL Pomeron intercept'

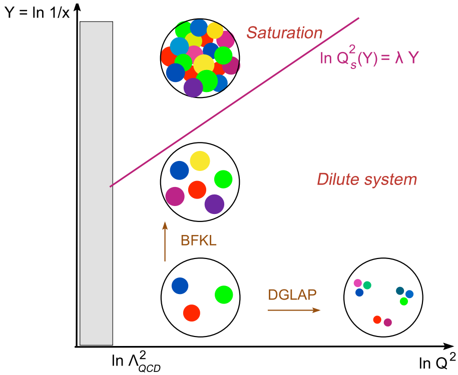




- BFKL: an evolution towards increasing density

Gluon evolution at small x





- BFKL: an evolution towards increasing density
- Non–trivial: not true for the DGLAP evolution !
  - the BFKL gluons have similar transverse momenta, hence similar transverse areas $\Longrightarrow$ they can overlap with each other
- The relevant quantity: not the gluon number, but ...

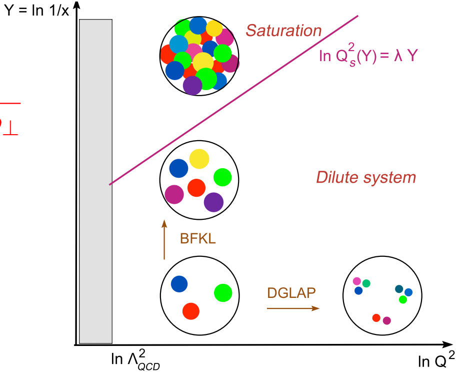
Color Glass Condensate

- The gluon occupation number (or 'packing factor')

$$n(x, \boldsymbol{k}_\perp) \equiv \frac{(2\pi)^3}{2(N_c^2-1)} \frac{\mathrm{d}N_{\rm gluon}}{\mathrm{d}Y\, \mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}^2\boldsymbol{b}_\perp}$$

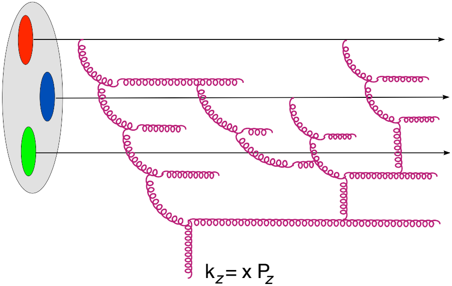
▷ $\boldsymbol{b}_\perp$: impact parameter in $\perp$ plane

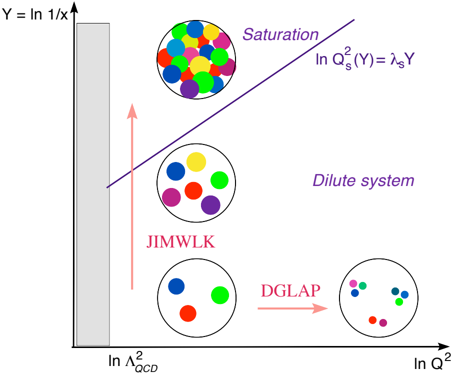
$$n(x, Q^2) \simeq \frac{\pi}{Q^2} \times \frac{xG(x,Q^2)}{\pi R^2}$$



- When $n \gtrsim 1$: gluons overlap, so they are coherent with each other
  - semi–classical description as a strong color field A_a^i : 'condensate'
  - during the scattering, they are frozen by Lorentz dilation, but randomly distributed due to quantum fluctuations: 'glass'

Gluon saturation





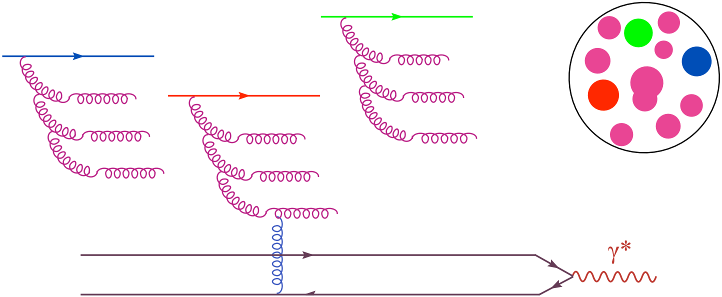
- $\alpha_s n \sim 1$: strong overlapping which compensates small coupling
- The evolution becomes non–linear :

  ▷ emissions + recombination $\Longrightarrow$ gluon saturation
- BFKL gets replaced by the non–linear Balitsky–JIMWLK equations

  Jalilian-Marian, Iancu, McLerran, Weigert, Leonidov, Kovner (97–00)

A cartoon of the evolution equations : BFKL

- $n(Y, Q^2)$: gluon occupation number
- Rapidity increment $Y \to Y + \mathrm{d}Y$: a probability $\alpha_s \mathrm{d}Y$ to emit an additional gluon out of any of the preexisting ones



$$\frac{\partial n}{\partial Y} \simeq \alpha_s n \quad \Longrightarrow \quad n(Y) \propto \mathrm{e}^{\omega \alpha_s Y}$$

- Valid so long as $n(Y, Q^2) \ll 1/\alpha_s$ (dilute system)

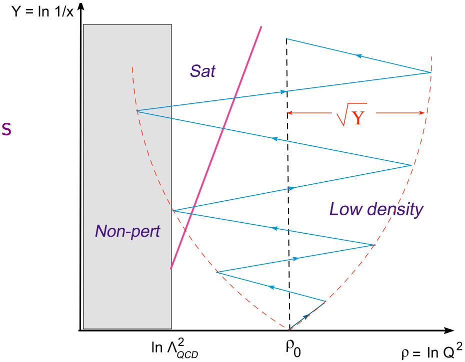
Conceptual difficulties

- Unitarity violation: $T \sim \alpha_s n$ cannot exceed 1
- Infrared diffusion : excursion through soft ($\sim \Lambda_{\rm QCD}$) momenta

  - The gluon emission vertex is non–local in $k_\perp$:

$$\partial_Y n = \alpha_s n + \alpha_s \partial_\rho^2 n$$

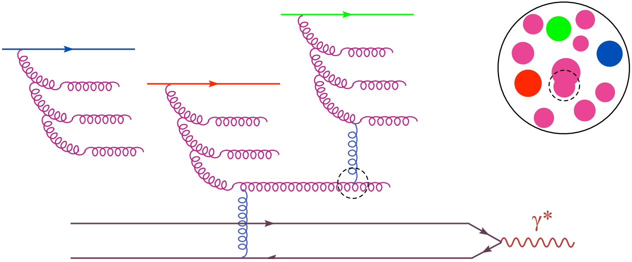
$\Longrightarrow$ diffusion in $\rho \equiv \ln k_\perp^2$



- Both problems are solved by gluon saturation

BK/JIMWLK

- High gluon density: recombination processes leading to saturation



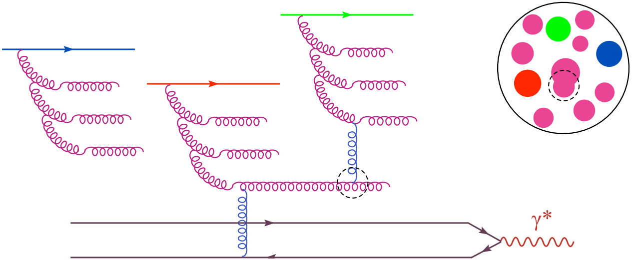
$$\frac{\partial n}{\partial Y} \simeq \alpha_s n - \alpha_s^2 n^2 = 0 \quad \text{when} \quad n \sim \frac{1}{\alpha_s} \gg 1$$

- Fixed point : the evolution stops when $\alpha_s n(Y, Q^2) \sim 1$
- The saturation condition involves Y and Q^2

  $\Longrightarrow$ saturation momentum $Q_s(Y)$

A classical stochastic process

$$\partial_Y n(\rho, Y) = \alpha_s \partial_\rho^2 n(\rho, Y) + \alpha_s n(\rho, Y) - \alpha_s^2 n^2(\rho, Y)$$



- Cartoon version of the Balitsky–Kovchegov equation
- FKPP equation for the 'reaction–diffusion' process ($A \rightleftharpoons 2A$)
  (Munier, Peschanski, 03; Iancu, Mueller, Munier 04; Pomeron loops ...)
- Mean field approximation (large–N_c) to the B–JIMWLK equations
- Known to next-to-leading-log accuracy (consistent with NLO BFKL)

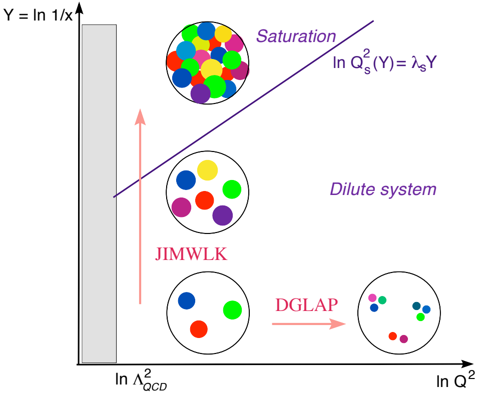
The saturation momentum

- The transverse momentum where saturation starts to be important

$$n(x, Q^2) = \frac{\pi}{Q^2} \times \frac{xG(x, Q^2)}{\pi R^2}$$

$$n(x, Q_s^2(x)) \sim \frac{1}{\alpha_s}$$

$$Q_s^2(x) \simeq \alpha_s \frac{xG(x, Q_s^2)}{\pi R^2} \sim \frac{1}{x^{\lambda_s}}$$



- Q_s is rapidly rising with $1/x$, i.e. with the center-of-mass energy :

$\lambda_s \simeq 0.2 \div 0.3$ at NLO accuracy *(Triantafyllopoulos, 2003)*

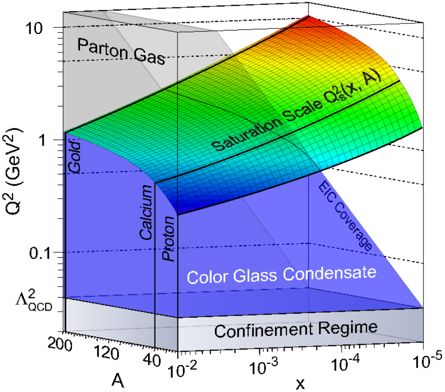
▷ the actual 'Pomeron intercept' in the presence of saturation

- The transverse momentum where saturation starts to be important

$$n(x,Q^2) = \frac{\pi}{Q^2} \times \frac{xG(x,Q^2)}{\pi R^2}$$

$$n\big(x,Q_s^2(x)\big) \sim \frac{1}{\alpha_s}$$

$$Q_s^2(x) \simeq \alpha_s \frac{xG_A(x,Q_s^2)}{\pi R_A^2} \sim \frac{A^{1/3}}{x^{\lambda_s}}$$



- ... and also with the atomic number A for a large nucleus ($A \gg 1$)

▷ $A^{1/3} \simeq 6$ for pA and AA collisions at RHIC and the LHC

The saturation momentum

- The transverse momentum where saturation starts to be important

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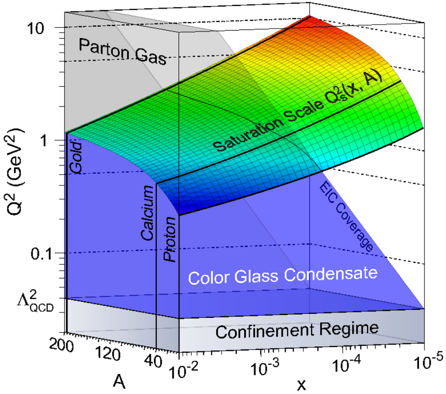

- $x \sim 10^{-5}$: $Q_s \sim 1$ GeV for proton and ~ 3 GeV for Pb or Au

  ▷ a semi–hard scale, at which perturbation theory is marginally valid

Gluon distribution & geometric scaling

- $Q_s^2(x) \propto$ the gluon density per unit transverse area
- $Q_s(x)$: the typical transverse momentum of the gluons with a given x

$$xG(x,Q^2) \,=\, \int \mathrm{d}^2 b_\perp \int^Q \mathrm{d}k_\perp \, k_\perp \, n(x, b_\perp, k_\perp)$$

$$n(Y,k_\perp) = \begin{cases} \dfrac{1}{\alpha_s} \ln \dfrac{Q_s^2(Y)}{k_\perp^2} & \text{for } \; k_\perp < Q_s(Y) \\ \\ \dfrac{1}{\alpha_s} \left(\dfrac{Q_s^2(Y)}{k_\perp^2} \right)^{\gamma_s} & \text{for } \; k_\perp > Q_s(Y) \end{cases}$$

- $\gamma_s \simeq 0.63$: anomalous dimension at saturation
- Geometric scaling : $n(Y,k_\perp) = F\big(k_\perp / Q_s(Y)\big)$

(Iancu, Itakura, McLerran; Mueller, Triantafyllopoulos; Munier, Peschanski, 02-03)

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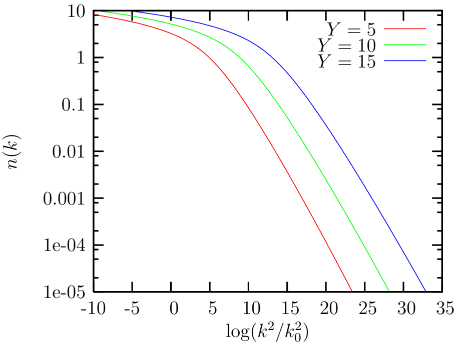
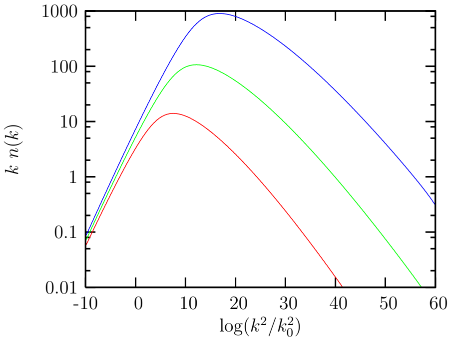




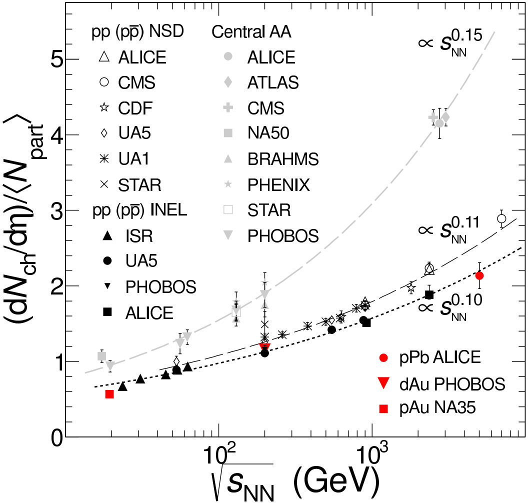
Multiplicity : energy dependence

- pp, pA, AA : the saturated gluons are released in the final state
- Particle multiplicity $\mathrm{d}N/\mathrm{d}\eta \propto xG(x, Q_s^2) \propto Q_s^2(x) \sim s^{\lambda_s/2}$

$$x \simeq \frac{k_\perp}{\sqrt{s}}$$

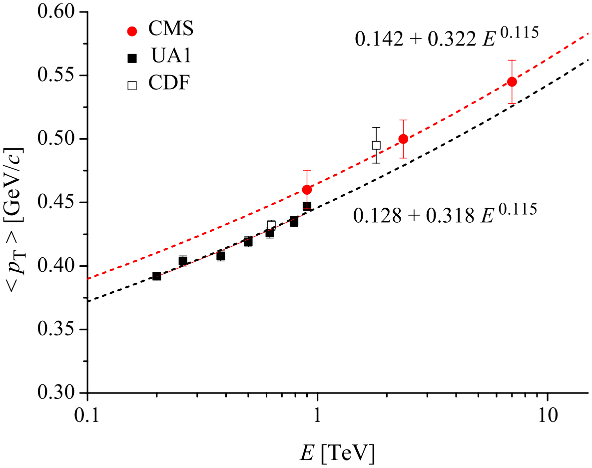
$$k_\perp \sim Q_s$$

$$\lambda_s \simeq 0.2 \div 0.3$$



Average transverse momentum in p+p

- Typical transverse momentum $\langle p_T \rangle \propto Q_s(x) \sim E^{\lambda_s/2}$ $(E \equiv \sqrt{s})$

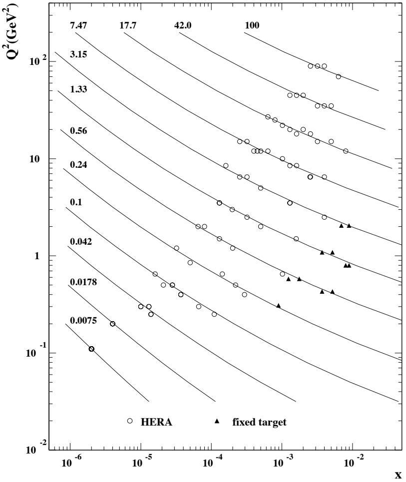


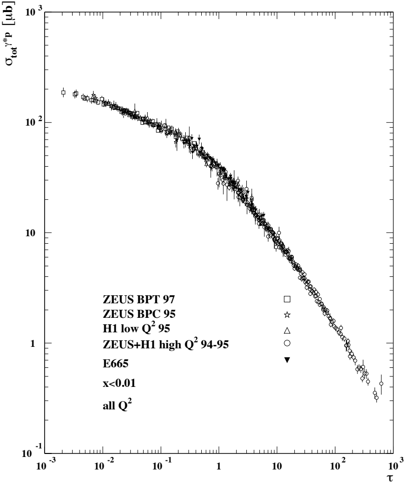
(McLerran and Praszalowicz, 2010)

Geometric scaling at HERA: F_2

- DIS cross–section at HERA *(Staśto, Golec-Biernat, Kwieciński, 2000)*

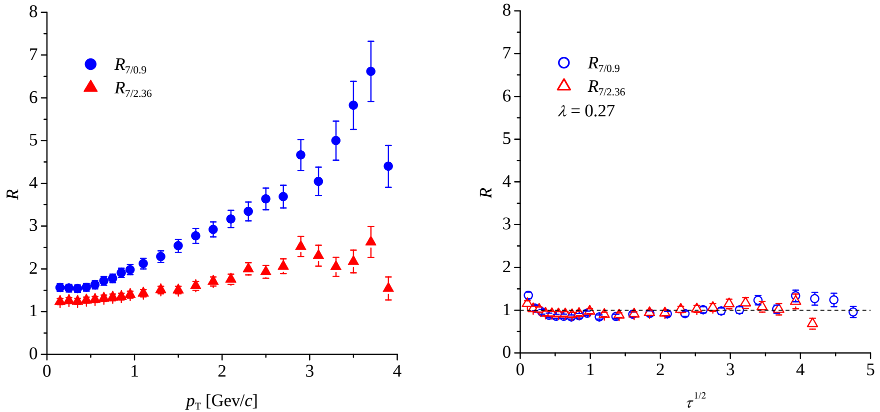
$$\sigma(x,Q^2)\ \text{vs.}\ \tau \equiv Q^2/Q_s^2(x) \propto Q^2/x^{0.3} \ : \ x \le 0.01, \quad Q^2 \le 450\ \text{GeV}^2$$



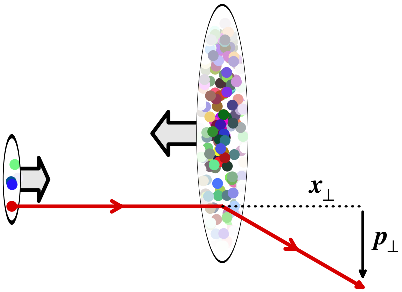


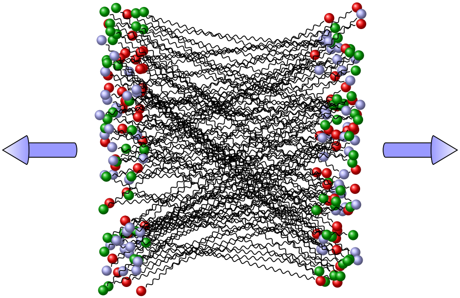
- Ratio between particle production at 2 different energies, s_1 and s_2

$$R_{s_1/s_2} = \frac{(\mathrm{d}N/\mathrm{d}\eta\,\mathrm{d}^2p_\perp)\big|_{s_1}}{(\mathrm{d}N/\mathrm{d}\eta\,\mathrm{d}^2p_\perp)\big|_{s_2}} \to 1 \text{ as a function of } \tau \equiv \frac{p_\perp^2}{Q_s^2(p_\perp/\sqrt{s})}$$

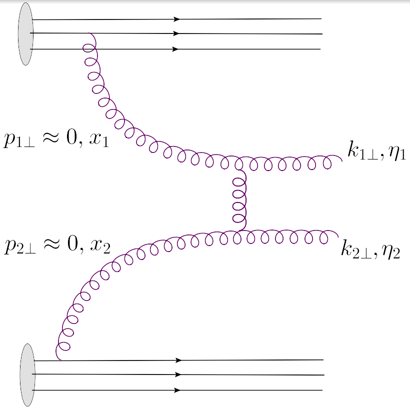


Particle production





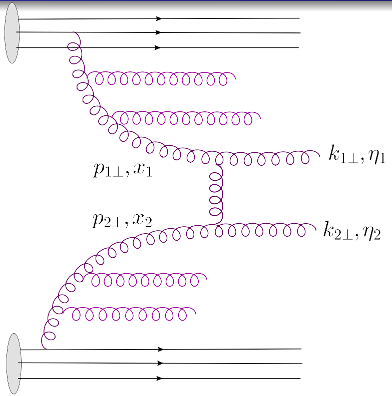
From collinear factorization ...



$$\frac{d\sigma}{d^2k_{1\perp}d^2k_{2\perp}d\eta_1 d\eta_2} = x_1G(x_1, Q^2)\, x_2G(x_2, Q^2)\, \delta^{(2)}(\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp})\frac{d\hat{\sigma}}{dk_\perp^2}$$

$$xG(x, Q^2) = \int^Q d^2\boldsymbol{p}\; \Phi(x, \boldsymbol{p}_\perp)\,, \quad \Phi(x, \boldsymbol{p}_\perp) \equiv x\,\frac{dN_{\rm gluon}}{dx d^2\boldsymbol{p}_\perp}$$

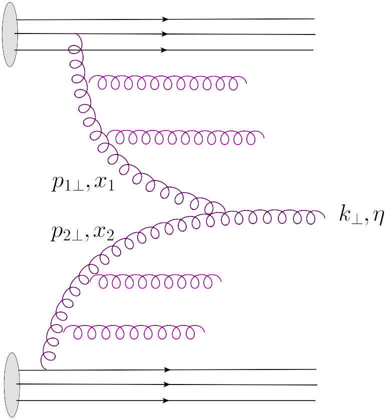
- Assumes $p_{1\perp},\, p_{2\perp} \approx \Lambda_{\rm QCD} \ll k_\perp$



- In reality $p_{1\perp}$, $p_{2\perp}$ $\sim$ Q_s can be comparable with $k_{i\perp}$

$$\frac{d\sigma}{d^2k_{1\perp}d^2k_{2\perp}d\eta_1 d\eta_2} = \int d^2\boldsymbol{p}_{1\perp} \int d^2\boldsymbol{p}_{2\perp}\ \delta^{(2)}(\boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp} - \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp})$$
$$\times\ \Phi(x_1, \boldsymbol{p}_{1\perp})\, \frac{d\hat{\sigma}}{dk_{1\perp}^2} \Phi(x_2, \boldsymbol{p}_{2\perp})$$

- Consistent with the BFKL evolution *(Catani and Hautmann, 1994)*



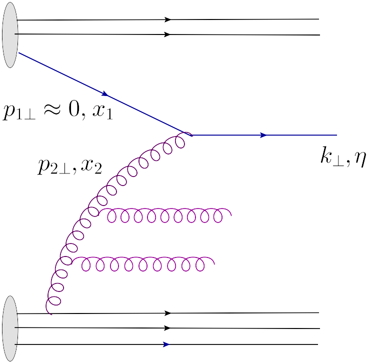
$$x_1 \sim \frac{k_\perp}{\sqrt{s}}\,\mathrm{e}^{\eta}$$

$$x_2 \sim \frac{k_\perp}{\sqrt{s}}\,\mathrm{e}^{-\eta}$$

- A parton with $k_\perp \lesssim Q_s$ can also be produced via the fusion of 2 initial partons : $gg \to g,\ qg \to q$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta} \simeq \frac{\alpha_s}{k_\perp^2}\int \mathrm{d}^2\boldsymbol{p}_\perp \Phi(x_1, \boldsymbol{p}_\perp)\,\Phi(x_2, \boldsymbol{k}_\perp - \boldsymbol{p}_\perp)$$

Forward quark production



$$x_{1,2} \sim \frac{k_\perp}{\sqrt{s}} \mathrm{e}^{\pm\eta}$$

$$\eta \sim 3 \div 4$$

$$\frac{x_1}{x_2} = \mathrm{e}^{2\eta} \sim 400 \div 3000$$

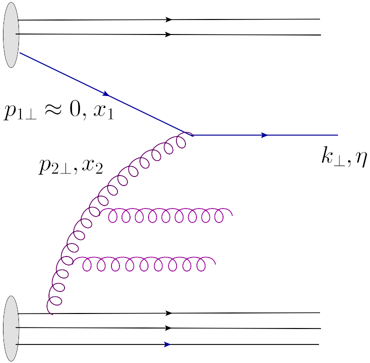
$$\text{e.g. } x_1 = 0.2 \ \& \ x_2 = 10^{-4}$$

- $p_{1\perp} \sim \Lambda_{\rm QCD} \ll k_\perp \sim Q_s(x_2) \Longrightarrow$ hybrid factorization

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta} \simeq \frac{\alpha_s}{k_\perp^2} x_1 q(x_1, k_\perp^2) \Phi(x_2, \boldsymbol{k}_\perp)$$

- So far though only linear evolution (BFKL) : no saturation effects

Forward quark production



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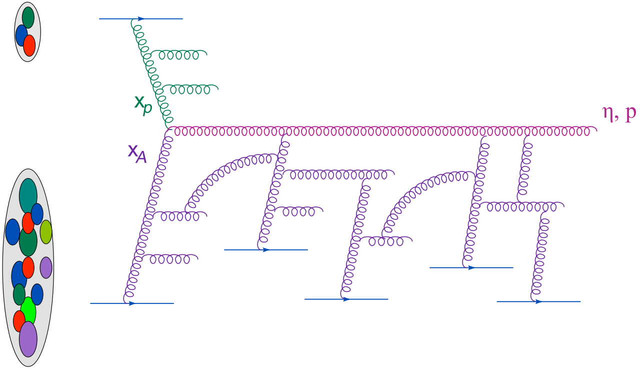
- $p_{1\perp} \sim \Lambda_{\rm QCD} \ll k_\perp \sim Q_s(x_2) \Longrightarrow$ hybrid factorization

$$\frac{\mathrm{d}N}{\mathrm{d}\eta} = \int \mathrm{d}^2\boldsymbol{k}_\perp \frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{k}_\perp \mathrm{d}\eta} \propto \int \frac{\mathrm{d}k_\perp^2}{k_\perp^4}$$

- Without saturation $\Phi(x_2, \boldsymbol{k}_\perp) \propto 1/k_\perp^2 \Longrightarrow \mathrm{d}N/\mathrm{d}\eta$ is divergent !

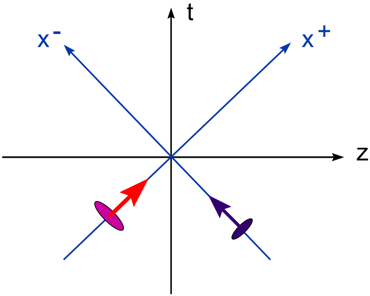
Towards CGC factorization



- k_T–factorization is not consistent with saturation/JIMWLK evolution
  - assumes single scattering (like collinear fact.) : "leading–twist"
  - unintegrated gluon distribution : a 2–point function
  - multiple scattering probes higher–points of the gluon distribution
  - multiple scattering and saturation are mixed under the evolution

Light–cone variables

- How to compute multiple scattering in QCD at high energy ?
- Convenient to use light–cone coordinates and momenta



$$x^{\pm} = \frac{1}{\sqrt{2}}(t \pm z)$$

$$p^{\pm} = \frac{1}{\sqrt{2}}(p_0 \pm p_z)$$

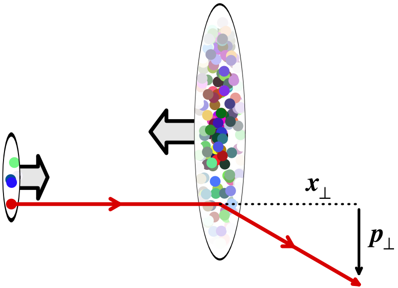
$$p^{\mu} = (p^+, p^-, \boldsymbol{p}_{\perp})$$

$$p \cdot x = p^+ x^- + p^- x^+ - \boldsymbol{p}_{\perp} \cdot \boldsymbol{x}_{\perp}$$

- Ultrarelativistic right mover :
  - $z \simeq t \implies x^- \simeq 0$ (Lorentz contraction) & $x^+ \simeq \sqrt{2}t$ (LC time)
  - $p_z \simeq p_0 \equiv E \implies p^{\mu} \simeq (p^+, 0, \boldsymbol{0}_{\perp})$ with $p^+ = \sqrt{2}E$
- Left mover: the roles of x^+ and x^- (or p^+ and p^-) get interchanged

Multiple scattering in pA : **Wilson lines**

- A quark (or gluon) from the proton scatters off the dense gluon distribution inside the nucleus: $\mathcal{L}_{\rm int}(x) = j_a^\mu(x) A_\mu^a(x)$



- the quark color current density

$$j_a^\mu(x) = g\bar{\psi}(x)\gamma^\mu t^a \psi(x)$$

- the quark S–matrix operator :

$$\hat{S} = \mathrm{T}\, \mathrm{e}^{\mathrm{i}\int \mathrm{d}^4 x\, \mathcal{L}_{\rm int}(x)}$$

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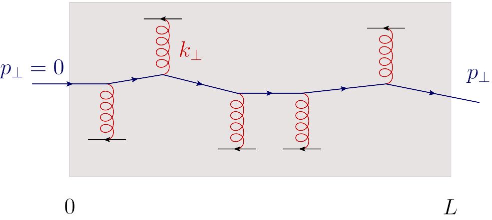

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- View the process in the nucleus rest frame : the quark is very energetic
- Quark energy $E \gg$ typical $k_\perp \lesssim Q_s \Longrightarrow$ small deflection angle $\theta \ll 1$

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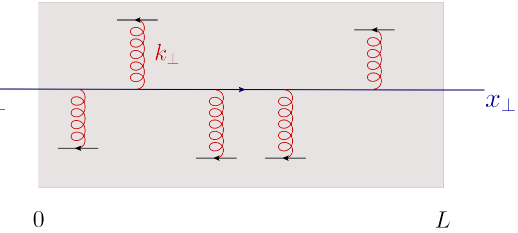

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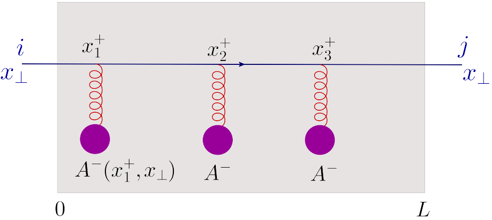
- View the process in the nucleus rest frame : the quark is very energetic
- Quark energy $E \gg$ typical $k_\perp \lesssim Q_s \Longrightarrow$ small deflection angle $\theta \ll 1$
- The quark transverse position is unchanged: eikonal approximation

$$j^\mu_a(x) \simeq \delta^{\mu +} g t^a \delta(x^-)\delta^{(2)}(\boldsymbol{x}-\boldsymbol{x}_0)$$

Multiple scattering in pA : Wilson lines

- A quark (or gluon) from the proton scatters off the dense gluon distribution inside the nucleus: $\mathcal{L}_{\rm int}(x) \,=\, j^\mu_a(x) A^a_\mu(x)$



- the quark color current density

$$j^\mu_a(x) \,=\, g\bar\psi(x)\gamma^\mu t^a \psi(x)$$

- the quark S–matrix operator :

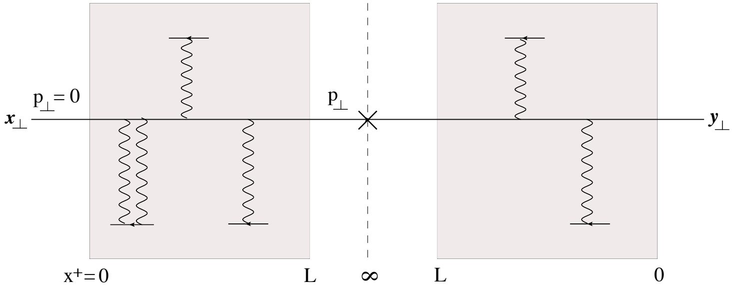
$$\hat S \,=\, {\rm T}\, {\rm e}^{{\rm i}\int {\rm d}^4x\, \mathcal{L}_{\rm int}(x)}$$

- View the process in the nucleus rest frame : the quark is very energetic
- Quark energy $E \gg$ typical $k_\perp \lesssim Q_s \Longrightarrow$ small deflection angle $\theta \ll 1$
- The S–matrix reduces to a Wilson line (color rotation)

$$\Psi_i(x_\perp) \,\to\, V_{ji}(x_\perp)\, \Psi_i(x_\perp), \quad V(x_\perp) = {\rm T} \exp\left\{ i \int {\rm d}x^+ A^-_a(x^+, x_\perp) t^a \right\}$$

Dipole picture

- The $p_\perp$–spectrum of the quark after crossing the medium $(\boldsymbol{r} = \boldsymbol{x} - \boldsymbol{y})$



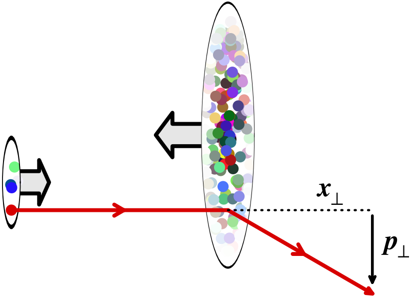
$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}} = \int \frac{\mathrm{d}^2\boldsymbol{r}}{(2\pi)^2}\, \mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}} \langle S_{\boldsymbol{xy}} \rangle\,, \qquad S_{\boldsymbol{xy}} \equiv \frac{1}{N_c}\,\mathrm{tr}\big(V_{\boldsymbol{x}} V^\dagger_{\boldsymbol{y}}\big)$$

▷ sum over the final color indices, average over the initial ones

▷ average over the distribution of the medium field A_a^-

- S–matrix for effective color dipole: $q\bar{q}$ pair in a color singlet state

CGC factorization for 'dilute–dense' (pA, forward pp)



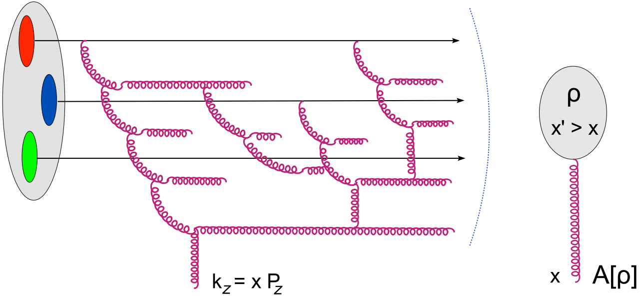
$$x_p = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{\eta}$$

$$x_A = \frac{p_\perp}{\sqrt{s}}\,\mathrm{e}^{-\eta}$$

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}\mathrm{d}\eta} = x_p\, q(x_p, Q^2) \int \frac{\mathrm{d}^2\boldsymbol{r}}{(2\pi)^2}\, \mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}} \langle S(\boldsymbol{r})\rangle_{x_A}$$

- The Fourier transform of the dipole S–matrix plays the role of a generalized unintegrated gluon distribution
  - includes multiple scattering in the eikonal approximation...
  - ... and saturation via the CGC average over the target wavefunction
  - it evolves according to BK equation (special case of JIMWLK)

The CGC weight function



- An effective theory for the small–x gluons:

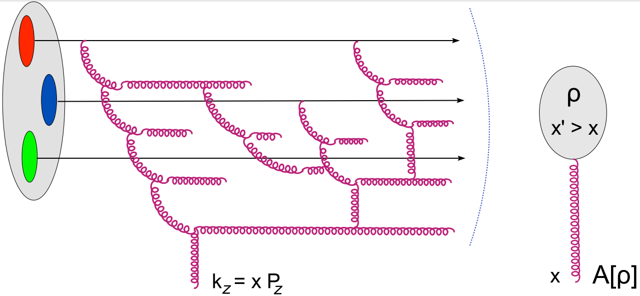
  ▷ classical color fields A_a^- radiated by randomly distributed color charges

$$D_\nu^{ab} F_b^{\nu\mu}(x) = \delta^{\mu -} \rho^a(x^+, \boldsymbol{x}_\perp) \qquad (D_\nu^{ab} = \partial_\nu^{ab} - g f^{abc} A_\nu^c)$$

- $W_Y[A]$: functional probability distribution for the color fields/charges

  ▷ information about all the n–point gluon correlations with $n \geq 2$

$$\langle S_{\boldsymbol{xy}} \rangle_Y = \int [\mathcal{D}A]\, W_Y[A]\, \frac{1}{N_c} \operatorname{tr}\big(V_{\boldsymbol{x}} V_{\boldsymbol{y}}^\dagger\big)[A]$$



- Functional evolution equation for $W_Y[A]$ with increasing $Y = \ln 1/x$

$$\frac{\partial}{\partial Y} W_Y[A] \;=\; H\, W_Y[A] \qquad \text{(JIMWLK)}$$

- Equivalent to an infinite hierarchy of non–linear equations for the correlations of products of Wilson lines *(Balitsky, 96)*

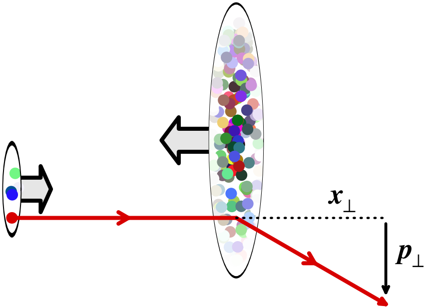
  ▷ a complete basis of high–energy S–matrices for dilute projectiles

$$S_{\boldsymbol{x}_1\boldsymbol{x}_2\cdots\boldsymbol{x}_{2n}} \;=\; \frac{1}{N_c}\,\mathrm{tr}\left(V_{\boldsymbol{x}_1}V^\dagger_{\boldsymbol{x}_2}\cdots V_{\boldsymbol{x}_{2n-1}}V^\dagger_{\boldsymbol{x}_{2n}}\right)$$

The nuclear modification factors

- Particle production in pA divided by particle production in pp scaled up by the number of independent binary collisions $A^{1/3}$

  ▷ the proton scatters off the $A^{1/3}$ nucleons at its own impact parameter



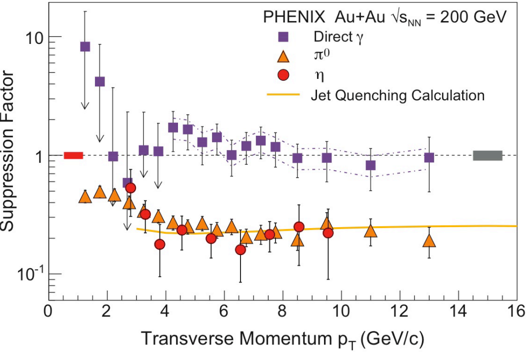
$$R_{pA} \equiv \frac{1}{A^{1/3}} \frac{\mathrm{d}N_{pA}/\mathrm{d}^2p_\perp \mathrm{d}\eta}{\mathrm{d}N_{pp}/\mathrm{d}^2p_\perp \mathrm{d}\eta}$$

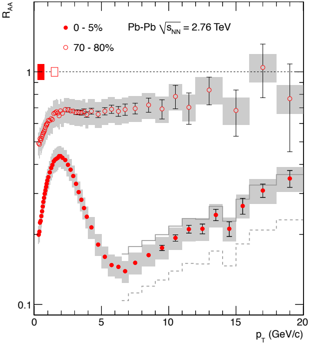
$$R_{AA} \equiv \frac{1}{A^{4/3}} \frac{\mathrm{d}N_{AA}/\mathrm{d}^2p_\perp \mathrm{d}\eta}{\mathrm{d}N_{pp}/\mathrm{d}^2p_\perp \mathrm{d}\eta}$$

- An important test of nuclear effects at both RHIC and the LHC

  ▷ $R_{AA} = 1$ if AA = incoherent superposition of pp collisions

- ... but for hadrons it is significantly different from 1 !



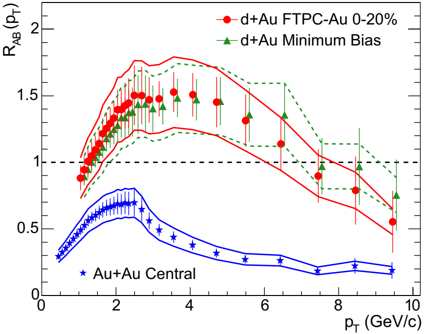


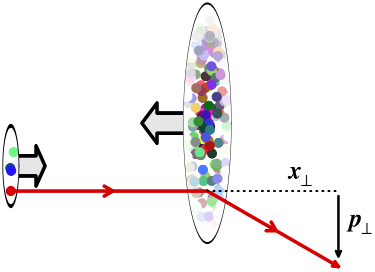
- As expected, $R_{AA} = 1$ for direct photons : no strong interactions
- Strong suppression for hadrons in AA (RHIC and LHC): $R_{AA} < 0.3$
- Possible explanations :
  - initial state effects: saturation in the nuclear wavefunctions
  - final state effects: interactions in the fireball created by the collision

The 'pA benchmark' : d+Au at RHIC

$$R_{\mathrm{d+Au}} \equiv \frac{1}{2A^{1/3}} \frac{\mathrm{d}N_{\mathrm{d+Au}}/\mathrm{d}^2p_\perp \mathrm{d}\eta}{\mathrm{d}N_{\mathrm{p+p}}/\mathrm{d}^2p_\perp \mathrm{d}\eta}$$

- One expects no fireball in d+Au $\Longrightarrow$ no final state interactions



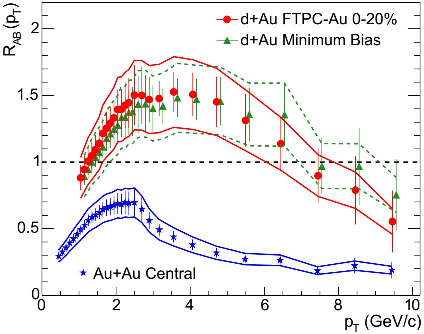


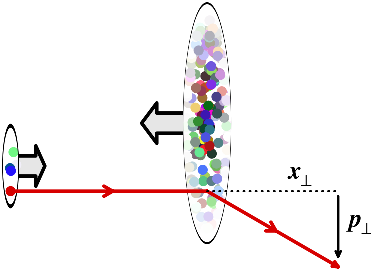
- No suppression, rather an enhancement at $\eta \simeq 0$: 'Cronin peak'
- The suppression seen in R_{AA} is a final state effect

The 'pA benchmark' : d+Au at RHIC

$$R_{\mathrm{d+Au}} \equiv \frac{1}{2A^{1/3}} \frac{\mathrm{d}N_{\mathrm{d+Au}}/\mathrm{d}^2p_\perp \mathrm{d}\eta}{\mathrm{d}N_{\mathrm{p+p}}/\mathrm{d}^2p_\perp \mathrm{d}\eta}$$

- One expects no fireball in d+Au $\Longrightarrow$ no final state interactions



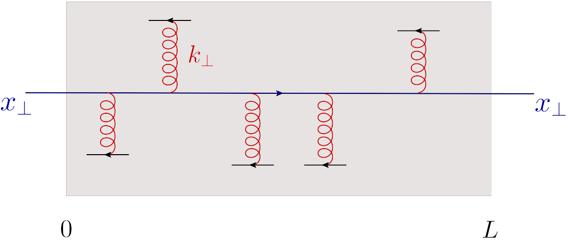


- No suppression, rather an enhancement at $\eta \simeq 0$: 'Cronin peak'
- Can one understand the Cronin peak within the CGC ?

Midrapidity: the Cronin peak

$$\frac{\mathrm{d}N}{\mathrm{d}^2\boldsymbol{p}\mathrm{d}\eta} = x_1 q(x_1, Q^2) \int \mathrm{d}^2\boldsymbol{r}\ \mathrm{e}^{-i\boldsymbol{p}\cdot\boldsymbol{r}} \langle S(\boldsymbol{r})\rangle_{x_2}$$

- d+Au collisions at RHIC: $\sqrt{s} = 200$ GeV, $p_\perp \sim 2$ GeV and $\eta \approx 0$
  - $x_1 = x_2 = 0.01 \Longrightarrow$ the proton is still dilute
  - nucleus : a collection of uncorrelated valence quarks (MV model)
  - independent scatterings off the valence quarks $\Longrightarrow$ random walk in $p_\perp$



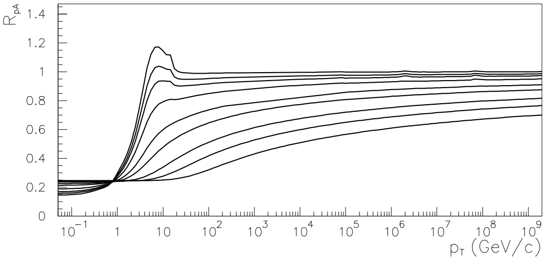
$$\tilde{S}_A(x_2, p_\perp) \simeq \frac{1}{\pi Q_s^2}\,\mathrm{e}^{-\frac{p_\perp^2}{Q_s^2}}$$

$$\langle p_\perp^2\rangle = Q_s^2(A) = A^{1/3} Q_0^2$$

  - the distribution in $p_\perp$ gets shifted towards harder values $\sim Q_s(A)$
- No such a shift in pp collisions $\Longrightarrow$ Cronin peak in R_{pA}

Forward rapidities: R_{pA} suppression

- Increasing $\eta > 0 \iff$ high–energy evolution in the target (p or A)
- Use BK equation for the evolution of the dipole S–matrix



$$R_{pA} = \frac{1}{A^{1/3}} \frac{\tilde{S}_A(x_2, p_\perp)}{\tilde{S}_p(x_2, p_\perp)}$$

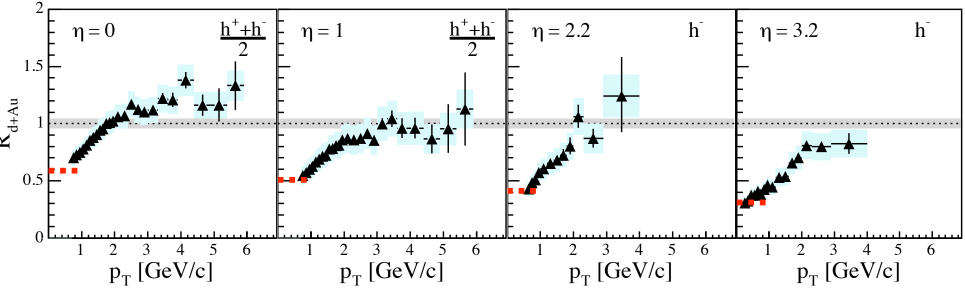
$$x_2 = \frac{p_\perp}{\sqrt{s}} \, \mathrm{e}^{-\eta}$$

η = 0, 0.05, 0.1, 0.2, 0.4, 0.6, 1, 1.4 and 2 *(BK equation: Albacete et al, 2003)*

- Rapid evolution with η : no Cronin peak for $\eta \gtrsim 0.4$
  - for $p_\perp \lesssim Q_s(A, x_A)$, the nucleus is already saturated $\Rightarrow$ no evolution
  - for $p_\perp \sim Q_s(A, x_A)$, the proton is still dilute $\Rightarrow$ rapid evolution
- The denominator (p) grows much faster than the numerator (A)

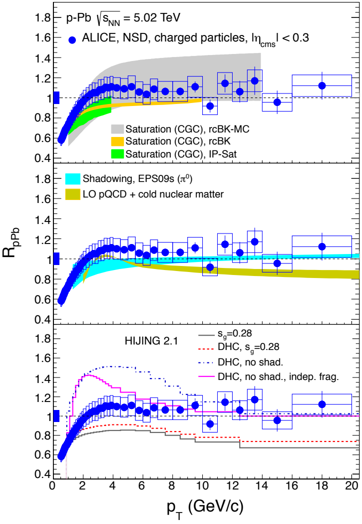
$R_{\rm d+Au}$ at RHIC : increasing η

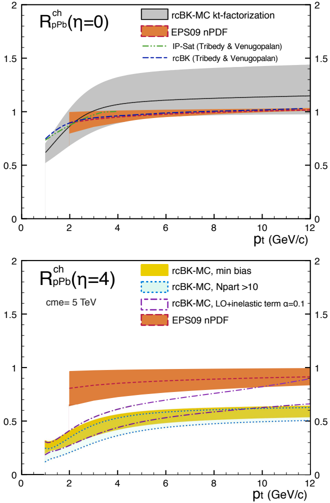
- This is in good agreement with the RHIC data (BRAHMS)



- The Cronin peak disappears already after one unit of rapidity !
- LHC : $x_1 \sim x_2 \sim 10^{-3}$ for $\eta = 0$
  - high-energy evolution is important already at mid rapidity
  - competition between multiple scattering in the nucleus and rapid evolution of the proton

R_{p+Pb} at the LHC for central rapidities

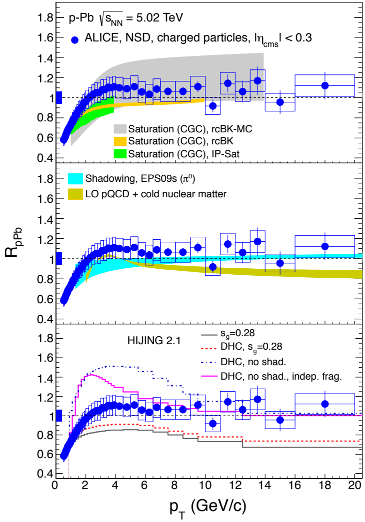


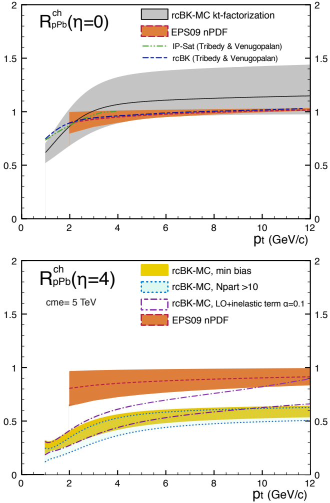


- No Cronin peak ... in agreement with the CGC expectations

  (Tribedy, Venugopalan; Rezaeian; Albacete, Dumitru, Fujii, Nara, 11-12)

$R_{\mathrm{p+Pb}}$ at the LHC for central rapidities

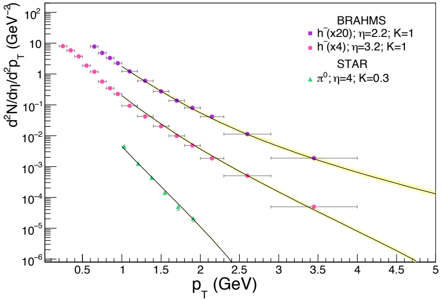


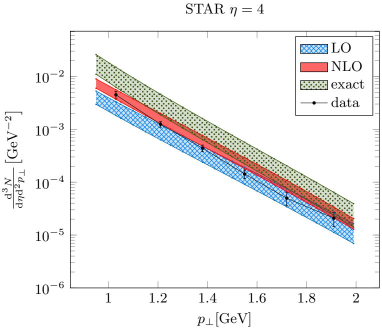


- Various models could be differentiated by going to forward rapidities
- This could be measured e.g. by LHCb (large η & semi–hard $p_\perp$)

Getting quantitative: d+Au at RHIC

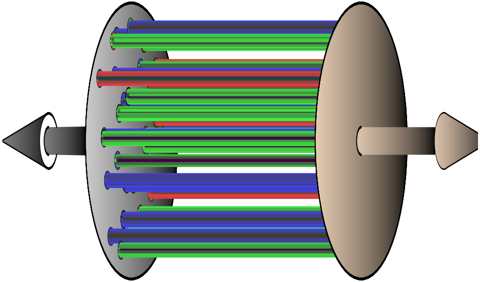
- One free parameter: the nuclear saturation momentum at $x_0 = 10^{-2}$

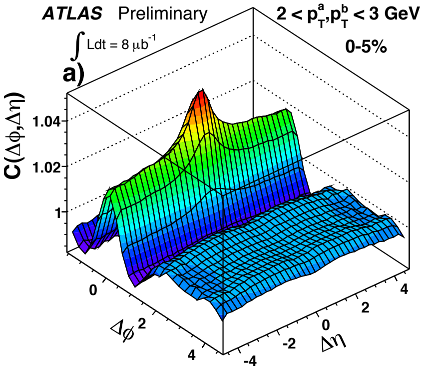




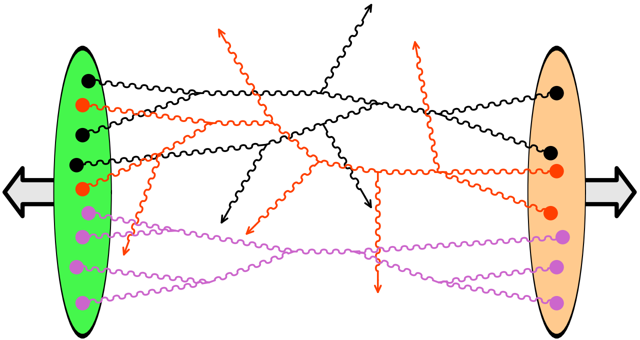
- Left: LO in the CGC *(Albacete and Marquet, 2010)*
  - ▷ *note K–factor $K = 0.3$ for π^0: normalization under predicted*
- Right: LO and NLO in the CGC vs. NLO in pQCD ('exact') *(Stasto, Xiao, Yuan, Zaslavski, 2014)*
  - ▷ *no K–factor anymore; CGC goes right; pQCD over predicts*

AA collisions : Glasma & the Ridge



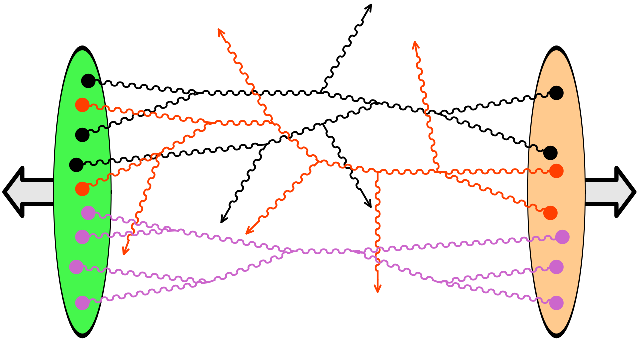


Nucleus–nucleus collisions



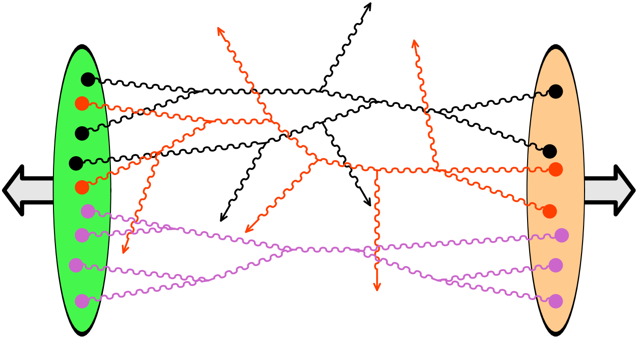
- "Dense–dense scattering" : much more complicated !
- Non–linear effects enter at all levels
  - in both incoming wavefunctions: gluon saturation
  - in the scattering process : multiple interactions
  - in the partonic medium created by the scattering: final–state interactions

Nucleus–nucleus collisions



- "Dense–dense scattering" : much more complicated !
- Non–linear effects enter at all levels
  - 2 CGC weight functions: $W_{Y_1}[\rho_1]$, $W_{Y_2}[\rho_2]$
  - in the scattering process : multiple interactions
  - in the partonic medium created by the scattering: final–state interactions

Nucleus–nucleus collisions



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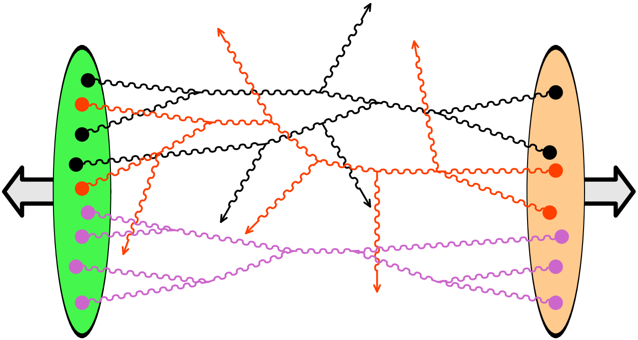
  - in the partonic medium created by the scattering: final–state interactions

Nucleus–nucleus collisions



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- Non–linear effects enter at all levels
  - 2 CGC weight functions: $W_{Y_1}[\rho_1],\ W_{Y_2}[\rho_2]$
  - classical Yang–Mills equations with 2 sources: $\rho_1,\ \rho_2$
  - the CGC provides only the initial state for the subsequent evolution of this partonic matter

The CGC factorization for 'dense–dense'

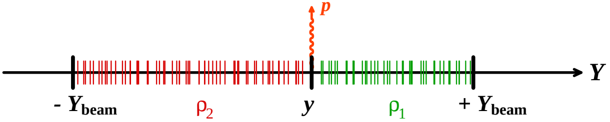
- Numerically solve classical YM equations with 2 sources (2D lattice)

$$D_\nu F^{\nu\mu}(x) = \delta^{\mu+}\rho_1(x) + \delta^{\mu-}\rho_2(x)$$

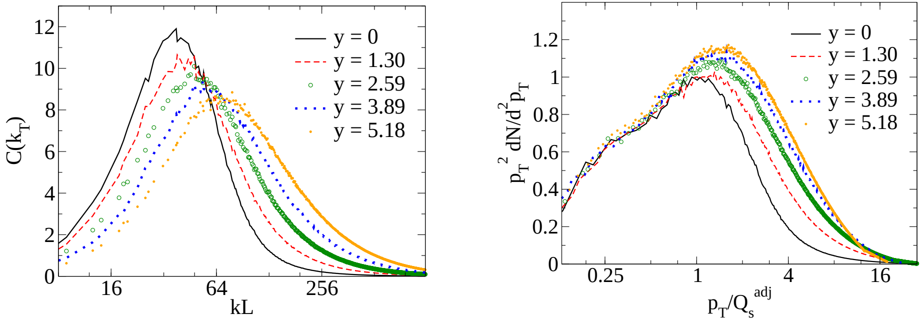
- Decompose the classical field A_a^μ in Fourier modes $\Longrightarrow$ gluon spectrum
- Average over ρ_1 and ρ_2 using the CGC distributions of the nuclei

$$\left\langle \frac{\mathrm{d}N}{\mathrm{d}Y\,\mathrm{d}^2p_\perp} \right\rangle = \int [\mathcal{D}\rho_1 \mathcal{D}\rho_2]\, W_{Y_{\mathrm{beam}-Y}}[\rho_1]\, W_{Y_{\mathrm{beam}+Y}}[\rho_2]\, \frac{\mathrm{d}N}{\mathrm{d}Y\,\mathrm{d}^2p_\perp}\bigg|_{\mathrm{class}}$$

▷ JIMWLK evolution from Y_{beam} up to the rapidity Y of the produced gluon



Gluon spectrum from classical Yang–Mills



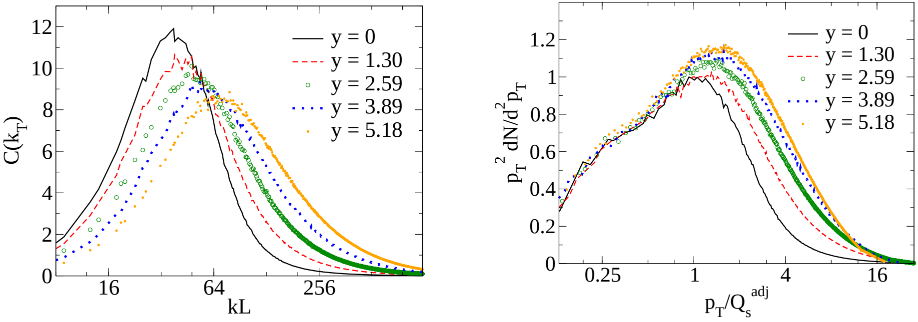
▷ *Numerical solutions to JIMWLK & CYM eqs. by T. Lappi (2011)*

▷ *Left: unintegrated gluon distribution for different values of* $Y = \ln(1/x)$

▷ *Right: spectrum of gluons produced in* AA *for different energies (*$y \propto \ln E$*)*

- Particle production at high energy can be computed from QCD ☺

Gluon spectrum from classical Yang–Mills



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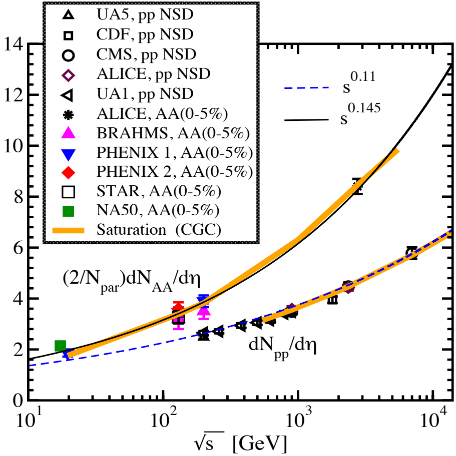
▷ *Right: spectrum of gluons produced in* AA *for different energies (*$y \propto \ln E$*)*

- Particle production at high energy can be computed from QCD ☺
- Hadron spectra can be modified by final state interactions ...
- ... but some gross features and special correlations will survive !

Recall: **Multiplicity : energy dependence**

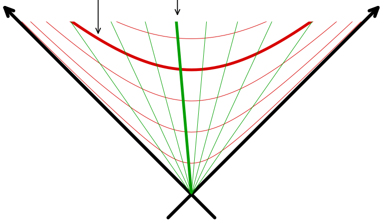
- Particle multiplicity $\mathrm{d}N/\mathrm{d}\eta \propto Q_s^2(A) \sim s^{\lambda_s/2}$



- Slight difference between energy growth in pp and AA
  (see Levin, Rezaeian, '11)

Boost invariance & longitudinal expansion

- The classical field is invariant under a boost along the collision axis
  - ▷ it depends upon proper time τ but not upon space–time rapidity η_s



$$\tau \equiv \sqrt{t^2 - z^2} = \sqrt{2x^+x^-}$$

$$\eta_s \equiv \frac{1}{2} \ln \frac{t+z}{t-z} = \frac{1}{2} \ln \frac{x^+}{x^-}$$

- Under a boost with velocity v_0

$$\eta_s \longrightarrow \eta_s + \beta \quad \text{with} \quad \tanh \beta = v_0$$

- Free streaming leading to longitudinal expansion *(Bjorken, 1983)*
  - ▷ particles propagate at the speed of light away from the interaction point

$$z \simeq v_z t \implies \eta_s \simeq \frac{1}{2} \ln \frac{1+v_z}{1-v_z} = -\ln \tan \frac{\theta}{2} = \eta$$

Boost invariance & longitudinal expansion

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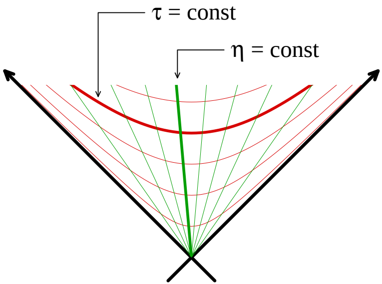
  - ▷ it depends upon proper time τ but not upon space–time rapidity η_s



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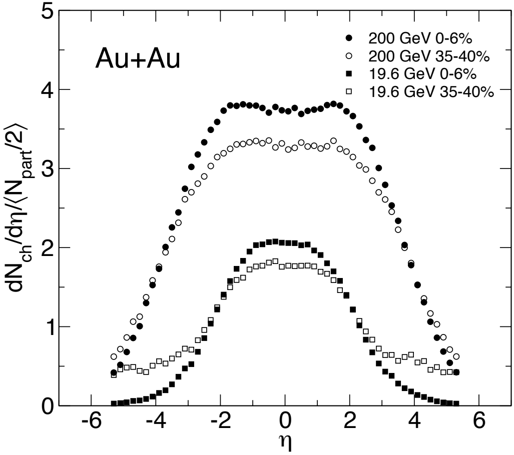
- Under a boost with velocity v_0

$$\eta_s \longrightarrow \eta_s + \beta \quad \text{with} \quad \tanh \beta = v_0$$

- Boost invariance $\Longrightarrow$ particle distribution is independent of η
  - ▷ particles propagate with the same probability along any direction θ
  - ▷ they separate from each other in the z direction
  - ▷ radial expansion remains negligible until $\tau \sim R_A$

Multiplicity : rapidity dependence

- RHIC (PHOBOS) data for $\mathrm{d}N_{\rm ch}/\mathrm{d}\eta$ as a function of η

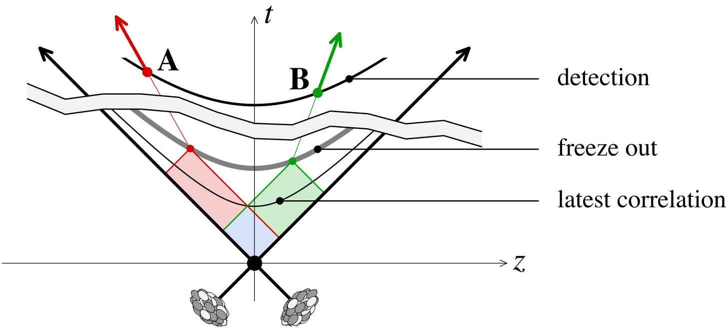


▷ flat in η around midrapidity : 'Feynman plateau'

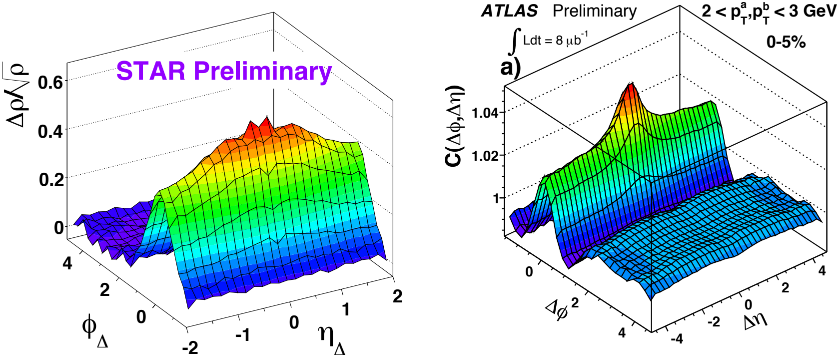
▷ for produced particles, $|\eta| \le \eta_{\rm beam}$

Long–range rapidity correlations probe early times

- Particles originating from the same interaction region are causally connected even if they make very different angles
- At late stages, they can be correlated with each other even if they have very different rapidities
- Vice–versa, the long–range correlations in rapidity are necessarily generated at early stages



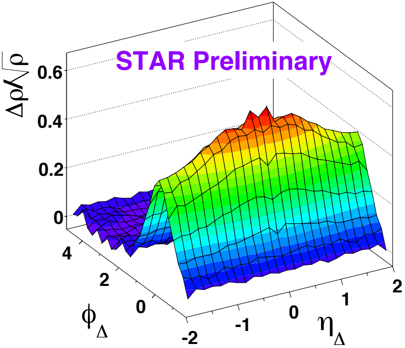
- A natural explanation for the 'ridge' :
  - di–hadron correlations long–ranged in $\Delta\eta$ & narrow in $\Delta\phi$
  - abundantly observed in AA collisions at RHIC and the LHC

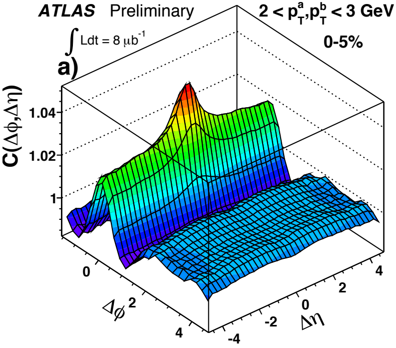


$$\mathcal{C}(\Delta\phi,\Delta\eta) \equiv \frac{\mathrm{d}N_{\mathrm{pair}}}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2} - \frac{\mathrm{d}N}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1}\,\frac{\mathrm{d}N}{\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2}$$

The Ridge in AA

- A natural explanation for the 'ridge' :
  - long–range correlations in $\Delta\eta$: boost invariance at early times
  - collimation in $\Delta\phi$ can be explained by radial flow

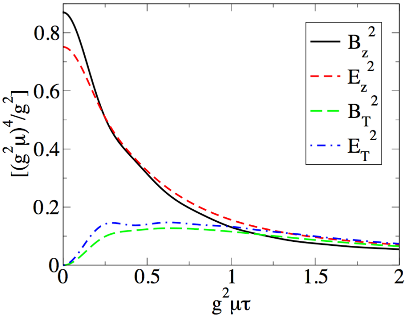


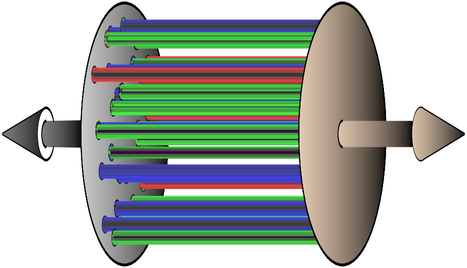


$$\mathcal{C}(\Delta\phi, \Delta\eta) \equiv \frac{\mathrm{d}N_{\text{pair}}}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2} - \frac{\mathrm{d}N}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1}\frac{\mathrm{d}N}{\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2}$$

Glasma

- Right after the collision, the chromo-electric and chromo-magnetic fields are purely longitudinal
- Flux tubes which extend between the recessing nuclei
  'glasma' (from 'glass' + 'plasma') *(McLerran and Lappi, 06)*

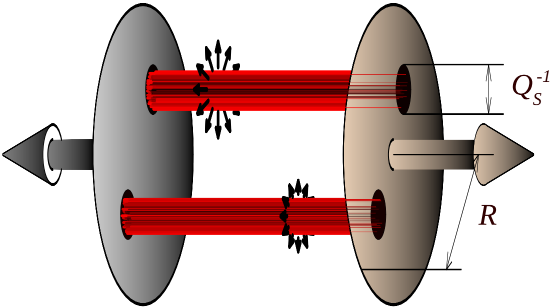




- At time $\tau \sim 1/Q_s$, the transverse fields are regenerated

From flux tubes to particles

- At time $\tau \sim 1/Q_s$, the glasma flux tubes break into particles (gluons)
- Gluons emitted from the same flux tube are correlated with each other



- correlation length in the transverse plane: $\Delta r_\perp \sim 1/Q_s$
- correlation length in rapidity : $\Delta\eta \sim 1/\alpha_s$
- to start with, this correlation is isotropic in $\Delta\Phi$

From flux tubes to particles

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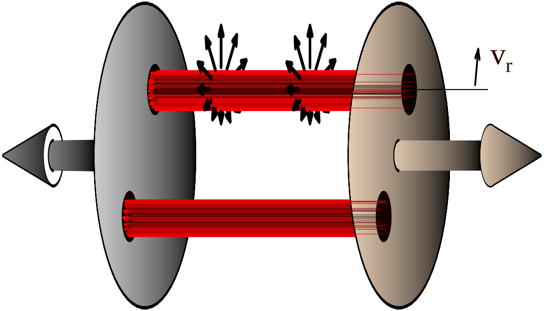
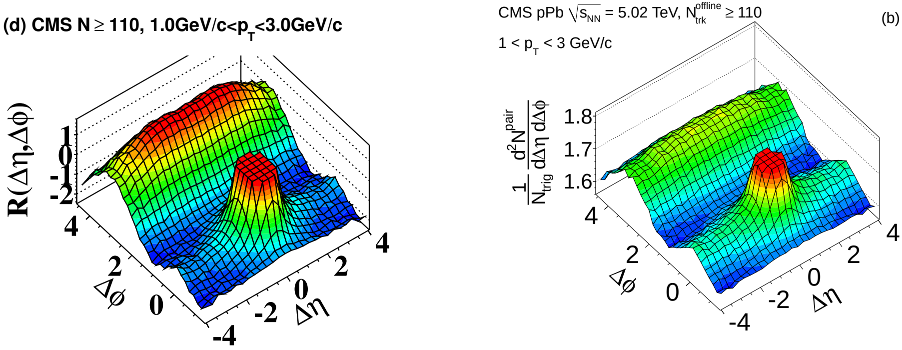


- correlation length in the transverse plane: $\Delta r_\perp \sim 1/Q_s$
- correlation length in rapidity : $\Delta\eta \sim 1/\alpha_s$
- in presence of radial flow, there is a bias leading to collimation in $\Delta\Phi$
  - ▷ more particles along the radial velocity $\boldsymbol{v}_r$ than perpendicular to it

The Ridge in pp and pA

- LHC : quite surprisingly, a ridge is also observed in p+p and p+A events with unusually high multiplicity



- What is the origin of the azimuthal collimation ?
- Can flow develop in such small systems (~ 1 fm) ?
- This might reflect the momentum correlations at early times (glasma)

The thermalization puzzle

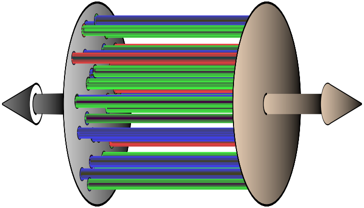
- Strong experimental evidence (RHIC, LHC) in favor of an intermediate phase of quark–gluon plasma in 'local thermal equilibrium'
  - the parton distribution is isotropic in momentum space and slowly varying in space and time; e.g.

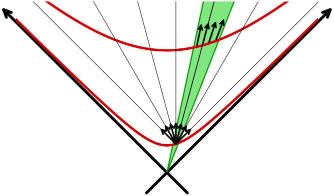
$$n(t, \mathbf{x}, \mathbf{p}) = \frac{1}{\mathrm{e}^{-p/T} \mp 1} \quad \text{where } \; T = T(t, \mathbf{x}) \; \text{ is slowly varying}$$

  - ... albeit there is no direct evidence for thermal distributions, like above
- Strongest evidence in that sense: the great success of nearly ideal hydrodynamics in describing collective phenomena like elliptic flow
  - requires small thermalization time: $\tau_0 \lesssim 1$ fm $\sim 10^{-23}$ secs
- This is very puzzling though
  - the early distribution is highly anisotropic ('glasma flux tubes')
  - to equilibrate, particles need to efficiently exchange 4–momentum
  - difficult to achieve for an expanding, weakly-coupled, system

The thermalization puzzle (2)

- Just after the collision, the partonic matter is highly anisotropic





  - the glasma flux tubes have ‘negative longitudinal pressure’ : they oppose to expansion (like a string of rubber)

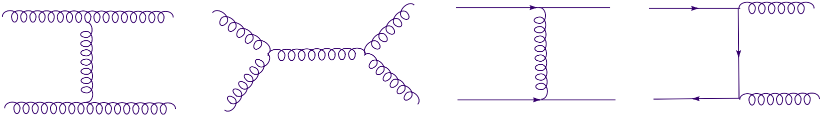
$$T_{\rm eq}=\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon/3 & 0 & 0 \\ 0 & 0 & \varepsilon/3 & 0 \\ 0 & 0 & 0 & \varepsilon/3 \end{pmatrix} \qquad T_{\rm initial}=\begin{pmatrix} \varepsilon & 0 & 0 & 0 \\ 0 & \varepsilon & 0 & 0 \\ 0 & 0 & \varepsilon & 0 \\ 0 & 0 & 0 & -\varepsilon \end{pmatrix}$$

  - in equilibrium: $P_T = P_L = \varepsilon/3$; in the early glasma: $P_T = \varepsilon = -P_L$

- The original anisotropy can be amplified by the longitudinal expansion

Thermalization in perturbation theory

- Particles can exchange energy and momentum through collisions.
- Weak coupling: the dominant mechanism is $2 \to 2$ elastic scattering



- Cross–section (σ) scales like $|\text{amplitude}|^2$, hence like $g^4 \sim \alpha_s^2$
- Mean free path (ℓ) = average distance between successive collisions

$$\ell \sim \frac{1}{\text{density} \times \sigma} \sim \frac{1}{\alpha_s^2}$$

- Typical equilibration time: $\tau_{\text{eq}} \sim \ell / v \sim 1/\alpha_s^2$
- Weakly coupled systems have large equilibration times ! ☹

The role of the strong fields

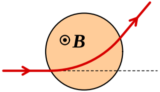
- Heisenberg's uncertainty principle requires

$$\text{mean free path } \ell \gtrsim \text{ de Broglie wavelength } \lambda \sim \frac{1}{p}$$

- In general, weakly interacting systems have $\ell \gg \lambda$
  - weakly coupled QGP, temperature T : $\lambda \sim 1/T$ while $\ell \sim 1/[\alpha_s^2 T]$

- However, the situation can change for a particle interacting with a strong electric, or magnetic, field, as in the glasma
  - domain of size Q_s^{-1} where the (chromo) magnetic field is $|\boldsymbol{B}| \sim Q_s^2/g$

$$\text{Lorentz force : } \quad \frac{\mathrm{d}\boldsymbol{p}}{\mathrm{d}t} = g\boldsymbol{v} \times \boldsymbol{B} \implies \dot{\theta} \sim \frac{gB}{p} \sim Q_s$$



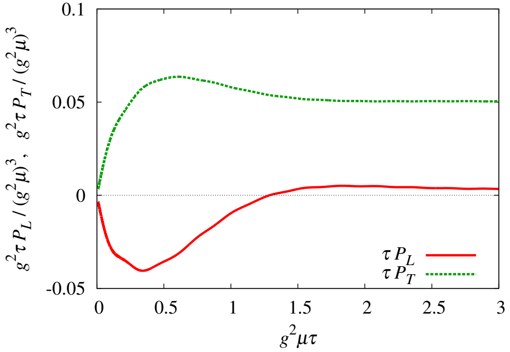
  - time spent in the domain $\tau \sim Q_s^{-1} \implies \Delta\theta \sim \mathcal{O}(1)$

- Mean free path $\ell \sim Q_s^{-1} \sim 1/p$: as low as permitted by Heisenberg

Thermalization at weak coupling & strong fields

(Epelbaum and Gelis, 2013)

- Numerical solution to classical Yang–Mills eq. confirms the anisotropy

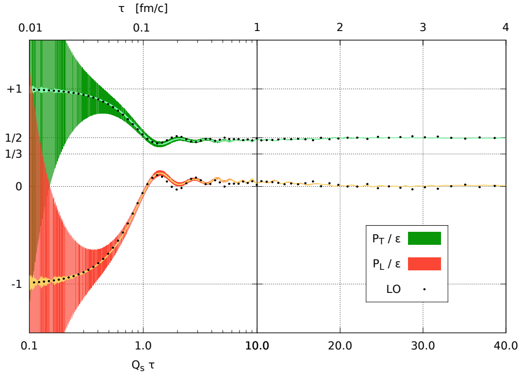


- the saturation momentum $Q_s = g^2\mu$ sets the scale
- $\tau\varepsilon = \tau\big(2P_T + P_L\big) \approx$ const. (longitudinal expansion)
- τP_L starts by being negative, then it becomes positive, but it remains much smaller than τP_T

(Epelbaum and Gelis, 2013)

- However, this (boost–invariant) classical solution is unstable under (rapidity–dependent) quantum fluctuations.
- The fluctuations can be added to the initial conditions

$\alpha_s = 8\,10^{-4}\ (g = 0.1)$

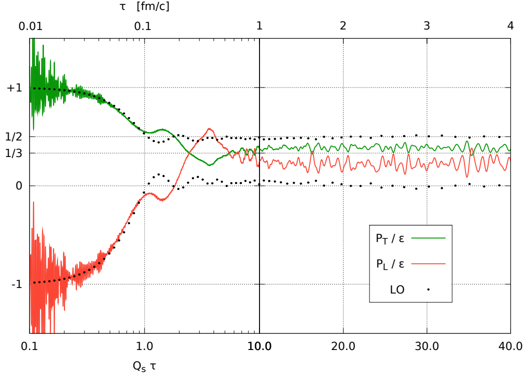


- for very small $g = 0.1$, the solution preserves boost invariance, as at LO

(Epelbaum and Gelis, 2013)

- However, this (boost–invariant) classical solution is unstable under (rapidity–dependent) quantum fluctuations.
- The fluctuations can be added to the initial conditions

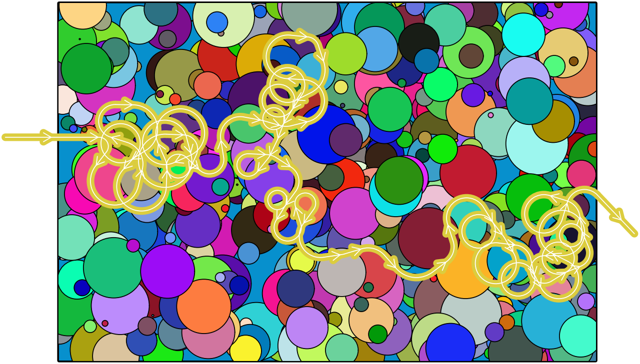
$\alpha_s = 2\,10^{-2}$ $(g = 0.5)$



- for $g \gtrsim 0.5$, it approaches isotropy: $P_L/P_T \simeq 0.7$ ☺

$$\frac{\text{viscosity}}{\text{entropy density}} \sim \frac{\text{mean free path}}{\text{de Broglie wavelength}} \gtrsim \hbar$$

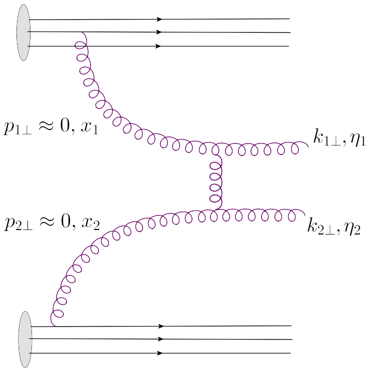
- Infinitely strong coupling (AdS/CFT) : $\eta/s = 1/4\pi$

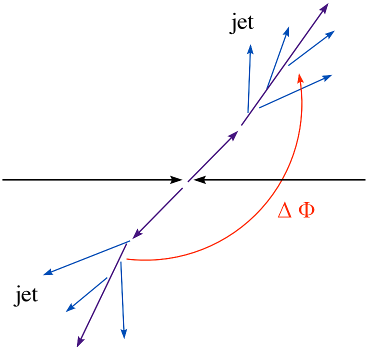


$$\frac{\eta}{s} \sim \frac{\ell}{\lambda} \sim \mathcal{O}(1)$$

(in units of $\hbar$)

- Glasma : strong classical Yang–Mills fields at weak coupling





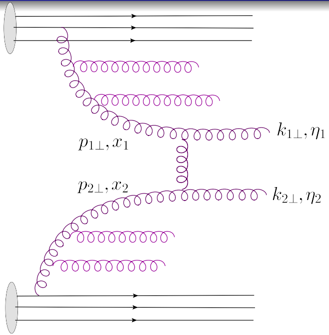
$$\frac{d\sigma}{d^2k_{1\perp}d^2k_{2\perp}d\eta_1 d\eta_2} = x_1 G(x_1, Q^2)\, x_2 G(x_2, Q^2)\, \delta^{(2)}(\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}) \frac{d\hat{\sigma}}{dk_\perp^2}$$

- Within collinear factorization : $\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp} = 0$

  ▷ a pair of hadrons propagating back–to–back in the transverse plane

- Their azimuthal distribution presents a peak at $\Delta\phi = \pi$

Di–hadrons with intrinsic $p_\perp$



- "Intrinsic $p_\perp$" (BFKL evolution, saturation) $\Longrightarrow$ $|\boldsymbol{k}_{1\perp} + \boldsymbol{k}_{2\perp}| \sim Q_s$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}^2k_{1\perp}\mathrm{d}^2k_{2\perp}\mathrm{d}\eta_1\mathrm{d}\eta_2} = \int \mathrm{d}^2\boldsymbol{p}_{1\perp} \int \mathrm{d}^2\boldsymbol{p}_{2\perp}\ \delta^{(2)}(\boldsymbol{p}_{1\perp} + \boldsymbol{p}_{2\perp} - \boldsymbol{k}_{1\perp} - \boldsymbol{k}_{2\perp})$$

$$\times\ \Phi(x_1, \boldsymbol{p}_{1\perp})\,\frac{\mathrm{d}\hat{\sigma}}{\mathrm{d}k_{1\perp}^2}\Phi(x_2, \boldsymbol{p}_{2\perp})$$

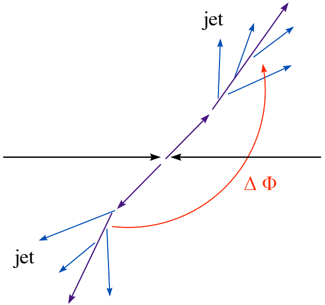
- The peak at $\Delta\phi = \pi$ acquires a broadening $\delta\phi \sim Q_s/k_\perp$

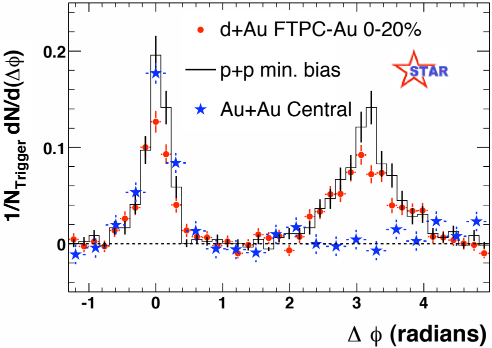
  $\triangleright$ a potential signature of saturation when $k_\perp \sim Q_s$

Di–hadron azimuthal correlations at RHIC

$$\mathcal{C}(\Delta\phi) \equiv \frac{\mathrm{d}N_{\rm pair}}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2} - \frac{\mathrm{d}N}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1}\,\frac{\mathrm{d}N}{\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2}$$

- Midrapidities $(\eta_1 \sim \eta_2 \simeq 0)$ and semi–hard $p_\perp \sim 1 \div 3$ GeV



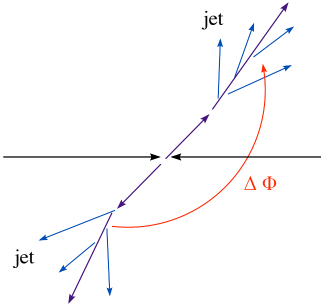


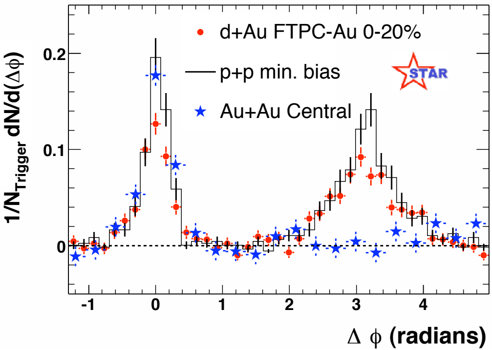
- p+p or d+Au : the peak at $\Delta\Phi = \pi$ is visible and equally pronounced
- Au+Au : strong suppression of the 'away peak' (final state effect)

Di–hadron azimuthal correlations at RHIC

$$\mathcal{C}(\Delta\phi) \equiv \frac{\mathrm{d}N_{\rm pair}}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2} - \frac{\mathrm{d}N}{\mathrm{d}^2p_{1\perp}\mathrm{d}\eta_1}\,\frac{\mathrm{d}N}{\mathrm{d}^2p_{2\perp}\mathrm{d}\eta_2}$$

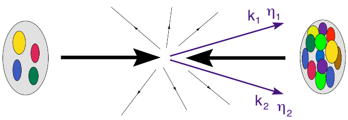
- Midrapidities $(\eta_1 \sim \eta_2 \simeq 0)$ and semi–hard $p_\perp \sim 1 \div 3$ GeV

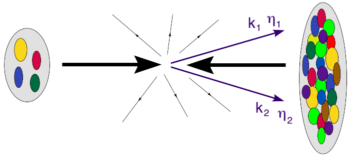


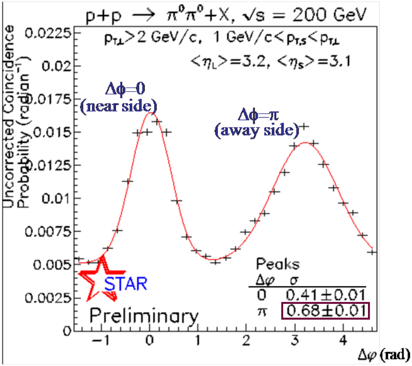


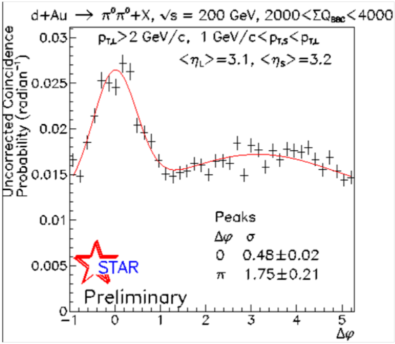
- Broadening in d+Au is controlled by jet fragmentation, like in p+p
- What happens if one moves to forward rapidities (larger $Q_s(A)$) ?

Forward rapidities: p+p vs. d+Au





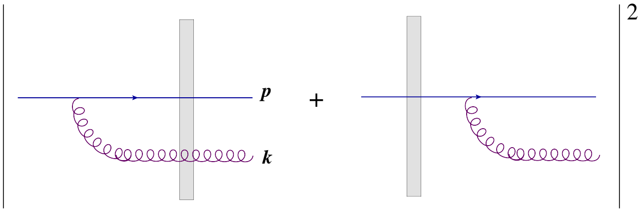




- Predicted by the CGC *(Marquet, 2007; Albacete and Marquet, 2010)*

Forward di–hadron production in pA (1)

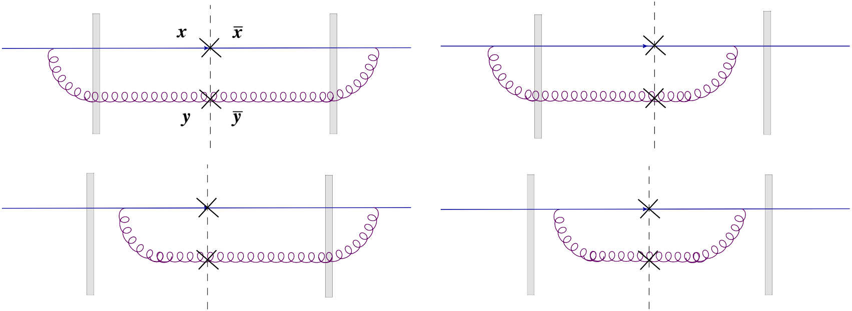
- A quark from the proton emits a gluon while scattering off the nucleus



▷ the gluon can be emitted either before, or after, the scattering with the 'shockwave' (Lorentz–contracted nucleus)

Forward di–hadron production in pA (1)

- A quark from the proton emits a gluon while scattering off the nucleus



▷ emissions in the DA (at $\boldsymbol{x}$, $\boldsymbol{y}$), absorptions in the CCA (at $\bar{\boldsymbol{x}}$, $\bar{\boldsymbol{y}}$)

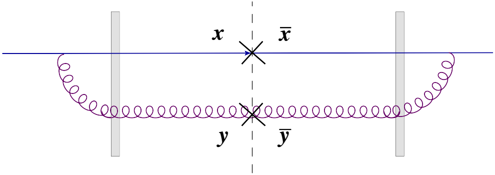
▷ Fourier transforms: $\boldsymbol{x}-\bar{\boldsymbol{x}} \to \boldsymbol{p}_1$ & $\boldsymbol{y}-\bar{\boldsymbol{y}} \to \boldsymbol{p}_2$

- Each parton (q, g) that crosses the shockwave acquires a Wilson line

$$U^{\dagger}(\boldsymbol{x}) = \mathrm{T}\exp\left\{ \mathrm{i}g\int \mathrm{d}x^+ A_a^-(x^+,\boldsymbol{x})\,T^a \right\}$$

The quadrupole

- Most complicated piece: a color quadrupole



$$\left\langle \left(U_{\bar{y}}U_{y}^{\dagger}\right)^{ab}\mathrm{Tr}\left[V_{x}^{\dagger}t^{b}t^{a}V_{\bar{x}}\right]\right\rangle_{Y}$$

V = quark , U = gluon

- Target average with CGC weight function $W_Y[A^-]$

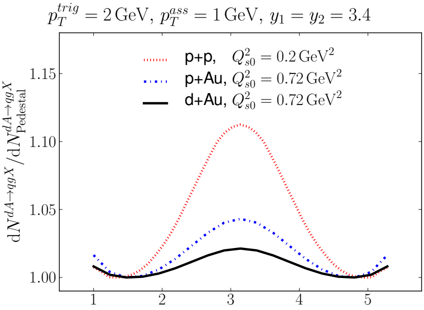
  ▷ can be computed by solving Balitsky–JIMWLK equations ... but it is hard *(see however Dumitru, Jalilian-Marian, Lappi, Schenke, Venugopalan, 2011)*

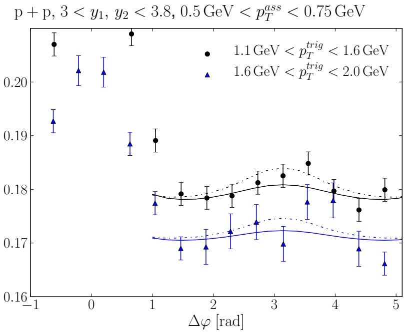
- Mean field approximation : a Gaussian Ansatz for $W_Y[A^-]$

  ▷ the quadrupole and all the higher n–point functions of the Wilson lines can be related to the color dipole, as obtained by solving the BK equation *(Iancu, Triantafyllopoulos, 2011)*

Di–hadron correlations in the MFA

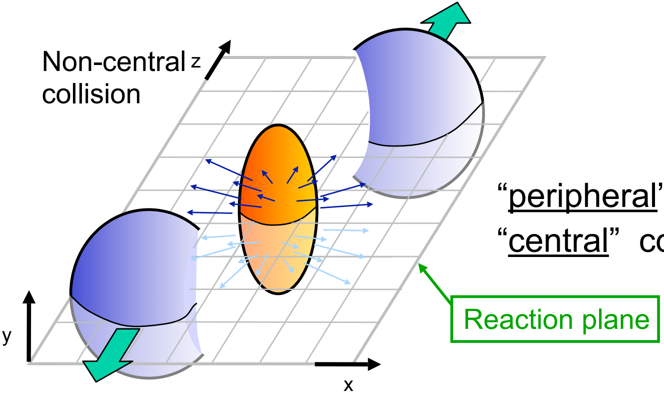
(Lappi and Mäntysaari, 2012; see also Stasto, Xiao, Yuan, 2011)





- left: different combinations projectile–target
- right: comparison with RHIC data (PHENIX, 2012)

The geometry of a HIC



"peripheral" collision ($b \sim b_{max}$)

"central" collision ($b \sim 0$)

Number of participants (N_{part}): number of incoming nucleons (participants) in the overlap region