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BFKL Phenomenology

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THE REGGE THEORY

The old Regge* theory

•Regge theory was the alternative to QFT for describing strong interactions in the 60's, 70's

•**Reggeon, Regge pole, Regge trajectory** were **really** familiar phrases

"Indeed, Regge theory has been included in good undergraduate textbooks for more than a decade"

E. Leader, Nature, 1978

"In no time at all, Regge pole (pole in the mathematical sense) had become a household expression. Indeed, there is the story of a party at which the charming wife of an American physicist, on being introduced to 'Regge', exclaimed:
<< A Mr. Pole, I'm so pleased to meet you at last >>"

E. Leader, Nature, 1978

*Tulio Regge

Tulio Regge, July 11, 1931, October 23, 2014

From a Theory Group communication email at CERN:

“He was a renowned scientist with important contributions to theoretical particle physics and gravity. His name is in particular known for ``Regge Poles'', which paved the way to string theory, as well as for ``Regge Calculus'', which deals with a discretization of space-time in gravity. There is also the ``Regge-Wheeler equation'' which arises in the study of black holes.”

He was awarded the Dirac Medal in 1996, the Pomeranchuk Prize in 2001.

The asteroid 3778 Regge was named after him.

The old Regge theory

Recall scattering in Quantum mechanics:

- Assume that there is a function F that controls the outcome of a collision between 2 particles, F is actually the scattering amplitude.

- In the simple case, $F = F(E, \theta)$, where E is the energy and θ is the scattering angle. F can be found by solving the Schroedinger eq.

- Generally, it is easier to study the collision at a fixed value of the angular momentum, l, l being positive integer.

- This way, one obtains a set of amplitudes $f_l(E)$, $l = 0, 1, 2, \ldots$ from which one could reconstruct $F(E, \theta)$.

The old Regge theory

Recall scattering in Quantum mechanics:

- Imagine now that the energy E is a complex number and each $f_l(E)$ is a complex function.

- Imagine further that the two colliding particles can bind together to form a bound state with angular momentum L and energy E_B.

- Then the Lth one from the $f_l(E)$ functions, that is $f_L(E)$ would be found to have a "pole" at $E = E_B$, that is a simple infinity $1/(E-E_B)$

- Then push the formalism further to allow the angular momentum to take non-integer values (this step taken 33 years after the original Regge paper).

The old Regge theory

Recall scattering in Quantum mechanics:

- More formally we are talking about the “partial wave expansion”:

$$\mathcal{A}_{ab\to cd}(s,t) = \sum_{l=0}^{\infty}(2l+1)\,a_l(t)\,P_l\,(1+2s/t)$$

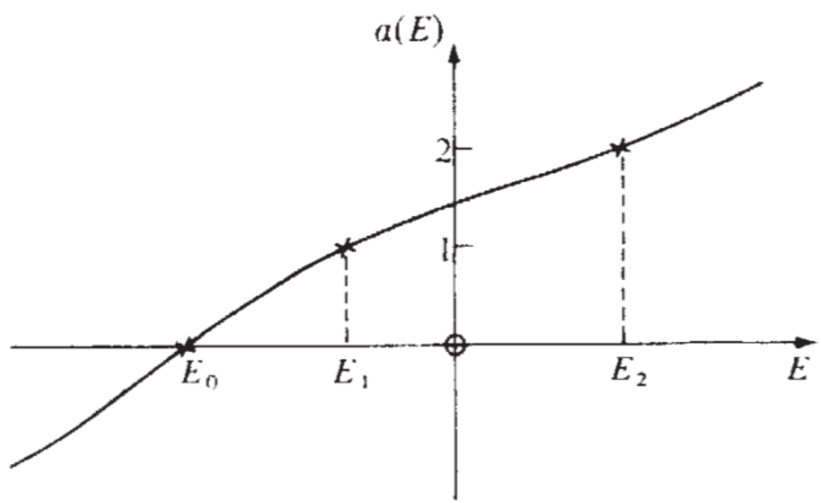
- Regge's idea: complexify energy and angular momentum and focus on the function $\alpha_{(l)}(t)$

- Gribov, Chew, Frautschi

Note that s, t and u are the Mandelstam variables and cos(θ) = 1 + 2 t/s

The old Regge theory

- The function $\alpha_{(l)}(t)$, the trajectory function, has the property that if an energy, E, exists such that $\alpha(E)$ is a positive integer L, then a bound state would exist at energy E with angular momentum, *l*=L



The old Regge theory

Why the previous result is so important?

1. A new way emerged for classifying bound states and resonances into families.

2. Instead of focussing on one bound state with a given angular momentum, say $l = 1$ with energy E_1 and then on another with $l = 2$ and energy E_2, one could focus on a single object, the "trajectory" function $\alpha(E)$ with the property that if a particular value of E causes $\alpha(E)$ to be equal to a positive integer L, then a bound state would exist at that energy E and with angular momentum $l = L$.

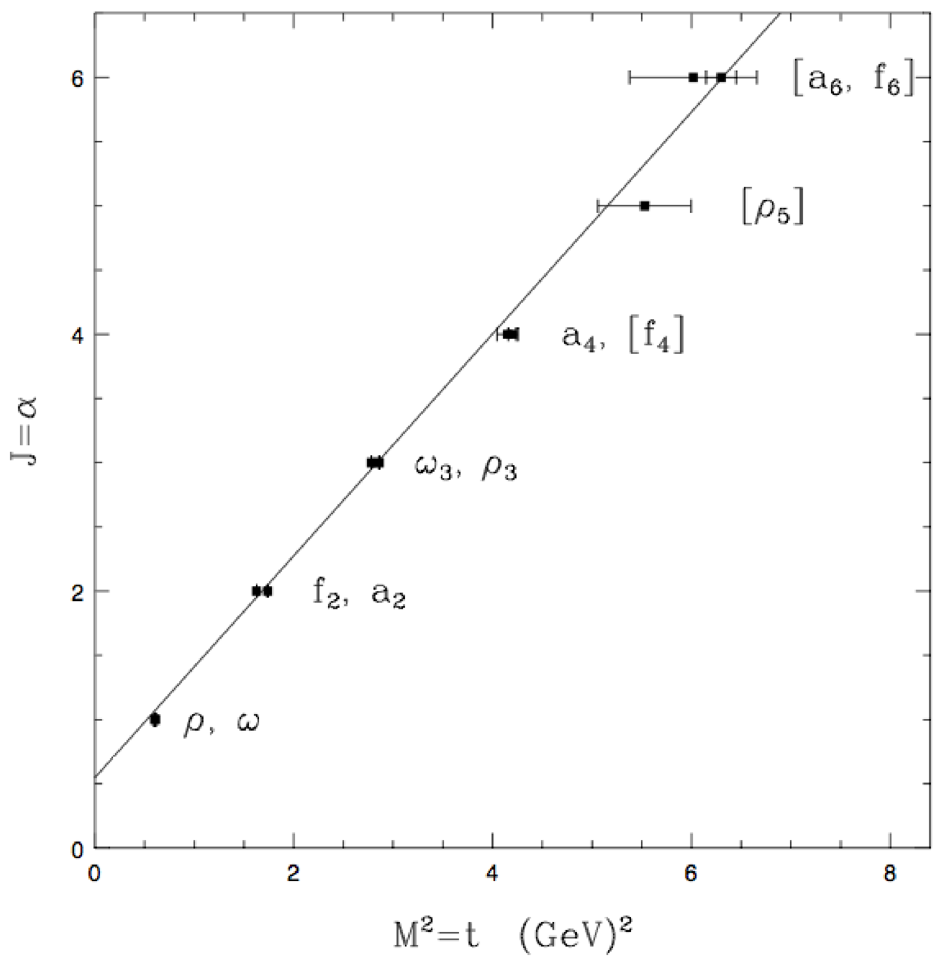
The old Regge theory

Chew and Frautschi ('61, '62) plotted the spins of low lying mesons against mass squared and noticed that they lie in a straight line.

$$\alpha(t) = \alpha(0) + \alpha' t$$

$$\begin{aligned} \alpha(0) &= 0.55 \\ \alpha' &= 0.86\ \mathrm{GeV}^{-2} \end{aligned}$$

$$\frac{d\sigma}{dt} \propto s^{(2\alpha(0) - 2\alpha' t - 2)}$$



How to use this fact?

The old Regge theory

- First, find a hadronic process that can be "characterized" by the particles that lie in the trajectory –same quantum numbers –
- Second, extrapolate the trajectory to negative values of t by using

$$\begin{aligned} \alpha(0) &= 0.55 \\ \alpha' &= 0.86 \text{ GeV}^{-2} \end{aligned}$$

into $\frac{d\sigma}{dt} \propto s^{(2\alpha(0)-2\alpha' t-2)}$

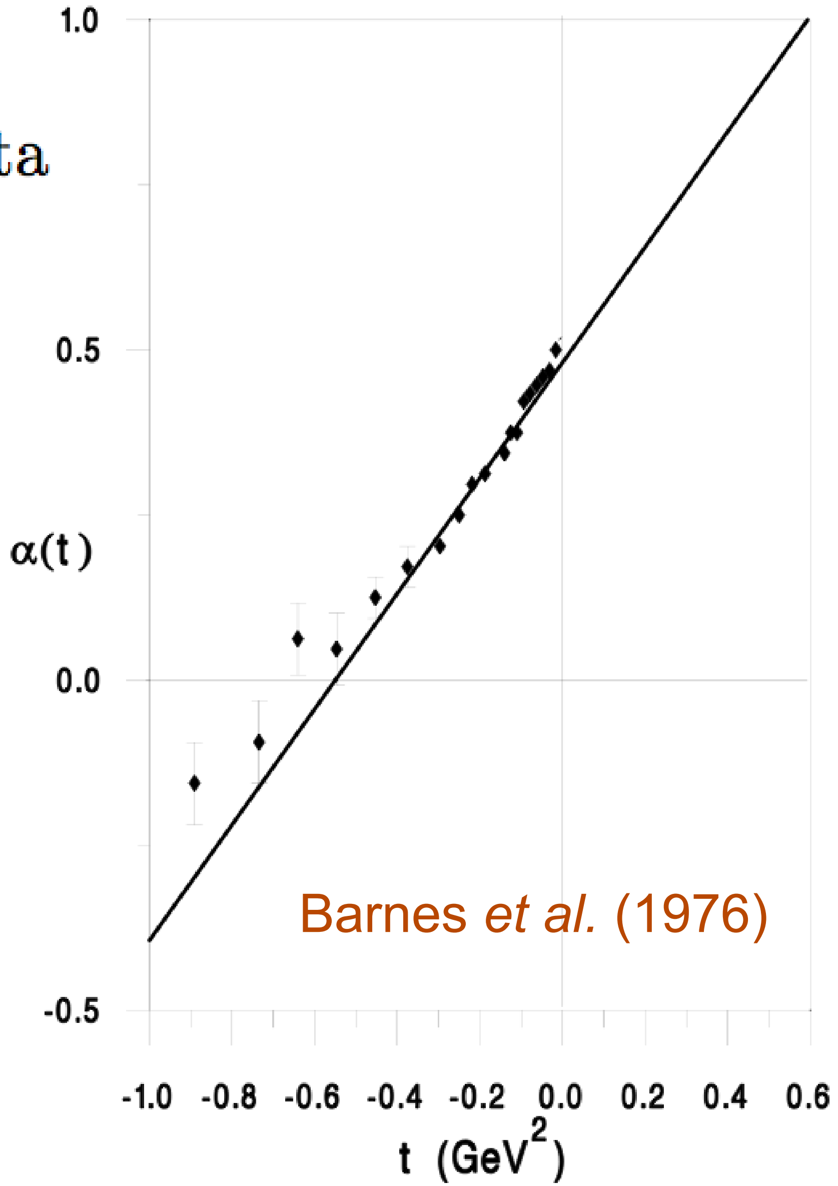
- Plot both the extrapolated trajectory and the experimental data

The old Regge theory

$\alpha(t)$ obtained from $\pi^- p \to \pi^0 n$ data

Characterized by Isospin=1
even Parity → → ρ mesons

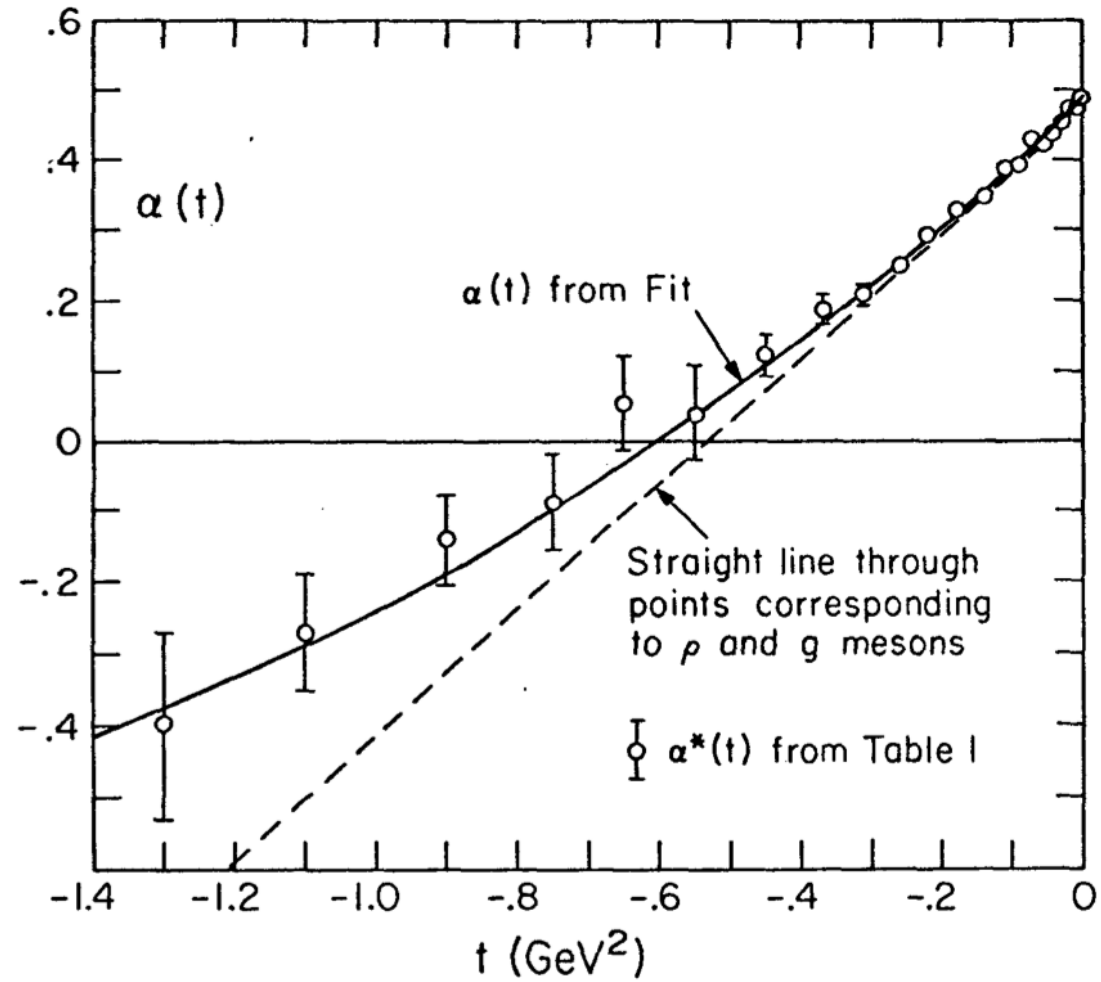
The interaction is mediated by the "exchange of a trajectory".



The old Regge theory

$\alpha(t)$ obtained from $\pi^- p \to \pi^0 n$ data

Barnes *et al.* (1976)



Actually, in the paper, they offer an effective trajectory, plotted with the continuous line whereas the dashed curve is the continuation of the ρ meson trajectory.

Toward the (soft) Pomeron

$$\sigma_{\text{tot}} \propto s^{(\alpha(0)-1)}$$

• The asymptotic behavior of the total cross section for a process can be obtained by the intercept of the Regge trajectory that dominates that process.

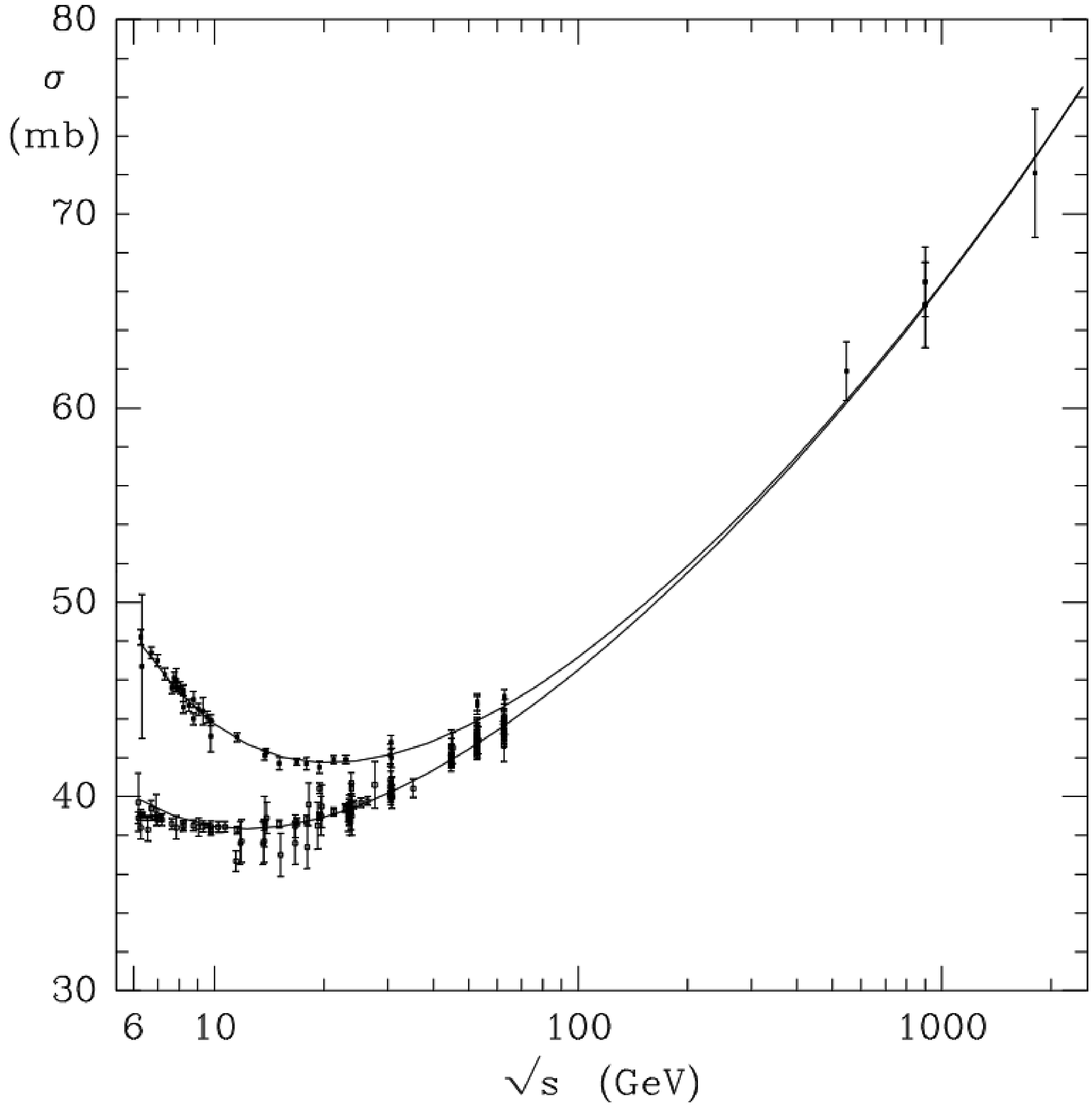
• Pomeranchuk and Okun &Pomeranchuk (1956) proved that if in a process there is charge exchange, then the cross section vanishes asymptotically.

• Pomeranchuk theorem: $\sigma_{\text{tot}}(ab) \underset{s\to\infty}{\simeq} \sigma_{\text{tot}}(a\bar{b})$

• Froissart-Martin bound: $\sigma_{\text{tot}} \leq C \ln^2 s\ , \quad \text{as } s \to \infty$

Toward the (soft) Pomeron

$$\sigma_{\rm tot} \propto s^{(\alpha(0)-1)}$$



But the total cross sections do rise with energy! This is not compatible with an exponent smaller than 1.

The Pomeron

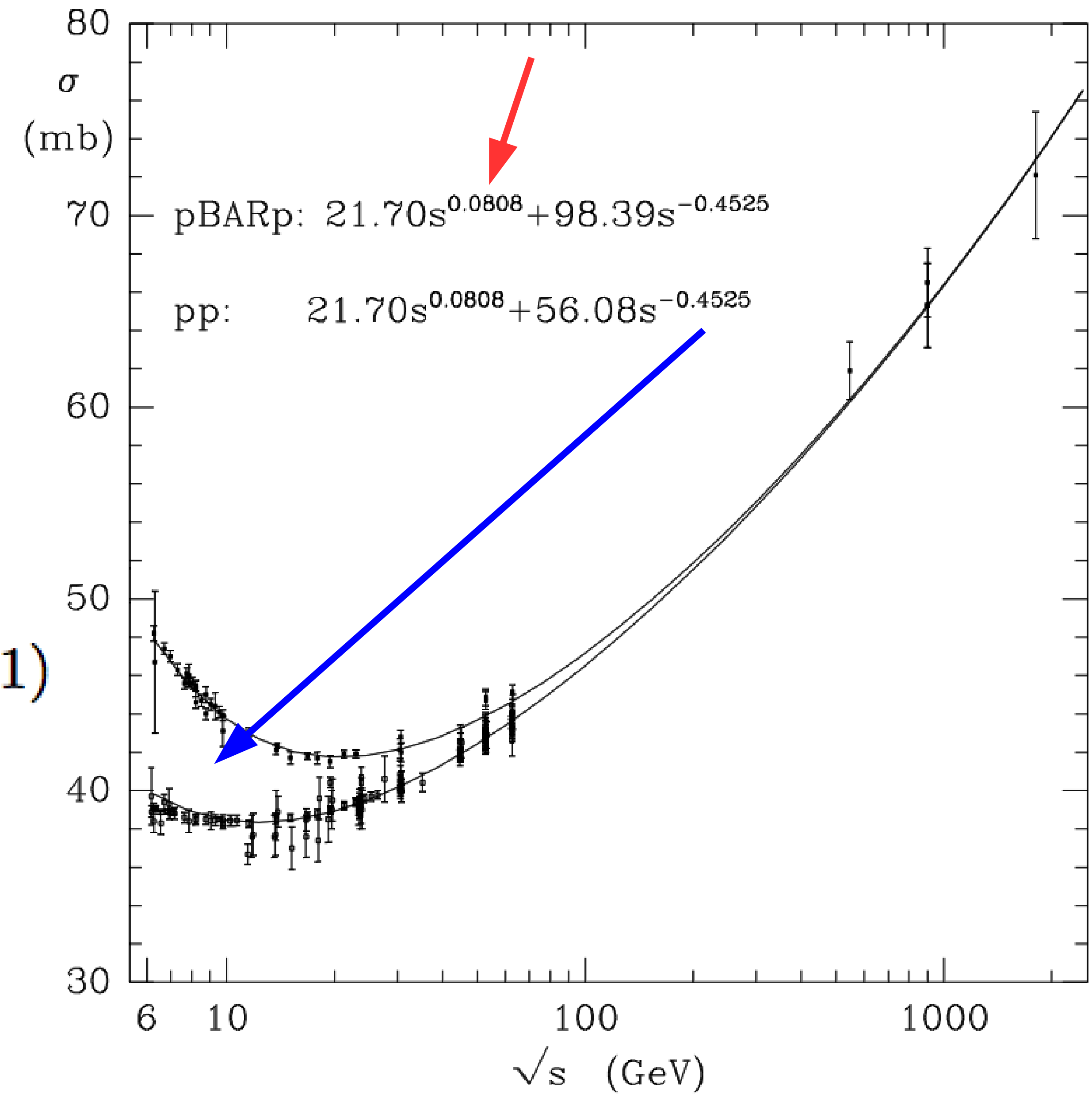
- Gribov introduced (1961) a Regge trajectory with intercept 1: the Pomeron (named after Pomeranchuk)
- It does not correspond to any known particle (glueballs?)
- It carries the quantum numbers of the vacuum, C-even, P-even, Charge 0, Isospin 0.
- Intercept consistent with fits (~1.08)

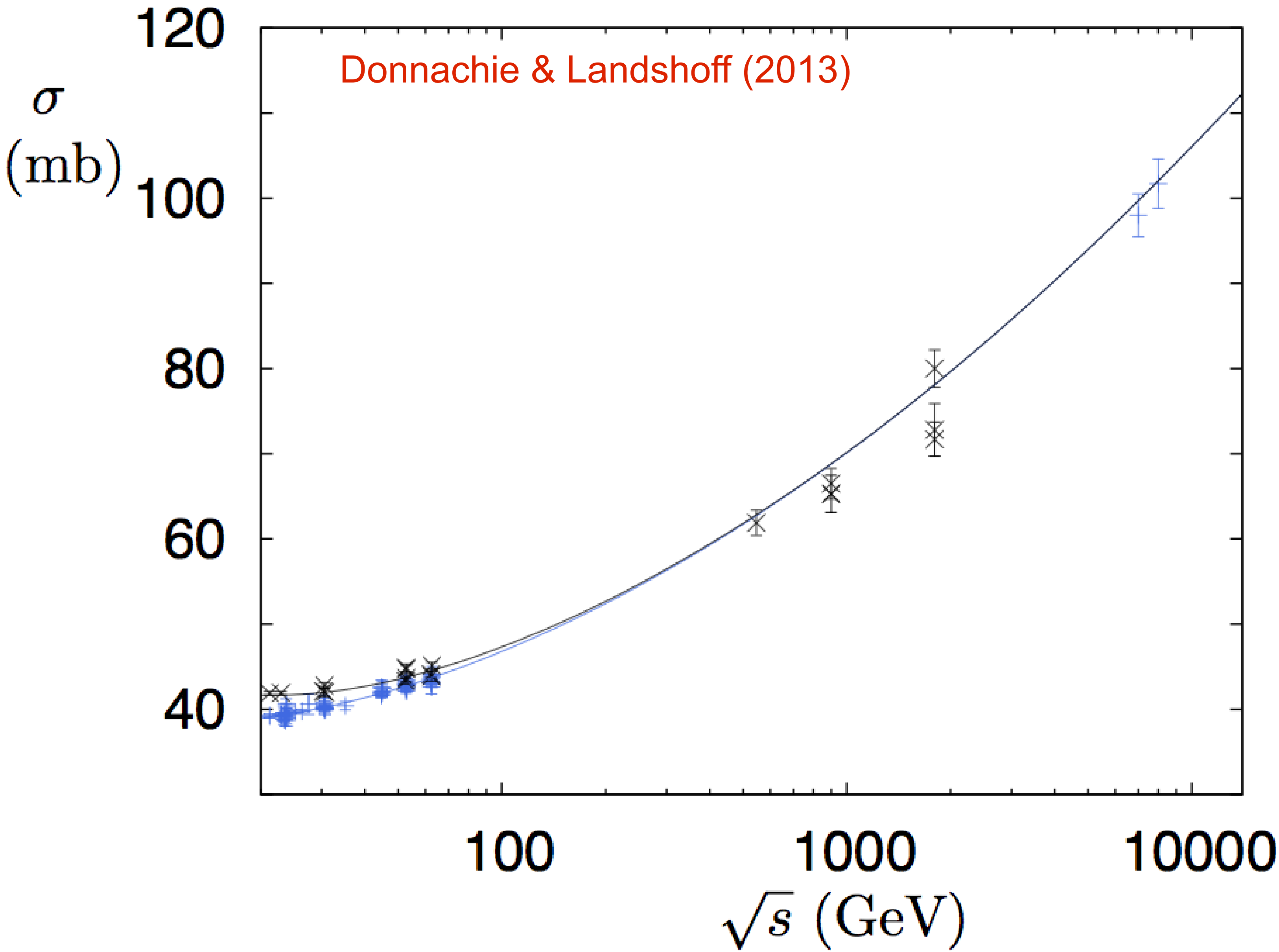
Toward the (soft) Pomeron

Fitting proton-proton & proton-antiproton scattering data.
NOTE: the parameters of the fit were determined before the measurements at Tevatron by using data below 100 c.o.m energy

$$\sigma_{\text{tot}} \propto s^{(\alpha(0)-1)}$$

Donnachie & Landshoff (1992)




Donnachie & Landshoff (2013)
σ (mb)
120
100
80
60
40
100
1000
10000
√s (GeV)

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See also a nice talk by Poghosyan: ***An introduction to Regge Field Theory***

BACK TO QCD: THE BFKL EQUATION

In the QCD era

- Let us leave the old Regge theory for now, we keep in our discussion though terms like "**Regge Limit**", "**trajectory**", "**exponential rise with energy**"
- We know that QCD is the fundamental theory for hadronic collisions
- What does QCD have to say about the rise of the scattering amplitudes at high energies
- Does the Pomeron fit within pQCD or is it of non-perturbative nature?
- The answers (or attempts to answers) to the above started in the 70's

The Balitsky-Fadin-Kuraev-Lipatov (BFKL) equation

This big adventure started almost 40 years ago

L. N. Lipatov, Sov. J. Nucl. Phys. **23** (1976) 338; **983 citations**

E. A. Kuraev, L. N. Lipatov, V. S. Fadin, Phys. Lett. B **60** (1975) 50, Sov. Phys. JETP **44** (1976) 443, Sov. Phys. JETP **45** (1977) 199. **2520 citations**

Ia. Ia. Balitsky, L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822. **2657 citations**

What we will cover and what not

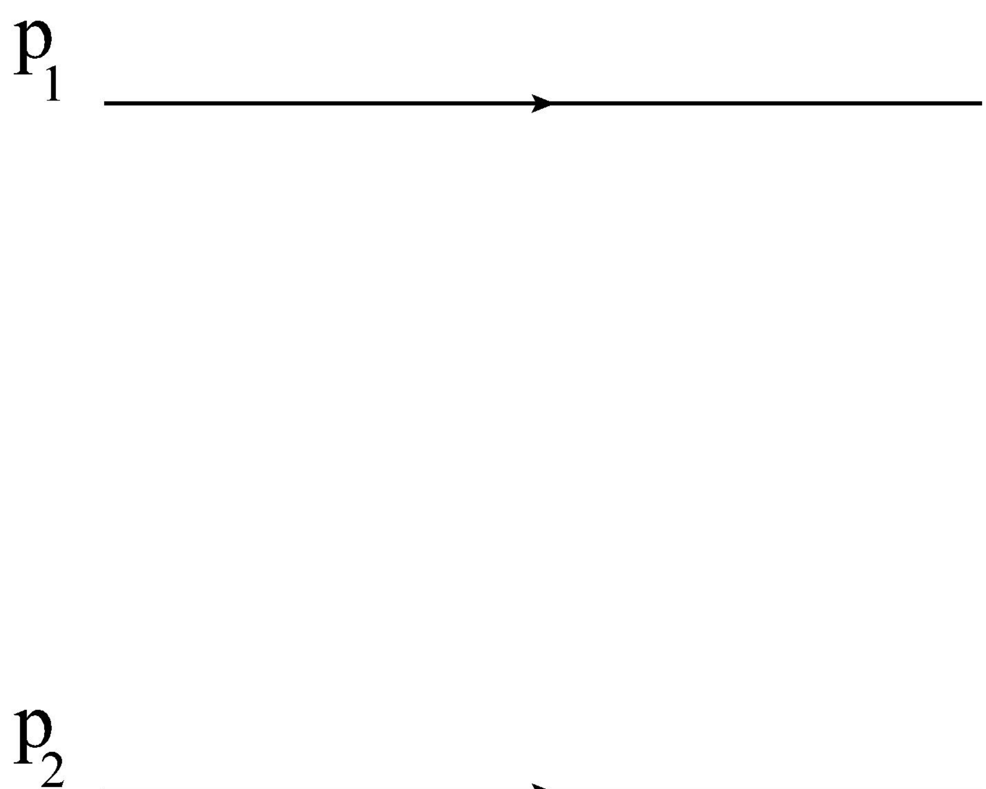
- Reggeization of the gluon
- LO BFKL equation
- Intro to methodology
- NLO BFKL
- Impact factors
- Issues/problems
- Phenomenology

- CCFM
- BK
- CGC
- Saturation
- … and many more

The aim is to show you that this is “good physics”, to prepare you for attending future small-x talks if not with anticipation at least with understanding.

If still things are not so clear, please do not despair, many great people found it hard too in the beginning.

BFKL how to, step 0



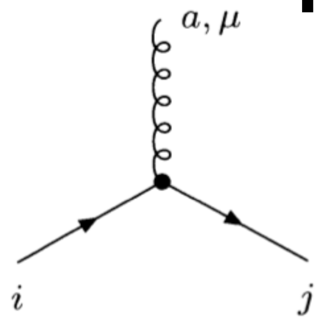
Start from the simplest q q scattering, with momenta p_1 and p_2

Remember that you will have to see an exponential rise to cross sections

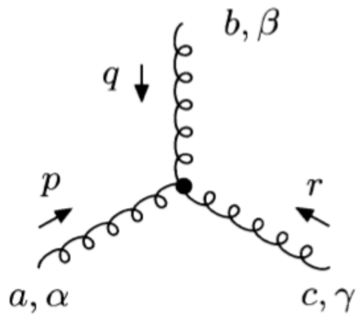
You will be hunting logarithms in s

The last two points are interconnected as we will soon see

Feynman rules for QCD

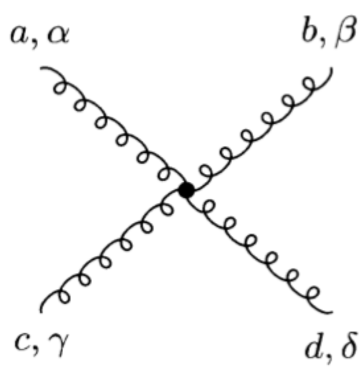


$$-\mathrm{i}g_s(t^a)_{ji}\gamma^\mu$$

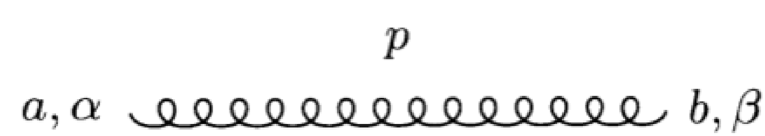


$$-g_s f_{abc}[(p-q)^\gamma g^{\alpha\beta} + (q-r)^\alpha g^{\beta\gamma} + (r-p)^\beta g^{\gamma\alpha}]$$

$$(p+q+r=0)$$



$$-\mathrm{i}g_s^2\{f_{eac}f_{ebd}[g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\delta}g^{\beta\gamma}]$$
$$+f_{ead}f_{ebc}[g^{\alpha\beta}g^{\gamma\delta} - g^{\alpha\gamma}g^{\beta\delta}]$$
$$+f_{eab}f_{ecd}[g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\beta\gamma}]\}$$



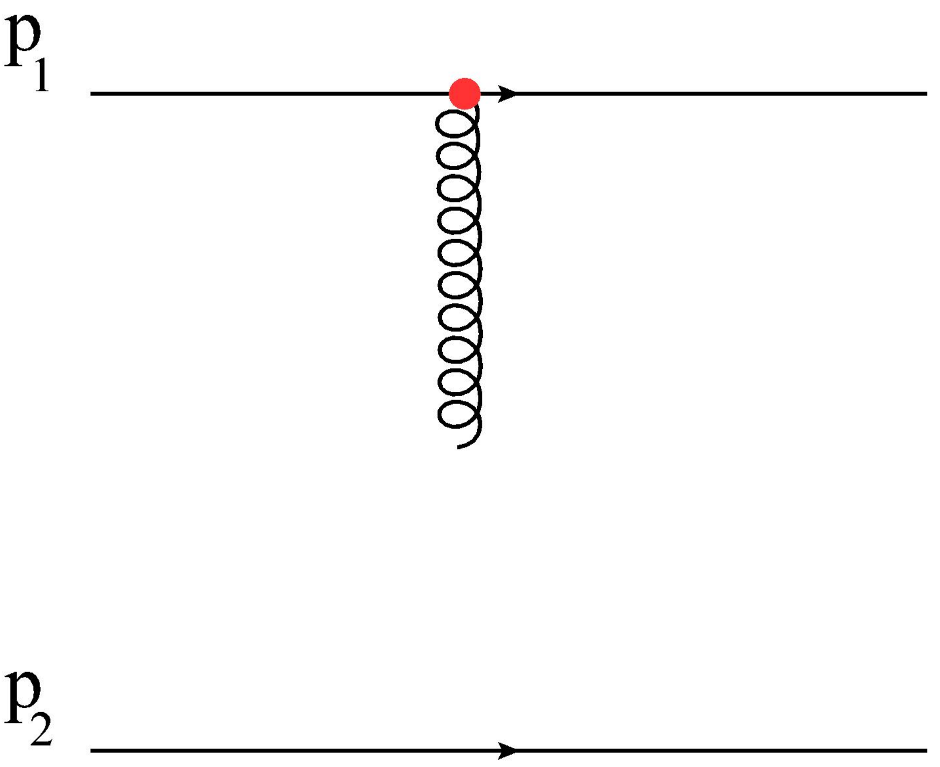
$$\Gamma^{ab}_{\alpha\beta}(p) = \delta^{ab}\left[-g_{\alpha\beta} + (1-\eta)\frac{p_\alpha p_\beta}{p^2+\mathrm{i}\epsilon}\right]\frac{\mathrm{i}}{p^2+\mathrm{i}\epsilon}$$

BFKL how to, still step 0

Assume that whatever is exchanged to the t-channel has mainly transverse components and also that it is much smaller than s. Actually the kinematical limit we are working in is s >> |t|, u~ -s

Then the quark-gluon vertex can be written as

$$-ig_s\bar{u}(p_1)\gamma_\mu u(p_1) = -2ig_s p_1^\mu$$

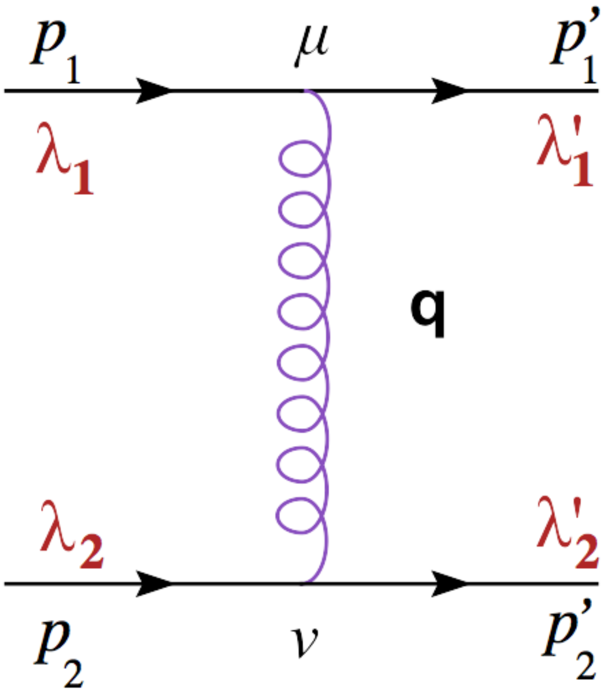


BFKL how to, step 1

$$q = \alpha\, p_1 + \beta\, p_2 + q_\perp$$

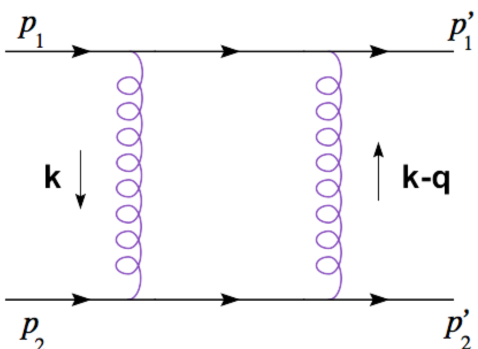
$$s = 2p_1p_2 \text{ and } t = q^2 = \alpha\beta s - \mathbf{q}^2$$

$$g_{\mu\nu} = \frac{2}{s}(p_{1\mu}p_{2\nu} + p_{1\nu}p_{2\mu}) + g_{\mu\nu\perp}$$

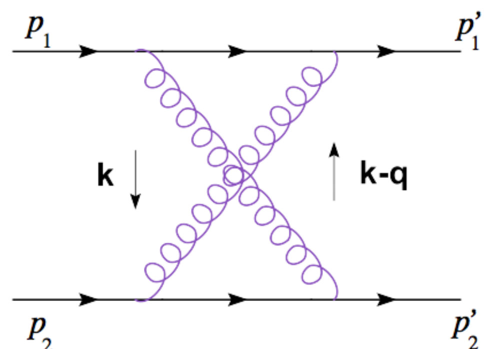


$$A^{(0)}(s,t) = 8\pi a_s t^{\alpha}_{ij} t^{\alpha}_{kl} \frac{s}{q^2} = 8\pi a_s t^{\alpha}_{ij} t^{\alpha}_{kl} \frac{s}{t}$$

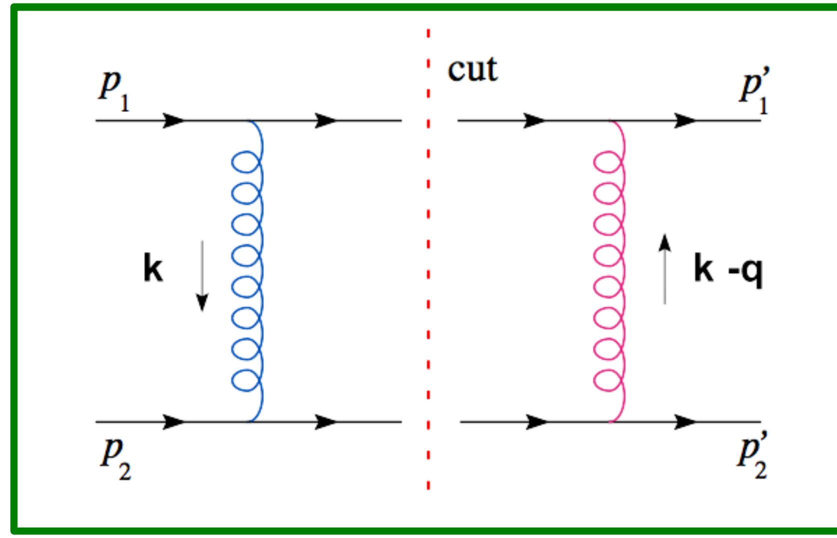
BFKL how to, step 2



(a)



(b)



$$ImA^{(1)}(s,t) = \frac{1}{2}\int d\Pi_2 A^{(0)}(s,k^2)A^{(0)\dagger}(s,(k-q)^2)$$

$$\int d\Pi = \int \frac{d^4k}{(2\pi)^2}\delta((p_1-k)^2)\delta((p_2+k)^2)$$

$$k = \alpha\, p_1 + \beta\, p_2 + k_\perp$$

$$d^4k = \frac{s}{2}d\alpha d\beta d^2\mathbf{k}\,.$$

$$A^{(0)}(s,k^2) \;=\; -8\pi a_s (t^\alpha_{mj}t^\alpha_{nl})\frac{s}{\mathbf{k}^2}$$

$$A^{(0)\dagger}(s,(k-q)^2) \;=\; -8\pi a_s (t^\beta_{mi}t^\beta_{nk})^*\frac{s}{(\mathbf{k}-\mathbf{q})^2}$$

$$\boxed{A^{(1)}(s,t) \;=\; -4\frac{\alpha_s^2}{\pi}\,(t^\alpha t^\beta)_{ij}\,(t^\alpha t^\beta)_{kl}\ln(\frac{s}{t})s\int\frac{d^2\mathbf{k}}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}}$$

Dispersion relations

BFKL how to, still step 2

$$\left.\begin{aligned} A^{(1)}(s,t) &= -\frac{16\pi\alpha_s}{N_c}\,(t^\alpha t^\beta)_{ij}\,(t^\alpha t^\beta)_{kl}\,\frac{s}{t}\,\ln(\frac{s}{t})\,\epsilon(t) \\ A^{(1)}_{cross}(s,t) &= -\frac{16\pi\alpha_s}{N_c}\,(t^\alpha t^\beta)_{ij}\,(t^\alpha t^\beta)_{kl}\,\frac{u}{t}\,\ln(\frac{u}{t})\,\epsilon(t) \end{aligned}\right\}\ \text{The two one-loop amplitudes}$$

after setting:
$$\epsilon(t) = \frac{N_c\alpha_s}{4\pi^2}\int -\mathbf{q}^2\,\frac{d^2\mathbf{k}}{\mathbf{k}^2(\mathbf{k}-\mathbf{q})^2}$$

Remember that we said we are hunting logs in s

$$\mathcal{A} = Re\mathcal{A} + iIm\mathcal{A} \sim \mathcal{B}\ln\frac{s}{t} = \mathcal{B}\ln\frac{s}{|t|} - i\pi\mathcal{B}$$

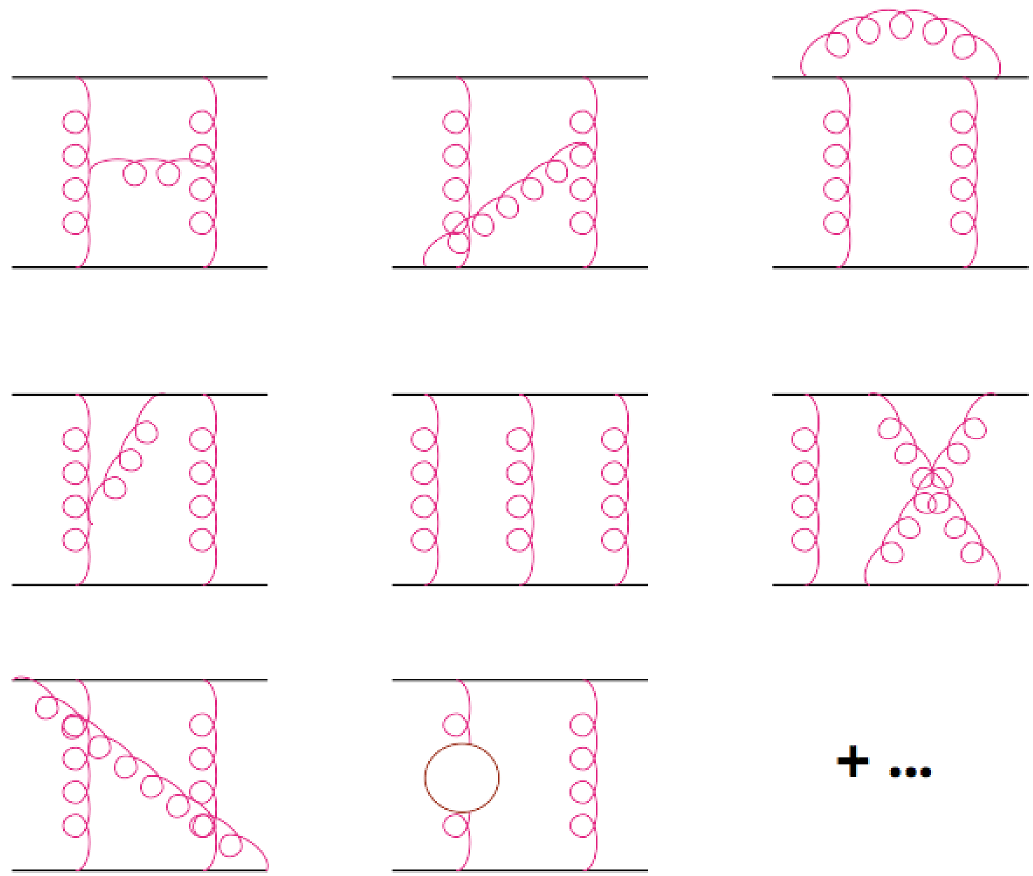
$$Re\mathcal{A} = -\tfrac{1}{\pi}\,Im\mathcal{A}\,\ln\tfrac{s}{|t|}$$

BFKL how to, step 2 final

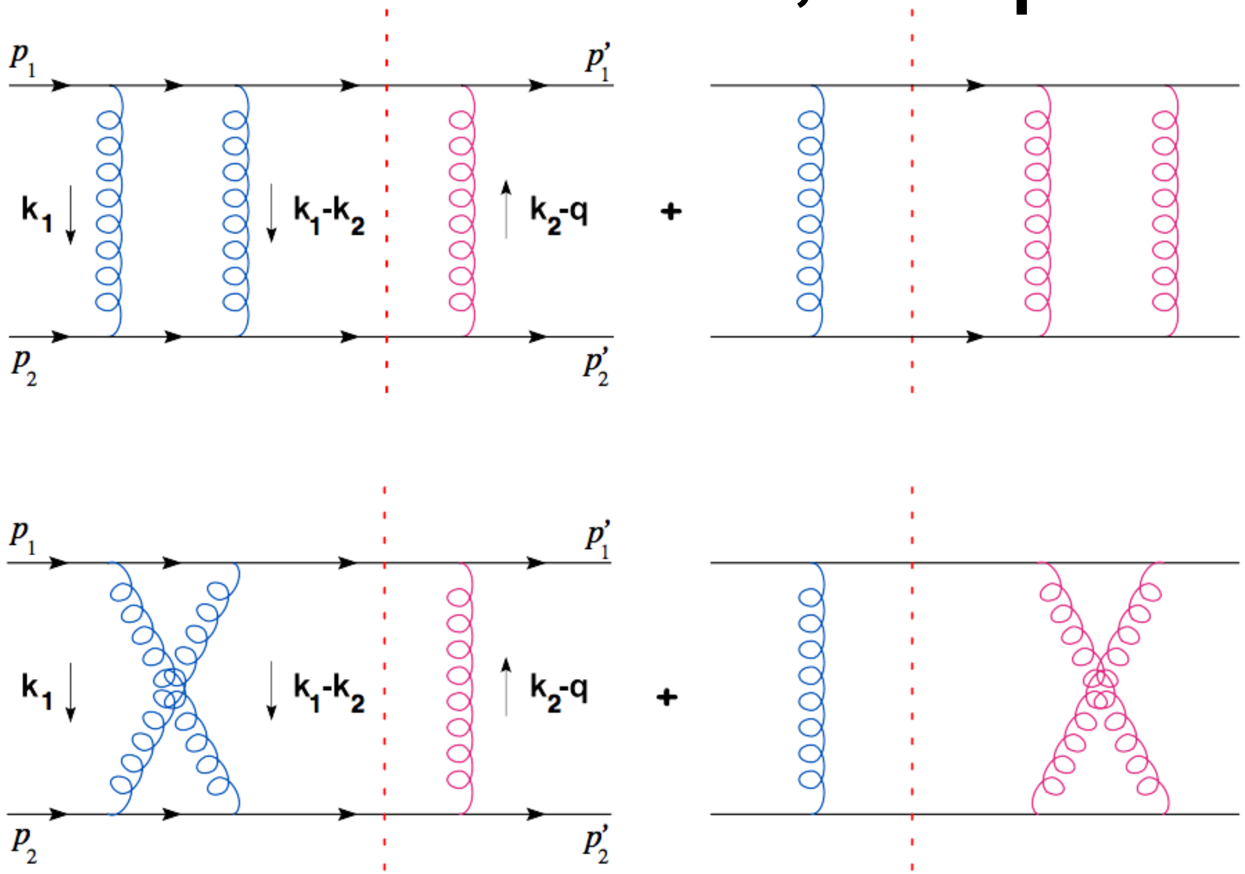
Putting together the two amplitudes for the one-loop, we obtain:

$$A_8^{(1)}(s,t) = 8\pi a_s t_{ij}^{\alpha} t_{kl}^{\alpha} \frac{s}{t} \ln\left(\frac{s}{|t|}\right) \epsilon(t) = A^{(0)} \ln\left(\frac{s}{|t|}\right) \epsilon(t)$$

These diagrams come in the next order



BFKL how to, step 3



$$A_8^{(2)}(s,t) = A^{(0)}(s,t)\,\frac{1}{2}\ln^2(\frac{s}{|t|})\,\epsilon^2(t)$$

Stay on the virtual contributions, real contributions will come afterward

BFKL how to, … pause...

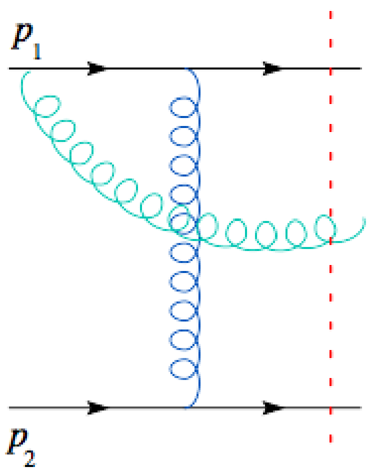
$$A_8(s,t) = A^{(0)}(s,t)\left(1 + \ln(\frac{s}{|t|})\,\epsilon(t) + \frac{1}{2}\ln^2(\frac{s}{|t|})\,\epsilon^2(t) + \ldots\right)$$

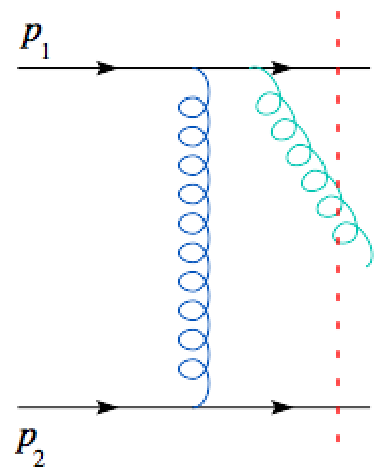
An ansatz seems natural: $$A_8(s,t) = A^{(0)}(s,t)\left(\frac{s}{|t|}\right)^{\epsilon(t)}$$

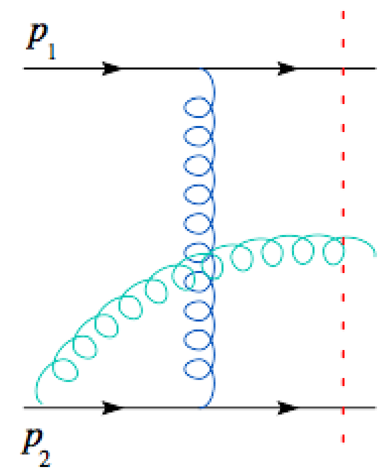
$$D_{\mu\nu}(s,q^2) = -i\frac{g_{\mu\nu}}{q^2}\boxed{\left(\frac{s}{\mathbf{k}^2}\right)^{\epsilon(q^2)}}$$

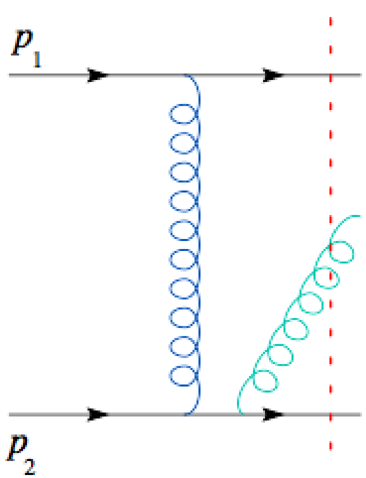
The reggeization of the gluon; Bootstrap equation

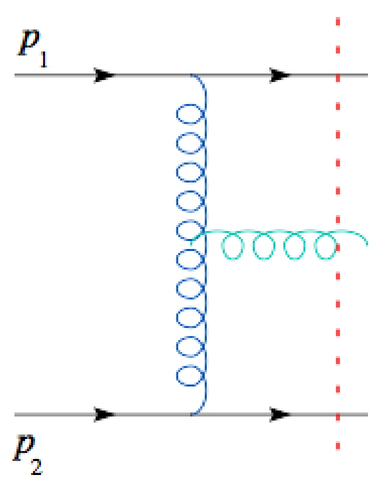
BFKL how to, next step, real corrections



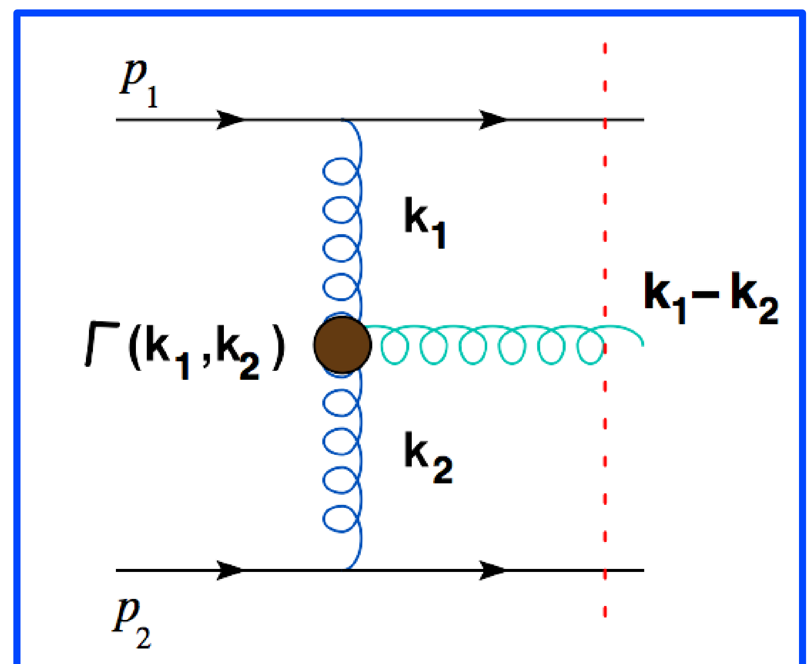








Lipatov's effective vertex



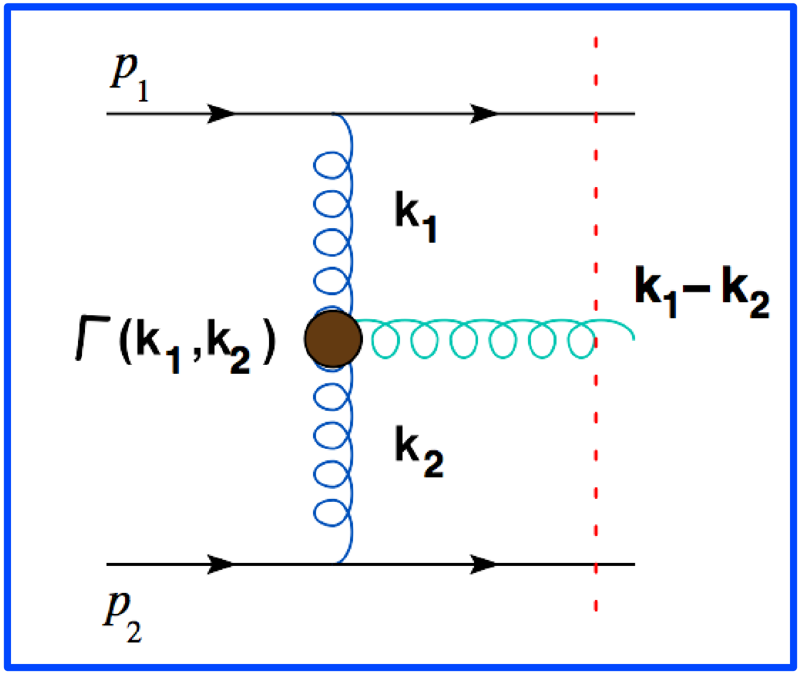
BFKL how to, next step, real corrections

$$k_1 = \alpha_1 p_1 + \beta_1 p_2 + k_{1\perp} \qquad 1 >> \alpha_1 >> \alpha_2$$

$$k_2 = \alpha_2 p_1 + \beta_2 p_2 + k_{2\perp}\,. \qquad 1 >> |\beta_2| >> |\beta_1|$$

$$\int d\Pi_3 = \frac{s^2}{4(2\pi)^5} \int d\alpha_1 d\alpha_2 d\beta_1 d\beta_2 d^2\mathbf{k}_1 d^2\mathbf{k}_2$$
$$\delta(-\beta_1(1-\alpha_1)s - \mathbf{k}_1^2)\ \delta(\alpha_2(1+\beta_2)s - \mathbf{k}_2^2)$$
$$\delta((\alpha_1-\alpha_2)(\beta_1-\beta_2)s - (\mathbf{k}_1-\mathbf{k}_2)^2)\,.$$

$$\int d\Pi_3 = \frac{1}{4(2\pi)^5 s} \underbrace{\int_{\mathbf{k}_2^2/s}^{1} \frac{d\alpha_1}{\alpha_1}}_{\ln\left(\frac{s}{\mathbf{k}_2^2}\right)} \int d^2\mathbf{k}_1 d^2\mathbf{k}_2$$



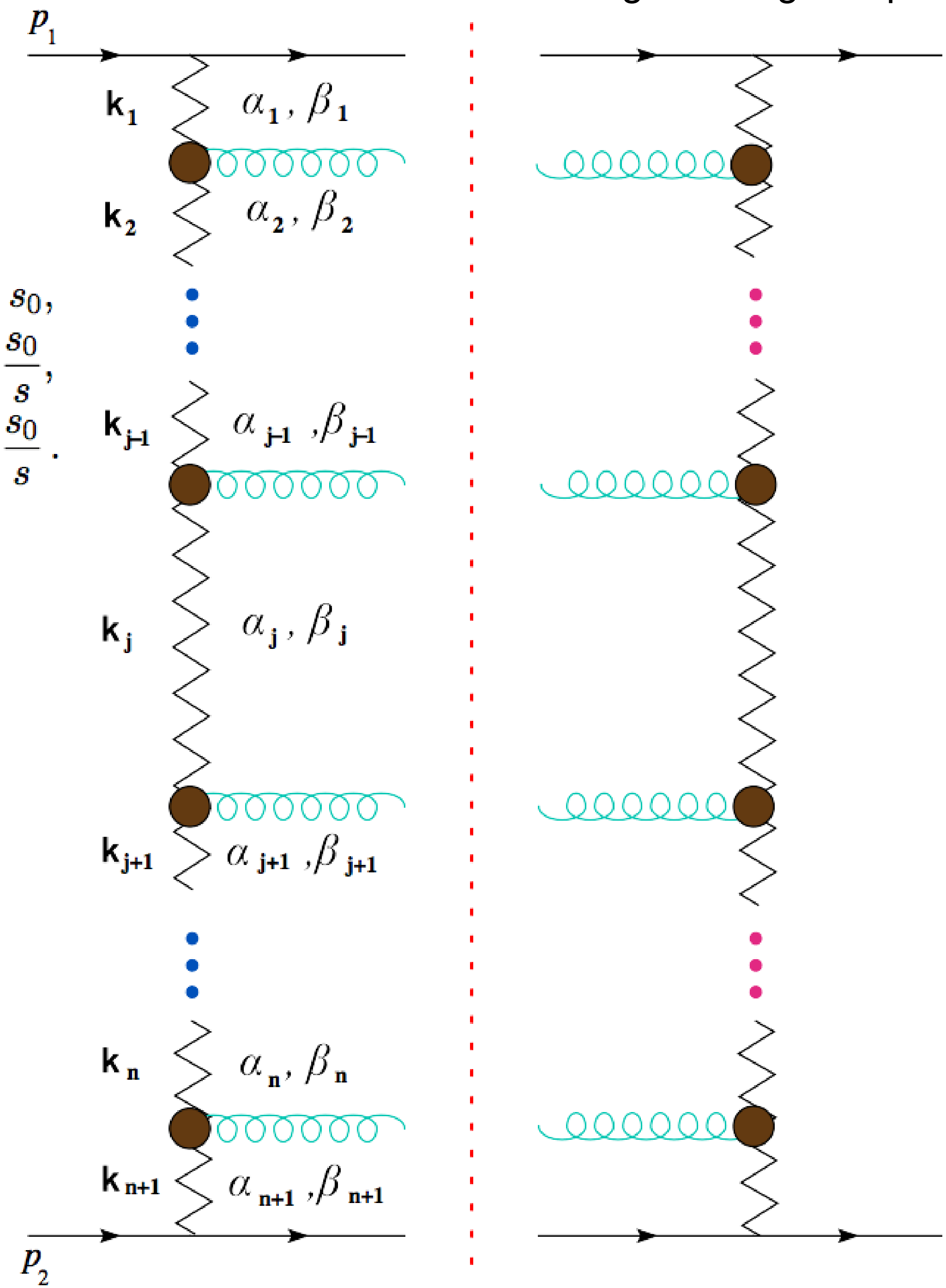
Almost there...

Now, time to iterate, set the t-channel gluons to reggeized gluons, use the conditions:

$$\mathbf{k}_1^2 \simeq \mathbf{k}_2^2 \simeq \ldots \mathbf{k}_i^2 \simeq \mathbf{k}_{i+1}^2 \ldots \simeq \mathbf{k}_n^2 \simeq \mathbf{k}_{n+1}^2 \gg \mathbf{q}^2 \simeq s_0,$$
$$1 \gg \alpha_1 \gg \alpha_2 \gg \ldots \alpha_i \gg \alpha_{i+1} \gg \alpha_{n+1} \gg \frac{s_0}{s},$$
$$1 \gg |\beta_{n+1}| \gg |\beta_n| \gg \ldots \gg |\beta_2| \gg |\beta_1| \gg \frac{s_0}{s}.$$

The (n+2)-body phase will now be much more complicated

Strong ordering in rapidity



Almost there... but before we have to do something with the phase space ...

$$\left.\begin{aligned}
\mathrm{d}\Pi_{n+2} = {} & \frac{s^{n+1}}{2^{n+1}\,(2\pi)^{3n+2}} \int \prod_{i=1}^{n+1} \mathrm{d}\alpha_i\,\mathrm{d}\beta_i\,\mathrm{d}^2\boldsymbol{k}_i \\
& \times \delta\left(-\beta_1(1-\alpha_1)s - \boldsymbol{k}_1^2\right)\,\delta\left(\alpha_{n+1}(1+\beta_{n+1})s - \boldsymbol{k}_{n+1}^2\right) \\
& \times \prod_{j=1}^{n} \delta\left((\alpha_j - \alpha_{j+1})(\beta_j - \beta_{j+1})s - (\boldsymbol{k}_j - \boldsymbol{k}_{j+1})^2\right) \ .
\end{aligned}\right\}\ \text{The (n+2)-body phase space}$$

After integrating over β_i we obtain:

$$\begin{aligned}
\mathrm{d}\Pi_{n+2} = {} & \frac{1}{2^{n+1}\,(2\pi)^{3n+2}} \prod_{i=1}^{n} \int_{\alpha_{i+1}}^{1} \frac{\mathrm{d}\alpha_i}{\alpha_i} \int_0^1 \mathrm{d}\alpha_{n+1} \\
& \times \prod_{j=1}^{n+1} \int \mathrm{d}^2\boldsymbol{k}_j\,\delta\left(\alpha_{n+1}s - \boldsymbol{k}^2\right) \ .
\end{aligned}$$

Mellin transform

$$\widetilde{f}(\omega) = \int_1^\infty \mathrm{d}\left(\frac{s}{s_0}\right) \left(\frac{s}{s_0}\right)^{-\omega-1} f(s) \tag{1}$$

$$f(s) = \frac{1}{2\pi\mathrm{i}} \int_C \mathrm{d}\omega \left(\frac{s}{s_0}\right)^{\omega} \widetilde{f}(\omega) \tag{2}$$

$$f(s) = \prod_{i=1}^{n} \int_{\alpha_{i+1}}^{1} \frac{\mathrm{d}\alpha_i}{\alpha_i}\, g_i\left(\frac{\alpha_{i-1}}{\alpha_i}\right) s_0\, \delta(\alpha_n s - s_0) \tag{3}$$

$$\widetilde{f}(\omega) = \prod_{i=1}^{n} \int_0^1 \mathrm{d}\rho_i\, \rho_i^{\omega-1}\, g_i\left(\frac{1}{\rho_i}\right) = \prod_{i=1}^{n} \widetilde{g}_i(\omega) \tag{4}$$

Finally the BFKL equation

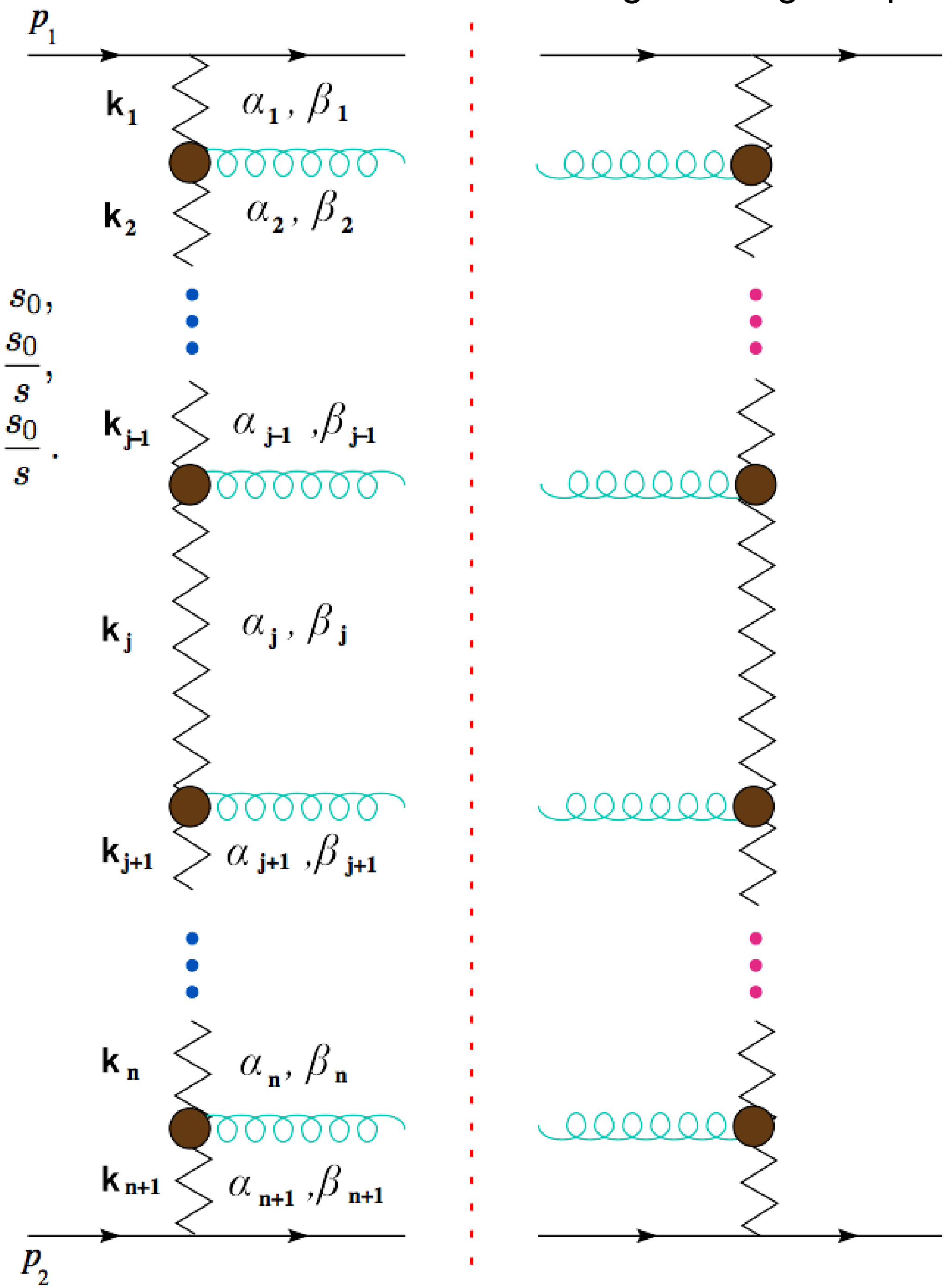
Again, time to iterate, set the t-channel gluons to reggeized gluons, use the conditions:

$$\mathbf{k}_1^2 \simeq \mathbf{k}_2^2 \simeq \ldots \mathbf{k}_i^2 \simeq \mathbf{k}_{i+1}^2 \ldots \simeq \mathbf{k}_n^2 \simeq \mathbf{k}_{n+1}^2 \gg \mathbf{q}^2 \simeq s_0,$$
$$1 \gg \alpha_1 \gg \alpha_2 \gg \ldots \alpha_i \gg \alpha_{i+1} \gg \alpha_{n+1} \gg \frac{s_0}{s},$$
$$1 \gg |\beta_{n+1}| \gg |\beta_n| \gg \ldots \gg |\beta_2| \gg |\beta_1| \gg \frac{s_0}{s}.$$

and after the Mellin transform to unfold the nested integrations over phase space, you finally get:

$$\boxed{\begin{aligned} \omega f_\omega(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) &= \delta^2(\mathbf{k}_1-\mathbf{k}_2) \\ &+\frac{\bar{\alpha}_s}{2\pi}\int d^2\mathbf{l}\Bigg\{\frac{-\mathbf{q}^2}{(\mathbf{l}-\mathbf{q})^2\mathbf{k}_1^2} f_\omega(\mathbf{l},\mathbf{k}_2,\mathbf{q}) \\ +\frac{1}{(\mathbf{l}-\mathbf{k}_1)^2}\Bigg(f_\omega(\mathbf{l},\mathbf{k}_2,\mathbf{q}^2) &- \frac{\mathbf{k}_1^2 f_\omega(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})}{\mathbf{l}^2+(\mathbf{k}_1-\mathbf{l})^2}\Bigg) \\ &+\frac{1}{(\mathbf{l}-\mathbf{k}_1)^2}\Bigg(\frac{(\mathbf{k}_1-\mathbf{q})^2\mathbf{l}^2 f_\omega(\mathbf{l},\mathbf{k}_2,\mathbf{q}^2)}{(\mathbf{l}-\mathbf{q})^2\mathbf{k}_1^2} \\ &-\frac{(\mathbf{k}_1-\mathbf{q})^2 f_\omega(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}^2)}{(\mathbf{l}-\mathbf{q})^2(\mathbf{k}_1-\mathbf{l})^2}\Bigg)\Bigg\}, \end{aligned}}$$

Strong ordering in rapidity



The BFKL equation for zero momentum transfer

$$\omega f_\omega(\mathbf{k}_1,\mathbf{k}_2) = \delta^2(\mathbf{k}_1-\mathbf{k}_2)$$
$$+\frac{\bar{\alpha_s}}{2\pi}\int\frac{d^2\mathbf{l}}{(\mathbf{l}-\mathbf{k}_1)^2}\left(f_\omega(\mathbf{l},\mathbf{k}_2)-\frac{\mathbf{k}_1^2 f_\omega(\mathbf{k}_1,\mathbf{k}_2)}{\mathbf{l}^2+(\mathbf{k}_1-\mathbf{l})^2}\right),.$$

We can rewrite the equation above very nicely as

$$\omega f_\omega(\mathbf{k}_1,\mathbf{k}_2) = \delta^2(\mathbf{k}_1-\mathbf{k}_2)+\int d^2\mathbf{l}\,\mathcal{K}(\mathbf{k}_1,\mathbf{l})\,f_\omega(\mathbf{l},\mathbf{k}_2)$$

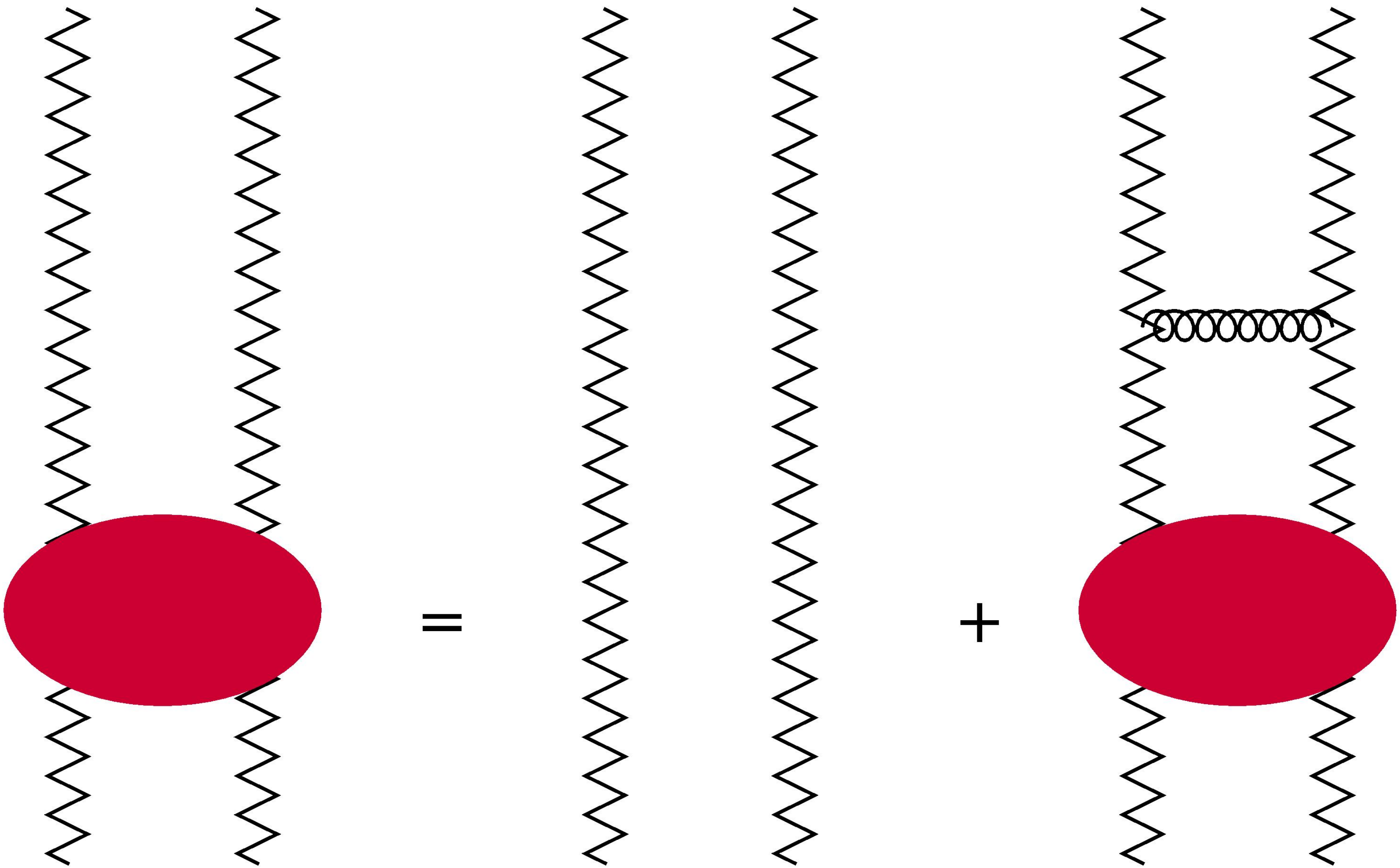
where $\mathcal{K}(\mathbf{k}_1,\mathbf{l})$ is the BFKL kernel:

$$\mathcal{K}(\mathbf{k}_1,\mathbf{l}) \;=\; \underbrace{2\epsilon(-\mathbf{k}^2)\,\delta^2(\mathbf{k}_1-\mathbf{l})}_{\mathcal{K}_{virt}} + \underbrace{\frac{N_c\alpha_s}{\pi^2}\,\frac{1}{(\mathbf{k}_1-\mathbf{k}_2)^2}}_{\mathcal{K}_{real}}\,.$$

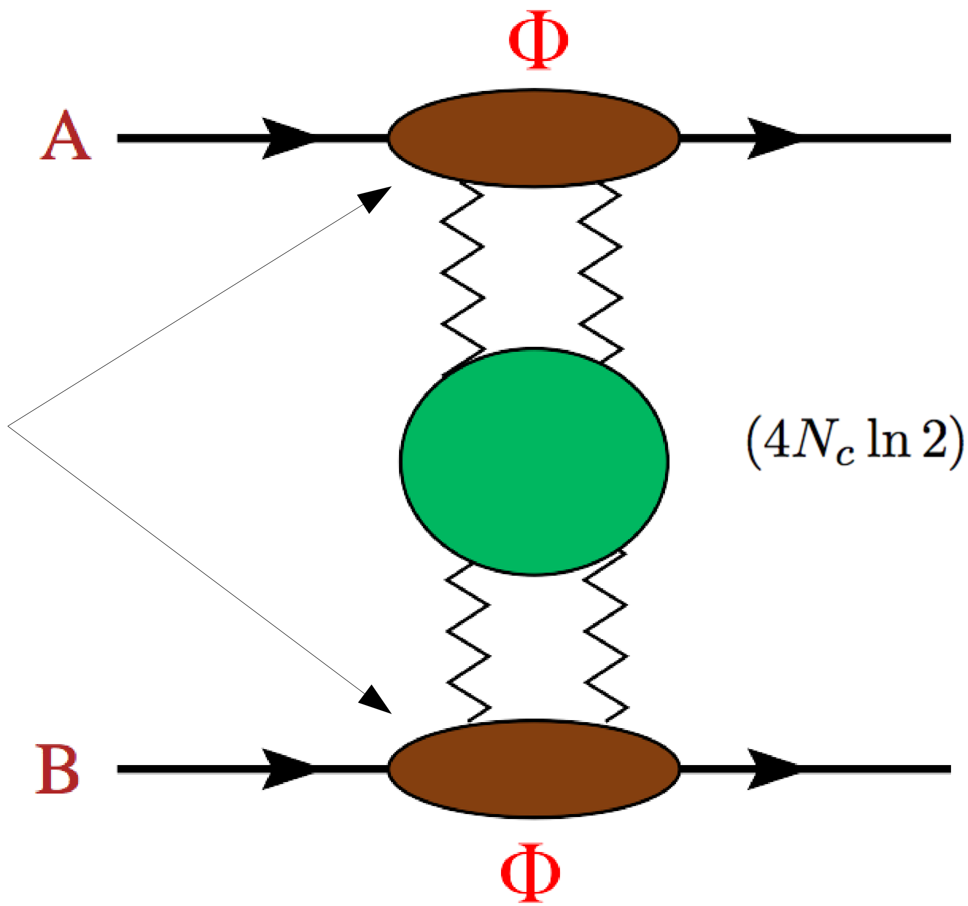
We can go back to s-space:

$$f(s,\mathbf{k}_1,\mathbf{k}_2,\mathbf{q}) \;=\; \frac{1}{2\pi i}\int_{c-i\infty}^{c+i\infty} d\omega\,\left(\frac{s}{s_0}\right)^{\omega} f_\omega(\mathbf{k}_1,\mathbf{k}_2,\mathbf{q})$$

$$\omega f_\omega(\mathbf{k}_1, \mathbf{k}_2) = \delta^2(\mathbf{k}_1 - \mathbf{k}_2) + \int d^2\mathbf{l}\, \mathcal{K}(\mathbf{k}_1, \mathbf{l})\, f_\omega(\mathbf{l}, \mathbf{k}_2)$$



A hadronic elastic amplitude



Impact factors... ?

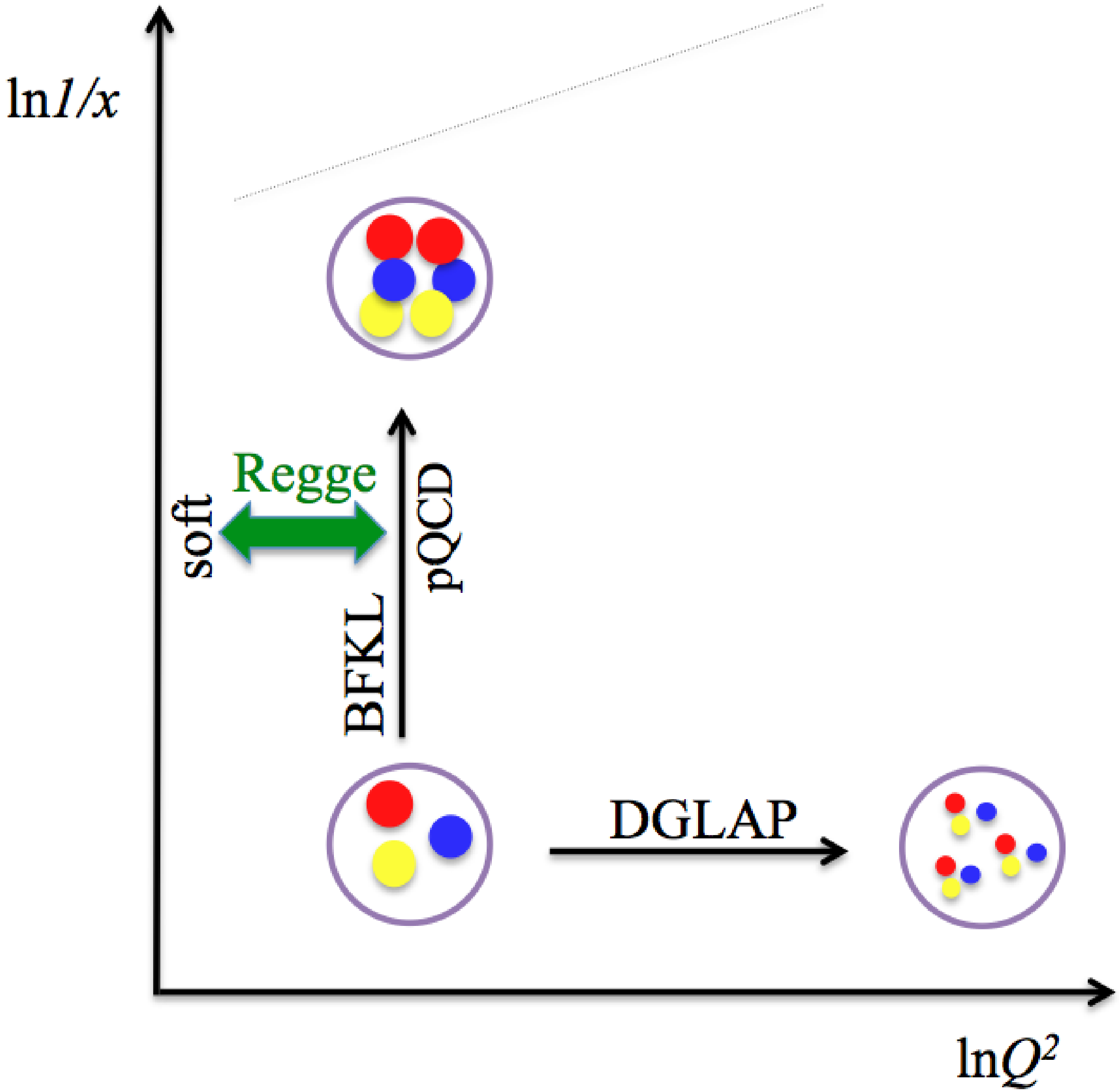
$(4N_c \ln 2)/\pi)\alpha_s \approx 0.5.\ (\alpha_s \sim 0.2)$

$$\mathcal{A}(s,t) = i\, s\, \mathcal{C} \int \frac{d^2\mathbf{k}_1}{(2\pi)^2}\, \frac{d^2\mathbf{k}_2}{(2\pi)^2}\, \Phi_A(\mathbf{k}_1, \mathbf{q})\, \frac{f(s, \mathbf{k}_1, \mathbf{k}_2, \mathbf{q})}{\mathbf{k}_2^2 (\mathbf{k}_1 - \mathbf{q})^2} \Phi_B(\mathbf{k}_2, \mathbf{q})$$

Let us make a summary here

- We have shown how to derive the BFKL equation at LO
- We find that the exchange of ladder-diagrams leads to an exponential rise for the total cross section
- The resummation of all these ladders gives an intercept ~ 0.5, perturbative Pomeron
- What is the connection between 'soft' and 'hard' Pomeron?
- Old ideas from Regge theory find accommodation -not in an always clear way- in QCD

A unifying picture



Check also the first lecture by Edmond for an alternative graph like this one

Bibliography II (very incomplete)

- Forshaw & Ross, “Quantum Chromodynamics and the Pomeron”
- Barone & Predazzi, High Energy Particle Diffraction
- Ioffe, Fadin & Lipatov, “Quantum Chromodynamics: Perturbative and Nonperturbative Aspects”
- Kovchegov & Levin, “Quantum Chromodynamics at High Energy”
- Many many review articles...