QCD, partons, and all that

A first look at proton structure (DIS)
quark-parton model (second look)

RGE, running coupling and the beta function

QCD effects in DIS (third look)
DGLAP equations, universal partons (PDFs)

LHC measurements

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Structure of the Proton

Hit it hard with "photon" probe radiated from high energy electron

1. e-nucleus scatt. -- scaling
   --scaling violation $\rightarrow$ substructure
   $\rightarrow$ n, p

2. e-proton scatt. -- scaling
   -- scaling violation $\rightarrow$ substructure
   $\rightarrow$ quarks

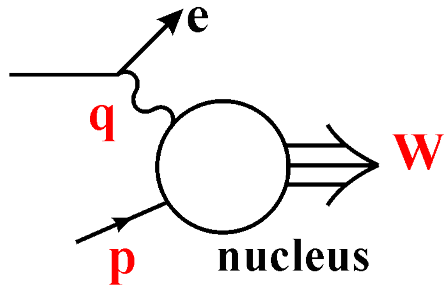
3. e-quark scatt. -- scaling
   -- scaling violation $\rightarrow$ substructure
   $\rightarrow$ no evidence, just more and more QCD scal. violat^{ns}

electron energy

First, a brief sketch of this interesting story

e-nucleus scatt.

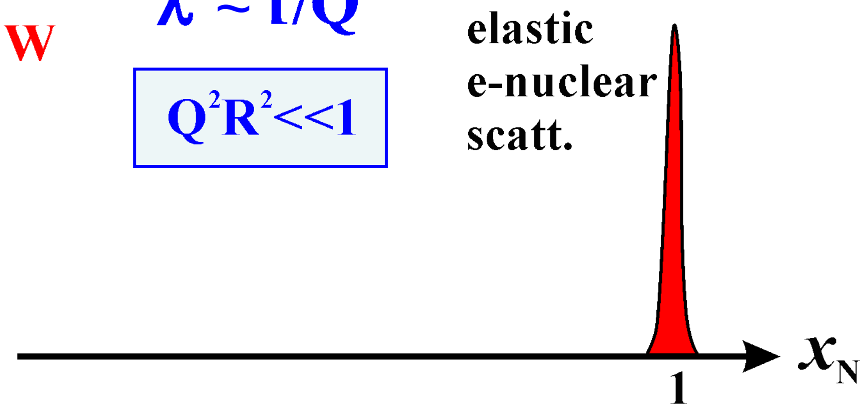
$$W^2 = (p+q)^2$$
$$= M^2 + 2p.q + q^2$$



$\lambda \sim 1/Q$

$\boxed{Q^2R^2 \ll 1}$

$(Q^2 \equiv -q^2)$



elastic
W=M

$$x_N \equiv \frac{Q^2}{2p.q} = 1$$

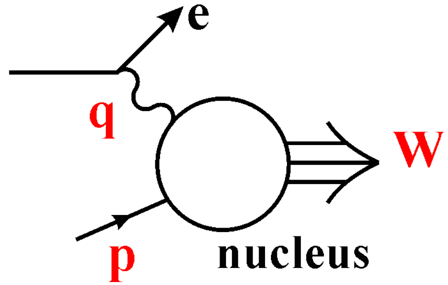
$$\left.\frac{Q^2}{2M\nu}\right|_{\text{lab.}}$$

scaling – independent of Q^2

Recall $M^2 = p^2$

e-nucleus scatt.

$$W^2 = (p+q)^2$$
$$= M^2 + 2p.q + q^2$$



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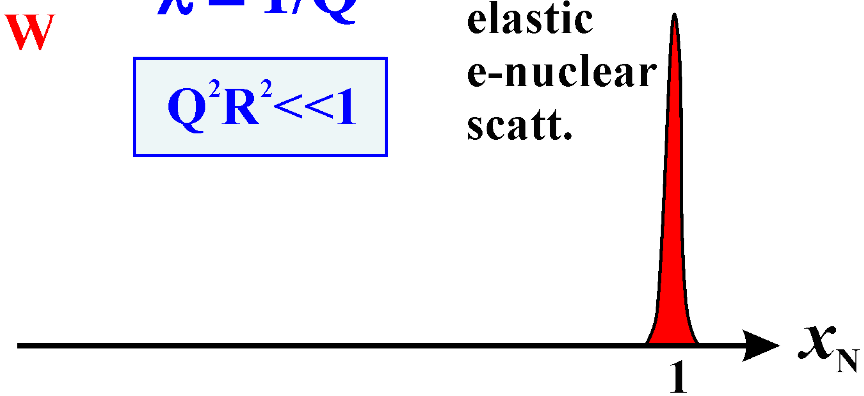

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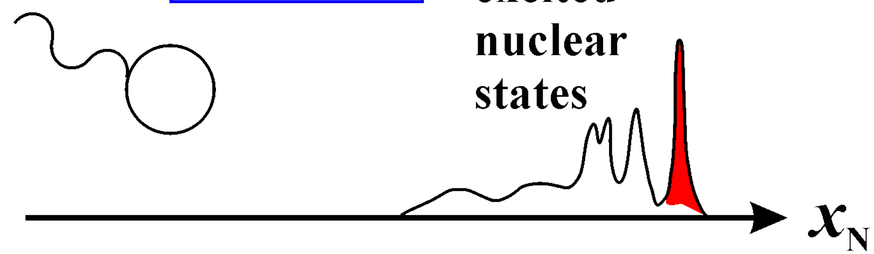


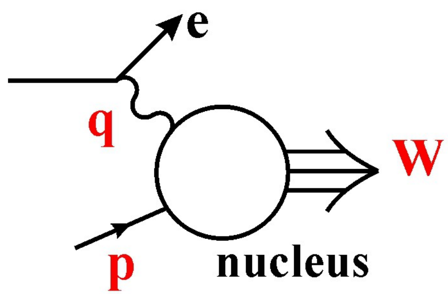
excit^{ns}
W>M

$$x_N < 1$$

scaling violation
– now dependent on Q^2

e-nucleus scatt.

$$W^2 = (p+q)^2 = M^2 + 2p.q + q^2$$



$(Q^2 \equiv -q^2)$

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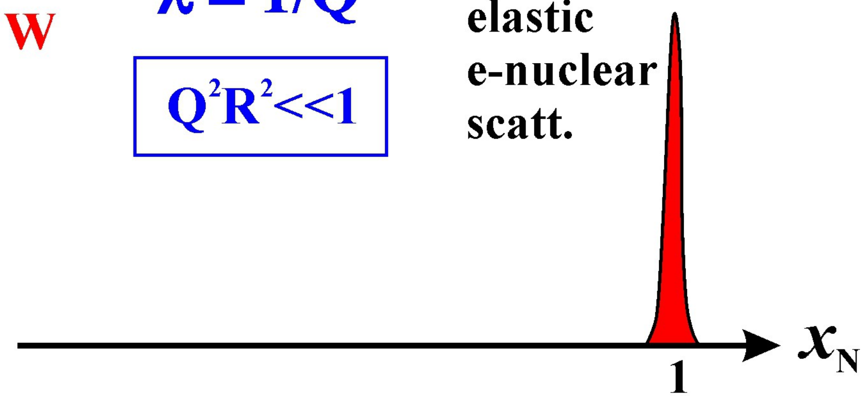
$\boxed{Q^2R^2 \ll 1}$

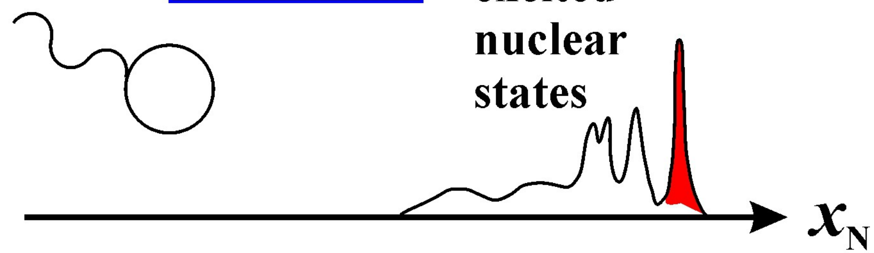


elastic
W=M

$$x_N \equiv \frac{Q^2}{2p.q} = 1$$

$$\left.\frac{Q^2}{2M\nu}\right|_{lab.}$$

$\boxed{Q^2R^2 \sim 1}$



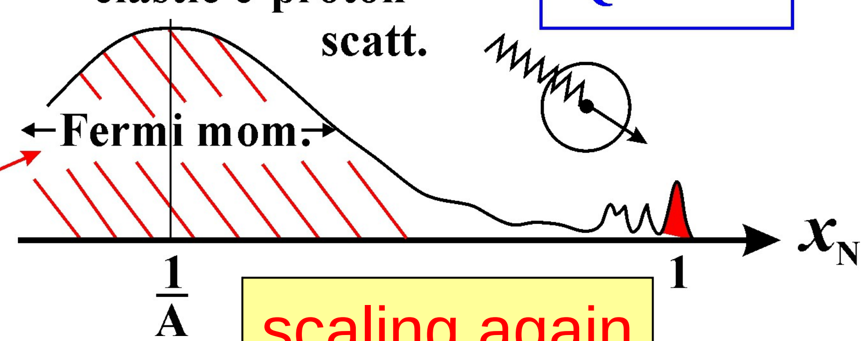
excit^{ns}
W>M

$$x_N < 1$$

e-proton
scatt:

$$x_N = \frac{m_p}{M}\left.\frac{Q^2}{2m_p\nu}\right|_{lab.} = \frac{1}{A}$$

$\boxed{Q^2R^2 \gg 1}$

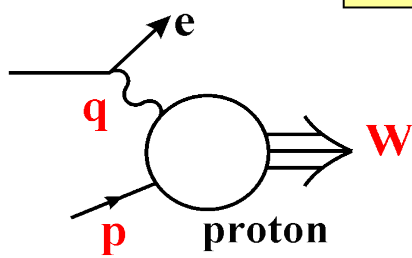


$\sigma \sim Z\sigma_{ep}$
shift: A=N+Z

scaling again

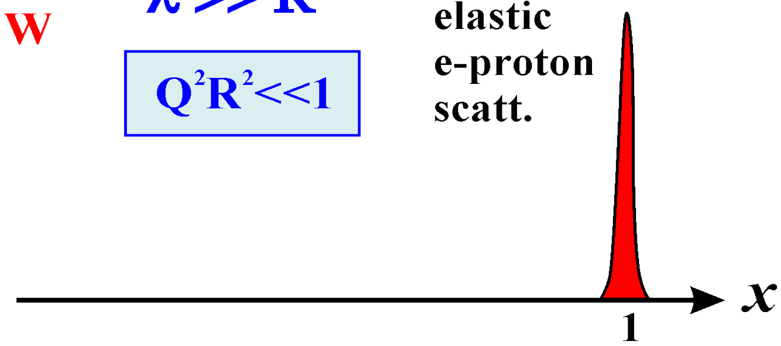
electron-proton scattering

$$W^2 = (p+q)^2$$
$$= M^2 + 2p.q + q^2$$
$$(Q^2 \equiv -q^2)$$



$\lambda >> R$

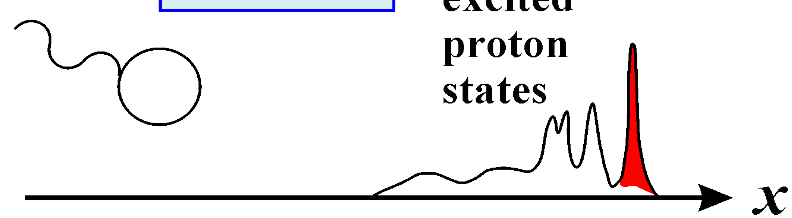
$Q^2R^2 << 1$



elastic
W=M

$$x \equiv \frac{Q^2}{2p.q} = 1$$

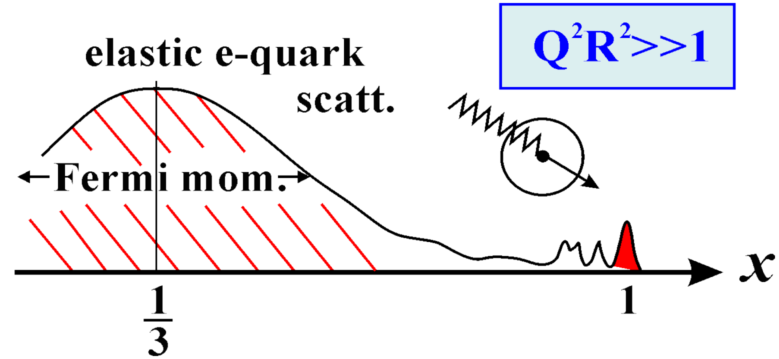
$Q^2R^2 \sim 1$



excit^{ns}
W>M

$$x < 1$$

$Q^2R^2 >> 1$



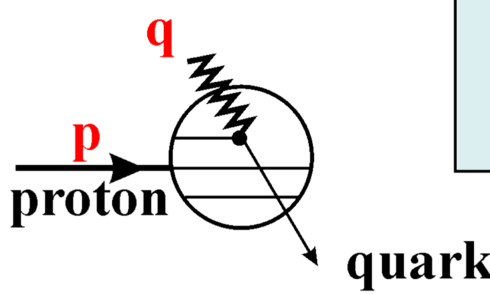
e-quark
scatt:

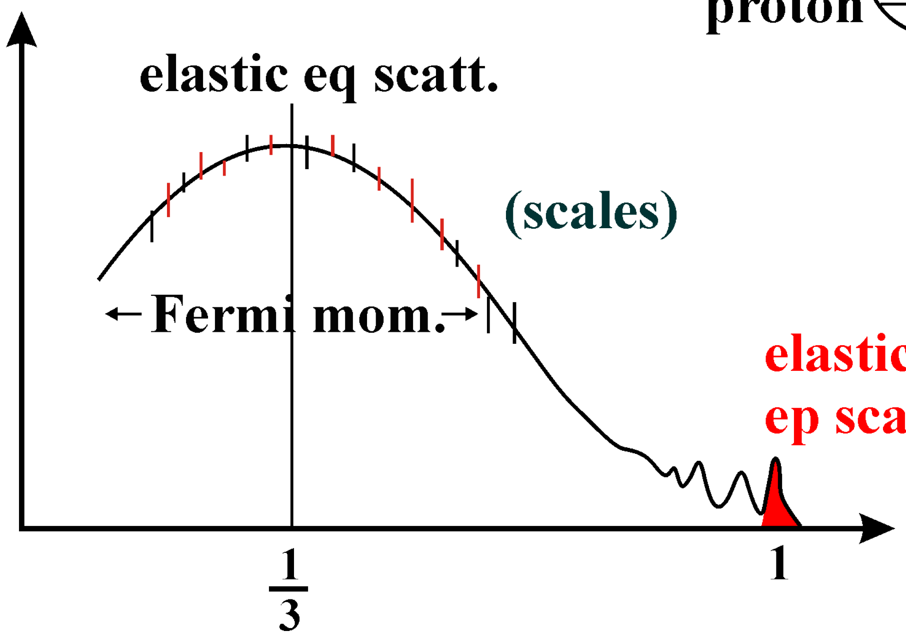
$$x = \frac{1}{3}\left(\frac{Q^2}{2p_q.q}\right) = \frac{1}{3}$$

$$p_q \sim \frac{1}{3}p$$

e - proton scatt.

Two variables
x, Q^2



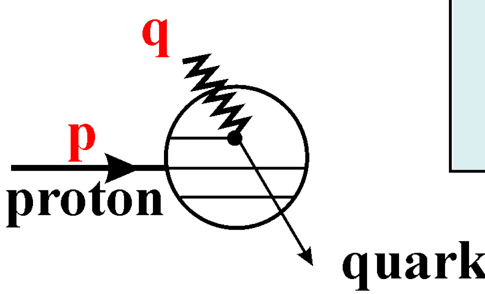


$$x = \frac{Q^2}{2p.q}$$

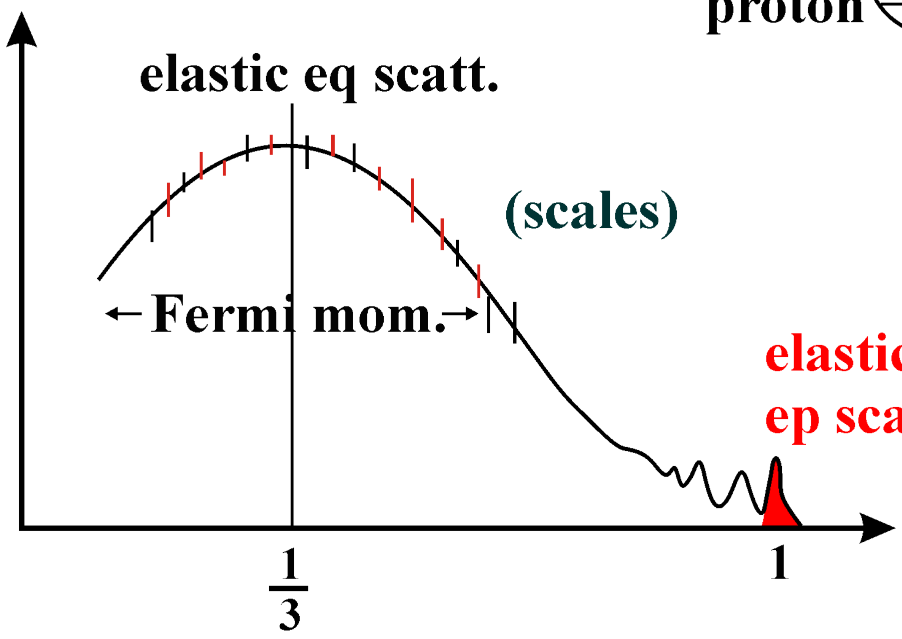
Bjorken scaling variable

e - proton scatt.

Two variables
x, Q^2



Known as deep inelastic scatt.



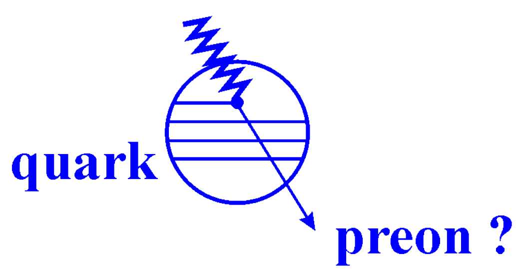
$$x = \frac{Q^2}{2p.q}$$

Bjorken scaling variable

Increase Q further

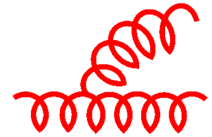
A replay ?

peak at $x = \frac{1}{12}$?



No evidence of q substructure

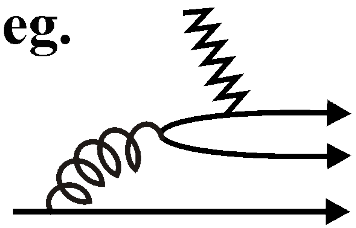
Instead of finding q substructure

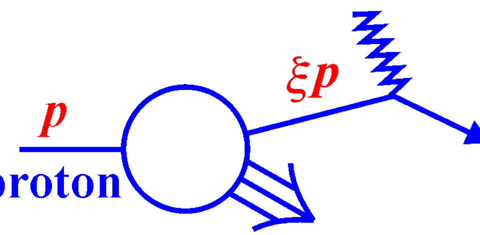


find QCD

(**compare QED** )

eg.





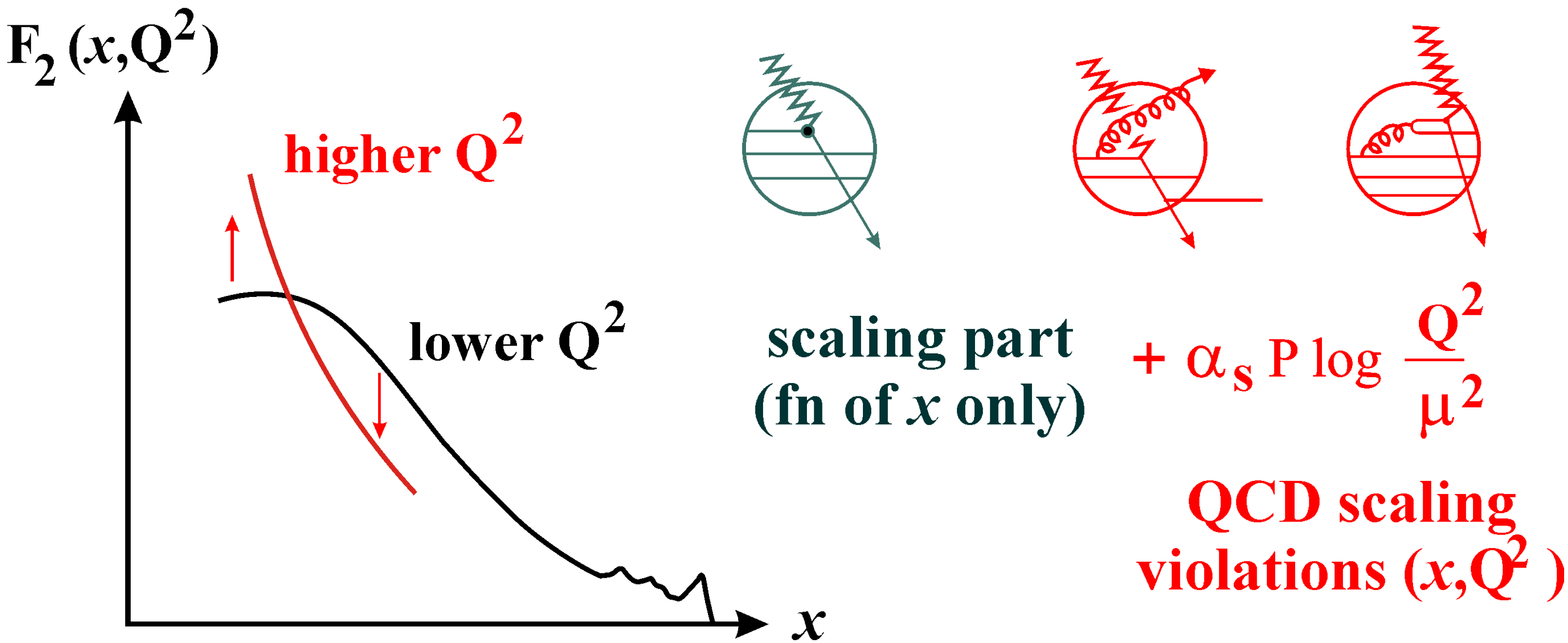
$$(\xi p + q)^2 \;=\; m_q^2 \;\simeq\; 0$$

$$2\xi p \cdot q - Q^2 \;\simeq\; 0$$

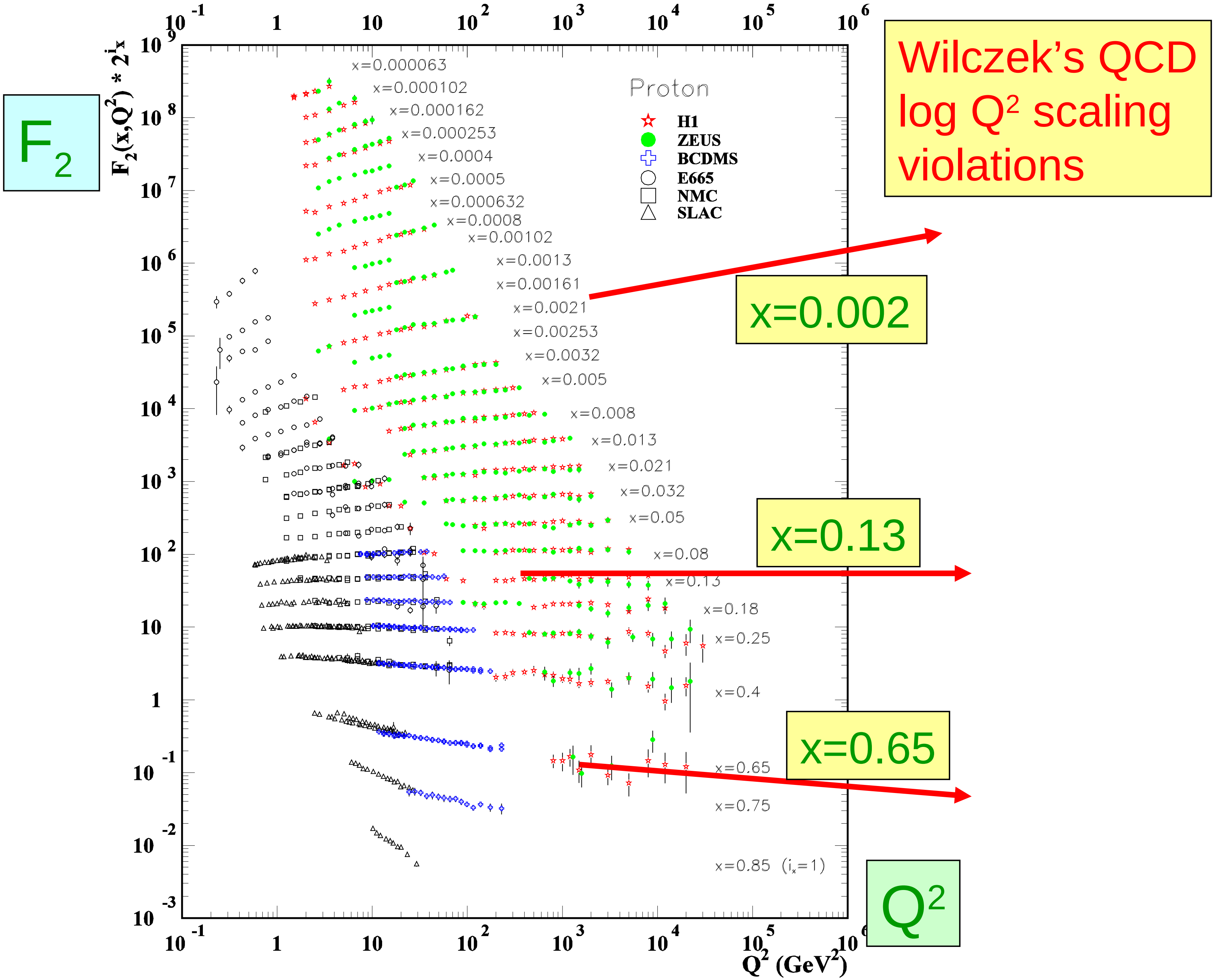
$$\xi \;=\; \frac{Q^2}{2p \cdot q} \;=\; x$$

Consequence :
as Q^2 increases, more and more partons are involved.
Each parton, on average, must have smaller *x*.

As Q^2 increases, each parton has, on average, smaller x

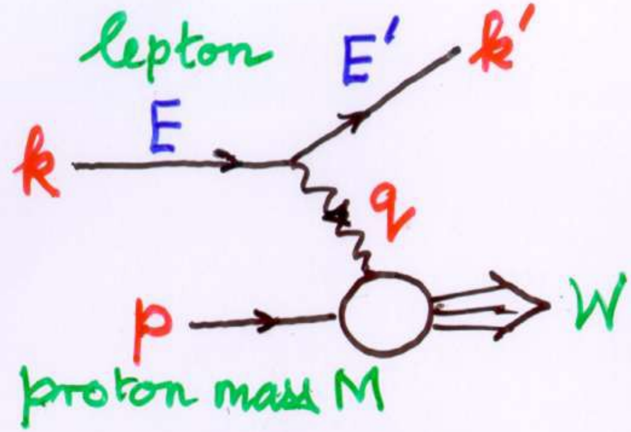


Famous experimentalist said to Wilczek :
You expect us to measure logarithms ! ?
Not in your lifetime, young man !



F2
Wilczek's QCD log Q² scaling violations
x=0.002
x=0.13
x=0.65
Q²
Proton
H1
ZEUS
BCDMS
E665
NMC
SLAC
F2(x,Q²) * 2^ix
Q² (GeV²)
x=0.000063
x=0.000102
x=0.000162
x=0.000253
x=0.0004
x=0.0005
x=0.000632
x=0.0008
x=0.00102
x=0.0013
x=0.00161
x=0.0021
x=0.00253
x=0.0032
x=0.005
x=0.008
x=0.013
x=0.021
x=0.032
x=0.05
x=0.08
x=0.13
x=0.18
x=0.25
x=0.4
x=0.65
x=0.75
x=0.85 (ix=1)

DIS invariant variables



$$Q^2 = -q^2$$

$$\nu = \frac{p\cdot q}{M} \underset{p\ \text{rest}}{=} E - E'$$

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2p\cdot q}$$

$$W^2 = (p+q)^2 = M^2 + \frac{Q^2}{x} - Q^2$$

elastic: $W^2 = M^2 \rightarrow x = 1$

inelastic: $W^2 > M^2 \rightarrow x < 1$

$$s = (k+p)^2 \simeq \frac{Q^2}{xy}$$

$$y = \frac{q\cdot p}{k\cdot p} \underset{p\ \text{rest}}{=} \frac{\nu}{E}$$

$$1-y = \frac{k'\cdot p}{k\cdot p} \underset{\text{c.m.}}{\simeq} \frac{1}{2}(1+\cos\theta)$$

e-quark

Deep ($Q^2 \gg M^2$) Inelastic ($W^2 \gg M^2$) Scatt.

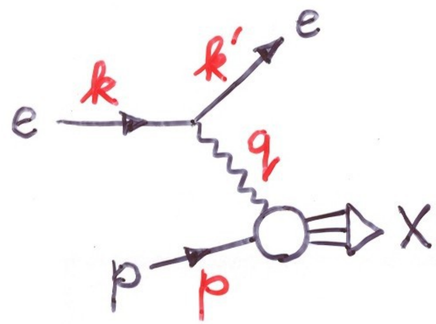
What exactly do experiments measure in DIS ?

Measure $F_i(x,Q^2)$

How many, $i=1,2\ldots$?

Deep Inelastic Lepton Scatt. (second look)

e.g. $ep \to eX$

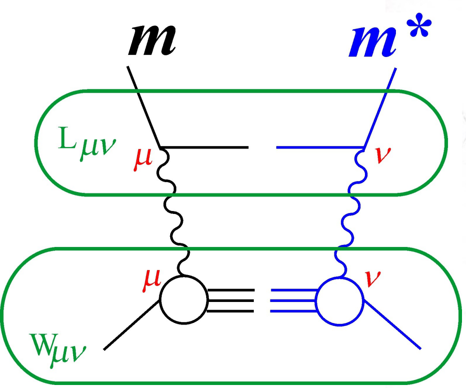


$Q^2 \equiv -q^2 \gg M^2$ "deep"

$\Big\}$ $W^2 = (p+q)^2 \gg M^2$ "inelastic"

(for moment assume $Q^2 \ll M_Z^2$ so γ exchange dominates)

unpolarised beams



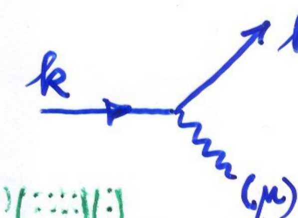
Spin-averaged matrix element squared:

$$\overline{|\mathcal{M}|^2} = \frac{e^4}{Q^4} L_{\mu\nu} W^{\mu\nu} (4\pi M)$$

lepton tensor

hadronic tensor

LEPTONIC



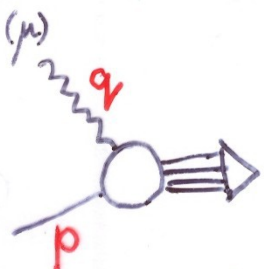
unpolarised

$$L_{\mu\nu} = \frac{1}{2} \sum_{\text{spins}} \left[\bar{u}(k')\gamma_\mu u(k)\right] \left[\bar{u}(k')\gamma_\nu u(k)\right]^*$$

$$= 2\left(k_\mu k'_\nu + k'_\nu k_\mu - k'\cdot k\, g_{\mu\nu}\right)$$

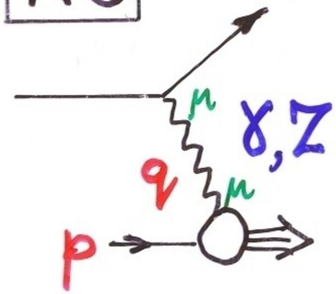
(HM 6.26)

HADRONIC



Most general form from $g^{\mu\nu}$, p^μ, q^μ

$\boxed{\text{NC}}$



$$|\mathcal{M}|^2 = \frac{e^4}{Q^4} L^{\mu\nu} W_{\mu\nu} (4\pi M)$$

$p \cdot p = M^2$ fixed

$\boxed{\begin{array}{l}\text{inv. variables in } W_{\mu\nu} \\ q\cdot q, \quad p\cdot q \\ \to Q^2, \; x = Q^2/2p\cdot q\end{array}}$

$$W_{\mu\nu} = \left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{q^2}\right) F_1(x,Q^2) + \frac{\hat{p}_\mu \hat{p}_\nu}{p\cdot q} F_2(x,Q^2) \underbrace{- i\varepsilon_{\mu\nu\alpha\beta} \frac{q^\alpha p^\beta}{2p\cdot q} F_3(x,Q^2)}_{\cancel{P}}$$

$q_\mu W_{\mu\nu} = 0$

$\left(\hat{p} \equiv p_\mu - \frac{p\cdot q}{q^2} q_\mu\right)$

tedious algebra $W^{\mu\nu}L_{\mu\nu}$

$$\boxed{\frac{d\sigma(e^\pm p \to e^\pm X)}{dx\, dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left\{Y_+ F_2 \mp Y_- xF_3 - y^2 F_L\right\}}$$

$$\left\{\begin{array}{l} Y_\pm \equiv 1 \pm (1-y)^2 \\ F_L = F_2 - 2xF_1 \end{array}\right.$$

$$\left.\begin{array}{l} F_2 = F_2^\gamma - v_e \eta F_2^{\gamma Z} + (v_e^2 + a_e^2)\eta^2 F_2^Z \\ xF_3 = \quad - a_e \eta x F_3^{\gamma Z} + 2 v_e a_e \eta\, x F_3^Z \end{array}\right\} \quad \eta = \frac{G_F M_Z^2}{2\sqrt{2}\pi\alpha}\left(\frac{Q^2}{Q^2+M_Z^2}\right)$$

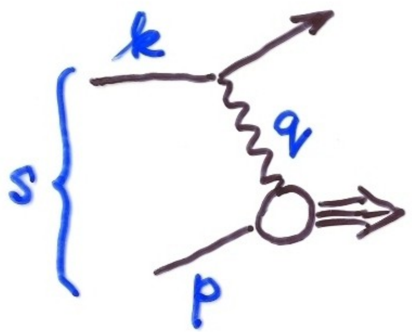
We will assume γ exchange only --- so $F_3=0$ --- come back to this later

$$F_L = F_2 - 2xF_1$$

$$\frac{d\sigma}{dx\,dQ^2}(ep \xrightarrow{\gamma} eX) = \frac{2\pi\alpha^2}{xQ^4}\left\{ \overbrace{\left(1+(1-y)^2\right)}^{Y_+} F_2(x,Q^2) - y^2 F_L(x,Q^2) \right\}$$

$\sqrt{s} \gg M$

$Q^2 \ll M_Z^2$



$$s = (k+p)^2 \simeq 2k\cdot p$$

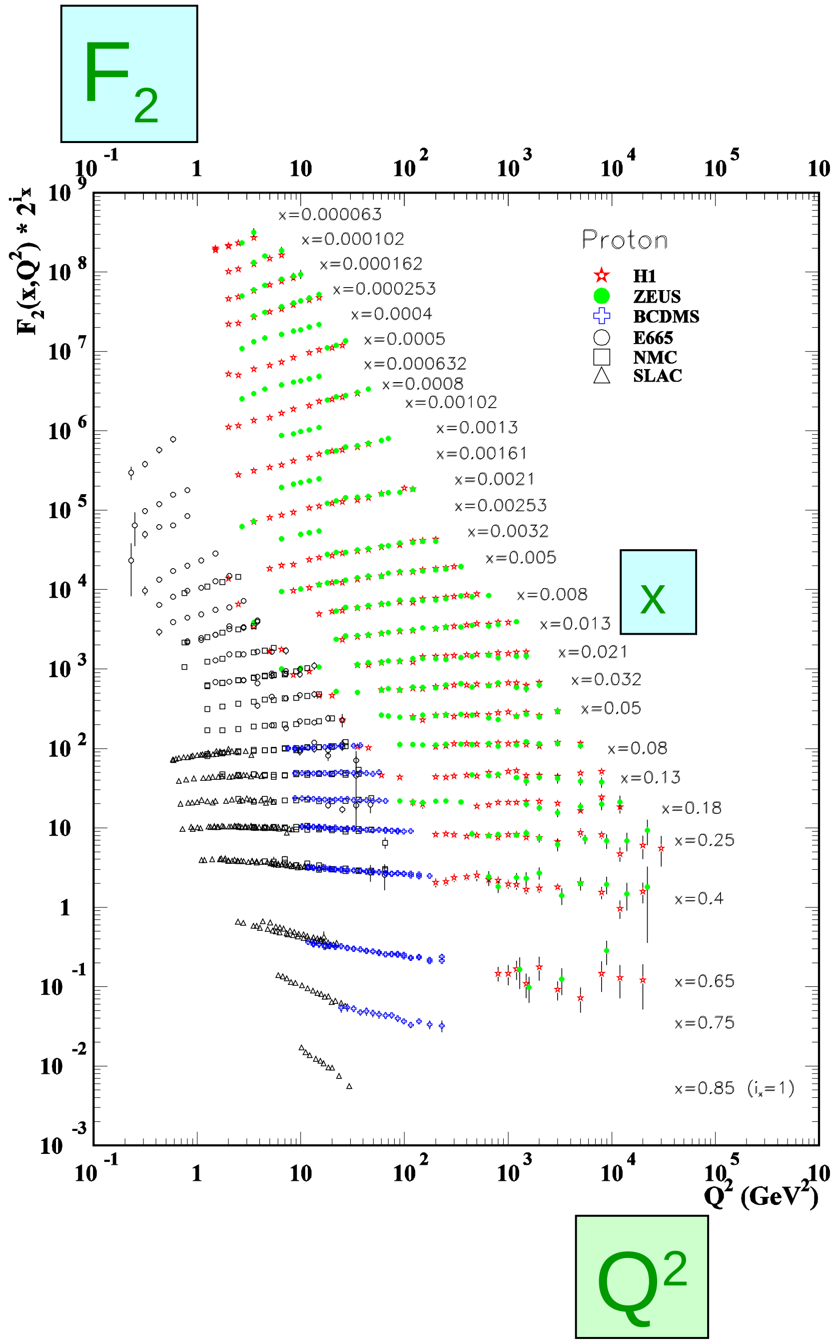
$$y = \frac{2p\cdot q}{s} = \frac{Q^2}{xs}$$

To measure both F_2 and F_L as functions of x and Q^2, we need y dep. – i.e. perform expts at different energies $\sqrt{s}$

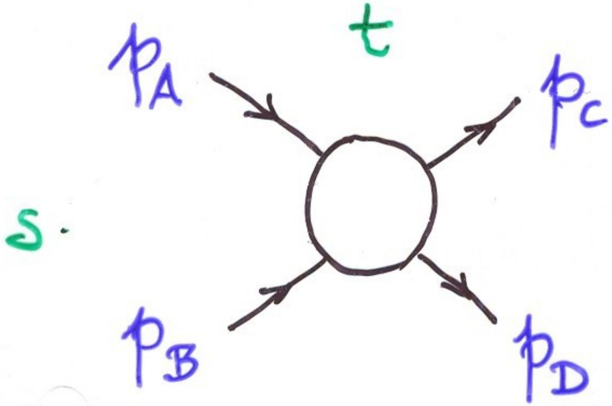
We will see $F_L \equiv F_2 - 2xF_1 \simeq 0$ so may use QCD to calculate F_L – and then expts. measure $F_2(x,Q^2)$.

What do the proton structure functions, $F_i(x,Q^2)$, measured in DIS, tell us?

Can we predict their values?



Mandelstam variables for $A+B \to C+D$



Lor. inv.

$$s = (p_A + p_B)^2 \qquad p_A \cdot p_B$$

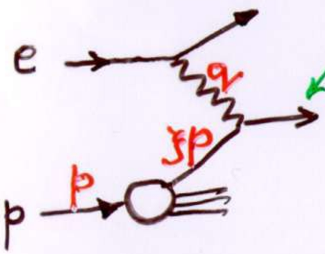
$$t = (p_A - p_C)^2 \qquad p_A \cdot p_C$$

$$u = (p_A - p_D)^2$$

$$s + u + t = m_A^2 + m_B^2 + m_C^2 + m_D^2$$

Quark Parton Model

Basic idea: in proton ∞-mom^{m} frame, $\gamma^* p$ interaction at large Q^2 can be expressed as sum of incoherent scatt. from pt-like quarks. Over short time scale $1/\sqrt{Q^2}$ photon sees state of non-interacting quarks. Hadronization occurs long after.

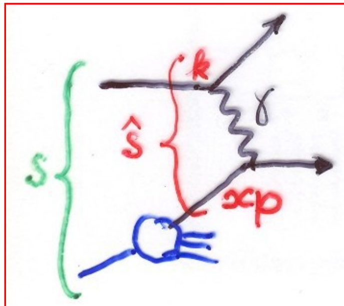


on-shell: $0 = (\xi p + q)^2 \simeq 2p\cdot q\,\xi - Q^2$. $\therefore \boxed{\xi = x}$

$$\boxed{\frac{d^2\sigma}{dx\,dQ^2} = \sum_q \int_0^1 d\xi \, f_q(\xi) \, \frac{d^2\hat{\sigma}_{eq}}{dx\,dQ^2}}$$

PDF $\rightarrow$ f_q is prob. of finding quark in proton with fraction ξ of its mom^{m}.

$$\frac{d\hat{\sigma}_{eq}}{d|\hat{t}|} = \frac{2\pi\alpha^2 e_q^2}{\hat{s}^2}\left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{t}^2}\right)^2$$



$$\hat{s} = (xp + k)^2 \simeq 2xp\cdot k \simeq xs$$

$$\hat{t} = -Q^2 = -xys$$

$$\hat{u} = -\hat{s} - \hat{t} = -x(1-y)s$$

$$\frac{d\hat{\sigma}_{eq}}{dx\,dQ^2} = \frac{2\pi\alpha^2 e_q^2}{Q^4}\left\{1 + (1-y)^2\right\}\delta(x-\xi)$$

Quark parton model

$$\therefore \quad \frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{x Q^4} \sum_q \int d\xi\, f_q(\xi)\, e_q^2\, \frac{x}{2} \left\{ 1 + (1-y)^2 \right\} \delta(x-\xi)$$

Insight into y dependence

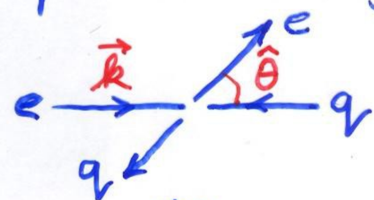
Recall:

$$\frac{d\hat{\sigma}_{eq}}{d\hat{t}} = \frac{2\pi\alpha^2 e_q^2}{\hat{t}^2} \left(\frac{\hat{s}^2 + \hat{u}^2}{\hat{s}^2} \right)$$

$$\downarrow$$

$$\left.\begin{aligned} \hat{s} &= xs \\ \hat{t} &= -xys \\ \hat{u} &= -x(1-y)s \end{aligned}\right\} \left(1 + (1-y)^2\right)$$

eq scatt in c.m. frame



$$\hat{s} \simeq 4\vec{k}^2$$

$$\hat{t} = -2\vec{k}^2 (1 - \cos\hat{\theta})$$

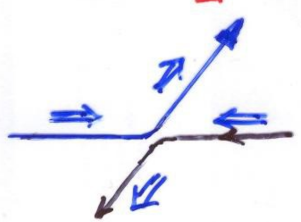
$$\hat{u} = -2k^2 (1 + \cos\hat{\theta})$$

$$\therefore \boxed{y = \tfrac{1}{2}(1 - \cos\hat{\theta})}$$

$y = 0$ forward scatt

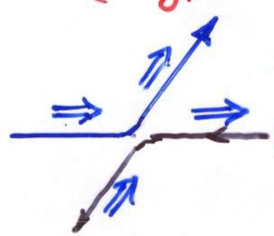
$y = 1$ backward scatt.

1



e, q same helicity

$(1-y)^2$



e,q opp. helicity

Must vanish for $\theta = \pi$ $(y=1)$

by conserv$^{\text{n}}$ of J_z

At HE

(fermion helicity conserved at gauge boson vertex) HM (6.37)

Quark parton model

{ Y_+ }

$$\therefore \quad \frac{d^2\sigma}{dx\,dQ^2} = \frac{4\pi\alpha^2}{x\,Q^4}\sum_q \int d\xi\, f_q(\xi)\, e_q^2\, \frac{x}{2}\left\{1+(1-y)^2\right\}\delta(x-\xi)$$

to be compared with general formula

ignore Z exch

$$\boxed{\frac{d\sigma(e^\pm p \to e^\pm X)}{dx\,dQ^2} = \frac{2\pi\alpha^2}{xQ^4}\left\{Y_+ F_2 \mp \cancel{Y_- xF_3} - y^2 F_L\right\}}$$

$$\begin{cases} Y_\pm \equiv 1 \pm (1-y)^2 \\ F_L = F_2 - 2xF_1 \end{cases}$$

$$\therefore \quad \boxed{\underbrace{F_2 = 2xF_1}_{\text{Callan-Gross relation}} = \sum_q \int_0^1 d\xi\, f_q(\xi)\, x\, e_q^2\, \delta(x-\xi) = \underbrace{\sum_q e_q^2\, x\, f_q(x)}_{\text{SCALING}}}$$

Callan-Gross relation : spin ½ quarks
(Spin 0 quarks: $F_1 = 0$)

SCALING

$F_i(x, \cancel{Q^2})$

Flavour sum rules

$$\text{proton} = \underset{\substack{\text{"valence" quarks} \\ \text{(carry q. nos.)}}}{uud} + \underset{\text{"sea" quarks}}{q\bar{q} \text{ pairs}}$$

When probed at scale Q all flavours $m_q \lesssim Q$ are active

Notation: $f_u(x) \equiv u(x) = u_v + u_{sea}$

$$f_{\bar{u}}(x) \equiv \bar{u}(x) = u_{sea}$$

$$\int_0^1 (u - \bar{u})\, dx = \int_0^1 u_v\, dx = 2$$

$$\int_0^1 (d - \bar{d})\, dx = \int_0^1 d_v\, dx = 1$$

<u>QPM</u>: $F_2 = \sum_q e_q^2\, x f_q(x)$

$$F_2^{ep} = x\left(\frac{4}{9}u + \frac{1}{9}d + \frac{1}{9}s + .. + \frac{4}{9}\bar{u} + \frac{1}{9}\bar{d} + \frac{1}{9}\bar{s} + \ldots\right)$$

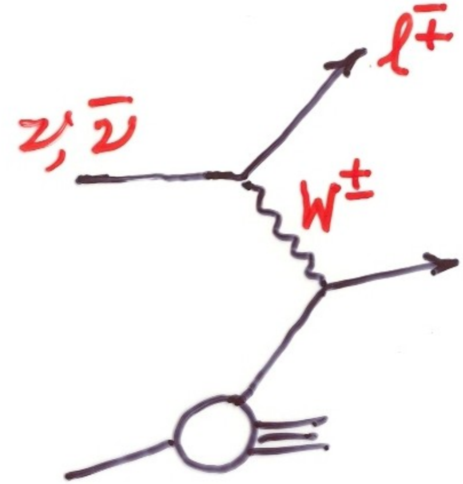
$$F_2^{en} = x\left(\frac{4}{9}d + \frac{1}{9}u + \frac{1}{9}s + \ldots + \frac{4}{9}\bar{d} + \frac{1}{9}\bar{u} + \frac{1}{9}\bar{s} + \ldots\right)$$

$(u \leftrightarrow d)$

$\not{P}$ contributions $\rightarrow$ then $F_3 \neq 0$

NC: $e^{\pm} p \xrightarrow{Z} e^{\pm} X$

CC: $e^{-} p \xrightarrow{W^{-}} \nu X$ or $\bar{\nu}_{\mu} p \xrightarrow{W^{-}} \mu^{+} X$



$$\frac{d\sigma^{\nu,\bar{\nu}}}{dx\,dQ^2} = \frac{G_F^2}{2\pi x}\left(\frac{M_W^2}{Q^2 + M_W^2}\right)^2 \left\{ Y_+ F_2^{\nu} \pm Y_- x F_3^{\nu} - y^2 F_L^{\nu} \right\}$$

$Q^2 \ll M_W^2$

$Y_+ = 1 + (1-y)^2$ $\quad$ $Y_- = 1 - (1-y)^2$

CC: $ep \to \nu X$ or $\nu p \to \mu X$

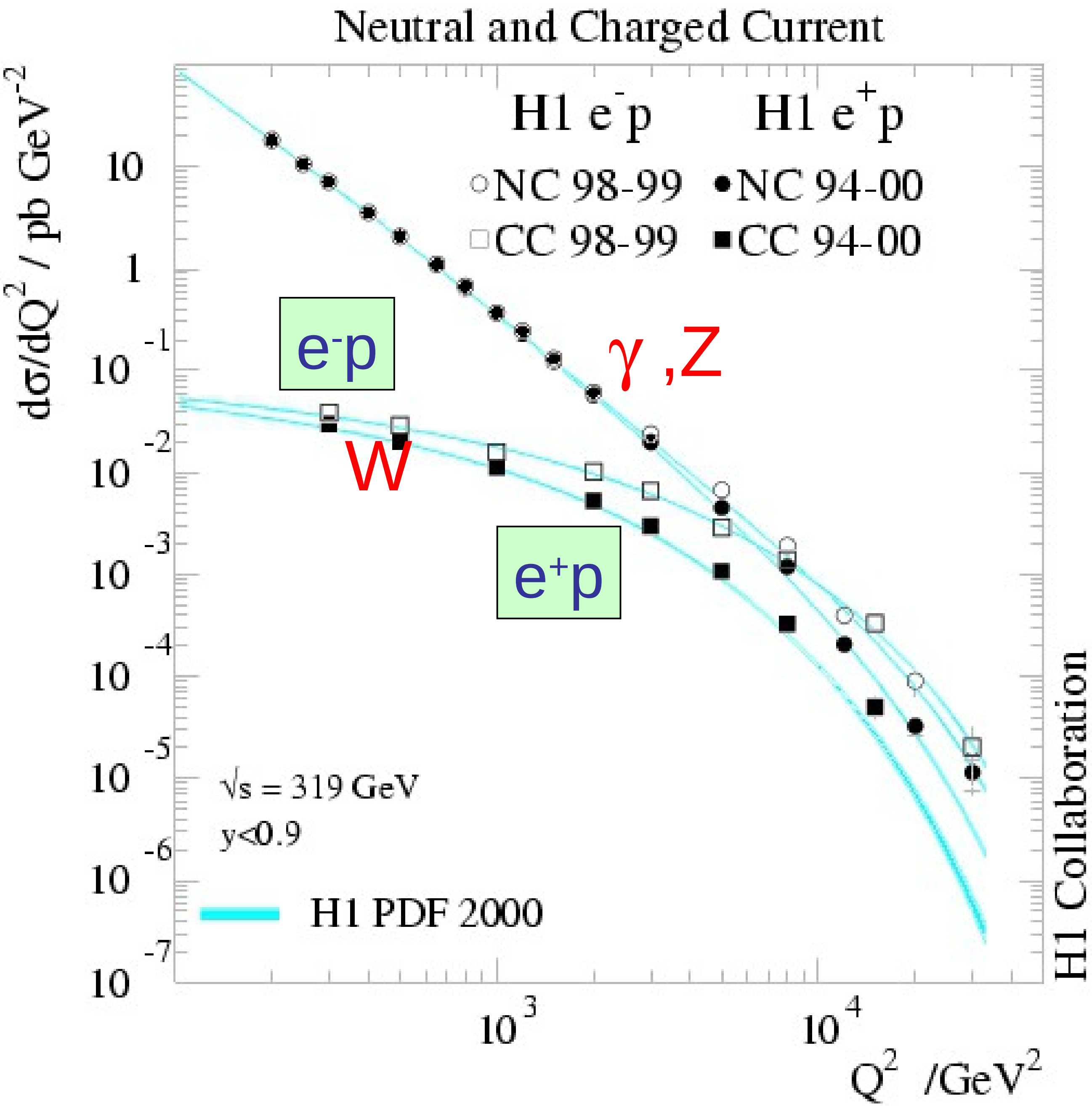
$$\boxed{\frac{d\sigma(e^\pm)}{dx\,dQ^2} = \frac{G_F^2}{2\pi x}\left(\frac{M_W^2}{Q^2+M_W^2}\right)^2 \underbrace{\left\{ Y_+ F_2^W \mp Y_- xF_3^W - y^2 F_L^W \right\}}_{\tilde{\sigma}}}$$

1	$(1-y)^2$		F_2^W	xF_3^W	F_L^W
$e^- u \to \nu d$ (L L, L L)	$e^- \bar{d} \to \nu \bar{u}$ (L R, L R)	$\}$	$x(u+\bar{d})$	$x(u-\bar{d})$	0
$e^+ \bar{u} \to \bar{\nu} \bar{d}$ (R R, R R)	$e^+ d \to \bar{\nu} u$ (R L, R L)	$\}$	$x(d+\bar{u})$	$x(d-\bar{u})$	0

$\theta_c = 0$

large x
$$\left\{\begin{array}{ll} \tilde{\sigma}^{CC}(e^-p) & \sim x u_{val} \\ \tilde{\sigma}^{CC}(e^+p) & \sim (1-y^2)\, x d_{val} \end{array}\right.$$

$$\therefore \tilde{\sigma}(e^-p) > \tilde{\sigma}(e^+p)$$



Neutral and Charged Current
H1 e⁻p
H1 e⁺p
NC 98-99
CC 98-99
NC 94-00
CC 94-00
e⁻p
e⁺p
γ ,Z
W
√s = 319 GeV
y<0.9
H1 PDF 2000
dσ/dQ² / pb GeV⁻²
Q² /GeV²
H1 Collaboration

$\boxed{\text{NC}: \quad ep \to eX}$

$$F_2(ep) = \sum_q e_q^2\, x\, q(x)$$

$$F_2(ep) = x\left(\frac{4}{9}u + \frac{4}{9}\bar{u} + \frac{1}{9}d + \frac{1}{9}\bar{d} + \ldots\right)$$

$$F_2(en) = x\left(\frac{4}{9}d + \frac{4}{9}\bar{d} + \frac{1}{9}u + \frac{1}{9}\bar{u} + \ldots\right)$$

$$F_2(eN) = \frac{5}{18}\, x\left(u + \bar{u} + d + \bar{d} + \ldots\right)$$

per nucleon

$$\boxed{F_2(eN) \simeq \frac{5}{18} F_2(\nu N)}$$

$\boxed{\text{CC}: \quad \nu p \to \mu X}$

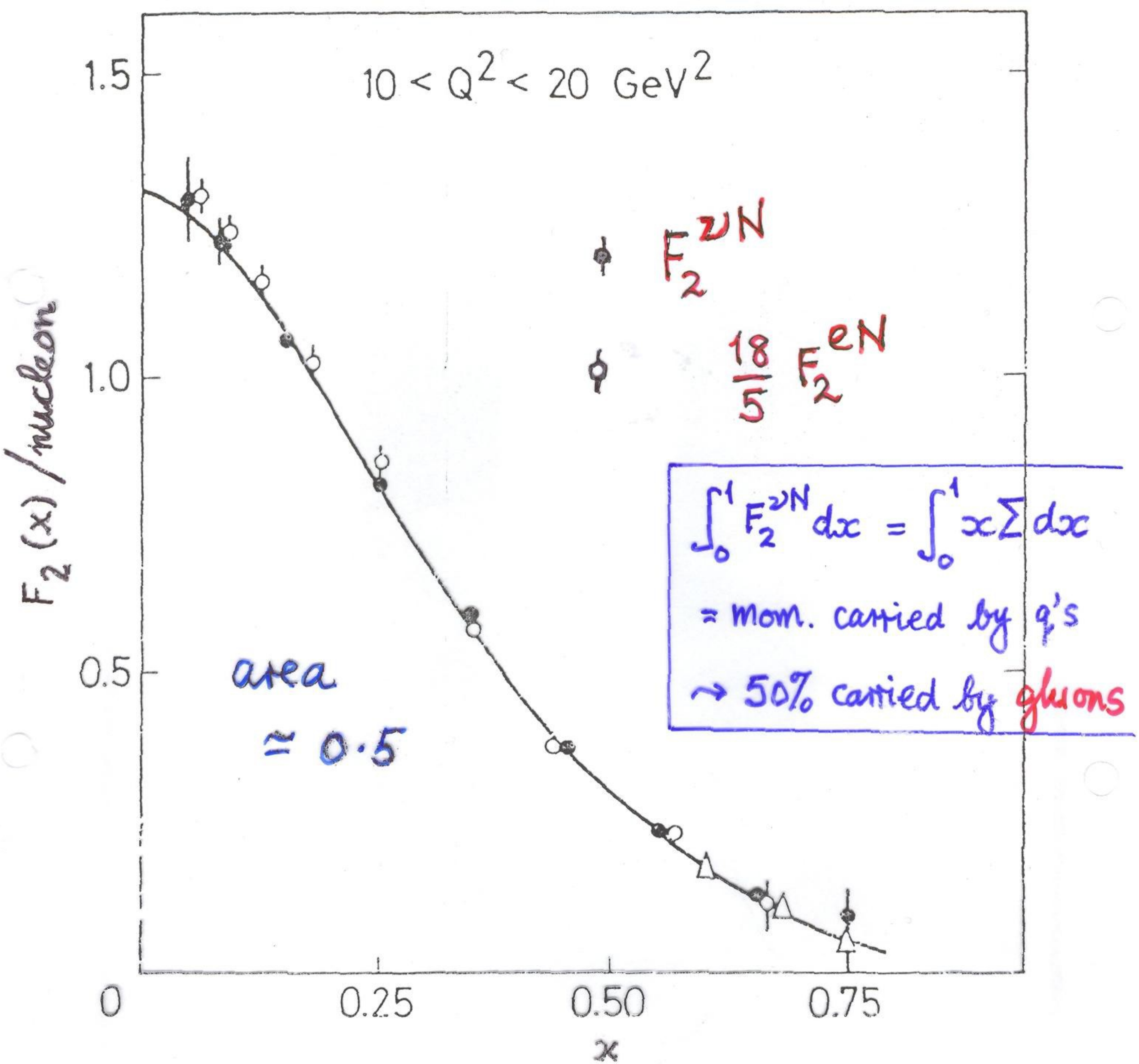
$$F_2(\nu p) = x(u + \bar{d} + \ldots)$$

$$F_2(\nu p) = x(u + \bar{d} + c + \bar{s})$$

$$F_2(\nu n) = x(d + \bar{u} + c + \bar{s})$$

$$F_2(\nu N) = x(u + \bar{u} + d + \bar{d} + 2c + 2s)$$

$\int F_2\, dx =$ mom. carried by quarks



$10 < Q^2 < 20\ \mathrm{GeV}^2$
$F_2(x)$ / nucleon
x
$F_2^{\nu N}$
$\frac{18}{5} F_2^{eN}$
area $\simeq 0.5$
$\int_0^1 F_2^{\nu N} dx = \int_0^1 x \Sigma\, dx$
= mom. carried by q's
→ 50% carried by gluons