Holography and Hadron Physics II

Dmitry Melnikov
International Institute of Physics, UFRN

New Trends – Natal
Oct'14

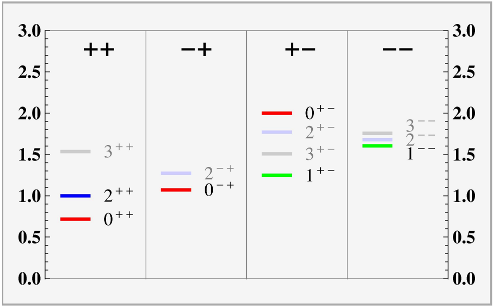
Plan

1. Holographic engineering
    - String theory and holographic correspondence
    - Gauge sector
    - Matter sector
    - Sakai-Sugimoto model
    - Klebanov-Strassler model
2. Holographic phenomenology
    - Glueballs (spectrum)
    - Mesons (spectrum/couplings)
    - Baryons (couplings/nuclear force/finite density)

Glueballs

Glueballs

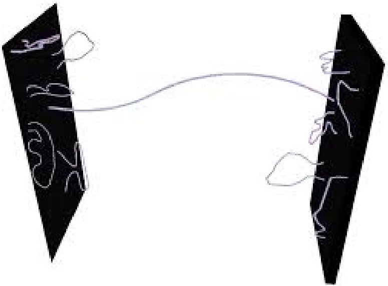
Lattice data



Spectrum of a pure glue $SU(3)$ YM was computed on a lattice by Morningstar and Peardon '99. SYM? Fermion sign problem

Glueballs

Closed strings



Closed strings don't have to be localized on branes. Glueballs are described by fluctuations of the bulk gravity fields

Glueballs

2^{++} state

Spectra of particles are given by the poles of the 2-point functions

$$2^{++}: \qquad \langle\, T_{\mu\nu}(p) T_{\rho\eta}(0)\, \rangle$$

In holography these can be computed as normal modes of the fluctuations of the gravity fields:

$$T_{\mu\nu} \qquad \longleftrightarrow \qquad g_{\mu\nu}$$

$$g_{\mu\nu} \qquad \longrightarrow \qquad g_{\mu\nu} + h_{\mu\nu}$$

Need to solve linearized gravity equations

$$\left(\frac{1}{\sqrt{-g}} \partial_z \sqrt{-g} g^{zz} \partial_z - m^2 \right) h_{ij} = 0$$

Glueballs

Type IIB SUGRA

For Klebanov-Strassler find the normal modes of the linearized eqns

$$
\begin{aligned}
R_{MN} &= \frac{1}{2}\partial_M\Phi\partial_N\Phi + \frac{1}{2}e^{2\Phi}\partial_M C_0 \partial_N C_0 + \frac{1}{4}\left(e^{-\Phi}H_{MPQ}H_N{}^{PQ} + e^{\Phi}\tilde{F}_{MPQ}\tilde{F}_N^{PQ}\right) \\
&\quad -\frac{1}{48}g_{MN}\left(e^{-\Phi}H_{PQR}H^{PQR} + e^{\Phi}\tilde{F}_{PQR}\tilde{F}^{PQR}\right) + \frac{1}{96}\tilde{F}_{MPQRS}\tilde{F}_N^{PQRS}
\end{aligned}
$$

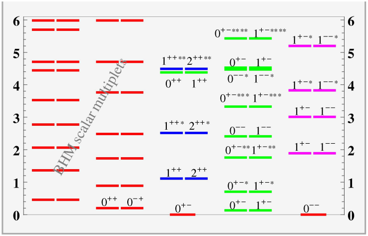
$$
\begin{aligned}
d\star d\Phi &= e^{2\Phi}dC_0 \wedge \star dC_0 - \frac{1}{2}e^{-\Phi}H_3\wedge\star H_3 + \frac{1}{2}e^{\Phi}\tilde{F}_3\wedge\star\tilde{F}_3 \\
d(e^{2\Phi}dC_0) &= -e^{\Phi}H_3\wedge\star\tilde{F}_3, \qquad d(e^{\Phi}\star\tilde{F}_3) = F_5\wedge H_3
\end{aligned}
$$

$$
\star\tilde{F}_5 = \tilde{F}_5, \qquad d\star(e^{-\Phi}H_3 - e^{\Phi}C_0\tilde{F}_3) = -F_5\wedge F_3
$$

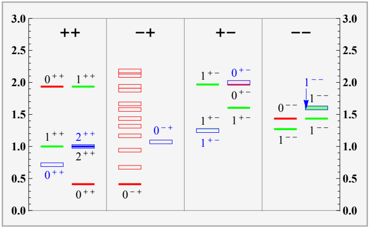
A bit simpler task for Witten-Sakai-Sugimoto model

Glueballs

Spectrum in Klebanov-Strassler model



m^2



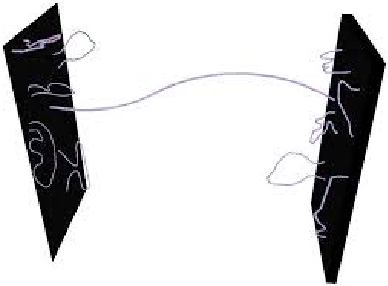
m: KS vs $SU(3)$

It would be nice to compare with lattice predictions

Mesons in Sakai-Sugimoto

Mesons

Open strings



Mesons come from the open string sector. They are fluctuations of the D-brane fields

Mesons

Effective action

DBI action

$$S_{\mathrm{DBI}} = -(2\pi\alpha')^{-9/2} \int_{D8} \mathrm{d}^9 x \, e^{-\phi} \, \mathrm{Tr} \sqrt{-\det(g_{ab} + 2\pi\alpha' \mathcal{F}_{ab})}$$

g_{ab} is an induced metric. Changing profile $x_4(u)$, changes g_{ab}

Chern-Simons actions

$$S_k = (2\pi\alpha')^{-9/2} \int_{D8} \mathrm{e}^{2\pi\alpha' \mathcal{F}} \wedge C_3$$

Mesons

Quadratic action for $N_f = 1$

$\mathcal{A}_a$, $a = 0, \ldots, 8$. Ignore non-singlets of $SO(5)$: $\mathcal{A}_I = 0$,

$$\mathcal{A}_\mu(x,z) = \sum_n B_\mu^{(n)}(x)\psi^{(n)}(z), \qquad \mathcal{A}_z(x,z) = \sum_n \varphi^{(n)}(x)\psi^{(n)}(z)$$

The quadratic part of the DBI action reads

$$S_{\mathrm{DBI}} = -\int \mathrm{d}^4x \left(\frac{1}{2}(\partial_\mu \varphi^{(0)})^2 + \sum_{n\geq 1} \frac{1}{4}(F_{\mu\nu}^{(n)})^2 + \frac{1}{2} m_n^2 M_{KK}^2 (B_\mu^{(n)})^2 \right)$$

where m_n^2 are eigenvalues of a 2^{nd} order diff operator for $\psi^{(n)}$

Similarly for $x_4(z)$

Mesons

Meson spectrum

- There is a massless pseudoscalar particle $\varphi^{(0)}$
- A set vector and axial vector mesons

$$m_1^2 = 0.67^{--}, \quad m_2^2 = 1.6^{++}, \quad m_3^2 = 2.9^{--}, \quad \ldots$$

- A tower of massive scalars

$$m_1^2 = 3.3^{--}, \quad m_2^2 = 5.3^{++}, \quad \ldots$$

Compare with known meson spectra

$$\frac{m_2^2}{m_1^2} = \frac{m_{a(1260)}^2}{m_\rho^2} = 2.4\,(2.51), \qquad \frac{m_2^2}{m_1^2} = \frac{m_{a_0(1450)}^2}{m_\rho^2} = 4.9\,(3.61)$$

Mesons

Effective action for $N_f > 1$

Spectrum of mesons does not change for the $U(N_f)$ case

Introducing $U(x) = P\exp\left(\int_{-\infty}^{\infty} \mathcal{A}_z(x,z)\mathrm{d}z\right)$ in the $\mathcal{A}_z = 0$ gauge

$$S_{\mathrm{DBI}} \propto \int \mathrm{d}^4x \mathrm{Tr}\ \left(A(U^{-1}\partial_\mu U)^2 + B[U^{-1}\partial_\mu U, U^{-1}\partial_\nu U]^2\right)$$

This is a Skyrme model Lagrangian provided that

$$f_\pi^2 = \frac{1}{54\pi^2} M_{KK}^2 \lambda N_c\,, \qquad e^2 = \frac{27\pi^7}{2b\lambda N_c}$$

Full Lagrangian also contains vector meson fields

Meson couplings

Restricting to the lightest vector meson, one can derive

$$S_{\mathrm{DBI}} = \int \mathrm{d}^4x \left[-a_\pi^2 \mathrm{Tr}\,(\partial_\mu \pi)^2 + a_{v^2}\left(\frac{1}{2}\mathrm{Tr}\,(\partial_\mu v_\nu - \partial_\nu v_\mu)^2 + m_v^2 \mathrm{Tr}\, v_\mu^2\right)\right.$$

$$\left. + a_{v^3}\mathrm{Tr}\,\left([v_\mu, v_\nu](\partial^\mu v^\nu - \partial^\nu v^\mu)\right) + a_{v\pi^2}\mathrm{Tr}\,\left([\partial_\mu \pi, \partial_\nu \pi](\partial^\mu v^\nu - \partial^\nu v^\mu)\right) + \ldots \right]$$

$$U(x) = \mathrm{e}^{2i\pi(x)/f_\pi}$$

All the couplings are fixed in terms of f_π and masses:

$$f_\pi^2 a_{v^3} a_{v\pi^2} = 0.72\,(1)\,, \qquad m_v^2 a_{v\pi^2}^2 f_\pi^2 = 1.3\,(2)$$

Baryons in Sakai-Sugimoto Model

Baryons

Strings and baryons

Baryon must correspond to a point where N_c strings can end (baryon vertex), joining N_c quarks on the opposite ends. Such objects must carry N_c units of charge. These can be D-branes wrapped on compact cycles. In a background with a flux of p-form through a p-cycle

$$\int_{C_p} F_p = N_c$$

p-brane wrapped on the cycle picks a charge N_c from the $U(1)$ field:

$$\int_{\mathrm{Dp}} a \wedge F_p = N_c \int a$$

To compensate the excess of charge on the compact manifold, it must be carried away by N_c strings

Baryons

Baryons and instantons

In Sakai-Sugimoto model baryons are D4-branes wrapping S^4

	0	1	2	3	4	5	6	7	8	9
$D4$	○	○	○	○	○					
$D4$	○						○	○	○	○
$D8/\overline{D8}$	○	○	○	○		○	○	○	○	○
	$\mathcal{M}^{1,3}$				S^1	R^+	S^4			

Strings that connect D4-baryon and D8-branes pull them towards each other. In the end D4-branes "dissolve" in the D8-brane and can be described as instantons of the gauge fields on the D8-branes

Baryons

Effective action

- DBI action – generalization of the relativistic particle action

$$S_{\mathrm{DBI}} = -\frac{1}{(2\pi)^p {\alpha'}^{\frac{p+1}{2}}} \int_{\mathrm{D}p} \mathrm{d}^{p+1}x \, e^{-\Phi} \mathrm{Tr} \sqrt{-\det(g + 2\pi\alpha' \mathcal{F})}$$

- Chern-Simons action – generalization of the Coulomb coupling

$$S_{\mathrm{CS}} = \int_{\mathrm{D}p} C \wedge \exp(2\pi\alpha' \mathcal{F})$$

Here $\mathcal{F} = \mathrm{d}\mathcal{A} + \mathcal{A} \wedge \mathcal{A}$ are $U(N)$ fields living on the Dp branes. For mesons we looked at $\mathcal{F} = 0$ background. Baryons are solitons

Baryons

Effective action ($N_f = 2$)

$$U(N_f) \text{ gauge fields:} \quad \mathcal{A}_M = A_M + \frac{1}{2}\widehat{A}_M$$

$$S = -\kappa \int \mathrm{d}^5x \, \frac{1}{2g^2_{\mathrm{YM}}(x_5)} \,\mathrm{Tr}\, F^2_{MN} + e \int \widehat{A} \wedge \mathrm{Tr}\, F \wedge F + S[\widehat{A}]$$

$$\kappa = \frac{\lambda N_c}{216\pi^3}\,, \qquad e = \frac{N_c}{15\pi^2}$$

This action is NLO in α' (λ^{-1})

Baryons

Dynamics

Baryons are subject to a Coulomb force mediated by $\hat{A}_\mu$ field and to a gravitational potential

$$\frac{1}{g_{\rm YM}^2(x_5)} \propto 1 + cM_{KK}^2 x_5^2 + O(M_{KK}^4 x_5^4)$$

It is consistent to look for solutions in $1/\lambda$ expansion. In the leading order baryon is a BPS (BPST) instanton. Interactions are $1/\lambda$ suppressed.

As a result of a competition between gravity force and Coulomb force, the size of the instanton-baryon stabilizes at

$$a \sim \frac{1}{\sqrt{\lambda} M_{KK}}$$

Baryons

Properties of baryons

- $m_\mathrm{B} \sim \lambda N_c$ – in the leading order baryons are pure YM instantons
- $\Pi \sim N_c$ – baryon interaction is $O(1/\lambda)$ correction to the rest energy.
- $a \sim 1/M_{KK}\sqrt{\lambda}$ – holographic baryons are very small

Consequences for holographic nuclear matter

- $\mathrm{K} \sim \frac{1}{m_\mathrm{B}} \ll \Pi$ – at finite density holographic baryons are crystals
- Small binding energy

Baryons

Baryon spectra

Instanton solutions depend on parameters (position, size and orientation). Plugging a solution in the effective action one finds the effective Hamiltonian for the parameters. Quantizing the Hamiltonian, one derives the spectrum

$$M = M_0 + \sqrt{\frac{(l+1)^2}{6} + \frac{2}{15} N_c^2} + \frac{2(n_\rho + n_z) + 2}{\sqrt{6}}$$

Here $n_z, n_\rho = 0, 1, 2, \ldots$ are quantum numbers related to the position in the holographic direction and the size

l is the orientation quantum number. It labels $(l/2, l/2)$ irreps of the $SO(4) \simeq SU(2) \times SU(2)/Z_2$

Baryons

Static properties

Using the techniques of Skyrme model various static properties can be computed: magnetic moments, charge radii, meson couplings. One can also derive baryon form-factors.

$$\langle r^2 \rangle_{I=0}^{1/2} \simeq 0.785\,\text{fm}\,(0.806), \qquad M_{KK} = 949\,\text{MeV}$$

$$g_{I=0} \simeq 1.68\,(1.76), \qquad M_N = 940\,\text{MeV}$$

$$\langle r^2 \rangle_{E,p}^{1/2} = \langle r^2 \rangle_{I=0}^{1/2}\,(0.875), \qquad M_{KK} = 949\,\text{MeV}$$

$$\langle r^2 \rangle_{E,n} = 0\,(-0.116), \qquad M_{KK} = 949\,\text{MeV}$$

Overall performance comparable is to other hadron physics models

Baryons

Walecka's model

In the effective model of a baryon coupled to massive vector (ω) and scalar (σ) fields the non-relativistic baryon-baryon potential reads

$$V = \frac{1}{4\pi}\left(g_\omega^2\,\frac{\mathrm{e}^{-m_\omega|x|}}{|x|} - g_\sigma^2\,\frac{\mathrm{e}^{-m_\sigma|x|}}{|x|}\right)$$

The interaction is attractive at large distances if $m_\sigma < m_\omega$ and repulsive at short distances if $g_\sigma < g_\omega$

Baryons

Holographic nuclear force

In Sakai-Sugimoto model $m_\sigma > m_\omega$, so repulsion will always win at large distances. How about the coupling?

$$S = -\frac{\lambda N_c}{216\pi^3} \int \mathrm{d}^4x\,\mathrm{d}z\,\frac{1}{2g_{\mathrm{YM}}^2(z)} \left(\mathrm{Tr}\,F_{\mu\nu}^2 + \frac{1}{2}\,\widehat{F}_{\mu\nu}^2\right) + \frac{1}{2}\,(\partial_\mu \Phi)^2 +$$

$$+\frac{N_c}{8\pi^2} \int \mathrm{d}^4x\,\mathrm{d}z\,\hat{A} \wedge \mathrm{Tr}\,F^2 + \frac{N_c}{6\pi^2} \int \mathrm{d}^4x\,\mathrm{d}z\,\Phi\mathrm{Tr}\,F^2$$

The net effect of Φ is to renormalize the vector meson coupling. The coupling g_σ is always smaller. No attractive nuclear force in Sakai-Sugimoto model

Phenomenology of KS model

Klebanov-Strassler Model

Adding flavor

Add N_f probe ($N_f \ll N_c$) D7 and anti-D7 "flavor" branes

Configuration in $R^{1,3} \times R^+ \times S^3 \times S^2$:

	0	1	2	3	4	5	6	7	8	9
$D3$	$\circ$	$\circ$	$\circ$	$\circ$						
$D5$	$\circ$	$\circ$	$\circ$	$\circ$					$\circ$	$\circ$
$D7/\overline{D7}$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$	$\circ$		

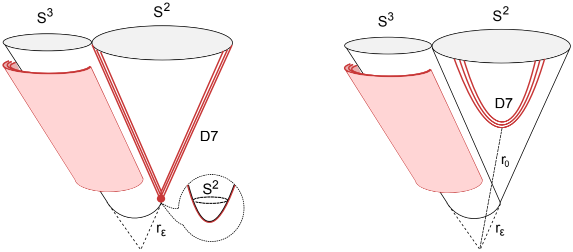
The D7 wrap S^3 and span a line in $R^+ \times S^2$. By symmetry transformation one can always restrict them to the equator of S^2

Stable embeddings correspond to U-shape configurations

Klebanov-Strassler Model with Flavor

Embedding

Effectively the embedding of D7-$\overline{\text{D7}}$ is again a U-shape configuration on a cigar



- $r_\epsilon \equiv \epsilon^{2/3}$ – sets the scale (M_{KK})
- r_0 – lowest point of the U-shape

Klebanov-Strassler Model

Spectrum

Closed string sector contains glueballs

$$m_{\text{gb}} = \frac{r_\epsilon}{\lambda\alpha'}$$

In the probe approximation we ignore corrections to glueball masses.

Typical meson mass scales as

$$m_{\text{meson}} \sim \frac{r_0}{\lambda\alpha'} = \frac{r_0}{r_\epsilon} m_{\text{gb}}$$

This is the result in the conformal limit. Corrections to this result in the KS case can be ignored if $r_0 \gg r_\epsilon$. If $r_0 \simeq r_\epsilon$, $m_{\text{meson}} \sim m_{\text{gb}}$ anyway

Klebanov-Strassler Model

Pseudo-Goldstone mode

In the KS background the conformal symmetry is broken explicitly. Therefore the Goldstone boson should acquire a small mass, different from the mass of other mesons

$$m_{\text{pG}} \sim \frac{r_\epsilon^3}{r_0^2 \lambda \alpha'} = \frac{r_\epsilon^2}{r_0^2} m_{\text{gb}}$$

This is similar to pions in QCD. If the quarks were massless QCD would have chiral symmetry $U(N_f)_L \times U(N_f)_R$, which is spontenously broken down to $U(N_f)$. Pions would be Goldstone bosons of the broken symmetry. Because of the quark mass, chiral symmetry is broken explicitly and pions are light pseudo-Goldstone particles.

Klebanov-Strassler Model

Summary

Spectrum in the $r_\epsilon \ll r_0$ case

- glueballs, including a pair of massless ones
- ordinary mesons $m \sim (r_0/r_\epsilon)m_{\text{gb}}$
- pseudo-Goldstone (σ) $m \sim (r_\epsilon/r_0)^2 m_{\text{gb}}$
- massless pions (fluctuations of A_z)

Taking r_0 large enough one can always have $m_\sigma < m_\omega$. However $r_0 \sim r_\epsilon$ case is more interesting

Klebanov-Strassler Model

Baryons

Baryon vertex corresponds to a D3 wrapping the S^3 of the conifold

$$\frac{1}{4\pi^2\alpha'}\int_{S^3} F_3 = M$$

M strings pull the b.v. to the D7. The energy of the configuration

$r_0 \gg r_\epsilon$ (non-antipodal)

$$E_{\text{total}} = \frac{M}{\alpha'}\left(Ar\log r + B|r_0 - r|\right)$$

gravity wins. b.v. is far from D7

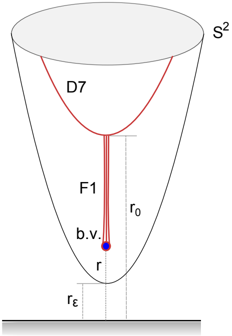
$r_0 \sim r_\epsilon$ (near antipodal)

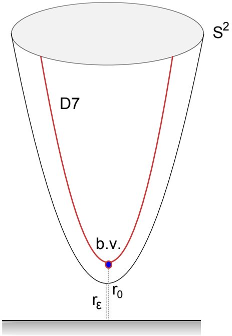
$$E_{\text{total}} = \frac{Mr_\epsilon}{\alpha'}\left(C + D|\tau_0 - \tau|\right)$$

strings win. b.v. dissolves in D7

Klebanov-Strassler Model

Strings vs Instantons





Non-antipodal and near antipodal regimes. Only in the second case one can describe baryons as flavor instantons at the D7. In both cases we can consider b.v. as sitting around the scale r_ϵ.

Klebanov-Strassler Model

Meson contribution in the non-antipodal case

σ

transverse fluctuations of the profile δr_0. Coupling to the strings is simply a variation of the string's length

$$S_{\text{string}}[\delta r_0] \sim \frac{M}{\alpha'} \int \mathrm{d}x^0 \, \delta r_0$$

ω

Vector meson coupling is a boundary term in the string action

$$M \int \hat{A}$$

$\hat{A}$ is the abelian field on the D7

Klebanov-Strassler Model

Meson contribution in the near-antipodal case

σ

Interaction of σ with baryon density is generated by the non-abelian term in the effective action

$$S_{\text{na}} \sim \frac{g_s M \alpha' r_\epsilon}{g_s \alpha'^2} \int \mathrm{d}x^0 \, \sigma$$

ω

Vector meson coupling arises from the CS term in the effective action

$$S_{\text{CS}} \sim M \int \mathrm{d}^4 x \, \mathrm{d}z \, \hat{A}_0 \frac{\epsilon^{ijk}}{2} \operatorname{Tr}\left(F_{ij} F_{kz}\right)$$

$$\sim M \int \mathrm{d}x^0 \, \hat{A}_0$$

Klebanov-Strassler Model

Contribution to the baryon-baryon potential

1. non-antipodal case

$$V \sim \pm \frac{1}{g_s \log r_0} \frac{\mathrm{e}^{-m_{\text{meson}}|x|}}{|x|}$$

   - g_s^{-1} times bigger than the glueball contribution

2. near-antipodal case

$$V = \pm \frac{g_{\text{meson}}^2}{4\pi g_s} \frac{\mathrm{e}^{-m_{\text{meson}}|x|}}{|x|}$$

   - Coupling constant of the scalar vanishes in the antipodal case. It is small in the near-antipodal regime

Klebanov-Strassler Model

Summary of the holographic nuclear force

- The large distance potential of the interaction through lightest meson exchanges:

$$V = \frac{1}{4\pi g_s}\left(g_\omega^2 \frac{e^{-m_\omega |x|}}{|x|} - g_\sigma^2 \frac{e^{-m_\sigma |x|}}{|x|}\right)$$

- In the non-antipodal regime force between two baryons can be made attractive by tuning the mass of the pseudo-Goldstone lighter than the mass of ω. Short distance repulsion is not guaranteed. No natural suppression for the binding energy
- In the near-antipodal regime the difference between mass of σ and ω is small. g_σ is suppressed – short distance repulsion. Small binding energy is possible

Conclusions

- String theory can model many interesting features of hadron physics. A lot of non-perturbative data can be computed from the first principles
- If one is to push to match with experimental results, one has to hide something under the rug
- Adopting a bottom-up approach, one can do as good as conventional hadron physics models
- This suggests that a mixture of the conventional and stringy approaches can be successful in addressing hadron physics problems